Politecnico di Torino – II Facoltà di Architettura
Corso di Istituzioni di Matematiche I

Esercizi sul calcolo delle derivate

1) Derivare le seguenti somme di funzioni

$$f_1(x) = x^5 - 7, \qquad f_2(x) = \sin x + 3x^8, \qquad f_3(x) = \sqrt{x} - \log x + 1,$$

$$f_4(x) = \arctan x + \frac{1}{x} - \frac{3}{x^3}, \qquad f_5(x) = x^{\frac{3}{4}} - x^{\frac{1}{2}}, \qquad f_6(x) = -2e^x + \frac{1}{2}\cos x$$

2) Derivare i seguenti prodotti di funzioni

$$f_1(x) = x \log x, \qquad f_2(x) = \sin x \cos x, \qquad f_3(x) = 2x^3 e^x,$$

$$f_4(x) = x \cos x \log x, \qquad f_5(x) = -3\sqrt{x}e^x \tan x, \qquad f_6(x) = \sqrt[3]{x^2} \sin x \arctan x$$

3) Derivare i seguenti rapporti di funzioni

$$f_1(x) = \frac{x}{x+1}, \qquad f_2(x) = \frac{\sin x}{x}, \qquad f_3(x) = \frac{\sqrt{x}}{\log x + 1},$$

$$f_4(x) = \frac{x^3 + 2x^2}{\arctan x}, \qquad f_5(x) = \frac{\sin x + \cos x}{x+2}, \qquad f_6(x) = \frac{x^3 + 2x - 4}{x^2 + x + 4}$$

4) Derivare le seguenti funzioni composte

$$f_1(x) = \sin \sqrt{x}, \qquad f_2(x) = \log(\arcsin x), \qquad f_3(x) = \sqrt{\log x + 1},$$

$$f_4(x) = e^{\sqrt[3]{x-2}}, \qquad f_5(x) = \cos(5x^3), \qquad f_6(x) = \sqrt{3 - 2x}$$

$$f_7(x) = (\log(\sin x))^4, \qquad f_8(x) = \tan(\sqrt{x}), \qquad f_9(x) = \sqrt[4]{x + \sin x}$$

5) Derivare le seguenti funzioni

$$f_1(x) = x \sin \sqrt{x}, \qquad f_2(x) = \frac{\log(1 + x^2)}{x+1}, \qquad f_3(x) = \sqrt{\frac{\log(x+1)}{x}},$$

$$f_4(x) = e^2 x \sin \sqrt{x}, \qquad f_5(x) = \cos(\frac{1}{x}), \qquad f_6(x) = \sqrt{\sin x^3}$$

$$f_7(x) = \log(\log x), \qquad f_8(x) = \frac{(\tan x)^2}{x^2 + x}, \qquad f_9(x) = \frac{\sin e^{-x}}{x}$$

Risultati

Esercizio 1.

$f_1' = 5x^4$ $\qquad\qquad$ $f_2' = \cos x + 24x^7$ $\qquad\qquad$ $f_3' = \frac{1}{2\sqrt{x}} - \frac{1}{x}$

$f_4' = \frac{1}{1+x^2} - \frac{1}{x^2} + 3\frac{3}{x^4}$ $\qquad$ $f_5' = \frac{3}{4}\frac{1}{\sqrt[4]{x}} - \frac{1}{2}\frac{1}{\sqrt{x}}$ $\qquad$ $f_6' = -2e^x - \frac{1}{2}\sin x$.

Esercizio 2.

$f_1' = \log x + 1$ $\qquad\qquad\qquad\qquad$ $f_2' = -\sin^2 x + \cos^2 x$

$f_3' = 6x^2 e^x + 2x^3 e^x$ $\qquad\qquad\qquad$ $f_4' = \cos x \log x - x\sin x \log x + \cos x$

$f_5' = -3\left(\frac{1}{2\sqrt{x}}e^x \tan x + \sqrt{x}e^x \tan x + \sqrt{x}e^x \frac{1}{\cos^2 x}\right)$

$f_6' = \frac{2}{3}\frac{1}{\sqrt[3]{x}}\sin x \arctan x + \sqrt[3]{x^2}\cos x \arctan x + \frac{1}{1+x^2}\sqrt[3]{x^2}\sin x$

Esercizio 3.

$f_1' = \frac{1}{(1+x)^2}$ $\qquad\qquad\qquad\qquad$ $f_2' = \frac{x\cos x - \sin x}{x^2}$

$f_3' = \frac{\frac{1}{2}\frac{1}{\sqrt{x}}(1+\log x) - \frac{1}{x}\sqrt{x}}{(1+\log x)^2}$ $\qquad\qquad$ $f_4' = \frac{(3x^2+4x)\arctan x - \frac{1}{1+x^2}(x^3+2x^2)}{(\arctan x)^2}$

$f_5' = \frac{(\cos x - \sin x)(x+2) - (\cos x + \sin x)}{(x+2)^2}$ $\qquad$ $f_6' = \frac{x^4 + 2x^3 + 10x^2 + 8x + 12}{(x^2+x+4)^2}$

Esercizio 4.

$f_1' = \frac{\cos\sqrt{x}}{2\sqrt{x}}$ $\qquad\qquad$ $f_2' = \frac{1}{\arcsin x\sqrt{1-x^2}}$ $\qquad\qquad$ $f_3' = \frac{1}{2x\sqrt{\log x + 1}}$

$f_4' = \frac{e^{\sqrt[3]{x-2}}}{3\sqrt[3]{x-2^2}}$ $\qquad\qquad$ $f_5' = -15x^2 \sin 5x^3$ $\qquad\qquad$ $f_6' = -\frac{1}{\sqrt{3-2x}}$

$f_7' = \frac{4(\log\sin x)^3 \cos x}{\sin x}$ $\qquad$ $f_8' = \frac{1}{\cos^2\sqrt{x}\,2\sqrt{x}}$ $\qquad\qquad$ $f_9' = \frac{1+\cos x}{4\sqrt[4]{(x+\sin x)^3}}$

Esercizio 5.

$f_1' = \sin\sqrt{x} + x\frac{\cos\sqrt{x}}{2\sqrt{x}}$ $\quad$ $f_2' = \frac{\frac{1}{1+x^2}2x(x+1) - \log(1+x^2)}{(x+1)^2}$ $\quad$ $f_3' = \frac{1}{2}\sqrt{\frac{x}{\log(x+1)}}\frac{\frac{x}{x+1} - \log(x+1)}{x^2}$

$f_4' = e^2\left(\sin\sqrt{x} + x\frac{\cos\sqrt{x}}{2\sqrt{x}}\right)$ $\quad$ $f_5' = \frac{1}{x^2}\sin\frac{1}{x}$ $\qquad\qquad$ $f_6' = \frac{3x^2 \cos x^3}{2\sqrt{\sin x^3}}$

$f_7' = \frac{1}{x\log x}$ $\qquad\qquad$ $f_8' = \frac{\frac{2(x+x^2)\tan x}{\cos^2 x} - (2x+1)\tan^2 x}{(x+x^2)^2}$ $\quad$ $f_9' = \frac{-e^{-x}\cos e^{-x} - \sin e^{-x}}{x^2}$