16 Nov

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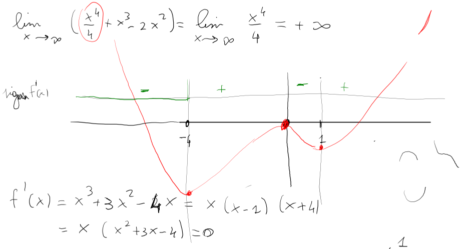
Studiare $f(x) = \frac{x^4}{4} + x^3 - 2x^2 = x^2 \left(\frac{x^2}{4} + x - 2\right)$

1) Dom f = $\mathbb{R}$

2) $\lim_{x \to \infty} \left(\frac{x^4}{4} + x^3 - 2x^2\right) = \lim_{x \to \infty} \frac{x^4}{4} = +\infty$



3) $f'(x) = x^3 + 3x^2 - 4x = x(x-1)(x+4)$

$= x(x^2 + 3x - 4) = 0$

$x = 0 \qquad x_{\pm} = -\frac{3}{2} \pm \frac{\sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} \begin{cases} 1 \\ -4 \end{cases}$

4) $f(-4) = \frac{(-4)^4}{4} + (-4)^3 - 2(-4)^2 = \cancel{4}^3 - \cancel{4}^3 - 2 \cdot 16 = -32$

$f''(x) = 3x^2 + 6x - 4 \qquad f''(0) = -4$

$f''(-4) = 3(-4)^2 - 24 - 4 = \overbrace{3 \cdot 16}^{48} - 24 - 4 > 0$

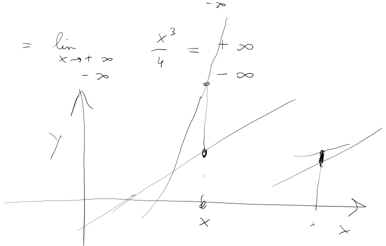
-4 è un minimo locale

$f(1) = \frac{x^4}{4} + x^3 - 2x^2 \Big|_{x=1} = \frac{1}{4} + 1 - 2 = -\frac{3}{4} < 0$

$f''(1) = 3 + 6 - 4 = 5 > 0$

5) $\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to +\infty, -\infty} \frac{\frac{x^4}{4} + x^3 - 2x^2}{x}$

$= \lim_{x \to +\infty, -\infty} \frac{x^3}{4} = +\infty, -\infty$



$$f(x) = (x^2+x+1)\, e^{-x}$$

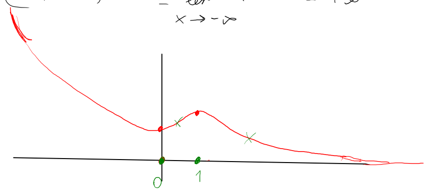
1) $\mathrm{Dom}\, f = \mathbb{R}$

2) $f(x) > 0 \ \forall x$ $\qquad x^2+x+1 > 0 \ \forall x$

$1^2 - 4 = -3 < 0$

3) $\lim_{x \to +\infty} \frac{x^2+x+1}{e^x} = \lim_{x \to +\infty} \frac{x^2}{e^x} = 0$

$\lim_{x \to -\infty} (x^2+x+1)\, e^{-x} = \lim_{x \to -\infty} x^2 e^{-x} = +\infty$



4) $f'(x) = \left((x^2+x+1)\, e^{-x} \right)' = e^{-x} \left[2x + 1 - (x^2+x+1) \right]$

$= e^{-x}(-x^2+x) = -e^{-x}(x-1)\,x = 0$ per $x = 0, 1$

$\mathrm{sign}(f'(x)) = \mathrm{sign}(-(x-1)\,x)$ $\qquad - \ \underset{0}{|} \ + \ \underset{1}{|} \ -$

$f(0) = 1$ $\qquad f(1) = \frac{3}{e} > 1$

5) $f''(x) = e^{-x}\left[x^2 - x - 2x + 1 \right] = e^{-x}\left[x^2 - 3x + 1 \right] = 0$

$x^2 - 3x + 1 = 0$ $\qquad x_{\pm} = \frac{3}{2} \pm \frac{\sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2} > 0$

$\frac{3+\sqrt{5}}{2} > 1$ ✓ $\qquad \frac{3+\sqrt{5}}{2} > \frac{3+1}{2} = 2 > 1$

Verifichiamo che $\frac{3-\sqrt{5}}{2} < 1$

$3 - \sqrt{5} < 2 \iff 1 - \sqrt{5} < 0$

$\Updownarrow$

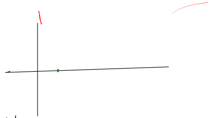
$1 < \sqrt{5}$ ✓

$f(x) = x^{\varepsilon} - \lg x = 0 \qquad 0 < \varepsilon < 1$

1) $\mathrm{Dom}\, f = \mathbb{R}_+$

2) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^{\varepsilon} \Big(1 - \underbrace{\frac{\lg x}{x^{\varepsilon}}}_{o(1)}\Big) = \lim_{x \to +\infty} x^{\varepsilon} = +\infty$

$\lim_{x \to 0^+} \big(\underbrace{x^{\varepsilon}}_{\to 0} - \lg x\big) = \lim_{x \to 0^+} - \lg x = +\infty$



$f'(x) = (x^{\varepsilon} - \lg x)' = \varepsilon\, x^{\varepsilon - 1} - \frac{1}{x} = 0$

$\varepsilon\, x^{\varepsilon} \frac{1}{x} - \frac{1}{x} = 0$

$\frac{1}{x}(\varepsilon x^{\varepsilon} - 1) = 0 \iff \varepsilon x^{\varepsilon} - 1 = 0$

$x^{\varepsilon} = \frac{1}{\varepsilon} \qquad x = \frac{1}{\varepsilon^{\frac{1}{\varepsilon}}}$

$\Big(\lim_{\varepsilon \to 0^+} \varepsilon^{-\frac{1}{\varepsilon}} = \lim_{\varepsilon \to 0} e^{-\frac{1}{\varepsilon} \lg \varepsilon} = e^{-\infty} = 0$

$\lim_{\varepsilon \to 0^+} \frac{\lg \varepsilon}{\varepsilon} = \lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon} = +\infty \Big)$

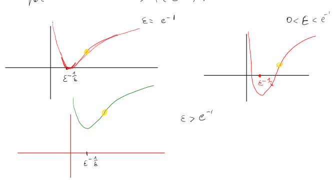
$f(\varepsilon^{-\frac{1}{\varepsilon}}) = (\varepsilon^{-\frac{1}{\varepsilon}})^{\varepsilon} - \lg \varepsilon^{-\frac{1}{\varepsilon}}$

$= \varepsilon^{-1} + \varepsilon^{-1} \lg \varepsilon = \varepsilon^{-1}(1 + \lg \varepsilon)$

$\lg \varepsilon + 1 = 0 \qquad \lg \varepsilon = -1 \qquad \varepsilon = e^{-1} \Rightarrow f(\varepsilon^{-\frac{1}{\varepsilon}}) = 0$

per $0 < \varepsilon < e^{-1} \Rightarrow f(\varepsilon^{-\frac{1}{\varepsilon}}) < 0$

per $e^{-1} < \varepsilon < 1 \Rightarrow f(\varepsilon^{-\frac{1}{\varepsilon}}) > 0$



$f' = \varepsilon\, x^{\varepsilon - 1} - x^{-1}$

$f'' = \varepsilon(\varepsilon - 1)\, x^{\varepsilon - 2} + x^{-2} =$

$= \frac{1}{x^2}\big(\varepsilon(\varepsilon - 1)\, x^{\varepsilon} + 1\big) = 0$

$-\varepsilon(1 - \varepsilon)\, x^{\varepsilon} + 1 = 0 \qquad \varepsilon(1 - \varepsilon)\, x^{\varepsilon} = 1 \qquad x^{\varepsilon} = \frac{1}{\varepsilon(1 - \varepsilon)}$

$x = \frac{1}{\varepsilon^{\frac{1}{\varepsilon}} (1 - \varepsilon)^{\frac{1}{\varepsilon}}} > \frac{1}{\varepsilon^{\frac{1}{\varepsilon}}}$

$f''(x) > 0$ per $x < \frac{1}{\varepsilon^{\frac{1}{\varepsilon}}(1 - \varepsilon)^{\frac{1}{\varepsilon}}}$, $f''(x) > 0$ per $x > \frac{1}{\varepsilon^{\frac{1}{\varepsilon}}(1 - \varepsilon)^{\frac{1}{\varepsilon}}}$