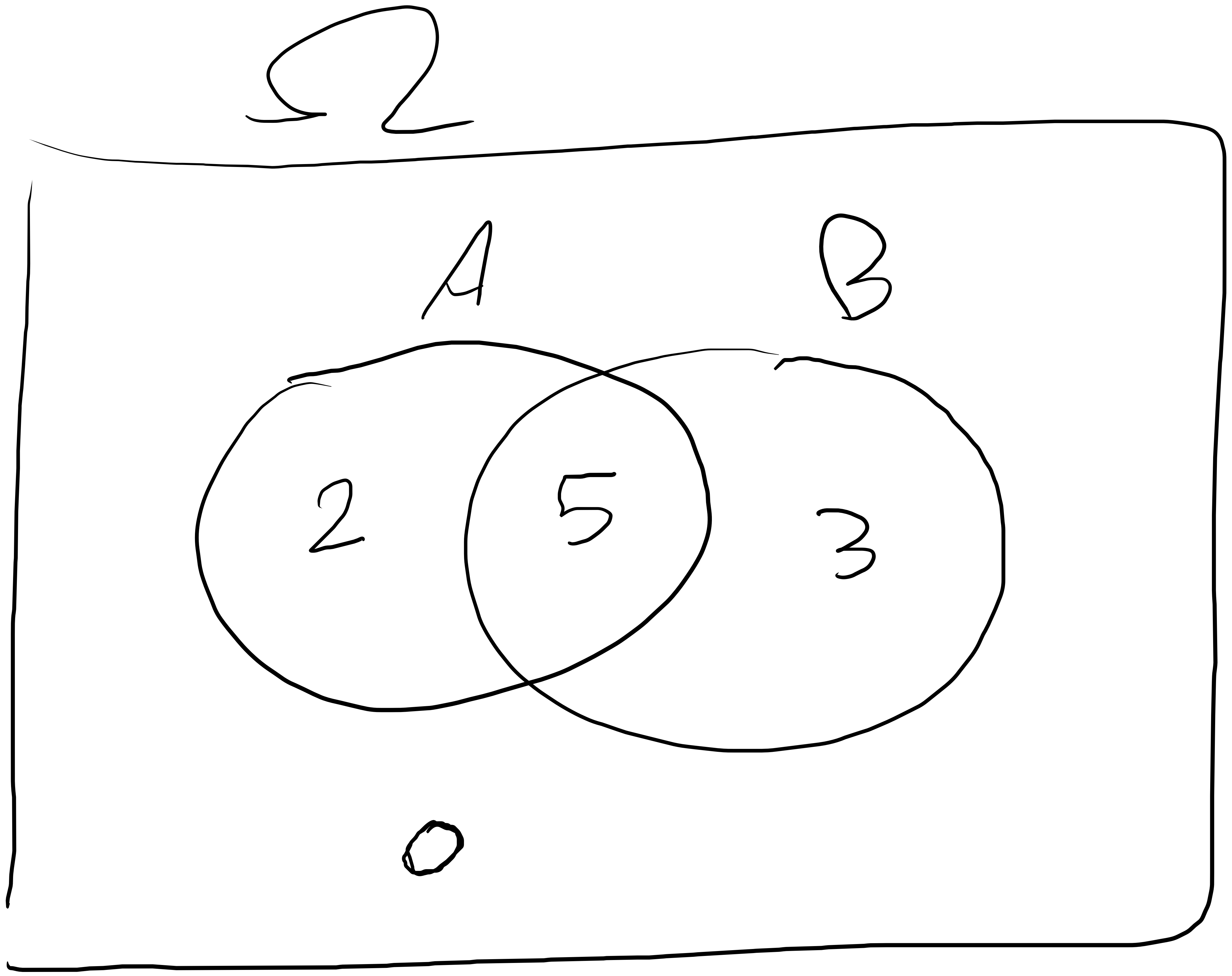


$$X = 2\,\mathbb{1}_A + 3\,\mathbb{1}_B$$

$$E[X] = 2P(A) + 3P(B)$$

$$X = 2\,\mathbb{1}_{A-B} + 3\,\mathbb{1}_{B-A} + 5\,\mathbb{1}_{A\cap B}$$

$$E[X] = 2P(A-B) + 3P(B-A) + \underbrace{5P(A\cap B)}_{2P(A\cap B) + 3P(A\cap B)}$$

$$= 2P(A) + 3P(B)$$

$X \quad x_1 \text{———} x_n$ (DISTINTI)

$\quad\;\; p_1 - - p_n$

$$X = \sum_i x_i \, 1_{\{X = x_i\}}$$

$$E[X] = \sum_i x_i \underbrace{P(X = x_i)}_{p_i}$$

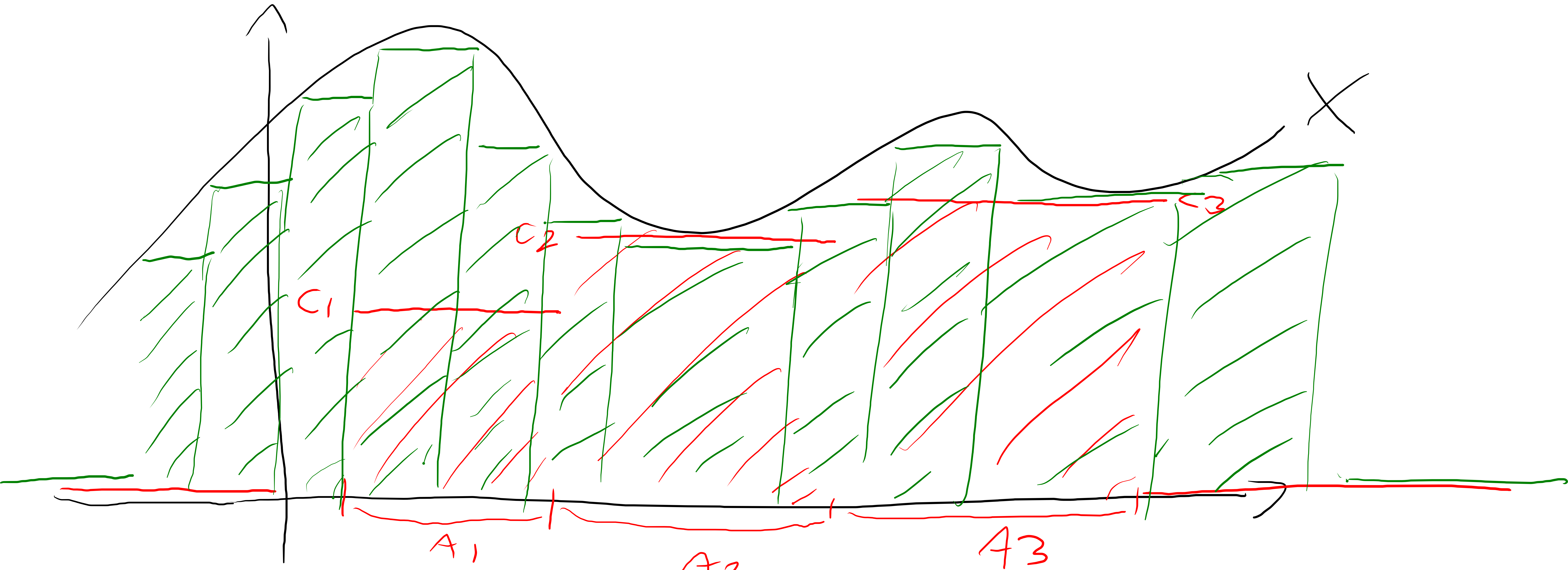
$X \sim$ BINOMIALE

$0, 1, \ldots, n$

$$P(X = i) = \binom{n}{i} p^i (1-p)^{n-i}$$

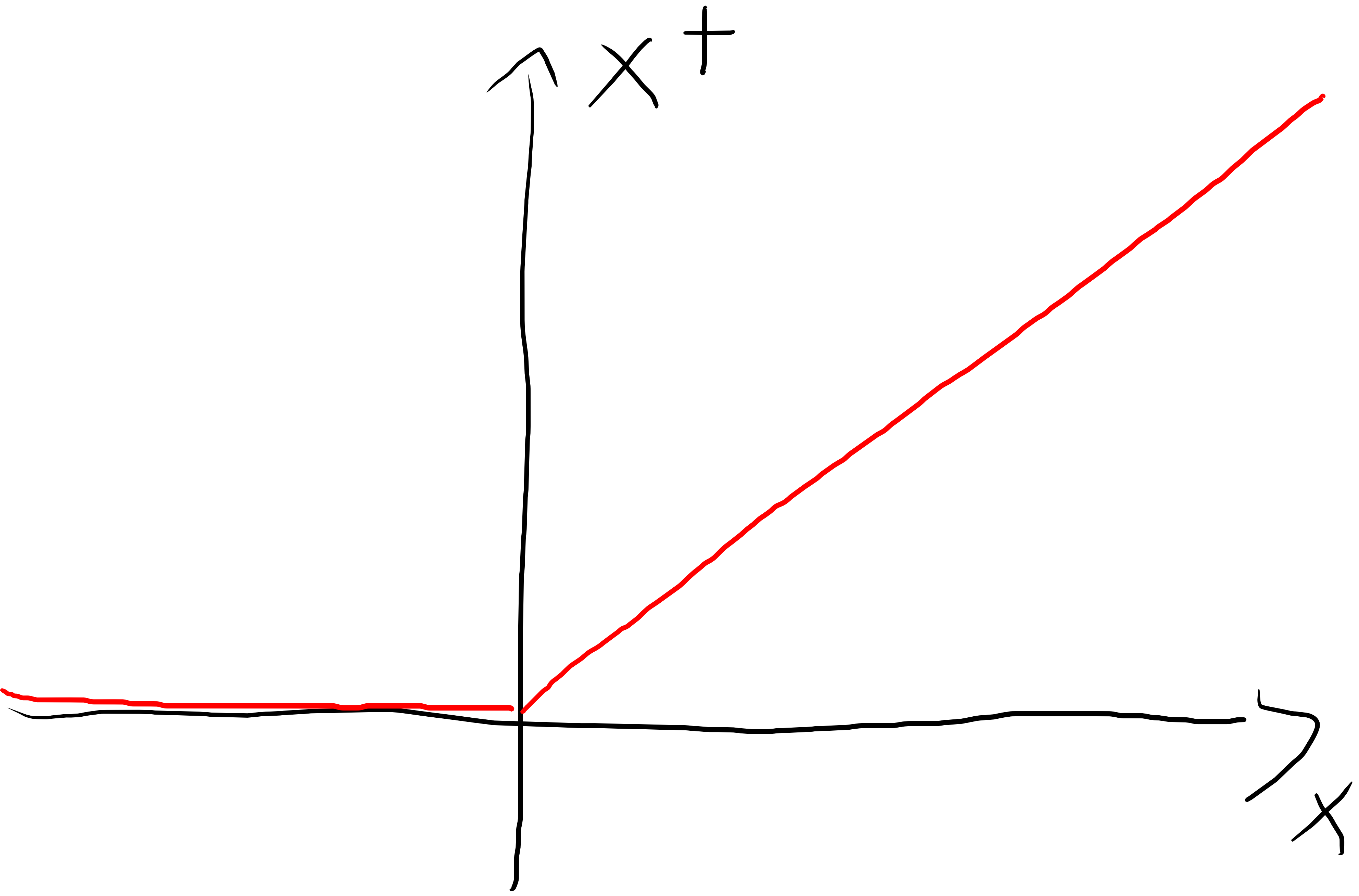
$$\sum_{i=0}^{n} i \cdot \binom{n}{i} p^i (1-p)^{n-i} = n \cdot p$$

$\Omega = \mathbb{R} \quad \mathcal{A} = \mathcal{B}$

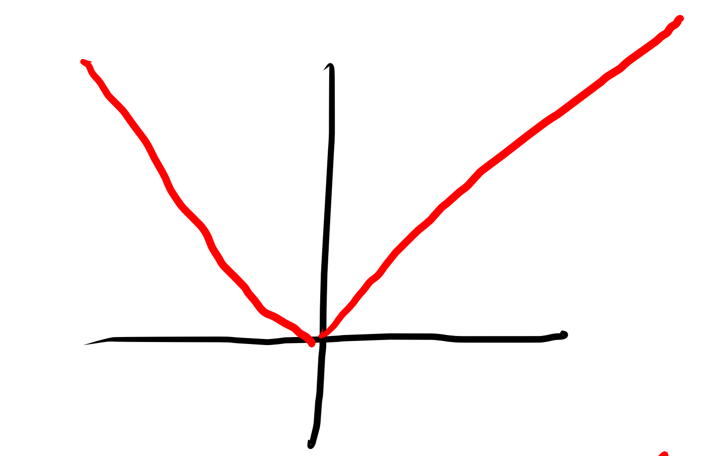


$Y = c_1 \mathbb{1}_{A} + c_2 \mathbb{1}_{A_2} + c_3 \mathbb{1}_{A_3}$

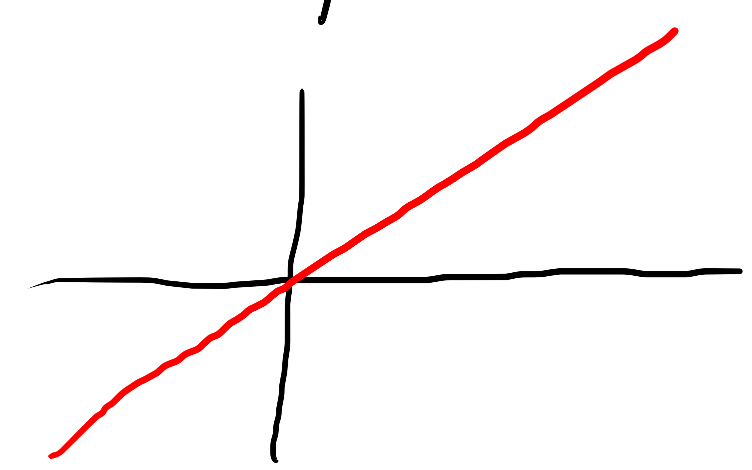
$E[Y] = c_1 P(A_1) + c_2 P(A_2)$
$+ c_3 P(A_3)$



$$x^+ + x^- = |x|$$



$$x^+ - x^- = x$$



$$0 \leq x^+ \leq |x|$$

$$0 \leq x^- \leq |x|$$

$|X| \geq X^+$

$\geq X^-$

QUINDI SE $E[|X|] < +\infty$

ALLORA $E[X^+] < +\infty$

$E[X^-] < +\infty$

SE $E[X^+] < +\infty$, $E[X^-] < +\infty$

ALLORA $E[|X|] = E[X^+ + X^-] = \underbrace{E[X^+]}_{< +\infty} + \underbrace{E[X^-]}_{< +\infty}$

$< +\infty$

$p = 1, \quad q = 2$

$$L^q = L^2 \subset L^p = L^1$$

DIMOSTRAZIONE DI $L^q \subset L^p$ SE $1 \leq p < q$

$$|X|^p = |X|^p \, 1_{\{|X| \leq 1\}} + |X|^p \, 1_{\{|X| > 1\}}$$

$$\leq 1 + |X|^q$$

SE $X \in L^q$ ALLORA $X \in L^p$ (DOMINAZIONE)

$X = 0$ Q.C. ALLORA $X \in L^1$, $E[X] = 0$

$P(X=0) = 1$

SE X SEMPLICE $X \begin{cases} 0 & p = 1 \\ x_1 \neq 0 & p_1 = 0 \\ x_2 \neq 0 & p_2 = 0 \\ \vdots & \vdots \\ x_n \neq 0 & p_n = 0 \end{cases}$ $E[X] = 0 \cdot 1 + \sum x_i p_i = 0$

SE $X \geq 0$, $X = 0$ Q.C.

$E[X] = \sup\{E[Y] \mid 0 \leq Y \leq X,\ Y \text{ SEMPLICE}\}$

ESEMPIO INCENDIO

$$X \begin{cases} \begin{cases} 0 & 50\% \\ 100\ m & 5\% \end{cases} \\ \text{UNIFORME TRA } 0 \text{ E } 100\ m \end{cases}$$

$$f(x) = \frac{0.45}{100m} \quad \text{SE} \quad 0 < x < 100\ m$$

$$E[X] = 0 \times 0.5 + 100m \times 0.05 + \int_0^{100m} x \cdot \frac{0.45}{100m}\, dx$$

$$= 5m + \frac{0.45}{100m}\left[\frac{x^2}{2}\right]_0^{100m} = 5m + \frac{0.45}{\cancel{100m}} \frac{(100m)^{\cancel{2}}}{2}$$

$$= 5m + 22.5\ m = 27.5\ m$$

$X_1, X_2 \geq 0$

$$E[\alpha X_1 + \beta X_2] = \alpha E[X_1] + \beta E[X_2]$$

VALE SE $\alpha \geq 0, \beta \geq 0$ o $\alpha \leq 0, \beta \leq 0$

$X \sim \text{PARETO}(\alpha, \lambda)$ $\quad \underline{\underline{\alpha > 0 \, , \, \lambda > 0}}$

$$F_X(x) = 1 - \left(\frac{\lambda}{\lambda + x}\right)^{\alpha} \quad x \geq 0 \qquad (0 \text{ SE } x < 0)$$

$$E[X^{\beta}] < +\infty \iff \beta < \alpha$$

$\alpha = 1.5 \qquad X \in L^1 \quad \text{MA} \quad X \notin L^2$

$\lambda = 1$ PER SEMPLICITÀ

f_X PDF DI X

$$E[X] = \int_0^{+\infty} x f_X(x)\,dx < +\infty \iff \alpha > 1$$

$$E[X^2] = \int_0^{+\infty} x^2 f_X(x)\,dx < +\infty \iff \alpha > 2$$

$$f_X(x) = F_X'(x) = \left(1 - \left(\frac{1}{1+x}\right)^{\alpha}\right)' = \left(1 - (1+x)^{-\alpha}\right)'$$

$$= -(-\alpha)(1+x)^{-\alpha-1} = \frac{\alpha}{(1+x)^{\alpha+1}}$$

$$E[X] = \int_0^{+\infty} x \cdot \frac{\alpha}{(1+x)^{\alpha+1}}\, dx = \alpha \int_0^{+\infty} \frac{x+1-1}{(1+x)^{\alpha+1}}\, dx$$

$$= \alpha \left(\int_0^{+\infty} (1+x)^{-\alpha} dx - \int_0^{+\infty} (1+x)^{-(\alpha+1)} dx \right)$$

$1 + x = u$

$dx = du$

$$= \alpha \left(\int_1^{+\infty} u^{-\alpha} du - \int_1^{+\infty} u^{-(\alpha+1)} du \right)$$

$$= \alpha \left(\left[\frac{u^{1-\alpha}}{1-\alpha} \right]_1^{+\infty} - \left[\frac{u^{-\alpha}}{-\alpha} \right]_1^{+\infty} \right)$$

SE

$\underline{\alpha \neq 1}$

$$\lim_{U \to +\infty} U^{1-\alpha} \quad \text{FINITO } (=0) \quad \text{SE } \alpha > 1$$

SE $\alpha > 1$

$$= \alpha \left(\frac{1}{\alpha - 1} - \left(- \frac{1}{-\alpha} \right) \right) = \alpha \left(\frac{1}{\alpha - 1} - \frac{1}{\alpha} \right)$$

$$= \cancel{\alpha} \, \frac{\alpha - (\alpha - 1)}{\cancel{\alpha}(\alpha - 1)} = \frac{1}{\alpha - 1}$$

SE Y SEMPLICE CON $0 \leq Y \leq X$

$\{Y > 0\} \subset \{X > 0\}$

MA $P(X > 0) = 0 \Rightarrow P(Y > 0) = 0$

CIOÈ $Y = 0$ Q.C.

QUINDI $E[Y] = 0$

DA CUI $\sup\{E[Y] \mid \_\ \_\_\ \_\} = 0 = E[X]$

SIA X QUALUNQUE, $X = 0$ Q.C.

ANCHE $|X| = 0$ Q.C.

E $\left.\begin{array}{l} 0 \leq X^+ \leq |X| \\ 0 \leq X^- \leq |X| \end{array}\right\} \Rightarrow \begin{array}{l} X^+ = 0 \quad \text{Q.C.} \\ X^- = 0 \quad \text{Q.C.} \end{array}$

$$E[X^+] = E[X^-] = 0$$

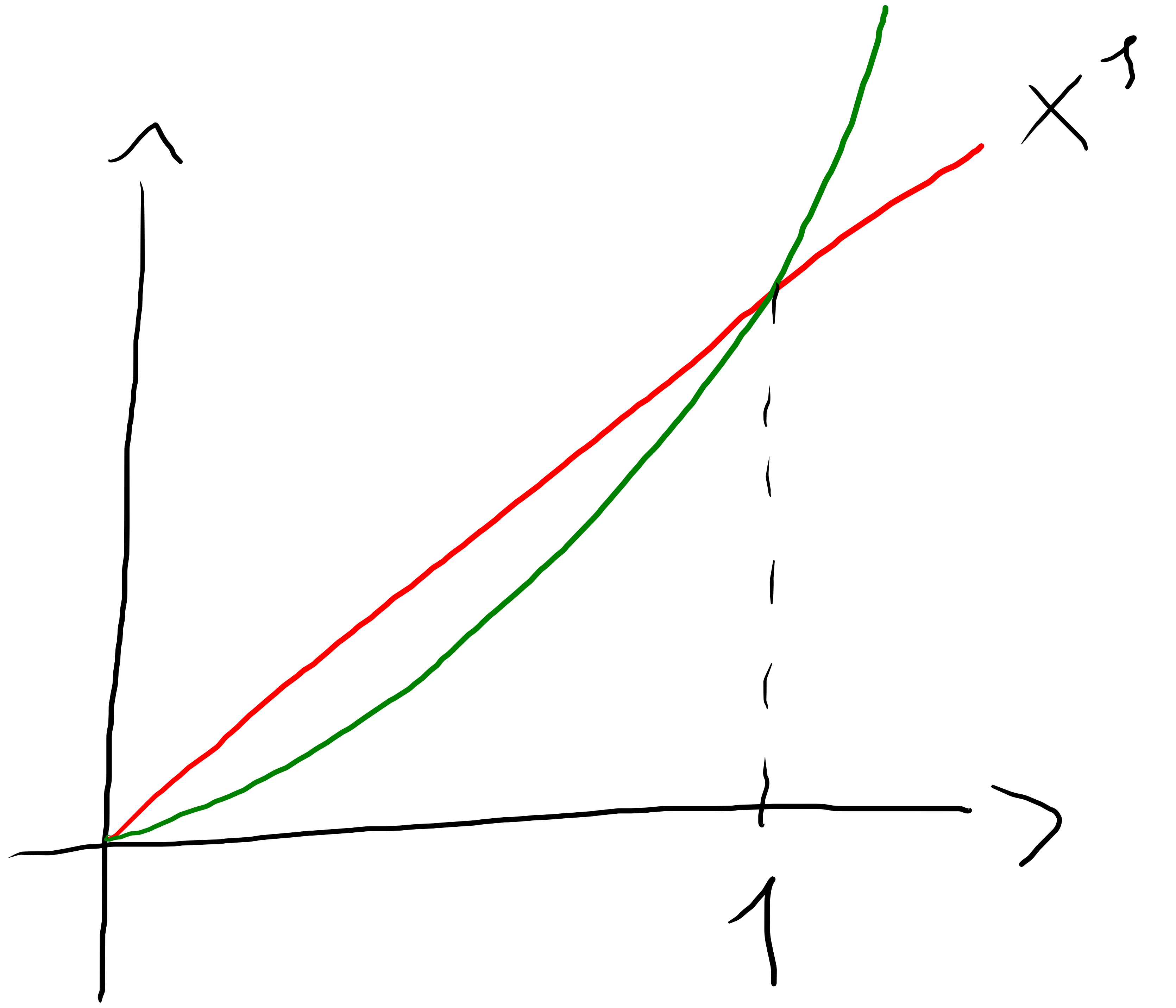
QUINDI $X \in L^1$, $E[X] = 0 - 0 = 0$

$A \in \mathcal{F} \quad P(A) = 0$

$$X 1_A(\omega) = \begin{cases} 0 & \omega \notin A \\ X(\omega) & \omega \in A \end{cases}$$

$\{X 1_A = 0\} \supset \overline{A} \quad \Rightarrow X 1_A = 0 \quad$ Q.C.

$\{X 1_A \neq 0\} \subset A$



x^p $\quad p < q$

x^q

$x \geq 0$

$x^p < x^q$ SE $x > 1$