Cyber-Physical Systems

Laura Nenzi

Università degli Studi di Trieste
I Semestre 2023

Lecture 15: Signal Temporal Logic

[Many Slides due to J. Deshmukh, A. Donzé]

Typical day in a control designer's life

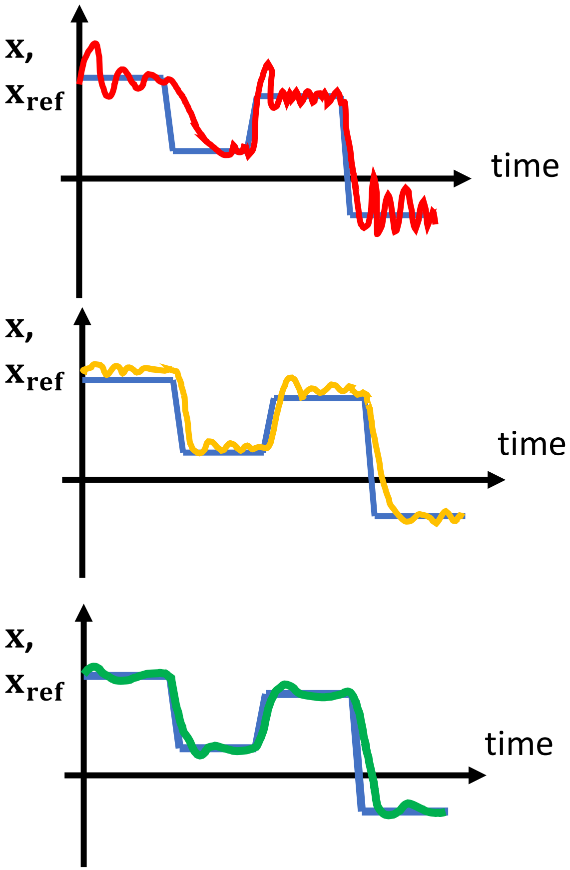
Check **transient response** of x when driving with highway 101 pattern with temperature around 40º C

Chief Engineer

Control Designer

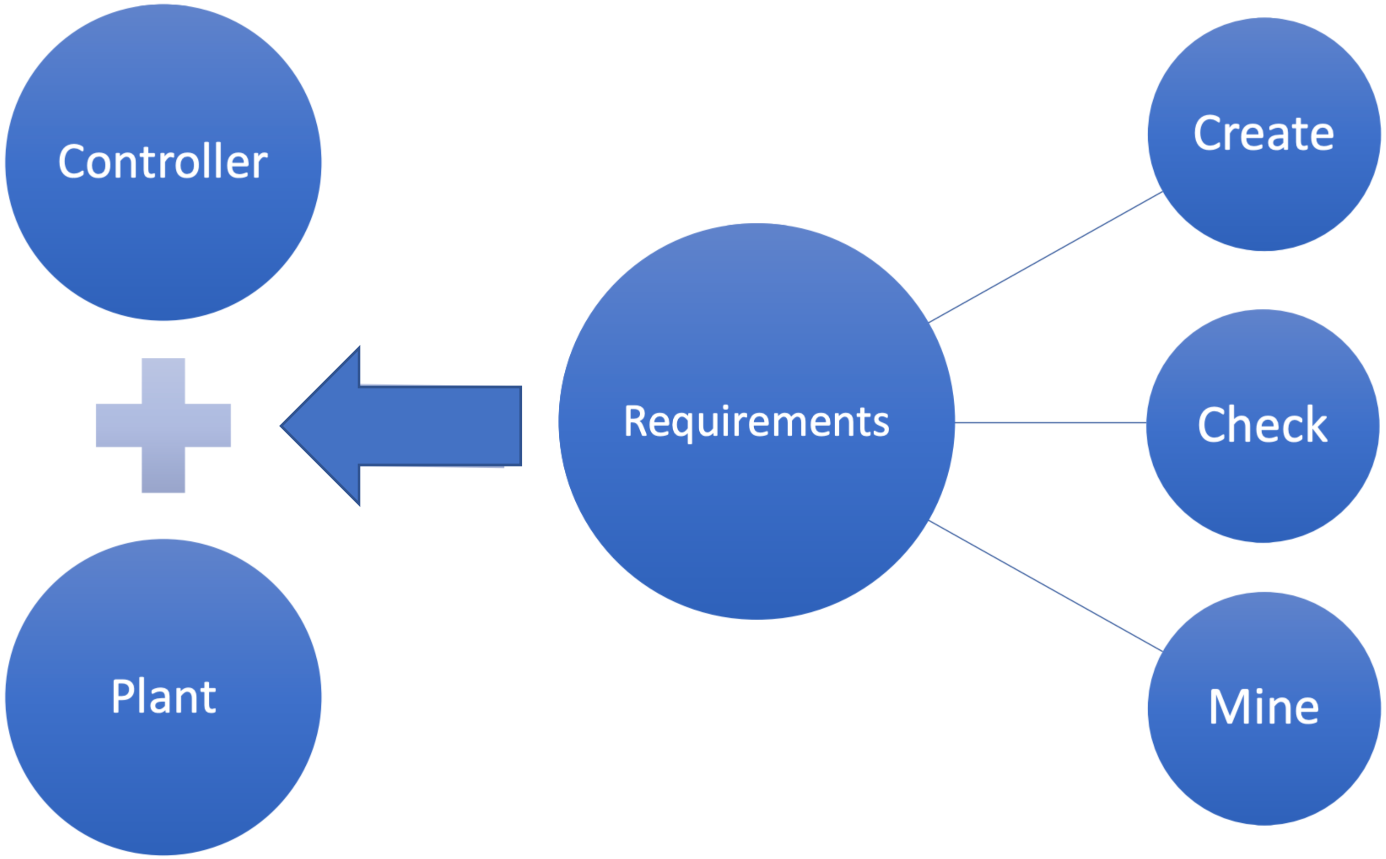
- Normally in natural language (ambiguous, error-prone)
- Sometime absent
- If you are LUCKY, they are written in English

Typical day in a control designer’s life



Requirements Driving Design

Requirements **formally** capture what it means for a system to operate correctly in its operating environment



Linear Temporal Logic (LTL) specification

It is a logic interpreted over infinite discrete-time traces

E.g. **For the next 3 days** the highest temperature will be below 75 degree and the lowest temperature will be above 60 degree

$X\,(p \wedge q) \wedge X\,X\,(p \wedge q) \wedge X\,X\,X\,(p \wedge q)$ with $p = T<75$, $q=T>60$

Metric Interval Temporal Logic (STL)

Invented by R. Alur, T.Feder, T.A. Henzinger (1991)
It extended LTL by adding **dense time intervals**:

$$G_{[0,3]}(\mathrm{p} \wedge q)$$

Signal Temporal Logic (STL)

Invented by D. Nickovic and O. Maler from Verimag (2004)
It extended MITL by having **signal predicates over real values as atomic formulas:**

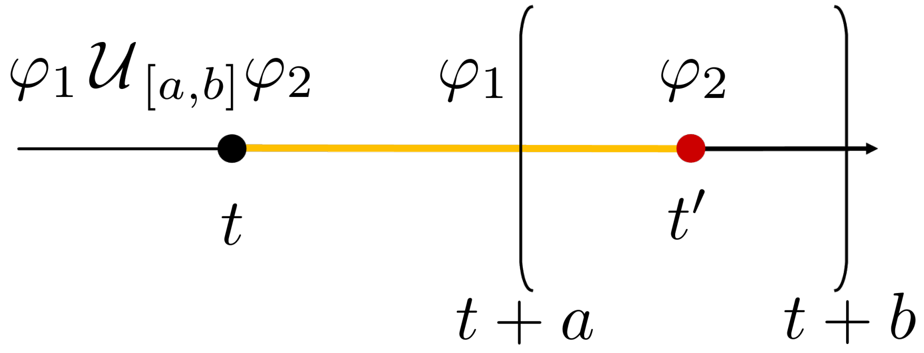
$$G_{[0,3]}(T < 75 \wedge T > 60)$$

STL Syntax

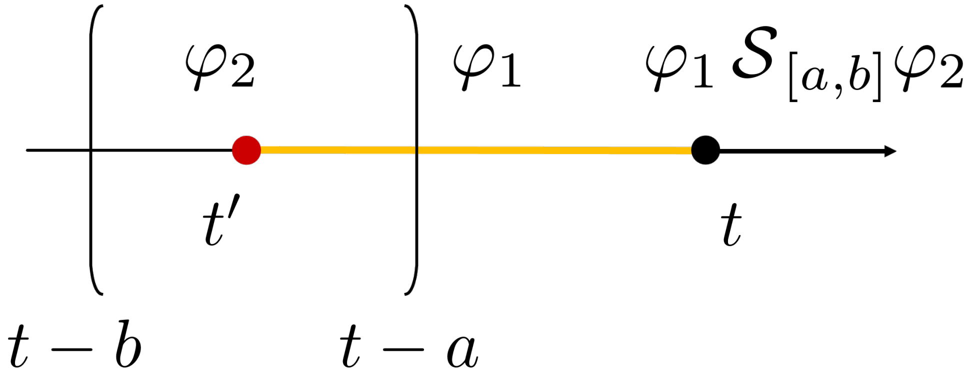
Syntax of STL			
$\varphi ::=$	$f(\mathbf{x}) \sim 0$	$\mid$	$f : \mathbb{D} \rightarrow \mathbb{R}$ is a function over the signal $\mathbf{x}$: $\mathbb{T} \rightarrow \mathbb{D}$,
			$\sim \in \{\leq, <, >, \geq, =, \neq\}$
	$\neg\varphi$	$\mid$	Negation
	$\varphi_1 \wedge \varphi_2$	$\mid$	Conjunction
	$\mathbf{F}_{[a,b]}\varphi$	$\mid$	At some **F**uture step in the interval $[a, b]$
	$\mathbf{G}_{[a,b]}\varphi$	$\mid$	**G**lobally in all times in the interval $[a, b]$
	$\varphi_1\ \mathbf{U}_{[a,b]}\ \varphi_2$	$\mid$	In all steps **U**ntil in interval $[a, b]$
	$\varphi_1\ \boldsymbol{S}_{[a,b]}\ \varphi_2$	$\mid$	In all steps **S**ince in interval $[a, b]$

Since and Until Operators

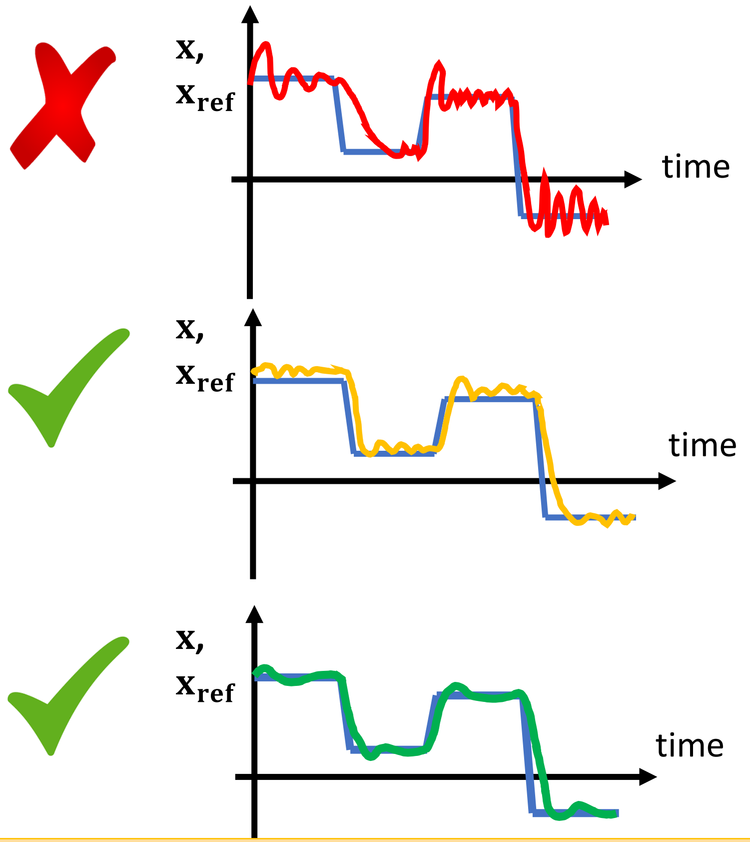
- Until



- Since



Can we express our engineer's requirements?

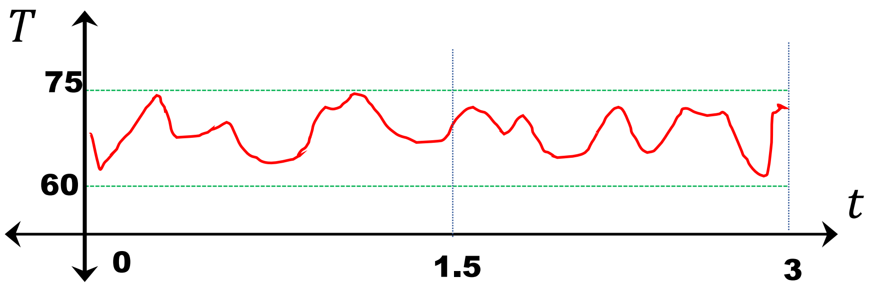


$$\varphi \equiv \mathrm{G}_{[0,10]}\left(\left|x - x_{ref}\right| < 0.05\right)$$

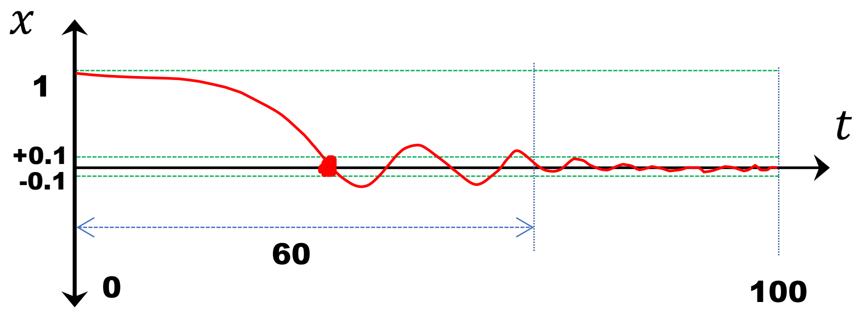
Expressing specifications in STL

$\mathbf{G}_{[0,3]}\,(60 < T < 75)$

Always between time 0 and 3



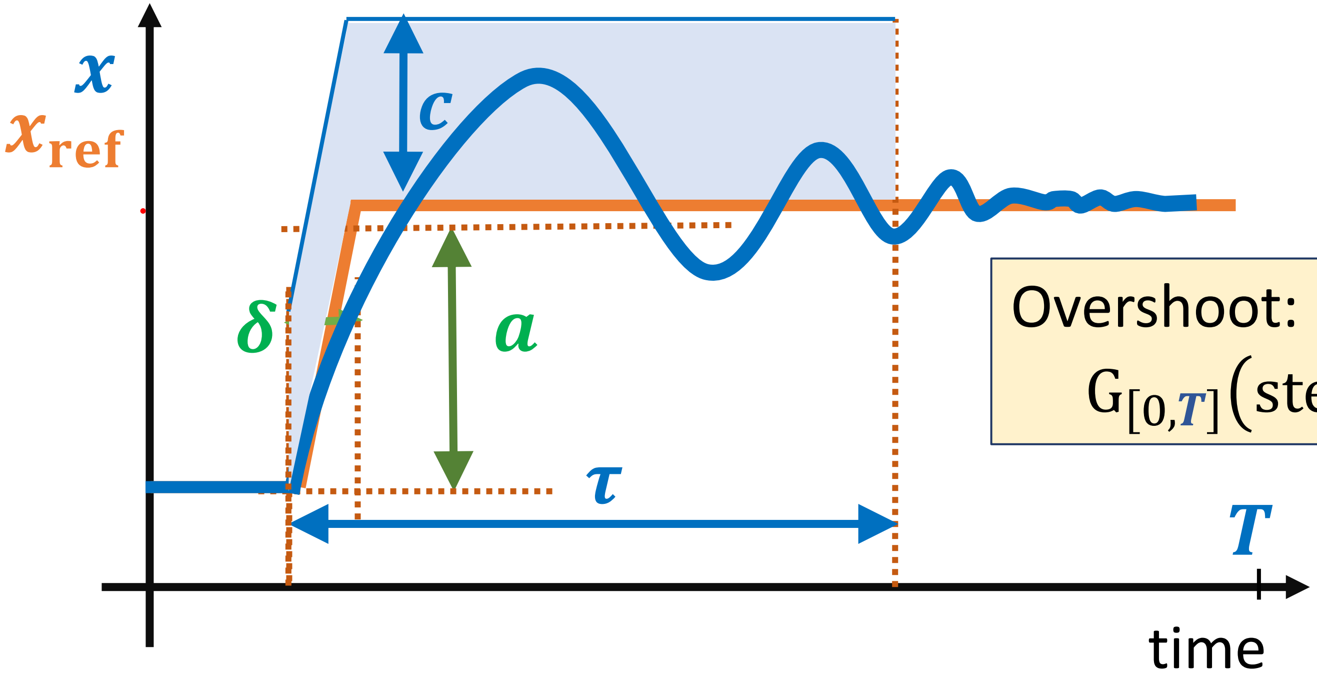
$\mathbf{F}_{[0,60]}(\mathbf{G}\,(|x| < 0.1)\,)$



Eventually at **some time** ***t*** between time 0 and 60

From that time ***t*,** always till the end of the signal trace

Example STL formulas: Overshoot



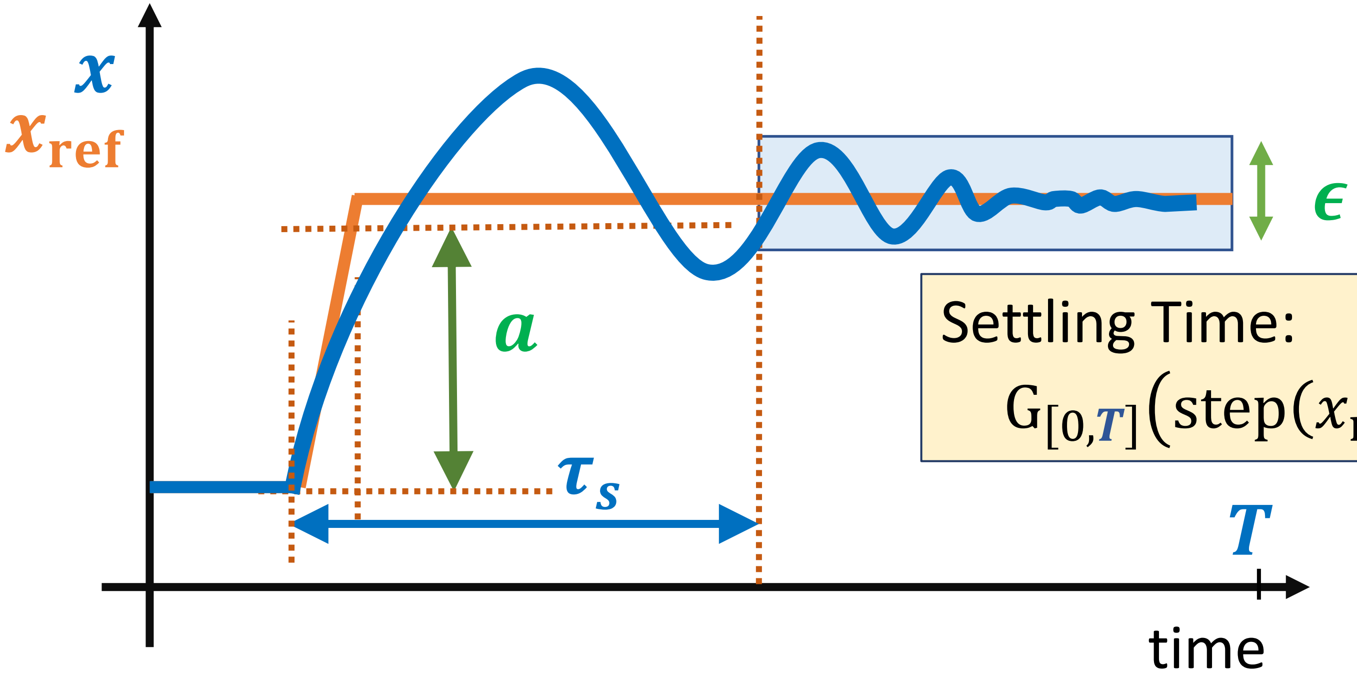
Step:

$$\text{step}(y, t) \coloneqq y(t+\tau) - y(t) > a$$

Overshoot:

$$\mathrm{G}_{[0,T]}\big(\text{step}(x_{\text{ref}}, t) \Rightarrow \mathrm{G}_{[0,\tau]}(x(t) - x_{\text{ref}}(t) < c)\big)$$

Example STL formulas: Settling Time



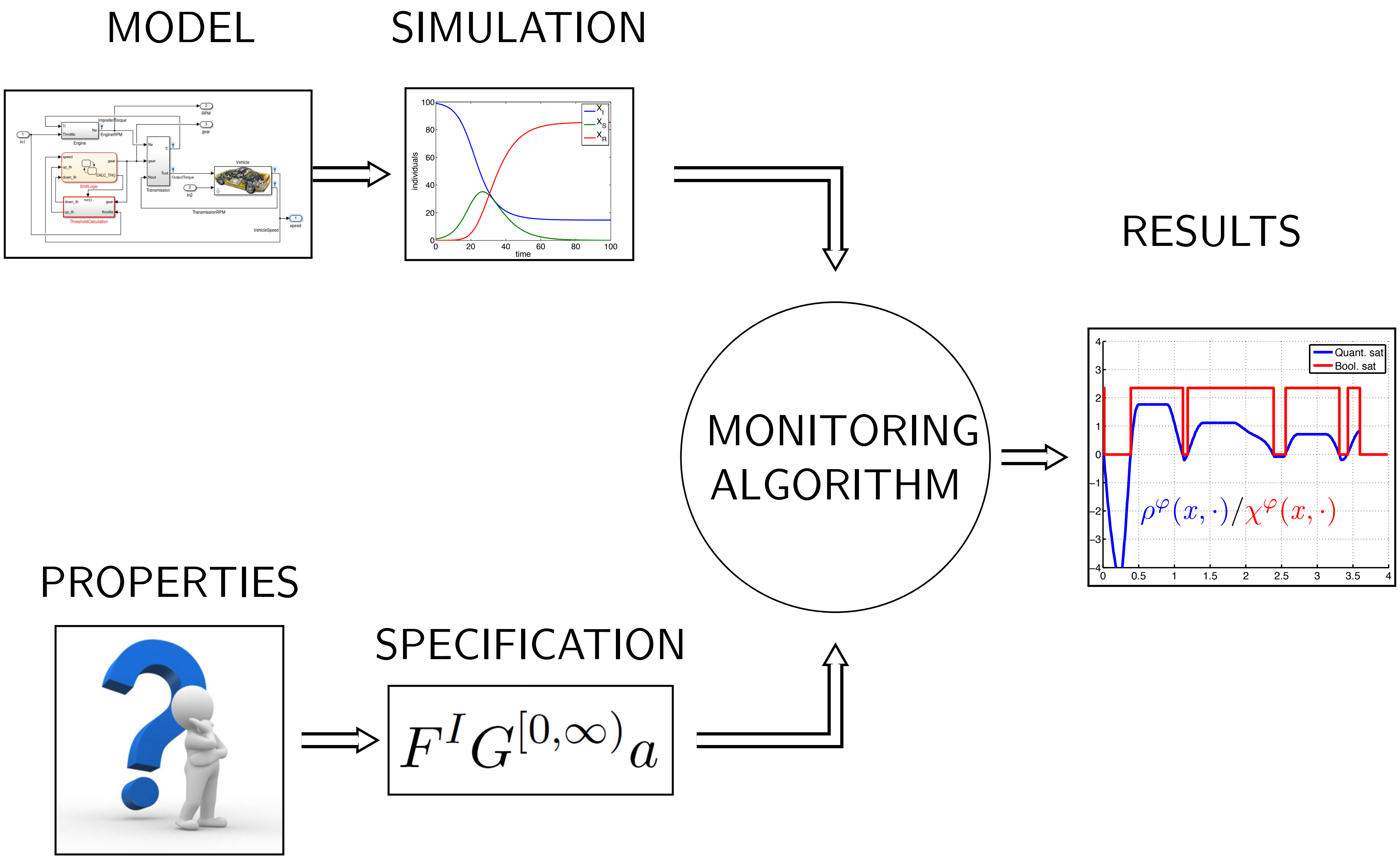
Step:

$$\text{step}(y, t) \coloneqq y(t+\delta) - y(t) > a$$

Settling Time:

$$G_{[0,T]}\left(\text{step}(x_{\text{ref}}, t) \Rightarrow G_{[\tau_s,\infty]}(|x(t) - x_{\text{ref}}(t)| < \epsilon)\right)$$

Specification-based Monitoring



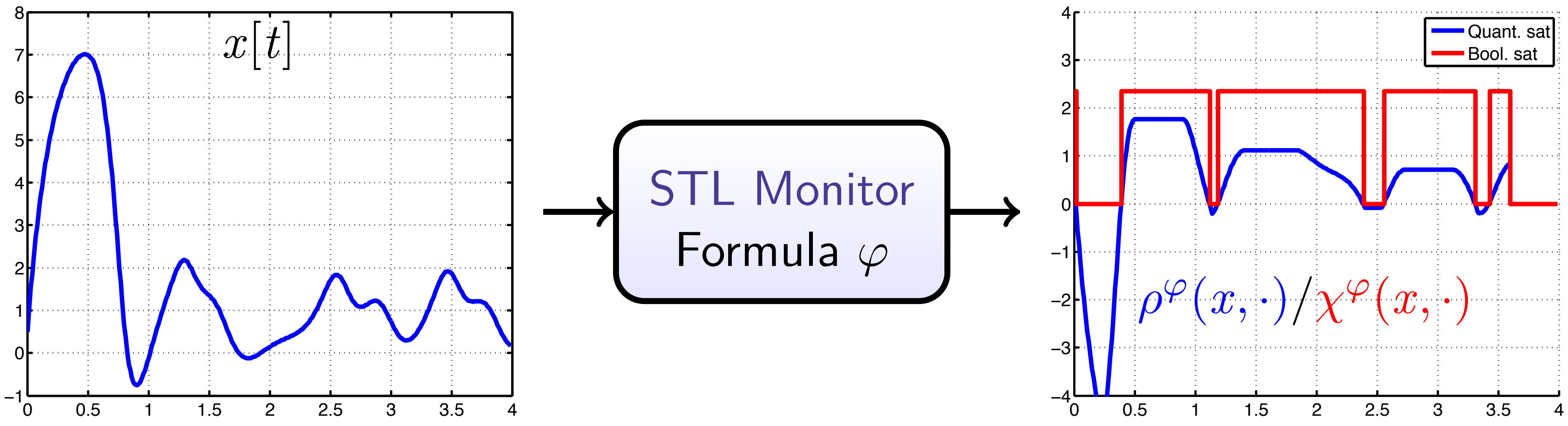
Specification-based Monitoring

Boolean Signal

$s_\varphi : [0, T] :\rightarrow \{0, 1\}$ s.t. $s_\varphi(t) = 1 \Leftrightarrow (\vec{x}, t) \models \varphi$

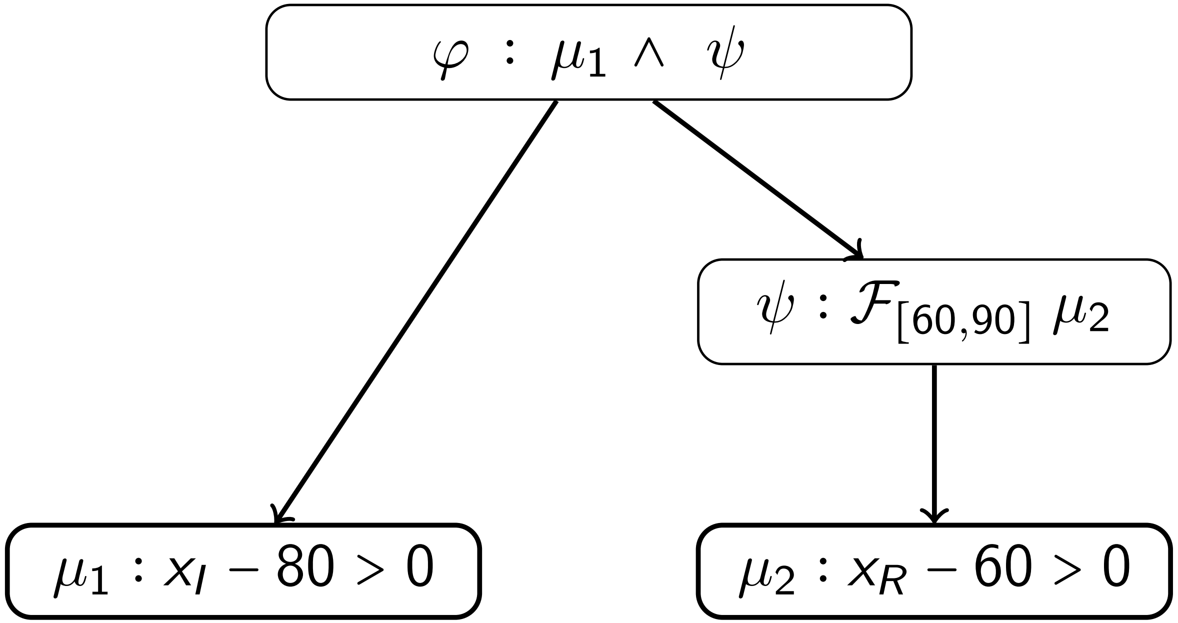
Quantitative Signal

$\rho_\varphi : [0, T] :\rightarrow \mathbb{R} \cup \{\pm\infty\}$ s.t. $\rho_\varphi(t) = \rho(\varphi, \vec{x}, t)$

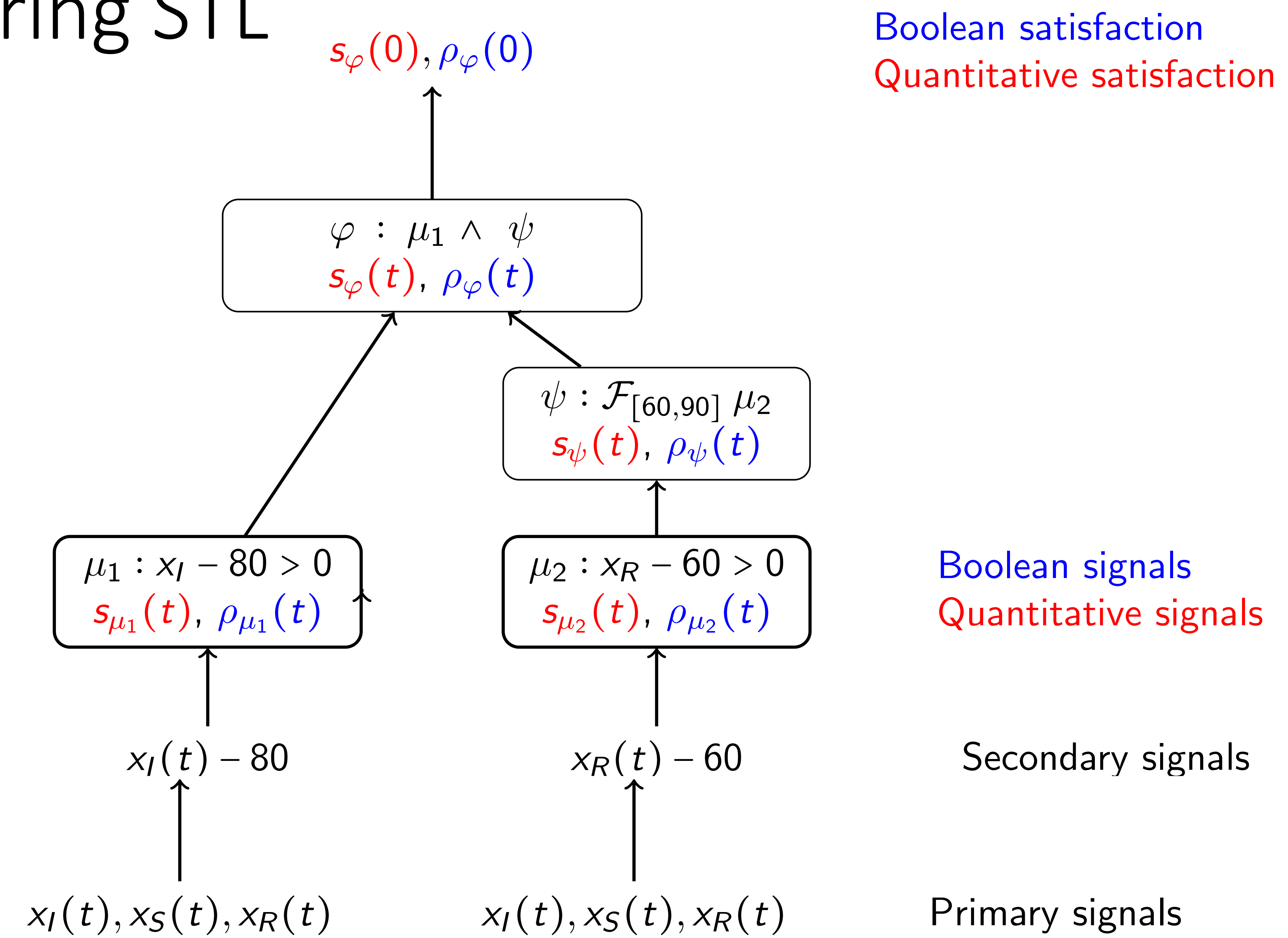


Monitoring STL

$\varphi : (x_I > 80) \wedge \mathcal{F}_{[60,90]} (x_R > 60)$



Monitoring STL



Recursive Boolean Semantics of STL

φ	$\mathrm{s}(\varphi, \mathbf{x}, t)$
$f(\mathbf{x}) \sim 0$	$f(\mathbf{x}(t)) \sim 0$, $\quad \sim \in \{\leq, <, >, \geq, =, \neq\}$
$\neg\varphi$	$\neg s(\varphi, \mathbf{x}, t)$
$\varphi_1 \wedge \varphi_2$	$\mathrm{s}(\varphi_1, \mathbf{x}, \mathrm{t}) \wedge s(\varphi_2, \mathbf{x}, \mathrm{t})$
$\mathbf{F}_{[a,b]}\varphi$	$\exists\tau \in [t+a, t+b]\ \ s(\varphi, \mathbf{x}, \tau)$
$\mathbf{G}_{[a,b]}\varphi$	$\forall\tau \in [t+a, t+b]\ \ s(\varphi, \mathbf{x}, \tau)$
$\varphi\ \mathbf{U}_{[a,b]}\ \psi$	$\exists\tau \in [t+a, t+b]\ \big(s(\psi, \mathbf{x}, \tau) \wedge \forall\tau' \in [t, \tau)\ \ s(\varphi, x, \tau')\big)$
$\varphi\ \mathbf{S}_{[a,b]}\ \psi$	$\exists\tau \in [t-a, t-b]\ \big(s(\psi, \mathbf{x}, \tau) \wedge \forall\tau' \in (\tau, t]\ s(\varphi, x, \tau')\big)$

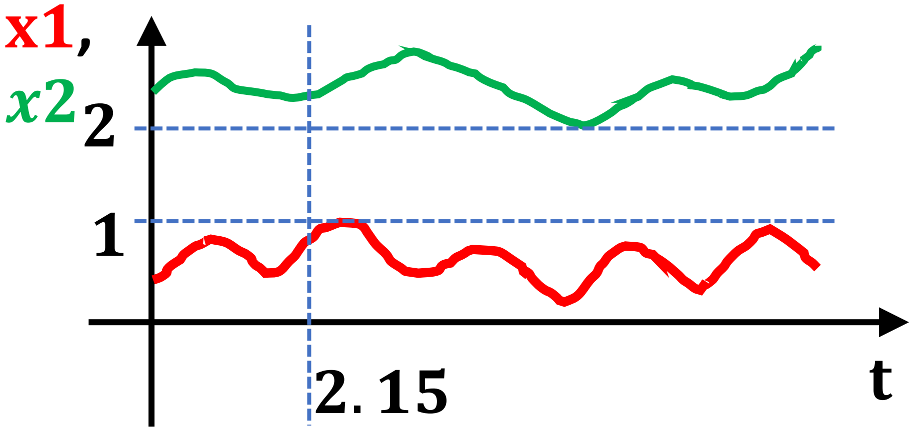
$$\mathrm{s}(\varphi, \mathbf{x}) = s(\varphi, \mathbf{x}, 0)$$

STL semantics

- Semantics of STL specified recursively over a signal $\mathbf{x}: \mathbb{T} \rightarrow \mathbb{D}$ at each time,

For each STL formula φ, here's how we define it's semantics:

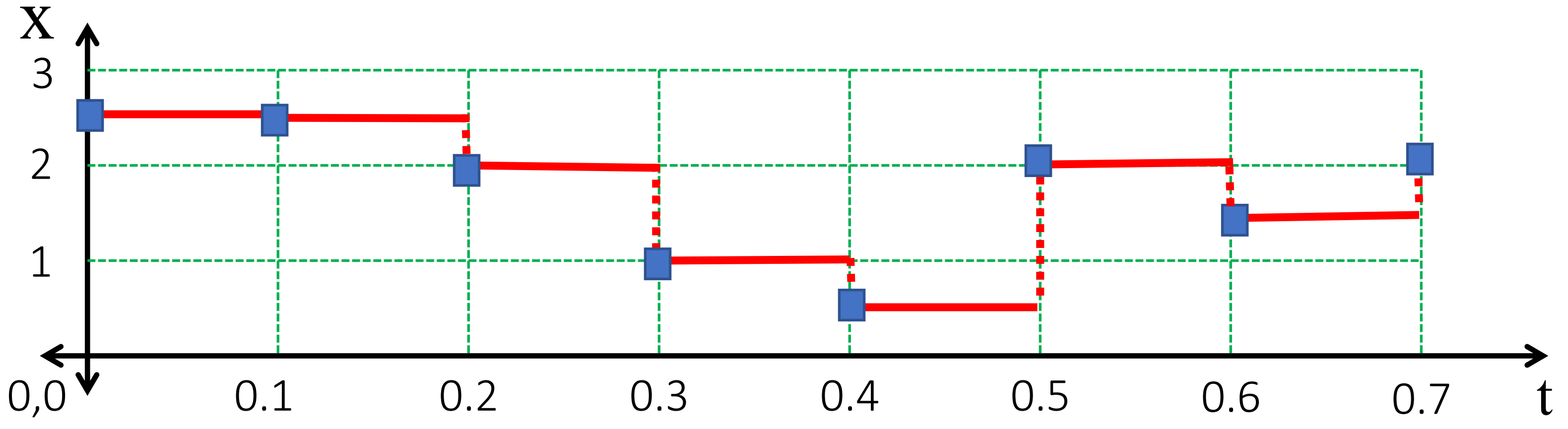
- If φ is the signal predicate $\mu = f(\mathbf{x}) > 0$, then
  $s(\varphi, \mathbf{x}, t) = \mathit{true}$ iff $f\big(\mathbf{x}(t)\big) > 0$



$$\mathbf{x} = (x1, x2)$$
$$\boldsymbol{f} = x2 - x1 - 1$$
$$s(f(\mathbf{x}) > 0, \mathbf{x}, 2.15)?$$

Recursive Boolean Semantics of STL

$\boldsymbol{\varphi} \equiv \mathbf{F}_{[0,0.2]}(\boldsymbol{x}(\boldsymbol{t}) \geq \mathbf{1.5})$



$x(t) - 1.5 > 0$	T	T	T	F	F	T	T	T
$\mathbf{F}_{[0,0.2]}\ \mu$	T	T	T	T	T	T		
$\mathbf{G}_{[0,0.7]}\mathbf{F}_{[0,0.2]}\ \mu$	T							

STL has quantitative semantics

- Quantitative semantics defined using the notion of a *Robust Satisfaction Value*, or *Robustness Value*
- Robustness ρ is a function that maps
  - a given trace $\mathbf{x}(t)$,
  - a formula φ,
  - and a time t

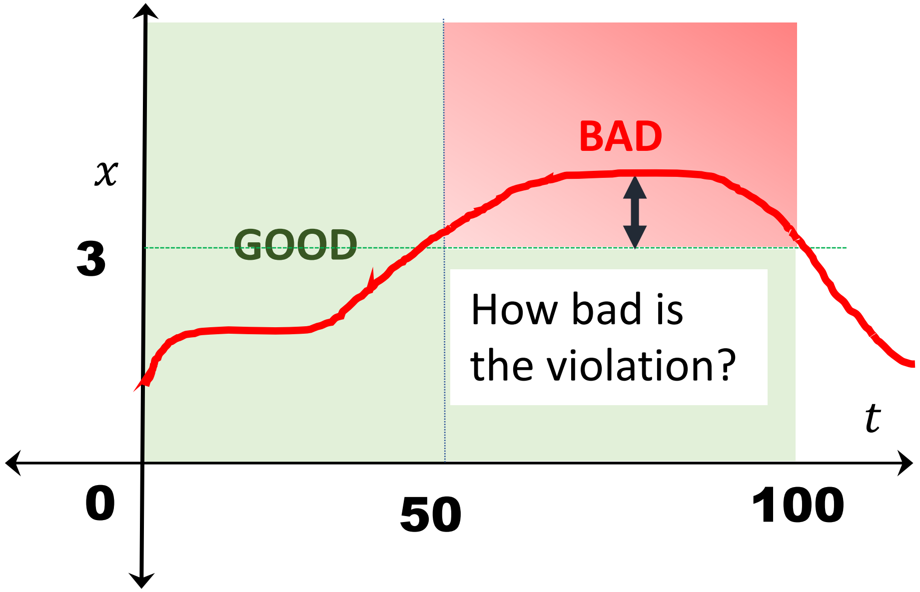
  to some real value
- We can interpret robustness as "distance to violation" of a given formula

Recursive Quantitative Semantics

φ	$\rho(\varphi, \mathbf{x}, t)$
$f(\mathbf{x}) > 0$, $f(\mathbf{x}) \geq 0$	$f(\mathbf{x}(t))$
$\neg\varphi$	$-\rho(\varphi, \mathbf{x}, t)$
$\varphi_1 \wedge \varphi_2$	$\min\big(\rho(\varphi_1, \mathbf{x}, \mathrm{t}), \rho(\varphi_2, \mathbf{x}, \mathrm{t})\big)$
$\mathbf{F}_{[a,b]}\varphi$	$\sup_{\tau \in [t+a, t+b]} \rho(\varphi, \mathbf{x}, \tau)$
$\mathbf{G}_{[a,b]}\varphi$	$\inf_{\tau \in [t+a, t+b]} \rho(\varphi, \mathbf{x}, \tau)$
$\varphi\ \mathbf{U}_{[a,b]}\ \psi$	$\sup_{\tau \in [t+a, t+b]} \left(\min\left(\rho(\psi, \mathbf{x}, \tau), \inf_{\tau' \in [t, \tau)} \rho(\varphi, \mathbf{x}, t)\right)\right)$

$$\rho(\varphi, \mathbf{x}) = \rho(\varphi, \mathbf{x}, 0)$$

Distance to violation/satisfaction



$$\mathbf{G}_{[50,100]}(x(t) < 3)$$

Property of Robust Satisfaction Signal

- Sign indicates satisfaction status (soundness):

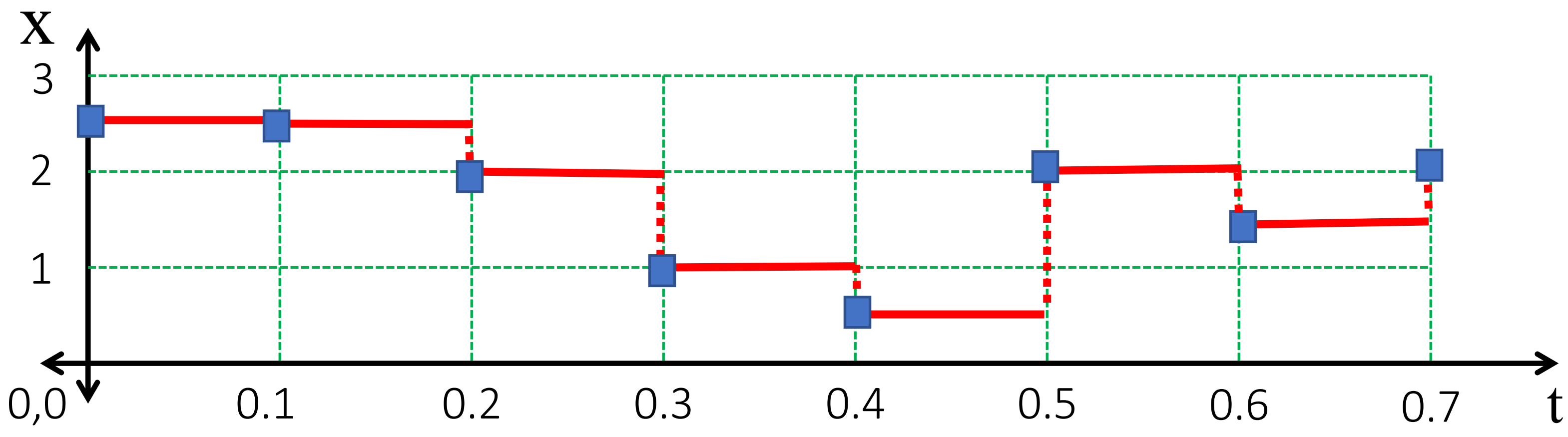
$$\rho(\varphi, \mathbf{x}, t) > 0 \;\Rightarrow\; \beta(\varphi, \mathbf{x}, t) = 1$$
$$\rho(\varphi, \mathbf{x}, t) < 0 \;\Rightarrow\; \beta(\varphi, \mathbf{x}, t) = 0$$

- Absolute value indicates tolerance (correctness)

$$\left|\left|\mathbf{x} - \mathbf{x}'\right|\right|_{\infty} < \rho(\varphi, \mathbf{x}, t) \;\Rightarrow\; \beta(\varphi, \mathbf{x}, t) = \beta(\varphi, \mathbf{x}', t)$$

Robustness computation example

$\boldsymbol{\varphi} \equiv \mathbf{G}_{[0,0.7]}\mathbf{F}_{[0,0.2]}(\boldsymbol{x}(\boldsymbol{t}) > \mathbf{1.5})$



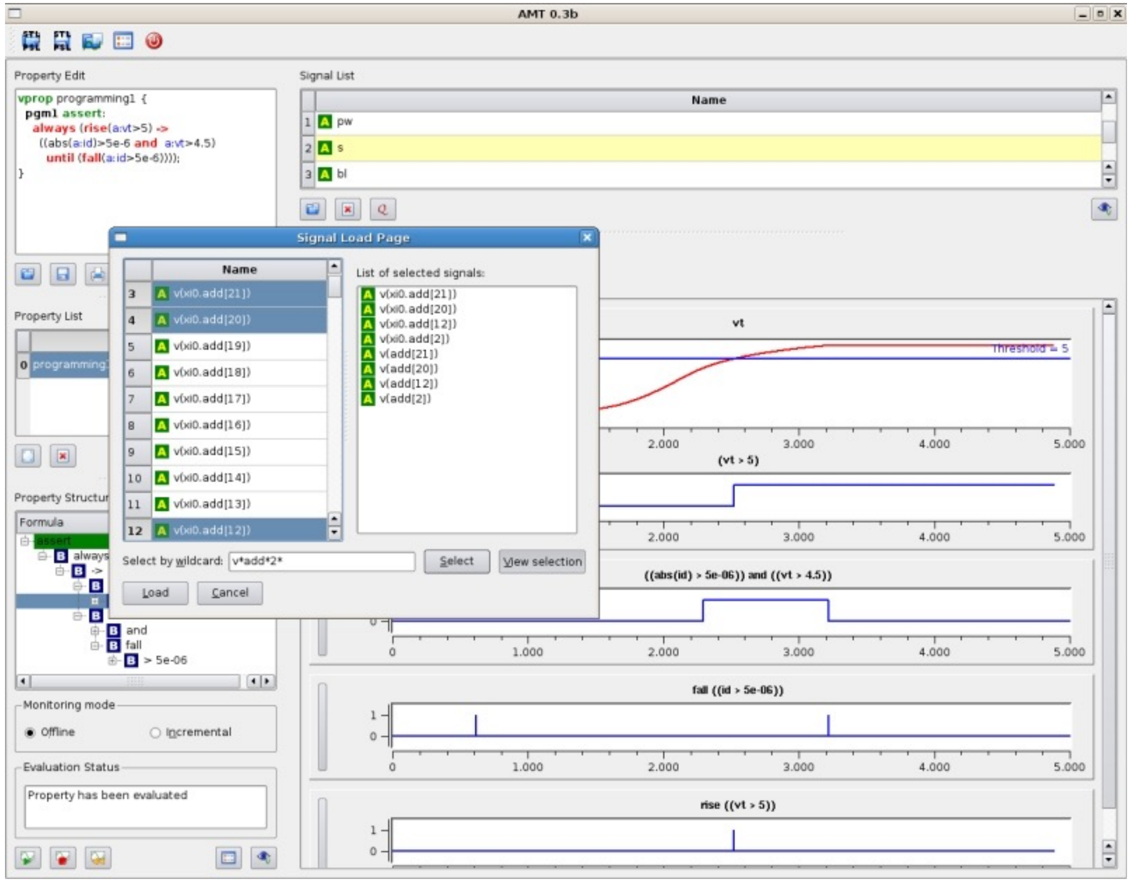
$x(t) - 1.5$	**1**	**1**	**0.5**	**-0.5**	**-1**	**0.5**	**0**	**0.5**
$\mathbf{F}_{[0,0.2]}\ \mu$	**1**	**1**	**0.5**	**0.5**	**0.5**	**0.5**		
$\mathbf{G}_{[0,0.7]}\mathbf{F}_{[0,0.2]}\ \mu$	**0.5**							

$f(x(t)) > 0$ at time t	$f(x(t))$
$F_{[a,b]}\ \varphi$ at time t	Maximum over robustness of φ for $t' \in t \oplus [a, b]$
$G_{[a,b]}\varphi$ at time t	Minimum over robustness of φ for $t' \in t \oplus [a, b]$

Analog Monitoring Tool (AMT)

http://www-verimag.imag.fr/DIST-TOOLS/TEMPO/AMT/content.html

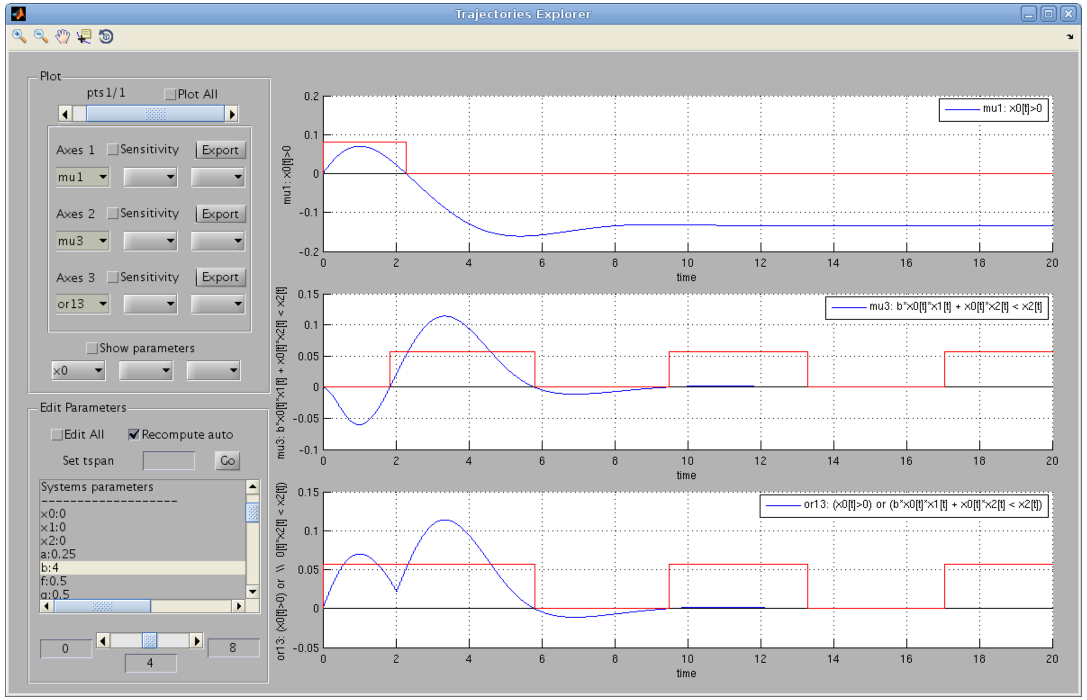
- Java toolbox
- STL with qualitative semantics
  - Correctness
- Offline monitoring



Breach

https://github.com/decyphir/breach

- MATLAB toolbox for
  - Simulation
  - Monitoring of temporal properties
  - Reachability



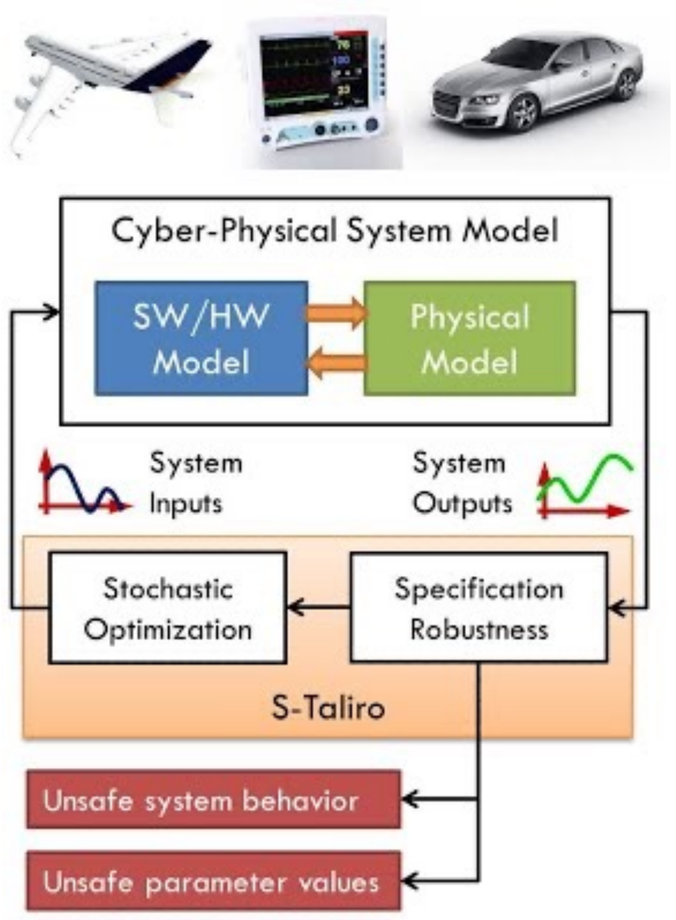
- STL with qualitative and quantitative semantics
  - Correctness
  - Robustness

- Offline and Online monitoring

S-TaLiRo

https://sites.google.com/a/asu.edu/s-taliro/s-taliro

- MATLAB toolbox for searching trajectories with minimal robustness
  - Randomized testing
    - Monte-Carlo simulation
    - Ant-colony optimization
    - Simulated annealing
    - Genetic algorithms
    - Cross enthopy
- MTL with quantitative semantics
  - Robustness
- Offline and Online monitoring



Moonlight

https://github.com/MoonLightSuite/MoonLight

- Java-toolbox + Matlab and Python interface for:
  - Monitoring of temporal properties
- STL + spatial operator with qualitative and quantitative semantics
  - Correctness
  - Robustness
- Offline monitoring

Bibliography

1. G. Fainekos, and G. J. Pappas. *Robustness of temporal logic specifications for continuous-time signals*. Theoretical Computer Science 2009.
2. Maler, Oded, and Dejan Nickovic. "Monitoring temporal properties of continuous signals." Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems. Springer, Berlin, Heidelberg, 2004. 152-166.
3. Donzé, Alexandre, and Oded Maler. "Robust satisfaction of temporal logic over real-valued signals." International Conference on Formal Modeling and Analysis of Timed Systems. Springer, Berlin, Heidelberg, 2010.