FORMULE ANALISI MATEMATICA

Trigonometria:

Formule degli archi associati

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\cos(\alpha);\quad \cos\left(\frac{\pi}{2}-\alpha\right)=\sin(\alpha)$$

$$\sin\left(\frac{\pi}{2}+\alpha\right)=\cos(\alpha);\quad \cos\left(\frac{\pi}{2}+\alpha\right)=-\sin(\alpha)$$

$$\sin(\pi-\alpha)=\sin(\alpha);\quad \cos(\pi-\alpha)=-\cos(\alpha)$$

$$\sin(\pi+\alpha)=-\sin(\alpha);\quad \cos(\pi+\alpha)=-\cos(\alpha)$$

$$\sin\left(\frac{3}{2}\pi-\alpha\right)=-\cos(\alpha);\quad \cos\left(\frac{3}{2}\pi-\alpha\right)=-\sin(\alpha)$$

$$\sin\left(\frac{3}{2}\pi+\alpha\right)=-\cos(\alpha);\quad \cos\left(\frac{3}{2}\pi+\alpha\right)=\sin(\alpha)$$

$$\sin(-\alpha)=-\sin(\alpha);\quad \cos(-\alpha)=\cos(\alpha)$$

Formule di somma degli angoli

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$

$$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$$

$$\tan(\alpha+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}\quad \text{dove}\quad \alpha,\ \beta,\ \alpha+\beta\neq\frac{\pi}{2}+k\pi,\ \ k\in\mathbb{Z}$$

$$\tan(\alpha-\beta)=\frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}\quad \text{dove}\quad \alpha,\ \beta,\ \alpha-\beta\neq\frac{\pi}{2}+k\pi,\ \ k\in\mathbb{Z}$$

Da cui discendono le formule di duplicazione

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

$$\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)} \quad \text{dove} \quad \alpha \neq \frac{\pi}{4} + k\frac{\pi}{2} \ \wedge \ \alpha \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

$$\sin(3\alpha) = 3\sin(\alpha) - 4\sin^3(\alpha)$$

$$\cos(3\alpha) = 4\cos^3(\alpha) - 3\cos(\alpha)$$

$$\tan(3a) = \frac{3tg(a) - tg^3(a)}{1 - 3tg^2(a)}$$

$$cotg(3a) = \frac{cotg^3(a) - 3cotg(a)}{3cotg^2(a) - 1}$$

Formule parametriche

$$\sin(\alpha) = \frac{2t}{1+t^2} \quad \text{dove} \quad t = \tan\left(\frac{\alpha}{2}\right) \text{ e } \alpha \neq \pi + 2k\pi$$

$$\cos(\alpha) = \frac{1-t^2}{1+t^2} \quad \text{dove} \quad t = \tan\left(\frac{\alpha}{2}\right) \text{ e } \alpha \neq \pi + 2k\pi$$

$$\tan(\alpha) = \frac{2t}{1-t^2} \quad \text{dove} \quad t = \tan\left(\frac{\alpha}{2}\right) \text{ e } \alpha \neq \frac{\pi}{2} + k\pi \ \wedge \ \alpha \neq \pi + 2k\pi$$

Formule di bisezione

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{1+\cos(\alpha)}} \quad \text{dove} \quad \alpha \neq \pi + 2k\pi, \quad k \in \mathbb{Z}$$

Formule di Werner

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$

Formule di prostaferesi

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Valori agli angoli notevoli delle funzioni trigonometriche

Angolo in gradi	Angolo in radianti	Valore del seno	Valore del coseno	Valore della tangente	Valore della cotangente
0°	0	0	1	0	$\pm\infty$
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18°	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
22° 30'	$\frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
36°	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
54°	$\frac{3}{10}\pi$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
67° 30'	$\frac{3}{8}\pi$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
72°	$\frac{2}{5}\pi$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
75°	$\frac{5}{12}\pi$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

Angolo in gradi	Angolo in radianti	Valore del seno	Valore del coseno	Valore della tangente	Valore della cotangente
72°	$\frac{2}{5}\pi$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
75°	$\frac{5}{12}\pi$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	$\pm\infty$	0
105°	$\frac{7}{12}\pi$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$-2-\sqrt{3}$	$\sqrt{3}-2$
108°	$\frac{3}{5}\pi$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1-\sqrt{5}}{4}$	$-\sqrt{5+2\sqrt{5}}$	$-\frac{\sqrt{25-10\sqrt{5}}}{5}$
112° 30'	$\frac{5}{8}\pi$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$-\frac{\sqrt{2-\sqrt{2}}}{2}$	$-1-\sqrt{2}$	$1-\sqrt{2}$
120°	$\frac{2}{3}\pi$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
126°	$\frac{7}{10}\pi$	$\frac{\sqrt{5}+1}{4}$	$-\frac{\sqrt{10-2\sqrt{5}}}{4}$	$-\frac{\sqrt{25+10\sqrt{5}}}{5}$	$-\sqrt{5-2\sqrt{5}}$
135°	$\frac{3}{4}\pi$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1
144°	$\frac{4}{5}\pi$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$-\frac{\sqrt{5}+1}{4}$	$-\sqrt{5-2\sqrt{5}}$	$-\frac{\sqrt{25+10\sqrt{5}}}{5}$
150°	$\frac{5}{6}\pi$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
157° 30'	$\frac{7}{8}\pi$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$	$1-\sqrt{2}$	$-\sqrt{2}-1$
162°	$\frac{9}{10}\pi$	$\frac{\sqrt{5}-1}{4}$	$-\frac{\sqrt{10+2\sqrt{5}}}{4}$	$-\frac{\sqrt{25-10\sqrt{5}}}{5}$	$-\sqrt{5+2\sqrt{5}}$
165°	$\frac{11}{12}\pi$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\sqrt{3}-2$	$-\sqrt{3}-2$
180°	π	0	-1	0	$\pm\infty$
195°	$\frac{13}{12}\pi$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
198°	$\frac{11}{10}\pi$	$\frac{1-\sqrt{5}}{4}$	$-\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
202° 30'	$\frac{9}{8}\pi$	$-\frac{\sqrt{2-\sqrt{2}}}{2}$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
210°	$\frac{7}{6}\pi$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
216°	$\frac{6}{5}\pi$	$-\frac{\sqrt{10-2\sqrt{5}}}{4}$	$-\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
225°	$\frac{5}{4}\pi$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1

Angolo in gradi	Angolo in radianti	Valore del seno	Valore del coseno	Valore della tangente	Valore della cotangente
225°	$\frac{5}{4}\pi$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1
234°	$\frac{13}{10}\pi$	$-\frac{\sqrt{5}+1}{4}$	$-\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
240°	$\frac{4}{3}\pi$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
247° 30'	$\frac{11}{8}\pi$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$	$-\frac{\sqrt{2-\sqrt{2}}}{2}$	$\sqrt{2}+1$	$\sqrt{2}-1$
252°	$\frac{7}{5}\pi$	$-\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1-\sqrt{5}}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
255°	$\frac{17}{12}\pi$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
270°	$\frac{3}{2}\pi$	-1	0	$\pm\infty$	0
285°	$\frac{19}{12}\pi$	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$-2-\sqrt{3}$	$\sqrt{3}-2$
288°	$\frac{8}{5}\pi$	$-\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$-\sqrt{5+2\sqrt{5}}$	$-\frac{\sqrt{25-10\sqrt{5}}}{5}$
292° 30'	$\frac{13}{8}\pi$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$-1-\sqrt{2}$	$1-\sqrt{2}$
300°	$\frac{5}{3}\pi$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
306°	$\frac{17}{10}\pi$	$-\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$-\frac{\sqrt{25+10\sqrt{5}}}{5}$	$-\sqrt{5-2\sqrt{5}}$
315°	$\frac{7}{4}\pi$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1
324°	$\frac{9}{5}\pi$	$-\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$-\sqrt{5-2\sqrt{5}}$	$-\frac{\sqrt{25+10\sqrt{5}}}{5}$
330°	$\frac{11}{6}\pi$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
337° 30'	$\frac{15}{8}\pi$	$-\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$1-\sqrt{2}$	$-1-\sqrt{2}$
342°	$\frac{19}{10}\pi$	$\frac{1-\sqrt{5}}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$-\frac{\sqrt{25-10\sqrt{5}}}{5}$	$-\sqrt{5+2\sqrt{5}}$
345°	$\frac{23}{12}\pi$	$\frac{\sqrt{2}-\sqrt{6}}{4}$	$\frac{\sqrt{2}+\sqrt{6}}{4}$	$\sqrt{3}-2$	$-2-\sqrt{3}$
360°	2π	0	1	0	$\pm\infty$

Alcune formule di trigonometria

$$\sin(arccos(\alpha)) = \sqrt{1-\alpha^2} = \cos(\arcsin(\alpha))$$

$$\arccos(\alpha) + \arcsin(\alpha) = \frac{\pi}{2} \qquad\qquad \text{arctg}(\alpha) + \text{arctg}\left(\frac{1}{\alpha}\right) = \frac{\pi}{2}$$

$$a\sin x + b\cos x = \sqrt{a^2+b^2}\sin\left(x + sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)\right) \qquad (\, solo\ se\ a > 0\,)$$

Funzioni iperboliche

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad ; \quad \cosh(x) = \frac{e^x + e^{-x}}{2} \quad ; \quad \tanh(x) = \frac{\sinh(x)}{\cosh(x)} \quad ; \quad \coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

$$\text{sech}(x) = \frac{1}{\cosh(x)} \quad ; \quad \text{csch}(x) = \frac{1}{\sinh(x)} \quad ; \quad settsinh(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

$$settcosh(x) = \ln\left(x + \sqrt{x^2-1}\right) \quad ; \quad settanh(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad ; \quad settcoth(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

$$settsech(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \quad ; \quad settcosech(x) = \ln\left(\frac{1 \pm \sqrt{1+x^2}}{x}\right)$$

Limiti notevoli:

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1 \quad ; \quad \lim_{f(x)\to 0} \frac{\ln(1+f(x))}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln(a)} \quad ; \quad \lim_{f(x)\to 0} \frac{\log_a(1+f(x))}{f(x)} = \frac{1}{\ln(a)}$$

con $a > 0,\ a \neq 1$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1 \quad ; \quad \lim_{f(x)\to 0} \frac{e^{f(x)} - 1}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln(a) \qquad ; \qquad \lim_{f(x)\to 0} \frac{a^{f(x)} - 1}{f(x)} = \ln(a)$$

con $a > 0$

$$\lim_{x\to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e \qquad ; \qquad \lim_{f(x)\to \pm\infty} \left(1 + \frac{1}{f(x)}\right)^{f(x)} = e$$

$$\lim_{x\to 0} \frac{(1+x)^c - 1}{x} = c \qquad ; \qquad \lim_{f(x)\to 0} \frac{(1+f(x))^c - 1}{f(x)} = c$$

con $c \in \mathbb{R}$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1 \qquad ; \qquad \lim_{f(x)\to 0} \frac{\sin(f(x))}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2} \qquad ; \qquad \lim_{f(x)\to 0} \frac{1-\cos(f(x))}{(f(x))^2} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\tan(x)}{x} = 1 \qquad ; \qquad \lim_{f(x)\to 0} \frac{\tan(f(x))}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{\arcsin(x)}{x} = 1 \qquad ; \qquad \lim_{f(x)\to 0} \frac{\arcsin(f(x))}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{\arctan(x)}{x} = 1 \qquad ; \qquad \lim_{f(x)\to 0} \frac{\arctan(f(x))}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{\sinh(x)}{x} = 1 \qquad ; \qquad \lim_{f(x)\to 0} \frac{\sinh(f(x))}{f(x)} = 1$$

$$\lim_{x \to 0} \frac{\cosh(x) - 1}{x^2} = \frac{1}{2} \quad ; \quad \lim_{f(x) \to 0} \frac{\cosh(f(x)) - 1}{(f(x))^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\tanh(x)}{x} = 1 \quad ; \quad \lim_{f(x) \to 0} \frac{\tanh(f(x))}{f(x)} = 1$$

$$\lim_{x \to 1^-} \frac{\arccos(x)}{\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)} = 1$$

Derivate delle funzioni elementari

$f(x) = \text{costante}$	$f'(x) = 0$ Dimostrazione derivata di una costante
$f(x) = x$	$f'(x) = 1$ Dimostrazione derivata di x
$f(x) = x^s, \ s \in \mathbb{R}$	$f'(x) = sx^{s-1}$ Dimostrazione derivata di una potenza
$f(x) = a^x$	$f'(x) = a^x \ln(a)$ Dimostrazione derivata dell'esponenziale
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \log_a(x)$	$f'(x) = \frac{1}{x \ln(a)}$ Dimostrazione derivata del logaritmo

$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$
$f(x) = \lvert x \rvert$	$f'(x) = \frac{\lvert x \rvert}{x}$ Dimostrazione derivata valore assoluto
$f(x) = \sin(x)$	$f'(x) = \cos(x)$ Dimostrazione derivata del seno
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$ Dimostrazione derivata del coseno
$f(x) = \tan(x)$ [non è elementare]	$f'(x) = \frac{1}{\cos^2(x)}$ Dimostrazione derivata della tangente
$f(x) = \cot(x)$ [non è elementare]	$f'(x) = -\frac{1}{\sin^2(x)}$ Dimostrazione derivata della cotangente
$f(x) = \arcsin(x)$	$f'(x) = \frac{1}{\sqrt{1-x^2}}$ Dimostrazione derivata dell'arcoseno

$f(x) = \arccos(x)$	$f'(x) = -\frac{1}{\sqrt{1-x^2}}$ Dimostrazione analoga alla precedente
$f(x) = \arctan(x)$	$f'(x) = \frac{1}{1+x^2}$ Dimostrazione derivata dell'arcotangente
$f(x) = \text{arccot}(x)$	$f'(x) = -\frac{1}{1+x^2}$ Dimostrazione analoga alla precedente
$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$	$f'(x) = \cosh(x)$ Dimostrazione: semplici conti
$f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$	$f'(x) = \sinh(x)$ Idem come sopra

Sviluppi di Taylor-McLaurin notevoli

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots + \frac{x^n}{n!} + o(x^n) \ \forall x \in \mathbb{R} \qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \forall x \in \mathbb{R}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots + \frac{(-1)^{n+1}}{n}x^n + o(x^n) \quad \text{per } |x|<1$$

$$\ln(1+x) = \sum_{n=1}^{+\infty} (-1)^{n+1}\frac{x^n}{n} \text{ per } |x|<1$$

$$\begin{aligned}(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6}x^3 + \cdots + \binom{\alpha}{n}x^n + o(x^n) \\ &= \sum_{n=0}^{\infty}\binom{\alpha}{n}x^n \quad \text{per } |x|<1\end{aligned}$$

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!}$$

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\sin(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1} \quad \forall x \in \mathbb{R}$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + o(x^{2n})$$

$$\cos(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n} \quad \forall x \in \mathbb{R}$$

$$\tan(x) = x + \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^6) \quad \text{per } |x|<\frac{\pi}{2}$$

$$\arcsin(x) = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + o(x^9) \quad \text{per } |x|<1$$

$$\arccos(x) = \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3}{40}x^5 - \frac{5}{112}x^7 - \frac{35}{1152}x^9 + o(x^9) \quad \text{per } |x|<1$$

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} + o(x^9) \text{ per } |x|<1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \forall |x|<1$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad \forall |x|<1$$

Integrali delle funzioni elementari

Integrali indefiniti fondamentali

$$\int f'(x)dx = f(x) + c$$

$$\int a\,dx = ax + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \text{, con } n \neq -1$$

$$\int \frac{1}{x} dx = \log|x| + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \left(1 + \tan^2 x\right) dx = \int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \left(1 + \cot^2 x\right) dx = \int \frac{1}{\sin^2 x} dx = -\cot x + c$$

$$\int \sinh x dx = \cosh x + c$$

$$\int \cosh x dx = \sinh x + c$$

$$\int e^x dx = e^x + c$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + c$$

$$\int a^x dx = \frac{a^x}{\log_e a} + c$$

Integrali notevoli

$$\int \frac{1}{\sin x} dx = \log\left|\frac{\tan x}{2}\right| + c$$

$$\int \frac{1}{\cos x} dx = \log\left|\frac{\tan x}{2} + \frac{\pi}{4}\right| + c$$

$$\int \frac{1}{\sqrt{1 - x^2}} dx = \begin{cases} arc\sin x + c \\ -arc\cos x + c \end{cases}$$

$$\int \frac{-1}{\sqrt{1 - x^2}} dx = \begin{cases} arc\cos x + c \\ -arc\sin x + c \end{cases}$$

$$\int \frac{1}{1 + x^2} dx = arc\tan x + c$$

$$\int \frac{1}{1 - x^2} dx = \frac{1}{2}\log\left|\frac{1 + x}{1 - x}\right| + c$$

$$\int \frac{1}{\sqrt{x^2 - 1}} dx = \log\left|x + \sqrt{x^2 - 1}\right| + c$$

$$\int \frac{1}{\sqrt{1 + x^2}} dx = \begin{cases} arc\sinh x + c \\ \log\left(x + \sqrt{1 + x^2}\right) + c \end{cases}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \log\left|x + \sqrt{x^2 \pm a^2}\right| + c$$

$$\int \sqrt{\left(x^2 \pm a^2\right)} dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right) + c$$

$$\int \sqrt{\left(a^2 - x^2\right)} dx = \frac{1}{2}\left(a^2 arc\sin\frac{x}{a} + x\sqrt{a^2 - x^2}\right) + c$$

$$\int \sin^2 x dx = \frac{1}{2}(x - \sin x \cos x) + c$$

$$\int \cos^2 x dx = \frac{1}{2}(x + \sin x \cos x) + c$$

$$\int \frac{1}{\cosh^2 x} dx = \int \left(1 - \tanh^2 x\right) dx + c = \tanh x + c$$

Integrali indefiniti riconducibili a quelli immediati

$$\int f^n(x)\cdot f'(x)dx = \frac{f^{n+1}(x)}{n+1}+c$$

$$\int \frac{f'(x)}{f(x)}dx = \log|f(x)|+c$$

$$\int f'(x)\cdot \cos f(x)dx = \sin f(x)+c$$

$$\int f'(x)\cdot \sin f(x)dx = -\cos f(x)+c$$

$$\int e^{f(x)}\cdot f'(x)dx = e^{f(x)}+c$$

$$\int a^{f(x)}\cdot f'(x)dx = \frac{a^{f(x)}}{\log_e a}+c$$

$$\int \frac{f'(x)}{\sqrt{1-f^2(x)}}dx = \begin{cases} \arcsin f(x)+c \\ -\arccos f(x)+c \end{cases}$$

$$\int \frac{f'(x)}{1+f^2(x)}dx = \arctan f(x)+c$$

$$\int \frac{dx}{Ax^2+B} = \frac{arctg\left(\sqrt{\frac{A}{B}}x\right)}{\sqrt{AB}}+c$$

Formule di integrazione ricorsiva

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$$

$$\int x^n \ln(x)dx = \frac{x^{n+1}\left((n+1)\ln(x)-1\right)}{(n+1)^2}+c$$

$$\int \sin^n(x)dx = -\frac{\cos(x)\sin^{n-1}(x)}{n}+\frac{n-1}{n}\int \sin^{n-2}(x)dx$$

$$\int \cos^n(x)dx = \frac{\sin(x)\cos^{n-1}(x)}{n}+\frac{n-1}{n}\int \cos^{n-2}(x)dx$$

$$\int \sin^n(x)\cos^m(x)dx = \begin{cases} -\dfrac{\sin^{n-1}(x)\cos^{m+1}(x)}{m+n}+\dfrac{n-1}{m+n}\displaystyle\int \sin^{n-2}(x)\cos^m(x)dx \\ \dfrac{\sin^{n+1}(x)\cos^{m-1}(x)}{m+n}+\dfrac{m-1}{m+n}\displaystyle\int \sin^{n}(x)\cos^{m-2}(x)dx \end{cases}$$

$$\int \frac{dx}{(1+x^2)^{n+1}} = \frac{x}{2n(1+x^2)^n}+\frac{2n-1}{2n}\int\frac{dx}{(1+x^2)^n}$$

Criteri di convergenza serie :

$$\text{consideriamo } S : \sum_{n=1}^{+\infty} a_n$$

S convergente => $\lim_{n\to+\infty} a_n = 0$ (se questo non è vero S non converge!)

Se $\exists m \quad t.c. \quad \forall n \geq m \quad a_n \geq 0$ (o $a_n \leq 0$) allora S non è indeterminata !

$$\sum_{n=1}^{+\infty} x^n = \frac{1}{1-x} \qquad (\,se -1 < x < 1\,) \qquad (div.\, se\ x \geq 1)\ (ind.\, se\ x \leq -1)$$

$$\sum_{n=1}^{+\infty} \frac{1}{n^p} \qquad (conv.\ se\ p > 1\,) \qquad (div.\, se\ p \leq 1)$$

Confronto : se $\exists m \quad t.c. \quad \forall n \geq m \quad b_n \geq a_n \geq 0$

$$\sum_{n=1}^{+\infty} b_n\ conv. \quad => \quad \sum_{n=1}^{+\infty} a_n \quad conv.$$

$$\sum_{n=1}^{+\infty} a_n\ div. \quad => \quad \sum_{n=1}^{+\infty} b_n \quad div.$$

Infinitesimi : sia $l = \lim_{n\to+\infty} n^p a_n$ e $\exists m \quad t.c. \quad \forall n \geq m \quad a_n \geq 0$

$$l \neq +\infty, \quad p > 1 \quad => \quad \sum_{n=1}^{+\infty} a_n \quad conv.$$

$$l \neq 0, \quad p \leq 1 \quad => \quad \sum_{n=1}^{+\infty} a_n = +\infty$$

Rapporto : sia $l = \lim_{n\to+\infty} \frac{a_{n+1}}{a_n}$ e $\exists m \quad t.c. \quad \forall n \geq m \quad a_n > 0$

$$l < 1 \quad => \quad \sum_{n=1}^{+\infty} a_n \quad conv.$$

$$l > 1 \quad => \quad \sum_{n=1}^{+\infty} a_n = +\infty$$

Radice : sia $l = \lim_{n\to+\infty} \sqrt[n]{a_n}$ e $\exists m \quad t.c. \quad \forall n \geq m \quad a_n \geq 0$

se $l < 1$ allora la serie converge

se $l > 1$ allora la serie diverge

Leibniz : sia $0 = \lim_{n\to+\infty} a_n$ e $\exists m \quad t.c. \quad \forall n \geq m \quad a_n \geq a_{n+1} \geq 0$

$$allora \sum_{n=1}^{+\infty} (-1)^n a_n \ converge$$

Integrale : sia $f(x)$ continua e decrescente in $[1; +\infty)$

$$allora \quad \sum_{n=1}^{+\infty} f(n) \quad converge \quad se\ e\ solo\ se \quad \int_1^{+\infty} f(x)dx \ \ converge$$

Confronto asintotico : Sia $a_n > 0\,, b_n > 0 \quad t.c. \ \exists \lim_{n\to+\infty} \frac{a_n}{b_n} = l \qquad allora$

$$0 \leq l < +\infty \ e \ \sum_{n=1}^{+\infty} b_n \ converge \quad ==> \quad \sum_{n=1}^{+\infty} a_n \quad converge$$

$$0 < l \leq +\infty \quad e \ \sum_{n=1}^{+\infty} b_n \ diverge \quad ==> \quad \sum_{n=1}^{+\infty} a_n \quad diverge$$

Disuguaglianza di Bernoulli :

$\forall n \epsilon \mathrm{N}_0, \forall x \geq -1 \qquad (1+x)^n \geq 1 + nx$

(molte di queste immagini sono state prese dal sito https://www.youmath.it/)