Image Processing for Physicists

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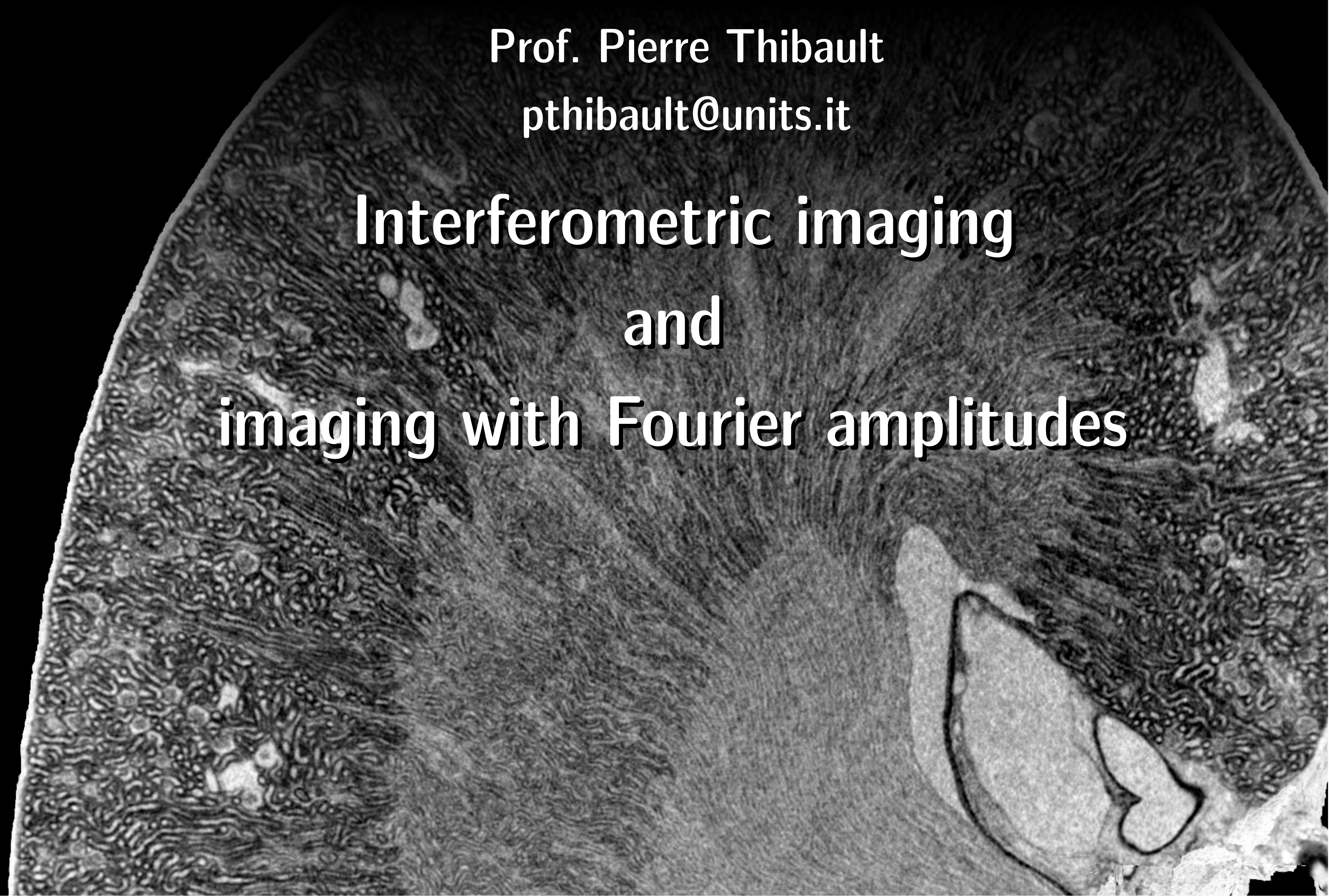
Interferometric imaging

and

imaging with Fourier amplitudes



Overview

- The phase problem
- Holography: on/off-axis
- Grating interferometric imaging
- Imaging using far-field amplitude measurements
  - Fourier transform holography
  - Coherent diffraction imaging
  - Ptychography

Wave propagation

$$\exp\left(i\pi \underbrace{u^2 \lambda z}_{\text{unitless number}}\right)$$

$\sim\sqrt{\lambda z}$ identical if

$$a_0^{-2}\lambda_0 z_0 = a_1^{-2}\lambda_1 z_1$$

$$\frac{a^2}{\lambda z} = f$$

Fresnel number

$f \ll 1$; far-field

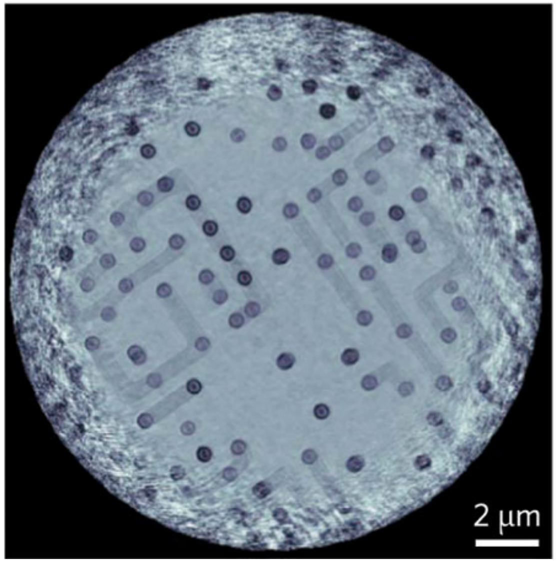
$f \gg 1$; near-field

a: here size of an aperture, but can be any characteristic length of interest

$f=1 \rightarrow a = \sqrt{\lambda z}$ ← characteristic length

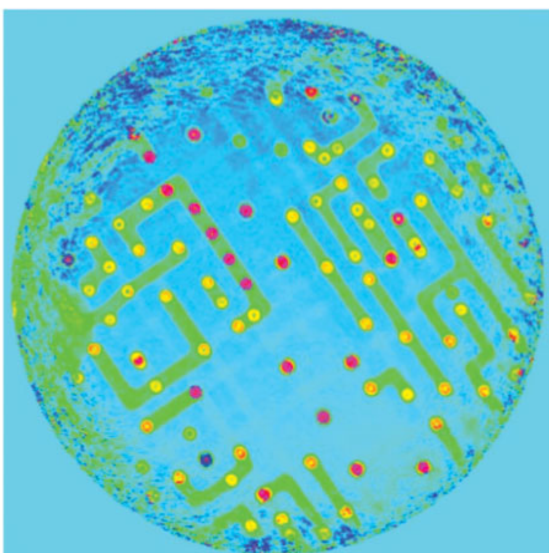
Complex-valued images

X-ray transmission image



Amplitude

attenuation of the wave

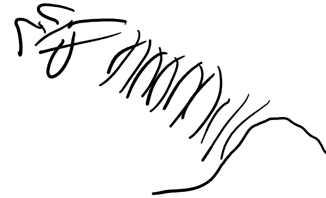


Phase

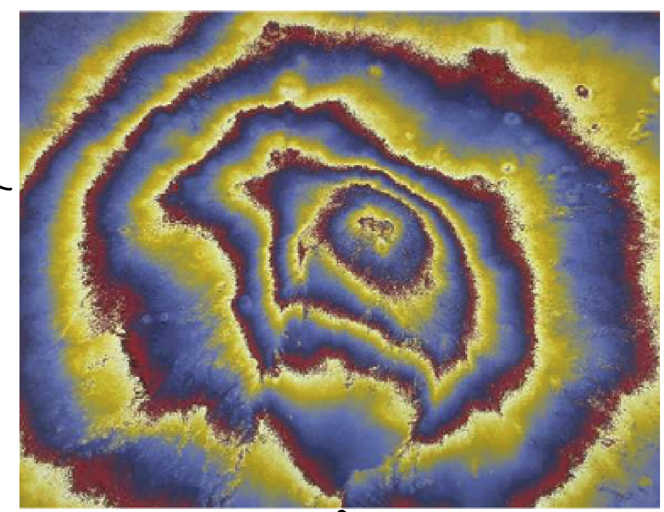
delay in wave phase

integrated circuit

SAR synthetic aperture radar



phase unwrapping



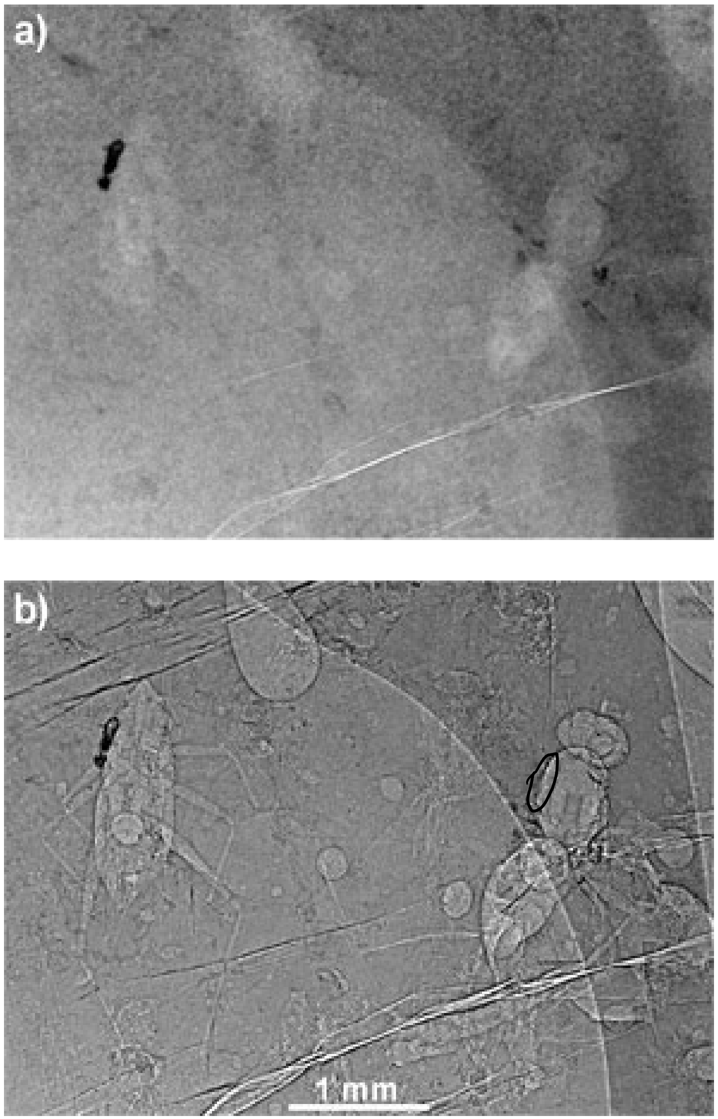
Phase

Phase-contrast

$|\psi|^2 = |A e^{i\phi}|^2 = A^2$

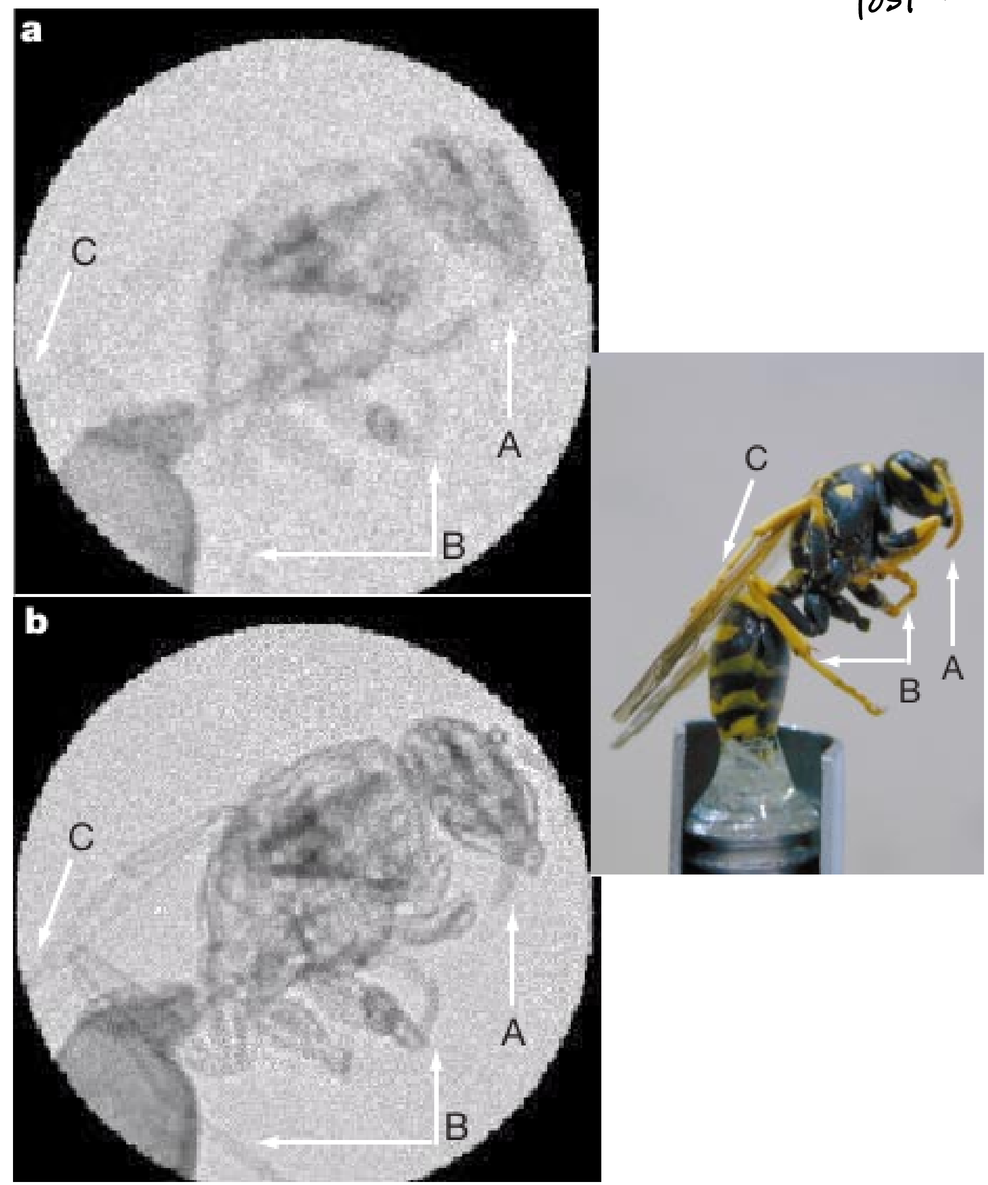
phase is lost !

Hard X-ray propagation-based phase contrast



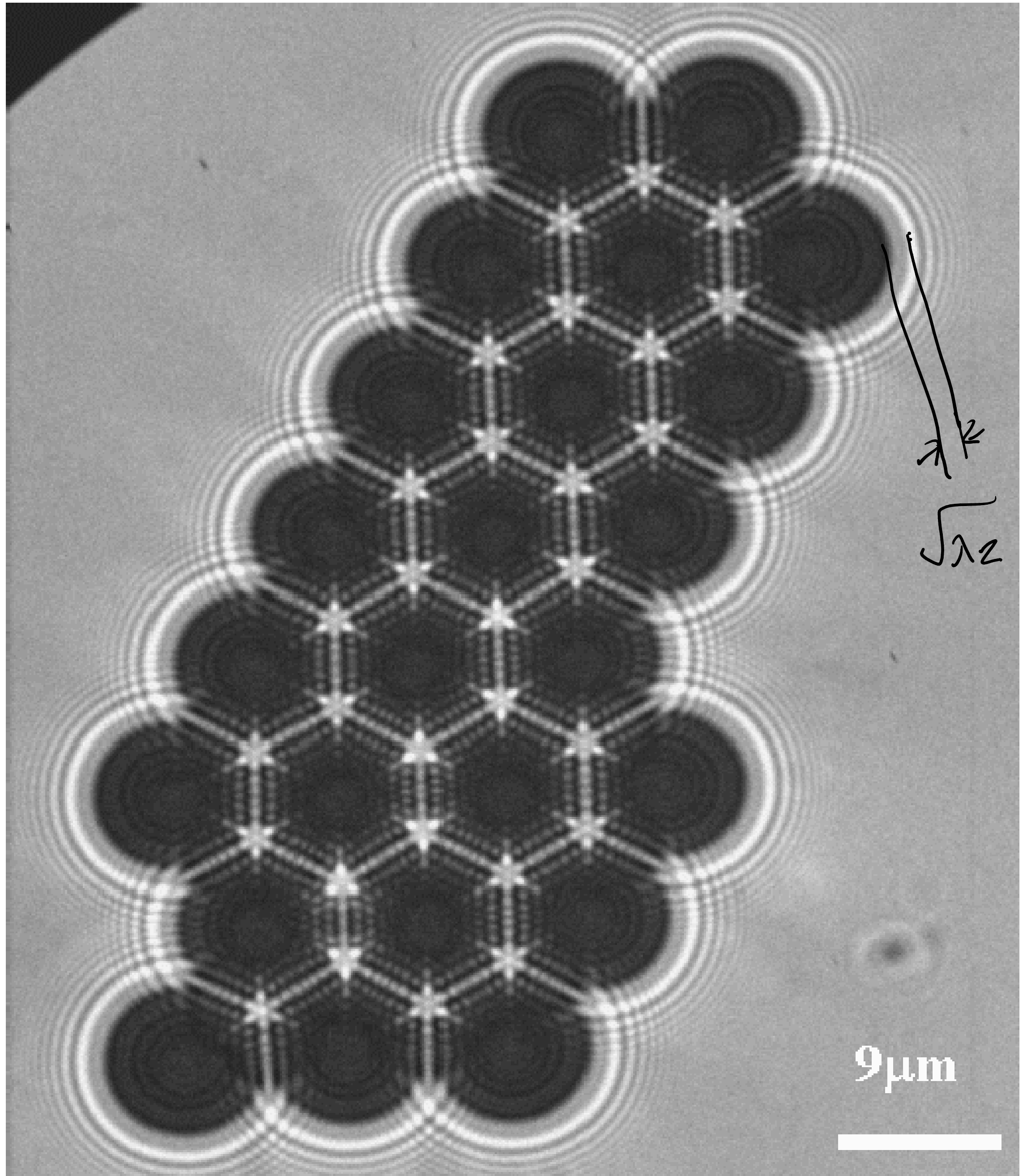
Source: www.esrf.eu/news/general/amber/amber/

Neutron phase contrast



Source: Allman et al. Nature **408** (2000).

In-line holography



"Deep" Fresnel regime, beyond near-field

$\sqrt{\lambda z}$

Source: Mayo et al. Opt Express **11** (2003).

In-line holography

Measure $I(\vec{r}) = |\psi(r;z)|^2$

If the illumination is a plane and monochromatic wave.

the transmission of the imaged object is weak:

$$\psi(\vec{r}; z=0) = A\left(1 + \vec{\varepsilon}(\vec{r})\right)$$

small perturbation of plane incident wave

$$I(\vec{r}) = \left|A\left(1+\varepsilon(\vec{r};z)\right)\right|^2 = |A|^2\Big(1 + \underbrace{\varepsilon(r;z)}_{} + \underbrace{\varepsilon^*(r;z)}_{\varepsilon(r;-z)} + \underbrace{\cancel{|\varepsilon(r;z)|^2}}_{\text{negligible}}\Big)$$

"twin image problem"

superposition of two images propagated by z and $-z$

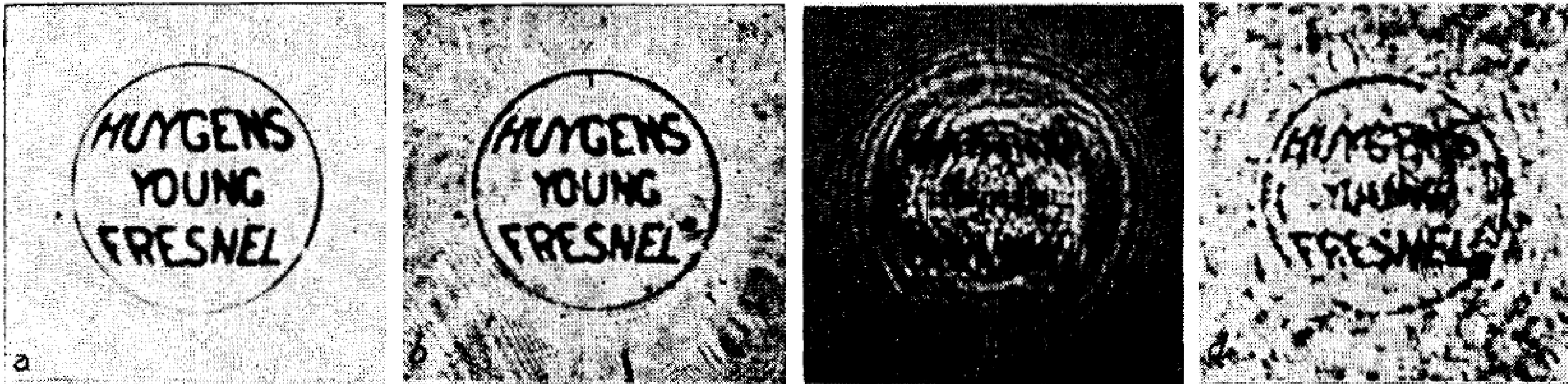
The phase problem

measure $I = |\psi|^2$ phases are lost

Sometimes: phase is quantity of interest

often: phase is auxiliary quantity for proper interpretation of wavefield.

In-line holography



D. Gabor, Nature **161**, 777-778 (1948).

mask

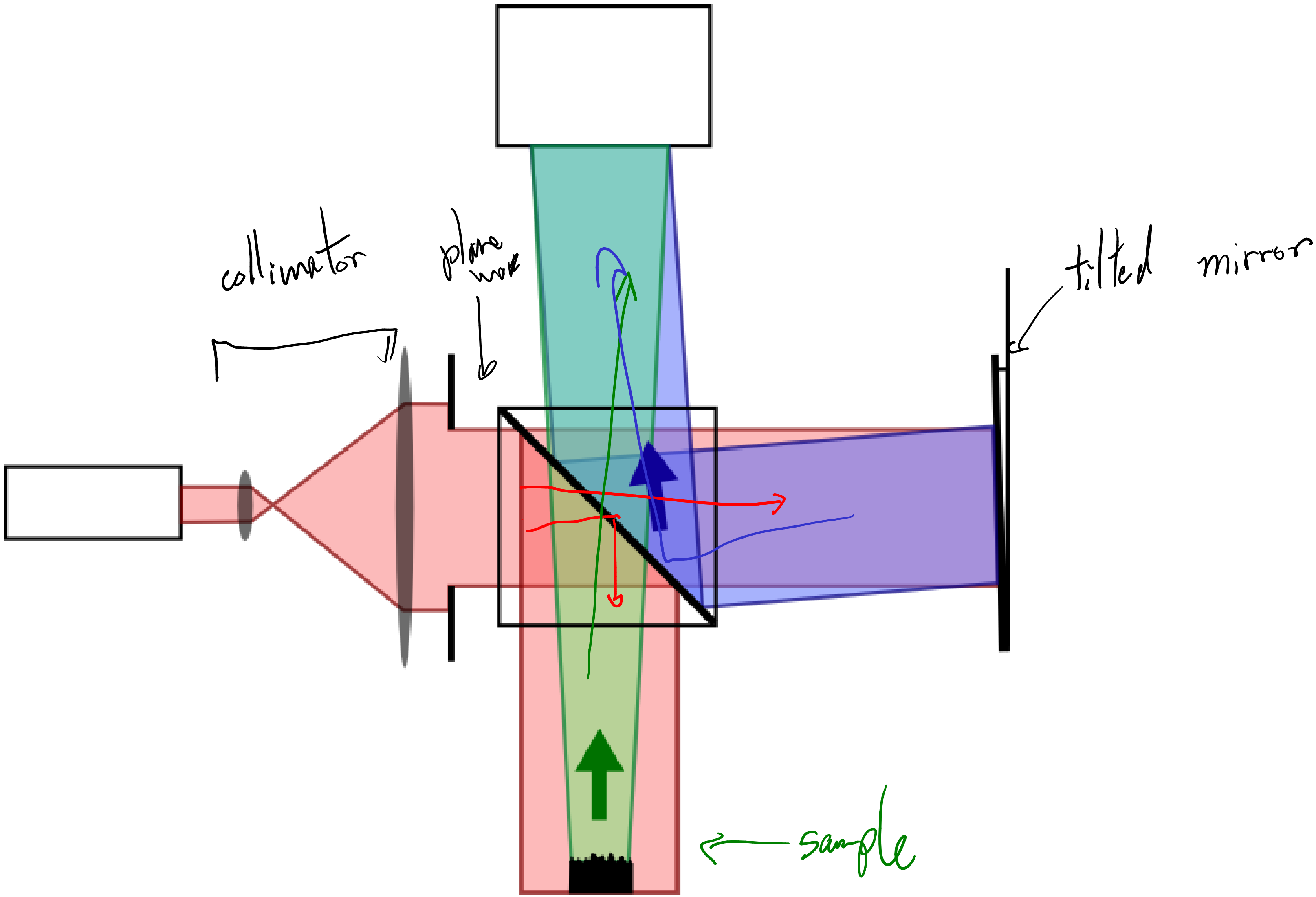
"in focus"

"after propagation"

propagated hologram

Digital in-line holography (DIH) : uses this principle mostly for particle tracking

Fringe interferometry



Twyman-Green interferometer

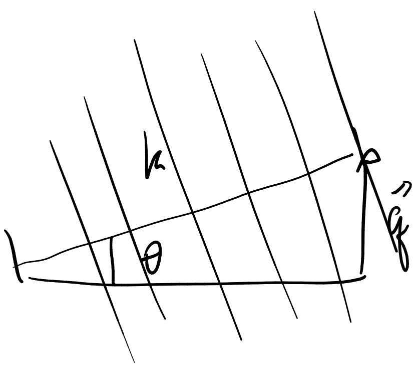
Fringe interferometry

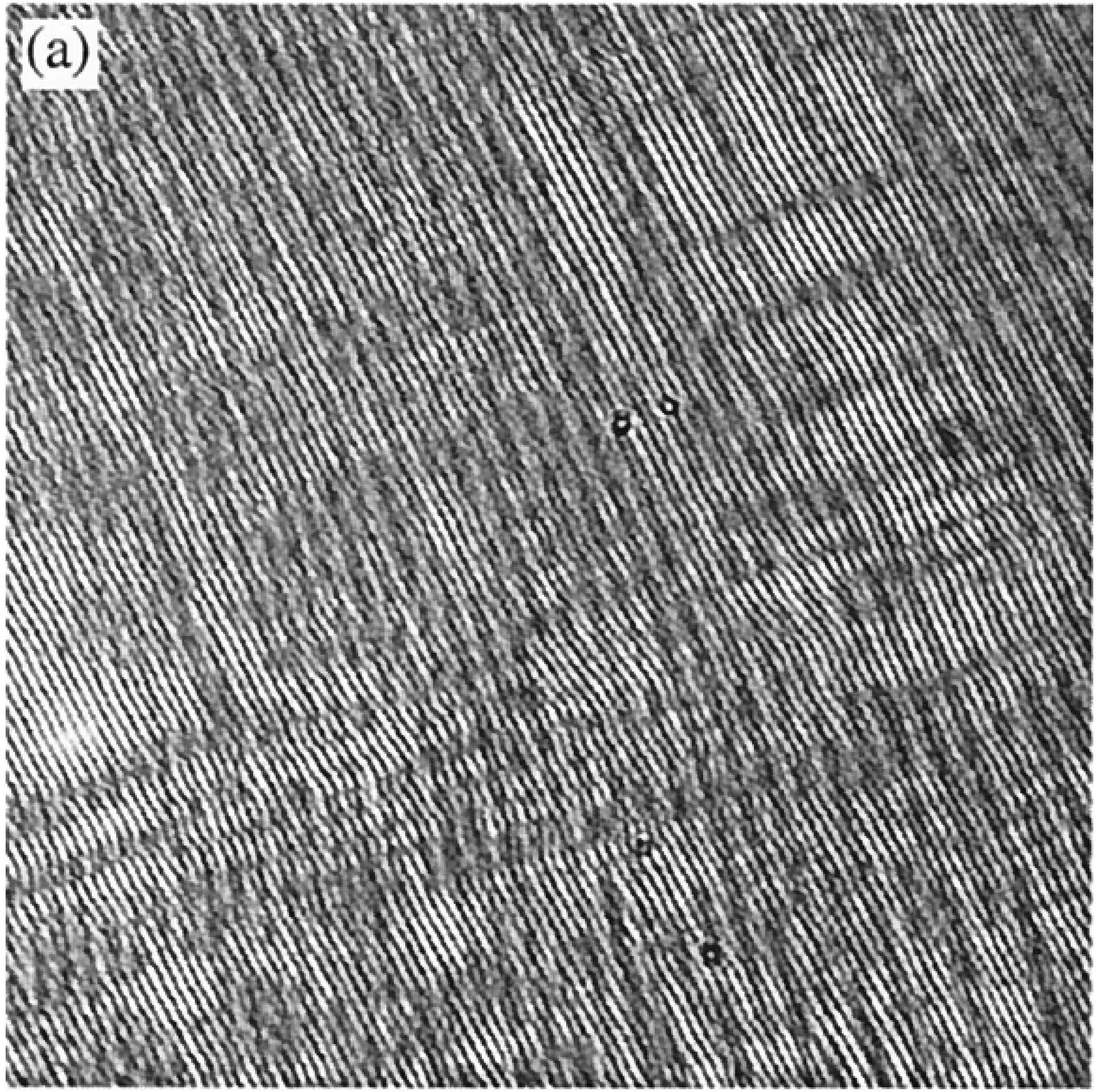
$1 + e^{i\vec{q}\cdot\vec{r}}$

along z

propagates at an angle

$\sin\theta = \frac{|q|}{|k|}$





Source: Cuche et al. Appl. Opt. **39**, 4070 (2000)

$I = \left|1 + e^{i\vec{q}\cdot\vec{r}}\right|^2$

$1 + e^{i\vec{q}\cdot\vec{r}} + e^{-i\vec{q}\cdot\vec{r}} + 1$

$2 + 2\cos(\vec{q}\cdot\vec{r})$

$2(1 + \cos(\vec{q}\cdot\vec{r}))$

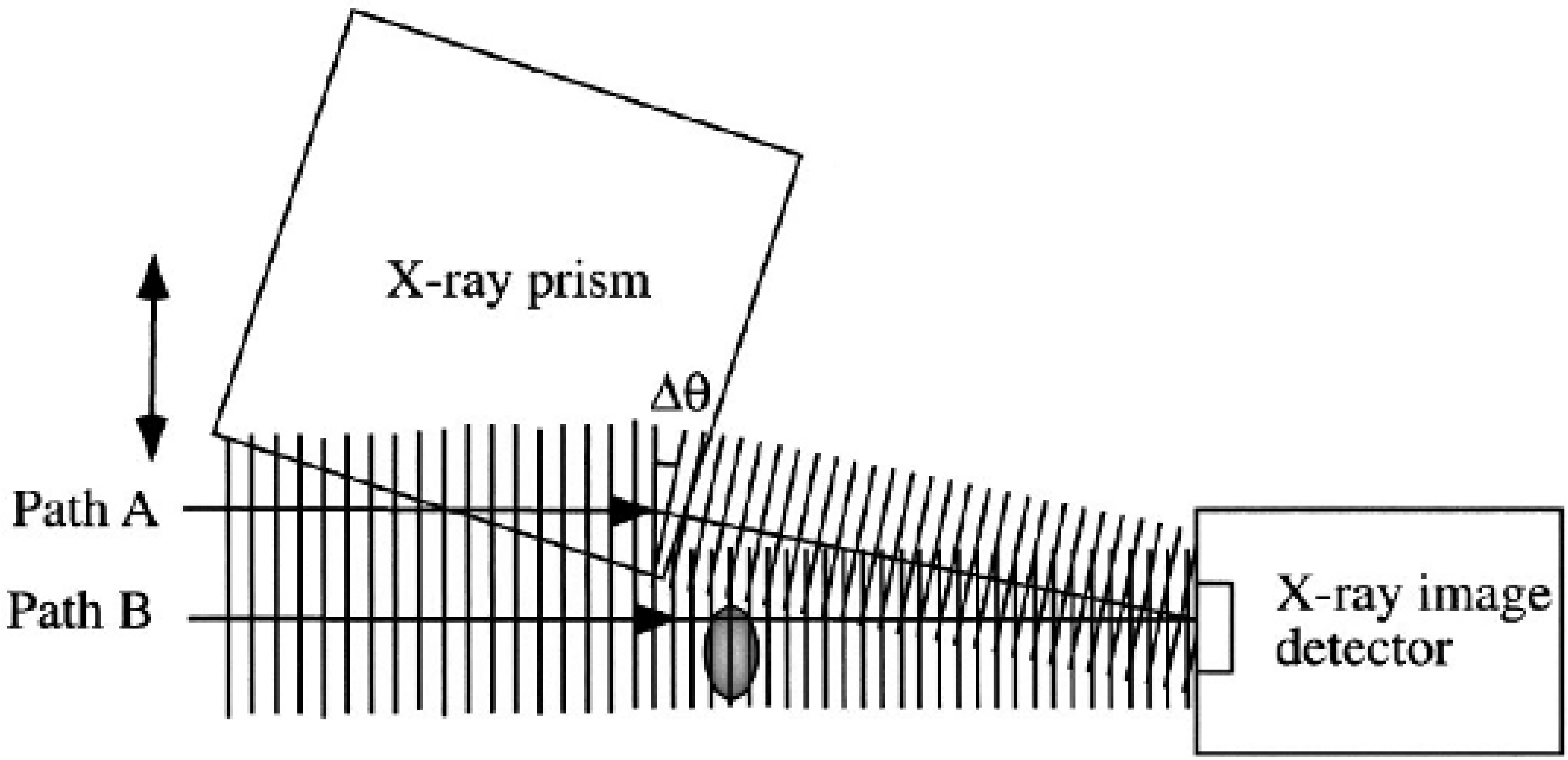
oscillates with spatial frequency

$\vec{u} = \vec{q}/2\pi$

for imaging:

$\left|e^{i\vec{q}\cdot\vec{r}} + a(\vec{r})\right|^2$

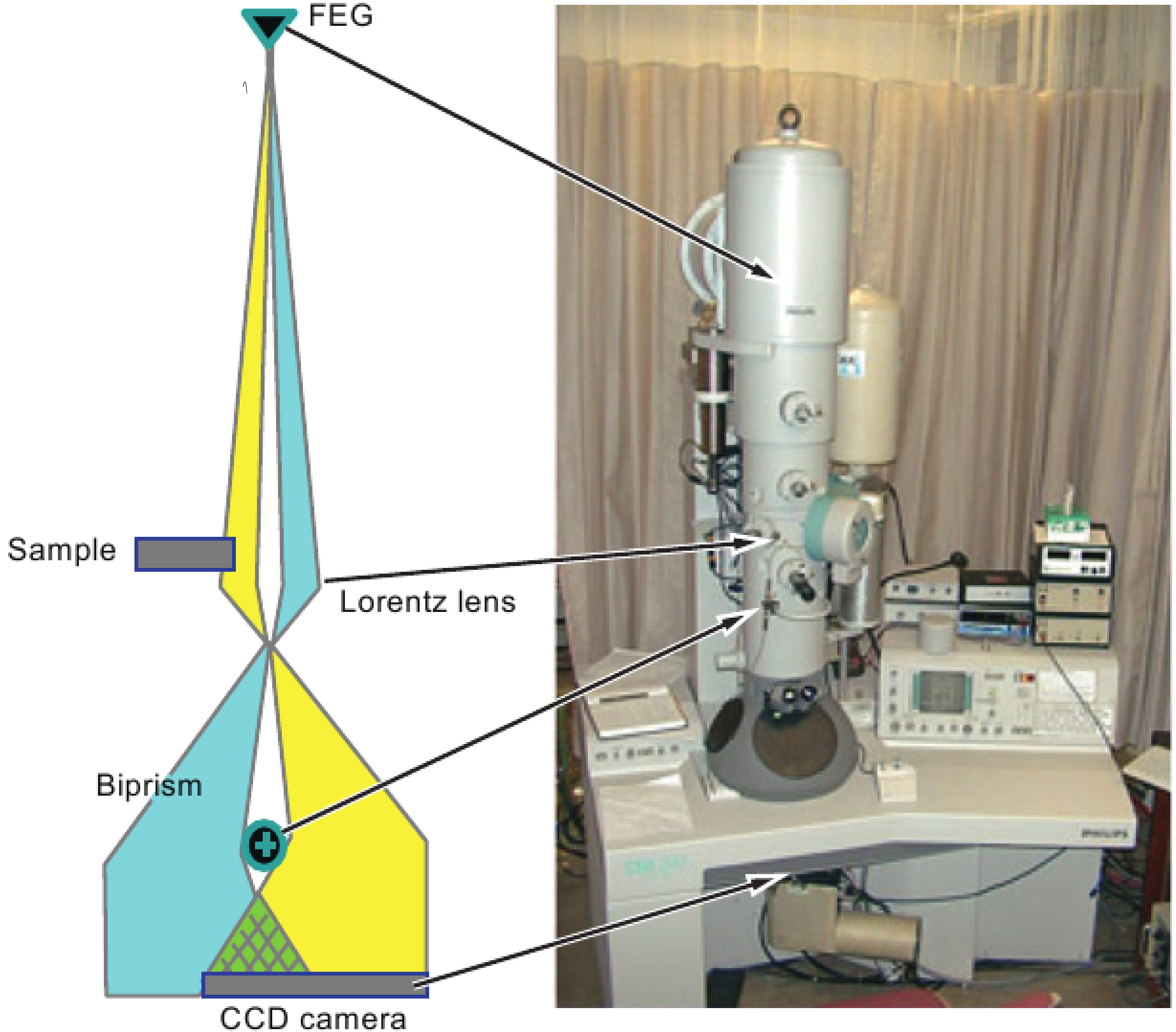
Off-axis X-ray holography



Source: Y. Kohmura, J. Appl. Phys. **96**, 1781-1784 (2004)

Off-axis electron holography

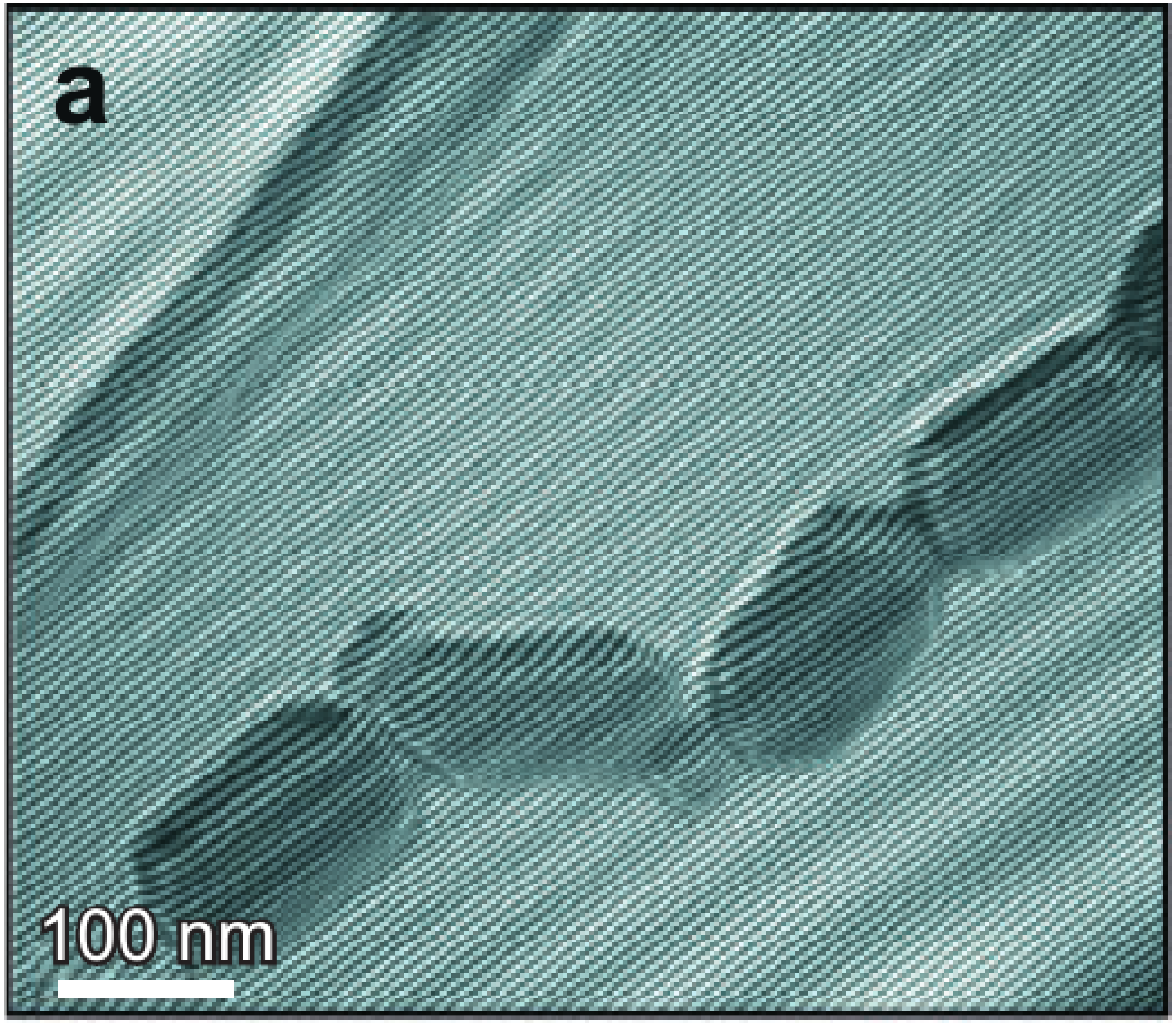
Electron microscopy



Hologram **Philips CM200 FEG**

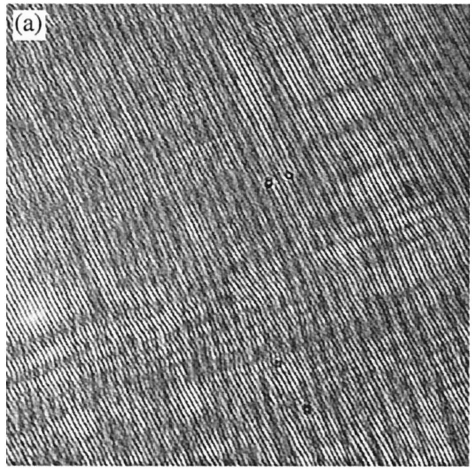
Source: M. R. McCartney, Ann. Rev. Mat. Sci. **37** 729-767 (2007)

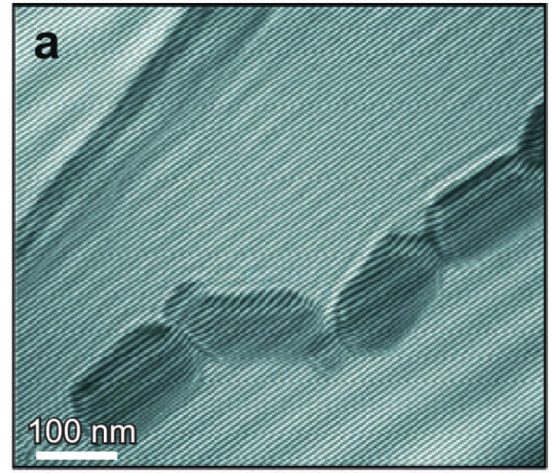
Off-axis electron holography



Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Fringe interferometry





$$\psi = \psi_o + \psi_r$$

object reference

attenuation

phase shift

$$\psi_r(\vec{r}) = A\, e^{i(\vec{q}\cdot\vec{r} + \varphi_r)}$$

$$\psi_o(\vec{r}) = A\, \underbrace{a(\vec{r})\, e^{i\varphi(\vec{r})}}_{\text{complex-valued transmission function}}$$

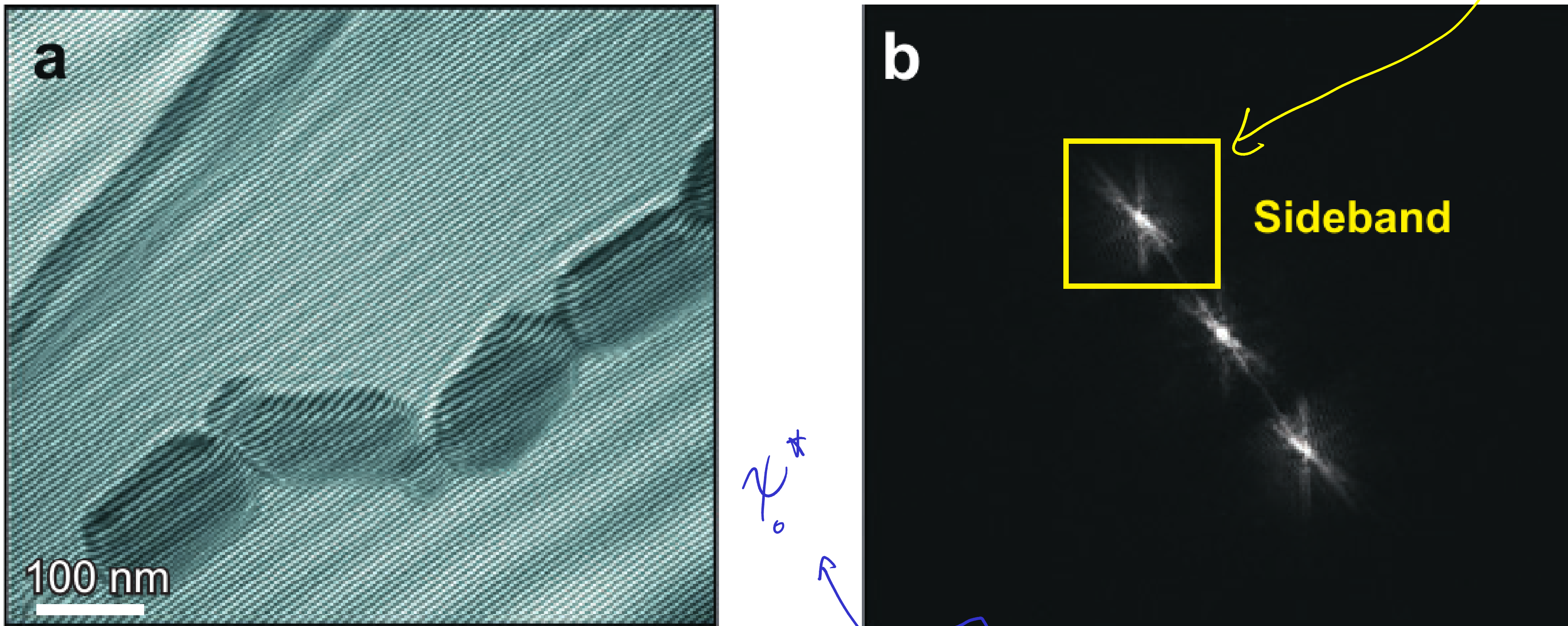
Measurement:

$$|\psi(\vec{r})|^2 = (\psi_o + \psi_r)(\psi_o^* + \psi_r^*)$$

$$= |A|^2 \left(a^2(r) + 1 + \underbrace{a(\vec{r})\, e^{i(\vec{q}\cdot\vec{r} - \varphi)} + a(\vec{r})\, e^{-i(\vec{q}\cdot\vec{r} - \varphi)}}_{2a(\vec{r}) \cos(\vec{q}\cdot\vec{r} - \varphi(\vec{r}))} \right)$$

Off-axis holography

cropping in Fourier domain = reducing resolution



ψ_0^*

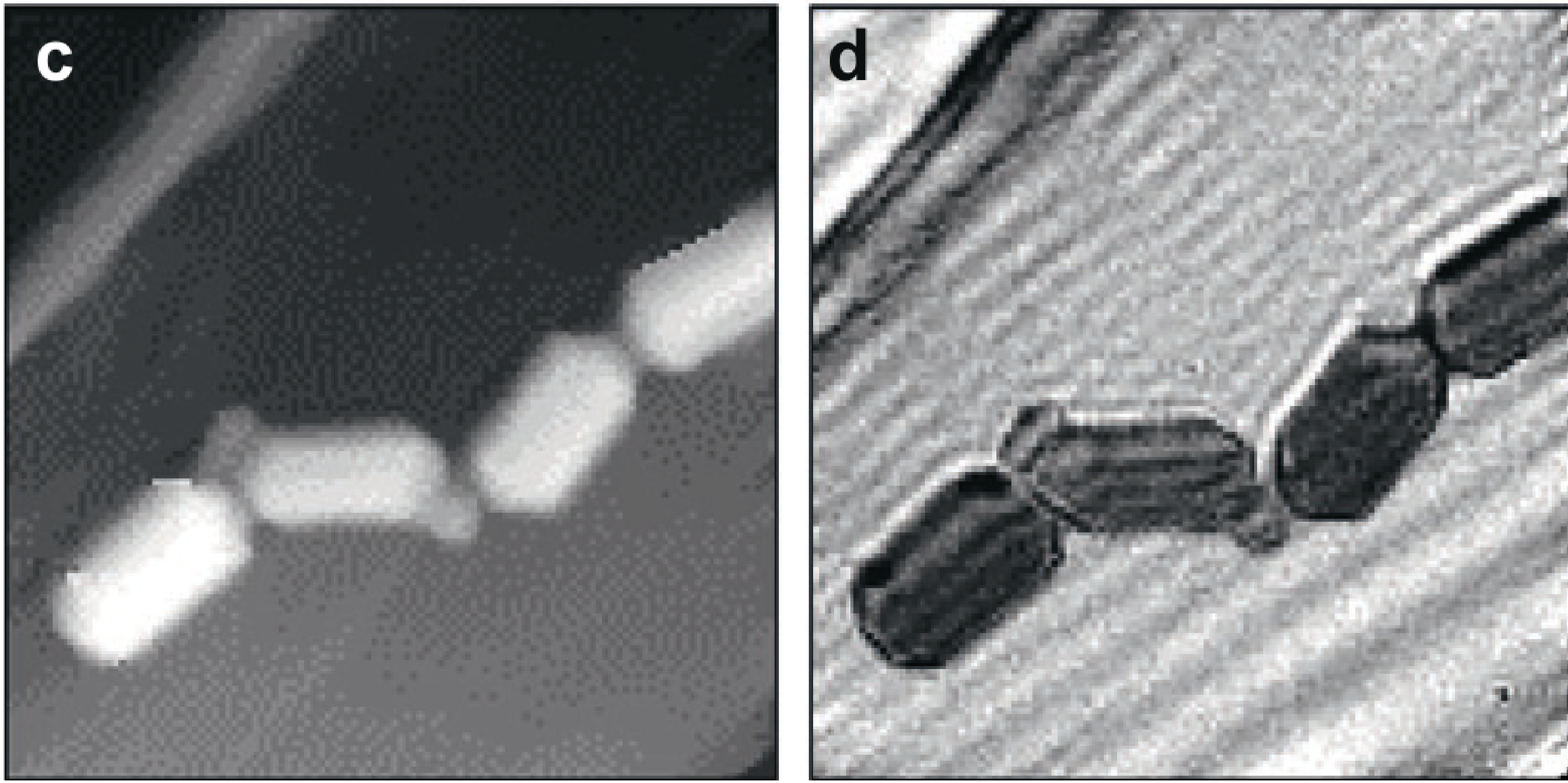
$$\mathcal{F}\{|\psi|^2\} = |A|^2 \Big[\underbrace{\mathcal{F}\{a^2(r)+1\}} + \underbrace{\mathcal{F}\{\boxed{a(\vec{r})\, e^{-i\varphi(\vec{r})}}\, e^{i\vec{q}\cdot\vec{r}}\}} + \mathcal{F}\{\underbrace{\boxed{a(\vec{r})\, e^{+i\varphi}}}_{\psi_0}\, e^{-i\vec{q}\cdot\vec{r}}\} \Big]$$

$$= \mathcal{F}\{\psi_0^*\}(\vec{u} - \vec{q}/2\pi) \qquad \mathcal{F}\{\psi_0\}(\vec{u} + \vec{q}/2\pi)$$

Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Off-axis holography

Price to pay to get phase & attenuation : resolution



phase

attenuation

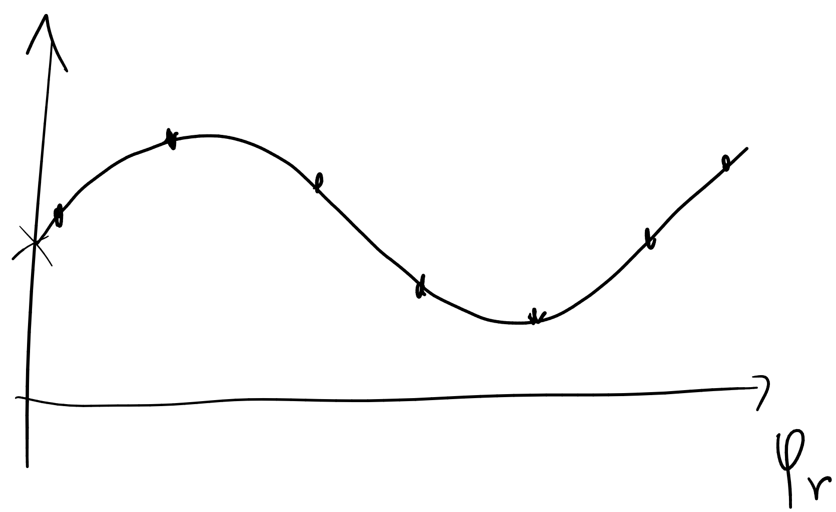
Source: M. R. McCartney, Annu. Rev. Mat. Sci. **37** 729-767 (2007)

Phase stepping

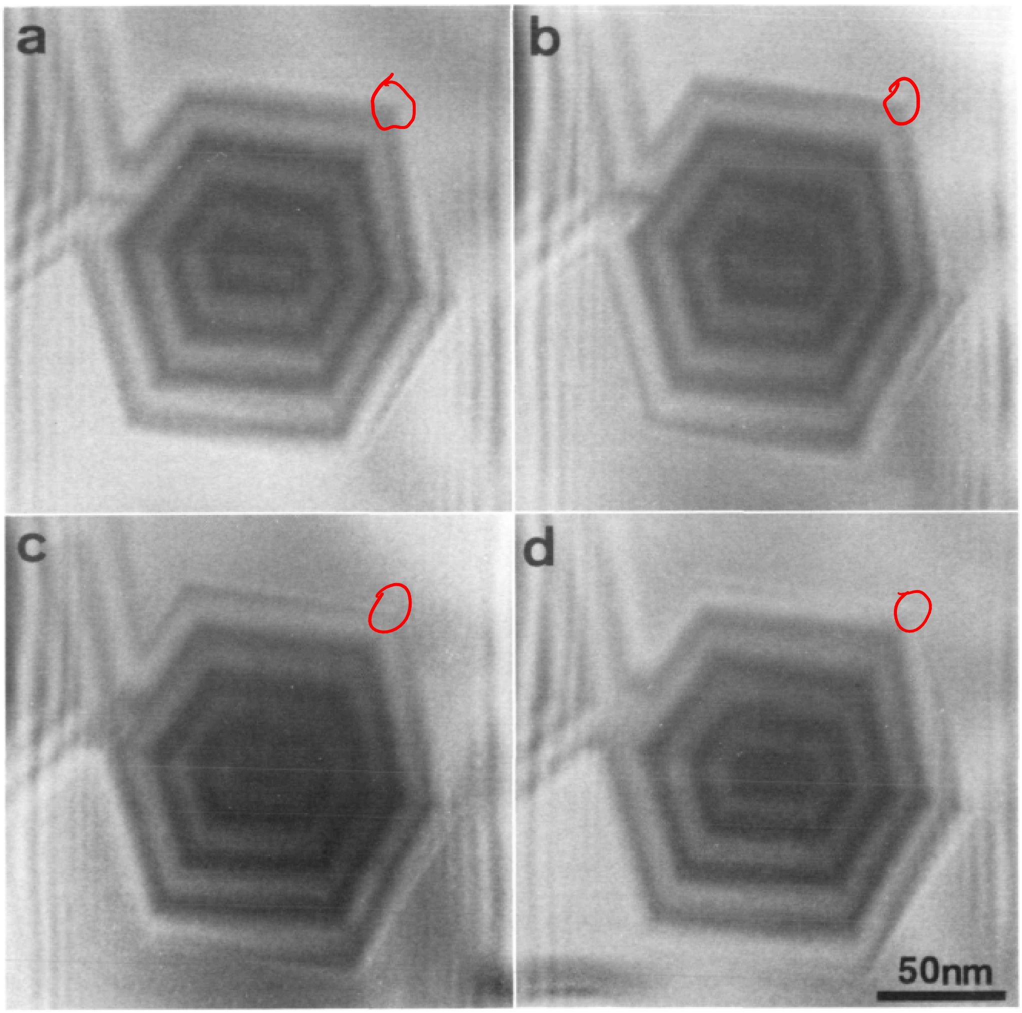
- Encoding phase **and** amplitude in a single image has a price: resolution

  $\rightarrow$ Take more than one image, changing the reference in each.

$$e^{i(\vec{q}\cdot\vec{r})} \qquad e^{i(\vec{q}\cdot\vec{r}+\frac{\pi}{2})} \qquad e^{i(\vec{q}\cdot\vec{r}+\pi)} \qquad \ldots \qquad e^{i(\vec{q}\cdot\vec{r}+\varphi_r)}$$



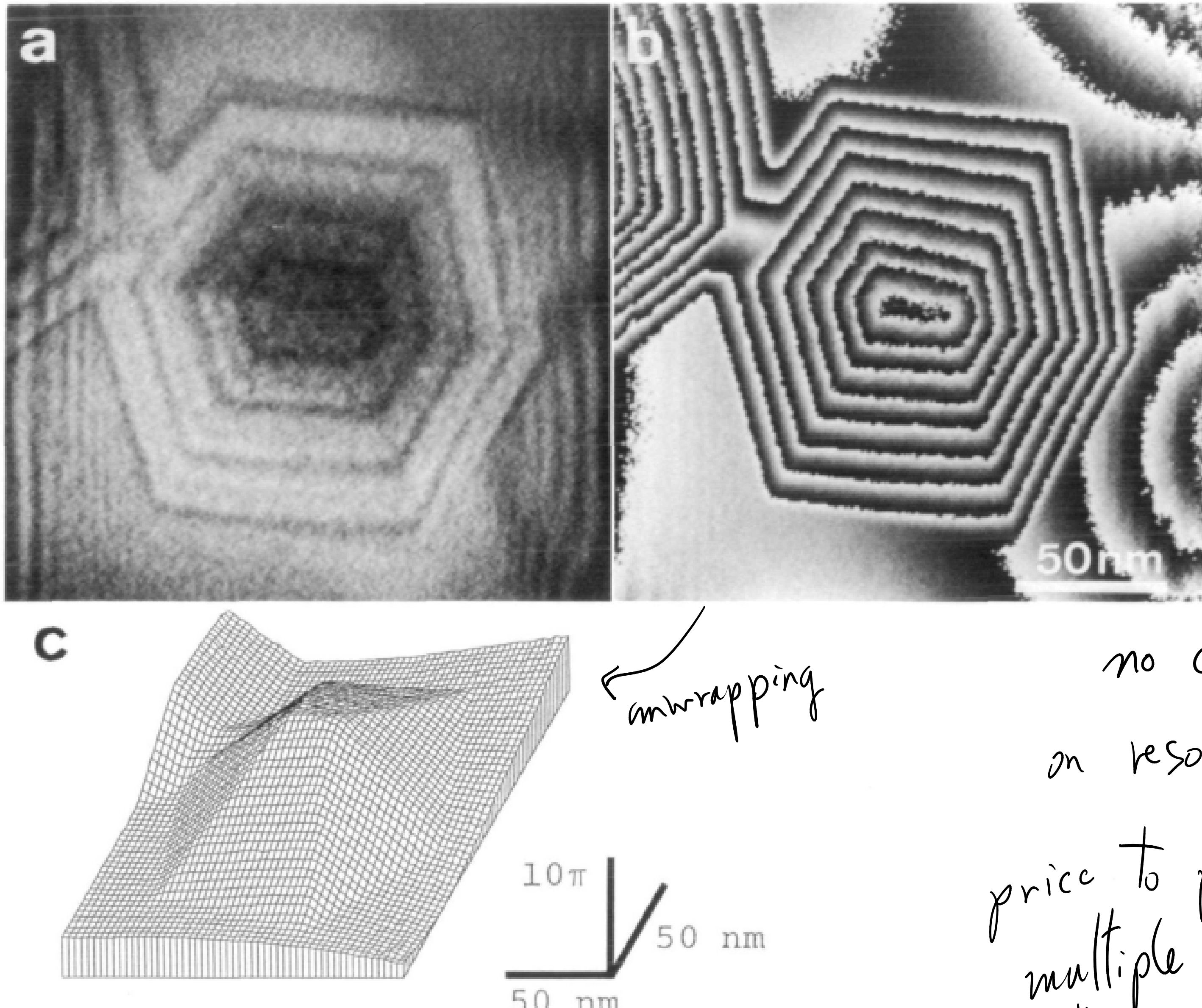
Fringe scanning



Electron microscopy

Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Fringe scanning



unwrapping

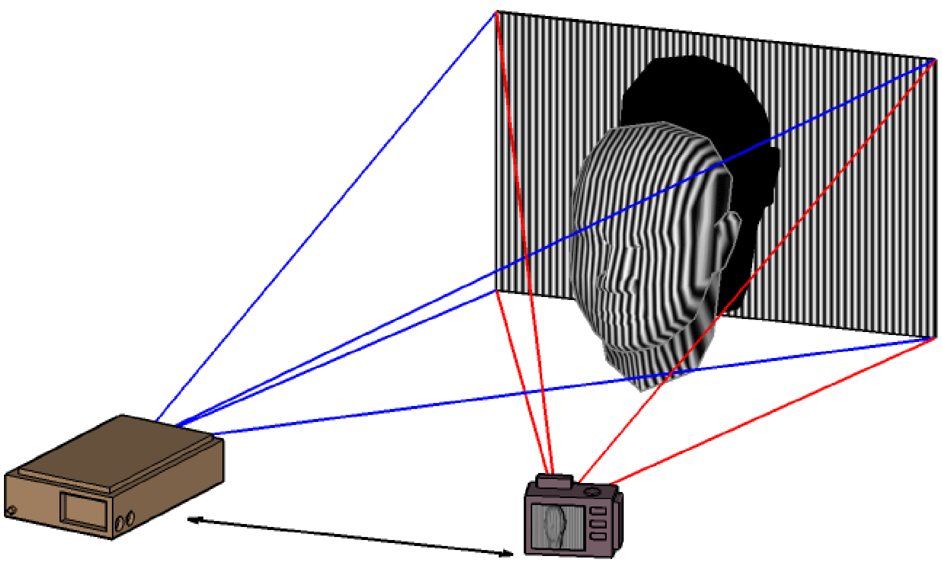
no compromise on resolution!

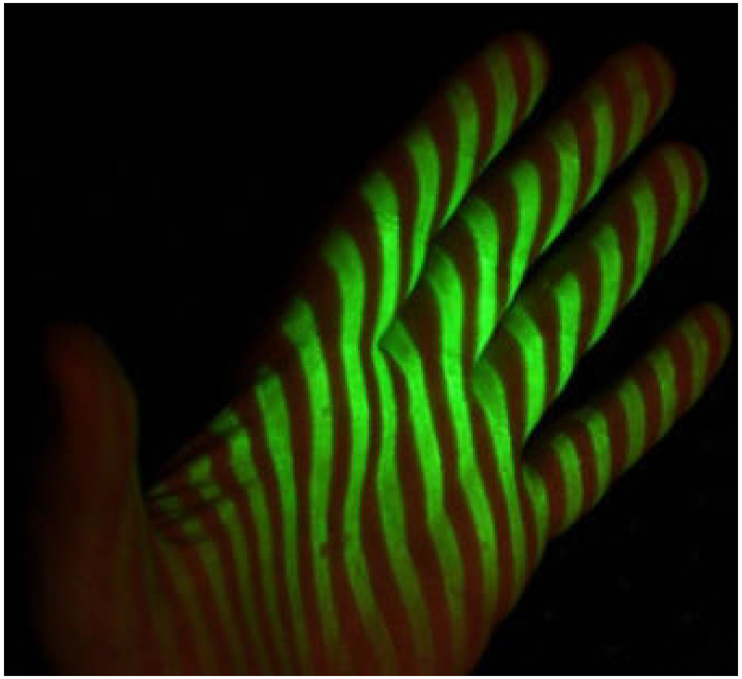
price to pay: multiple measurements with assumption that the sample is static

Source: K. Harada, J. Electron Microsc. **39** 470-476 (1990)

Structured light sensing

- Project a structured light pattern onto sample
- Distortions of light pattern allow reconstruction of sample shape





Phase unwrapping

- Phase is measured only in the interval [0, 2π)
- Physical phase shifts (which can be larger) are wrapped on this interval

  $\rightarrow$ Any multiple of 2π is possible

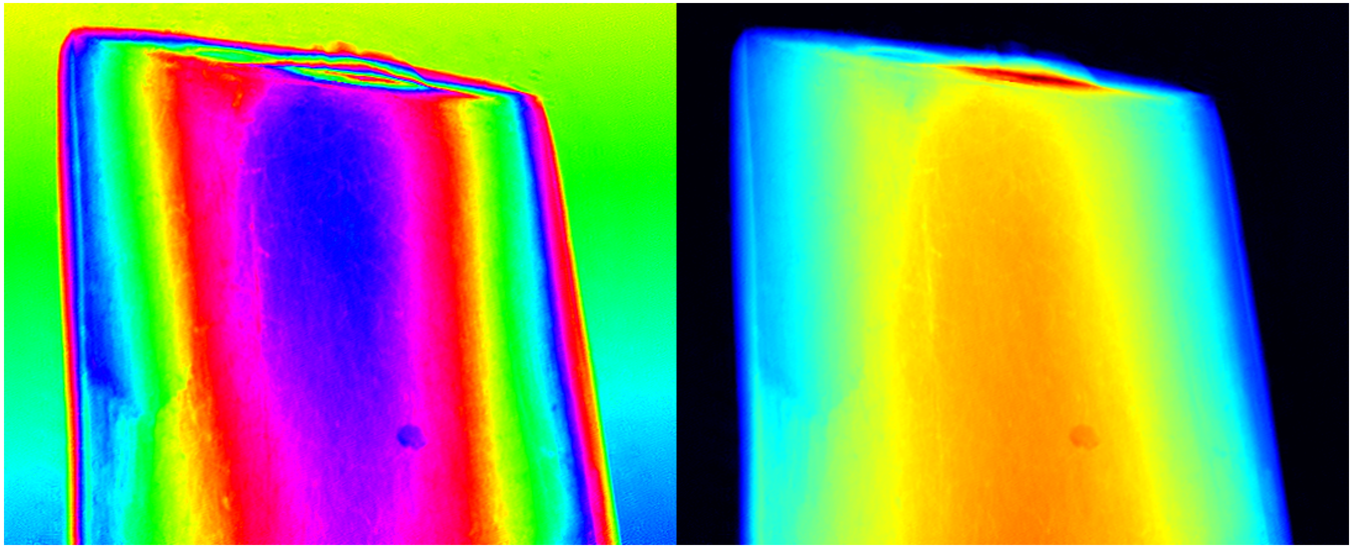
- Unwrapping: use correlations in the image to guess the total phase shift.
- Main difficulties:
  - aliasing: phase shifts are too rapid for the image sampling
  - noise: produces local singularities (vortices)
- Many strategies exist

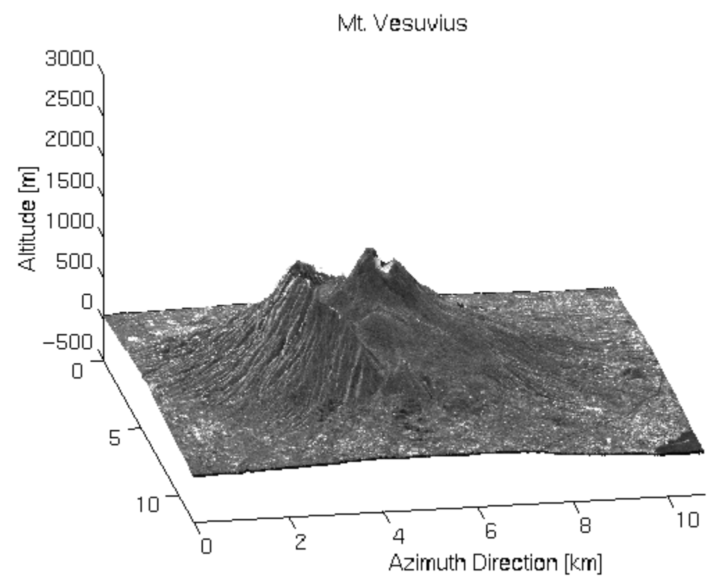
  → path following methods

  → identify phase vortices and connect them

Complex-valued images

Phase unwrapping

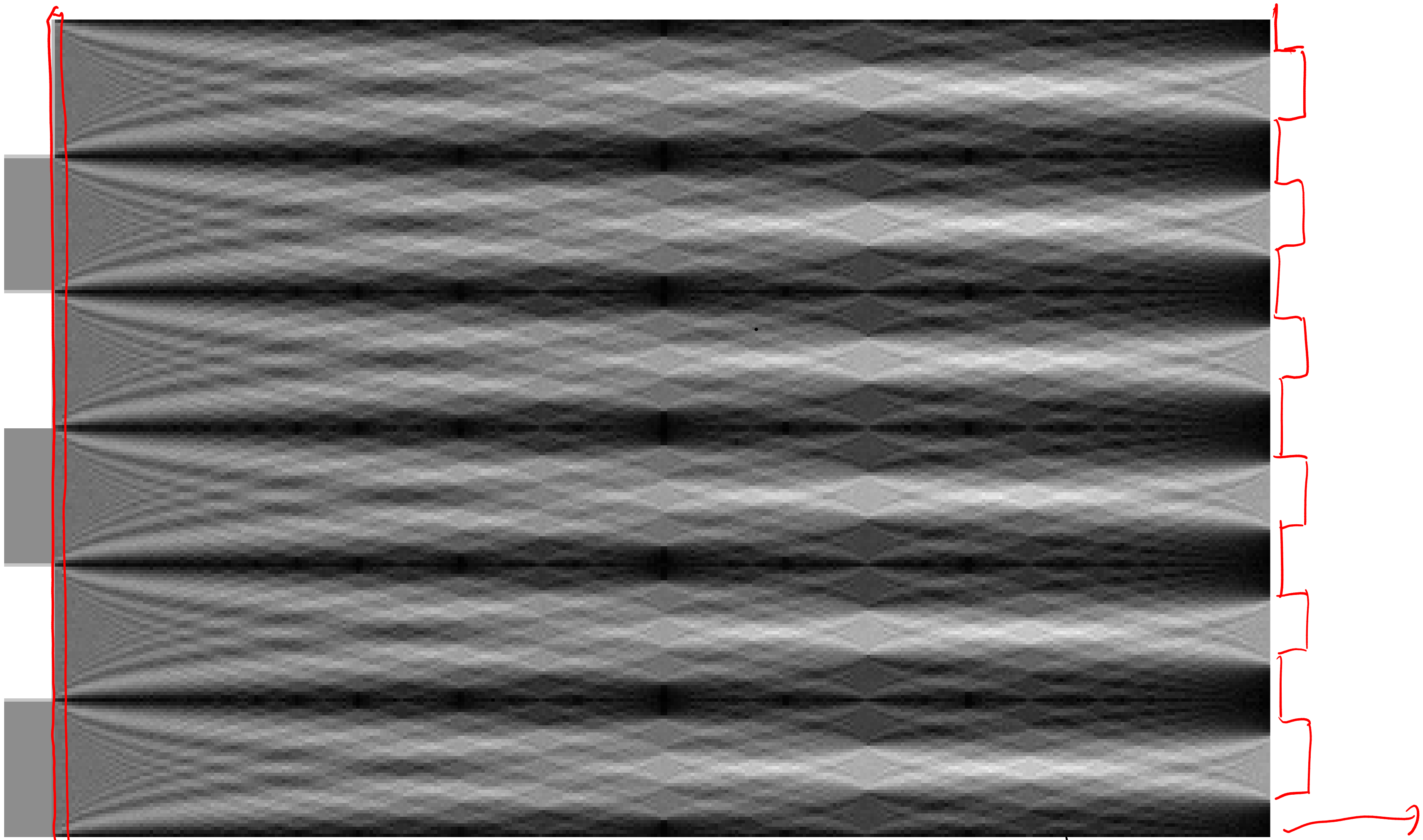




Source: http://earth.esa.int/workshops/ers97/program-details/speeches/rocca-et-al/

Grating interferometry

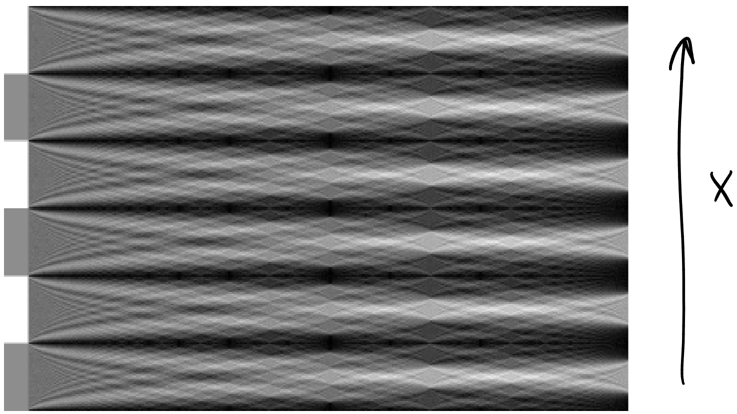
Diffraction from a grating

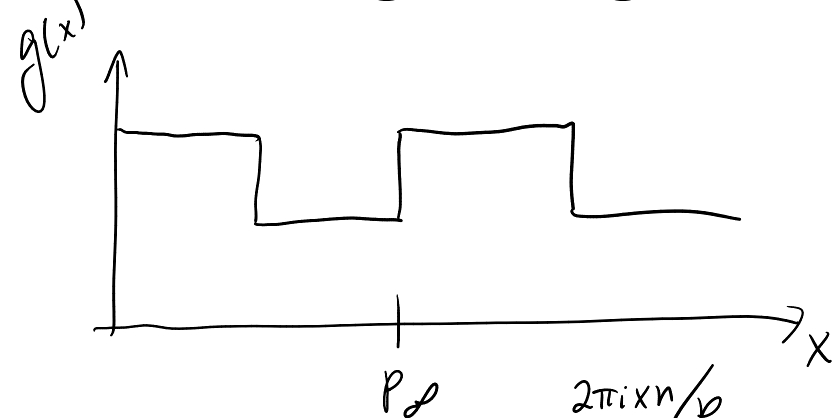


periodicity in grating propagation: Talbot effect

Grating interferometry

Diffraction from a grating

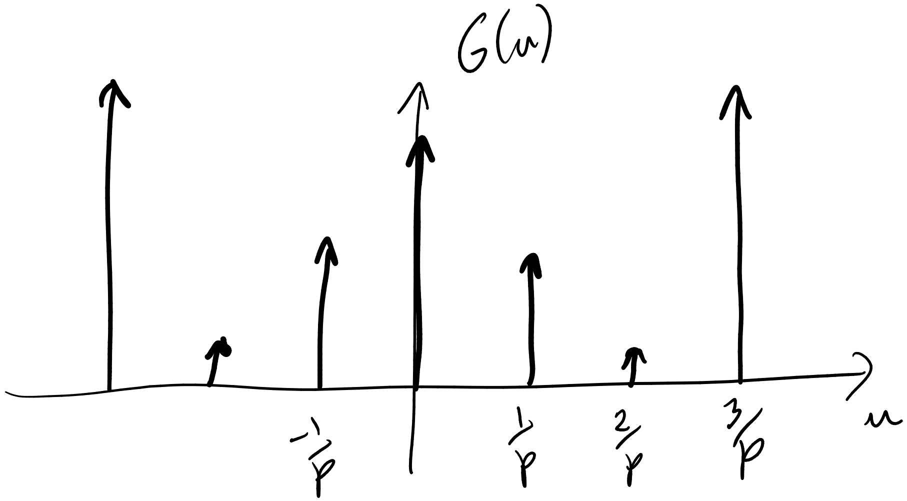


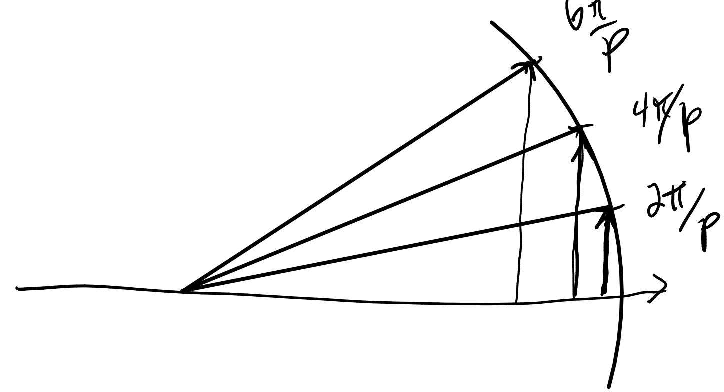


$$g(x) = \sum_{-\infty}^{\infty} g_n e^{2\pi i x n/p}$$

$$\Downarrow$$

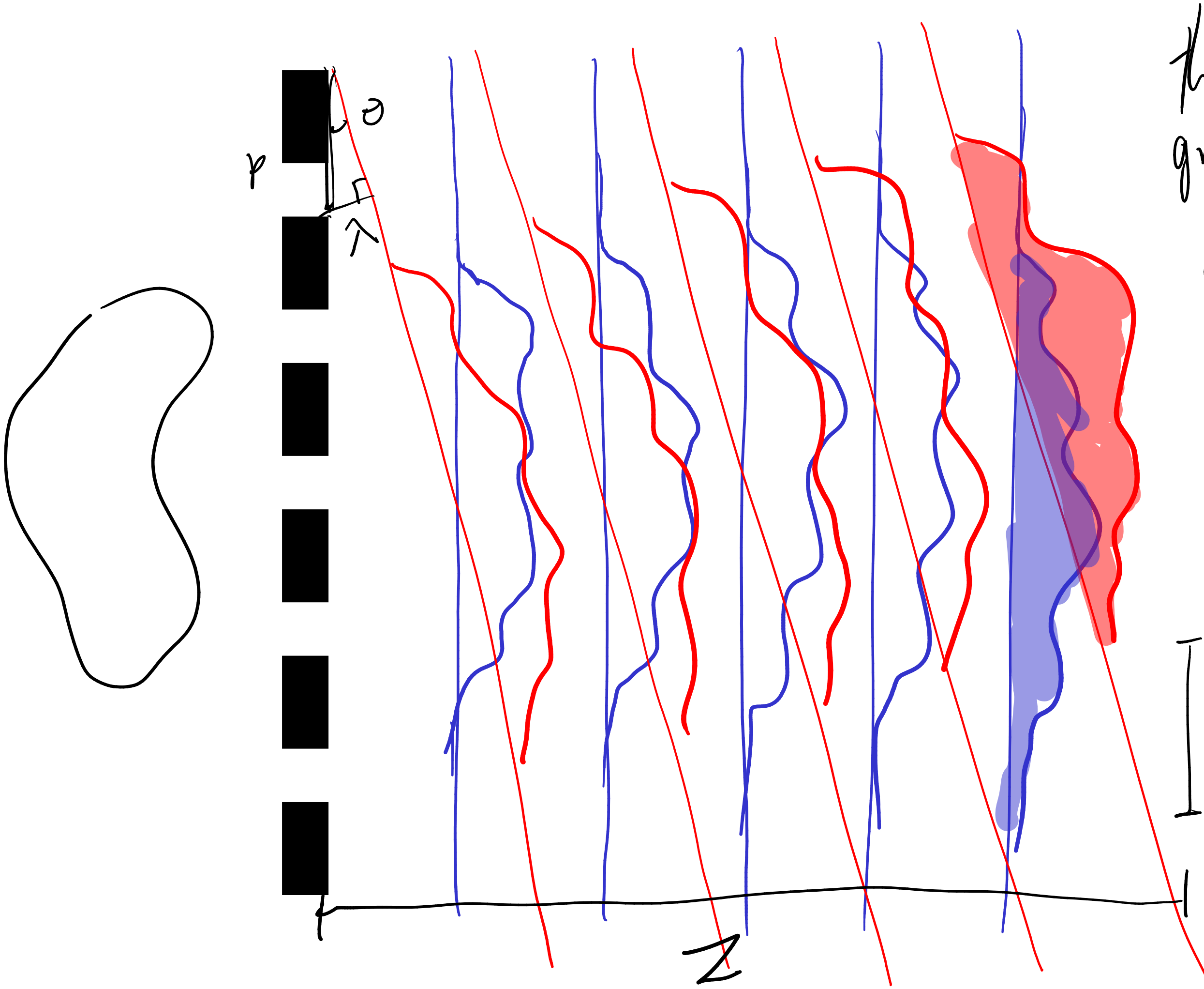
$$G(u) = \sum_{n=-\infty}^{\infty} g_n \, \delta\left(u - \frac{n}{p}\right)$$





Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.



the wavefield after the grating is duplicated and carried in different directions

$$\sin\theta = \frac{\lambda}{p}$$

$$s = \sin\theta \, z = \frac{\lambda z}{p}$$

Grating interferometry

Observing the interference between two (slightly offset) copies of the same sample.

e.g. if only orders ± 1 are relevant

$$\psi(\vec{r};z) = \psi_0\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right) e^{2\pi i x/p} + \psi_0\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right) e^{-2\pi i x/p}$$

$$\psi_0 = a e^{i\varphi}$$

$$I = |\psi(\vec{r};z)|^2 = \underbrace{a^2\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right) + a^2\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)}_{\simeq 2a^2(\vec{r})} + 2\overbrace{a\left(\vec{r} + \frac{z\lambda}{p}\hat{x}\right)a\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)}^{\simeq a^2(\vec{r})}$$

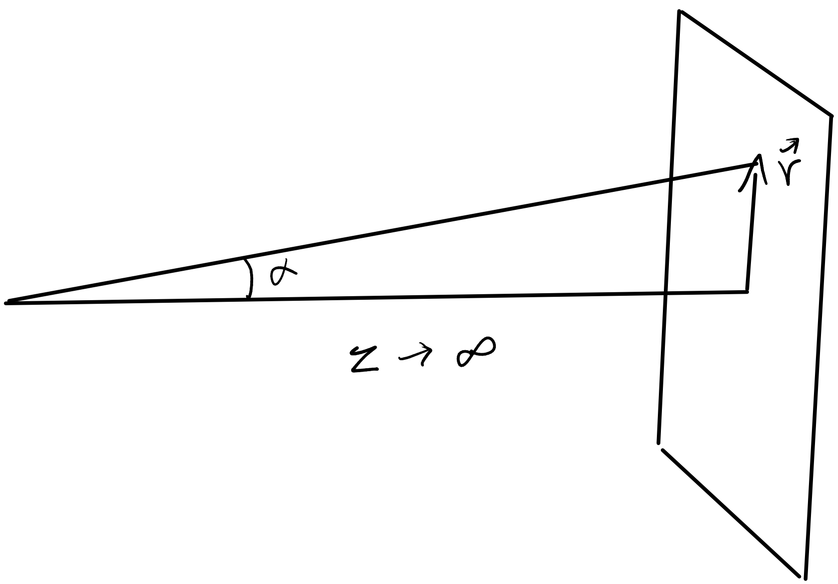
$$\cos\left[\underbrace{\varphi\left(\vec{r} + \frac{\lambda z}{p}\hat{x}\right) - \varphi\left(\vec{r} - \frac{\lambda z}{p}\hat{x}\right)}_{\frac{\lambda z}{p}\frac{\partial}{\partial x}\varphi} + \frac{4\pi x}{p}\right]$$

$$\simeq 2a^2(\vec{r})\left[1 + \cos\left(\frac{\lambda z}{p}\nabla\varphi\cdot\hat{x} + 4\pi\, x/p\right)\right]$$

"differential phase contrast"

Far-field diffraction

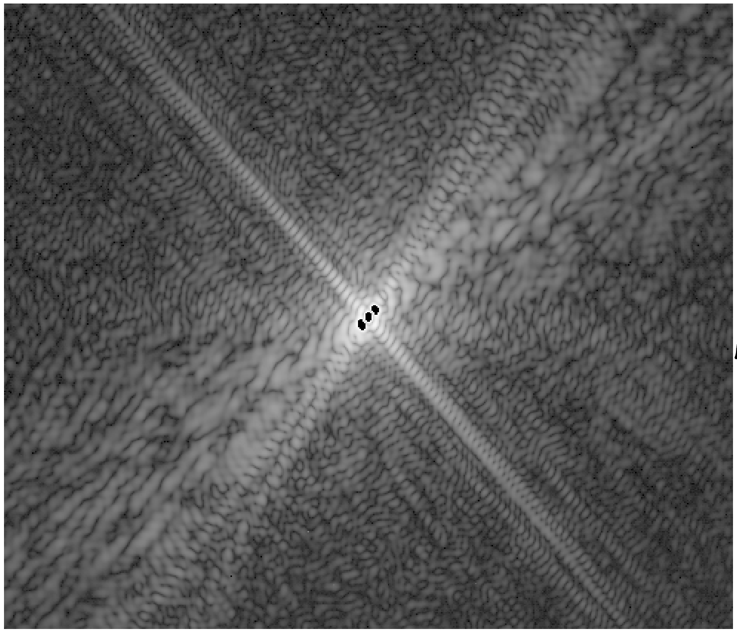
The Fraunhofer regime

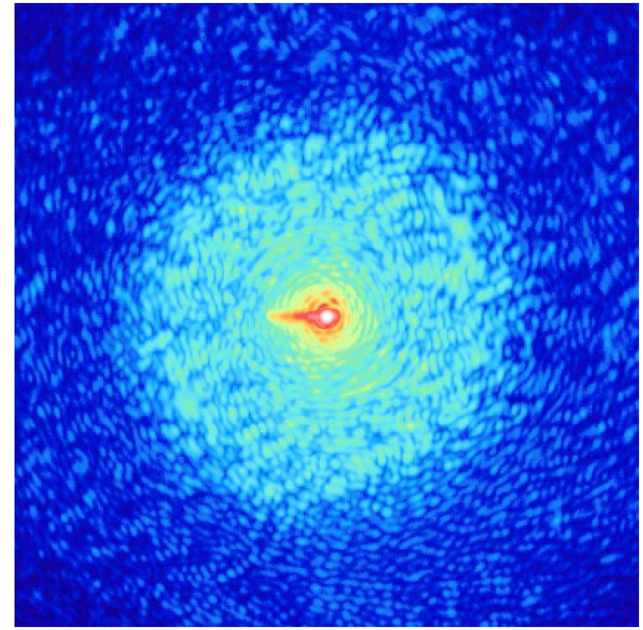


$$\frac{\vec{r}}{z} = \frac{\vec{q}}{k} = \frac{2\pi \vec{u}}{2\pi/\lambda} = \lambda \vec{u}$$

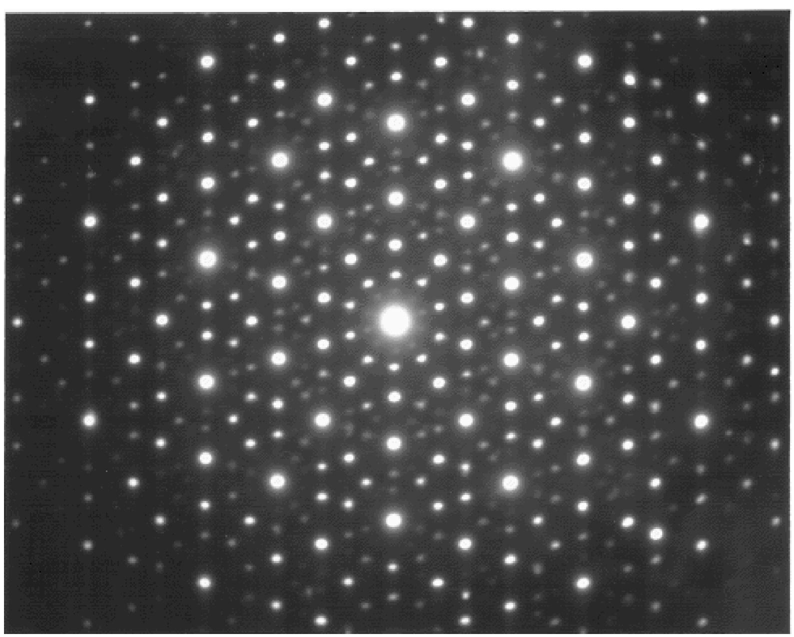
$$\left| \psi(\vec{r}; z \to \infty) \right|^2 \propto \left| \mathcal{F}\psi \right|^2 = I(\vec{u})$$

Diffraction patterns

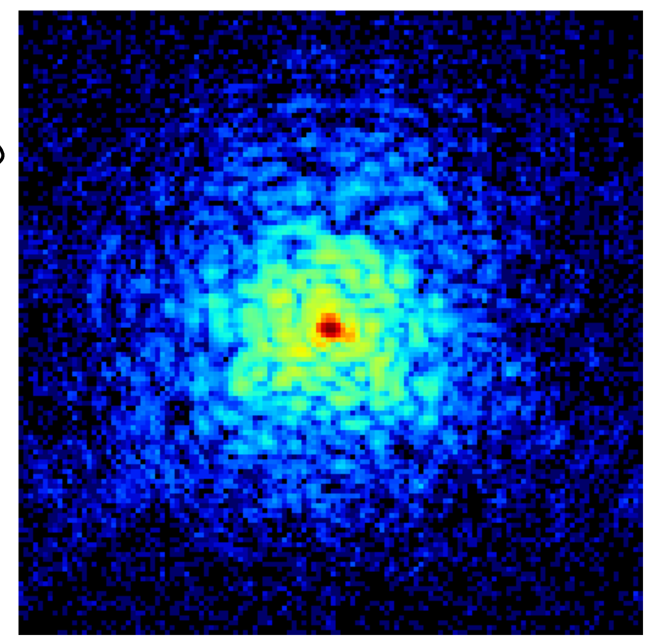




speckles



Bragg peaks



Diffraction and autocorrelation

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \mathcal{F}^{-1}\{\psi(\vec{u})\,\psi^*(\vec{u})\} = \psi(\vec{r}) \circledast \psi(\vec{r})$$

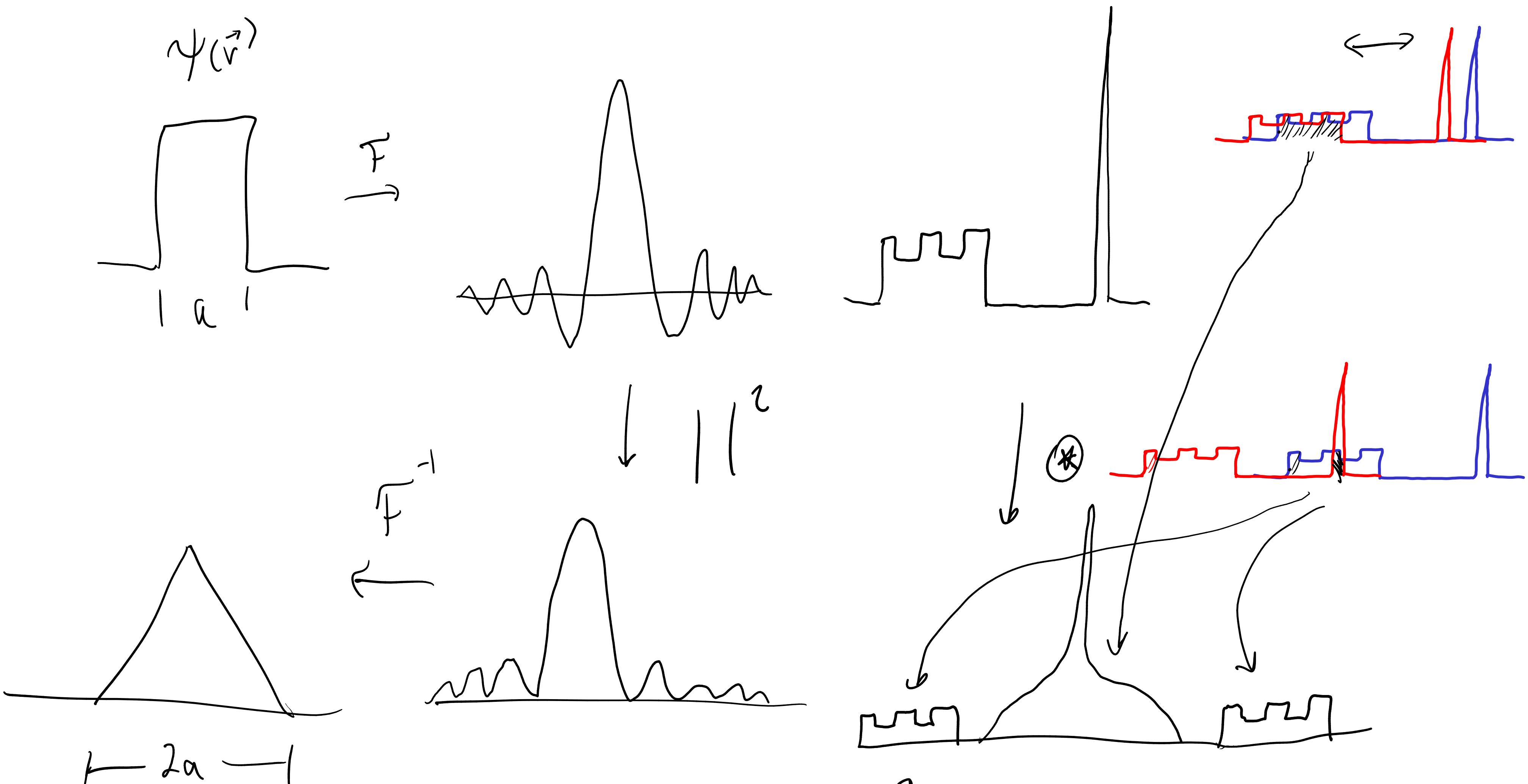


image of original feature!

by design: comes from Dirac-like feature

Fourier transform holography

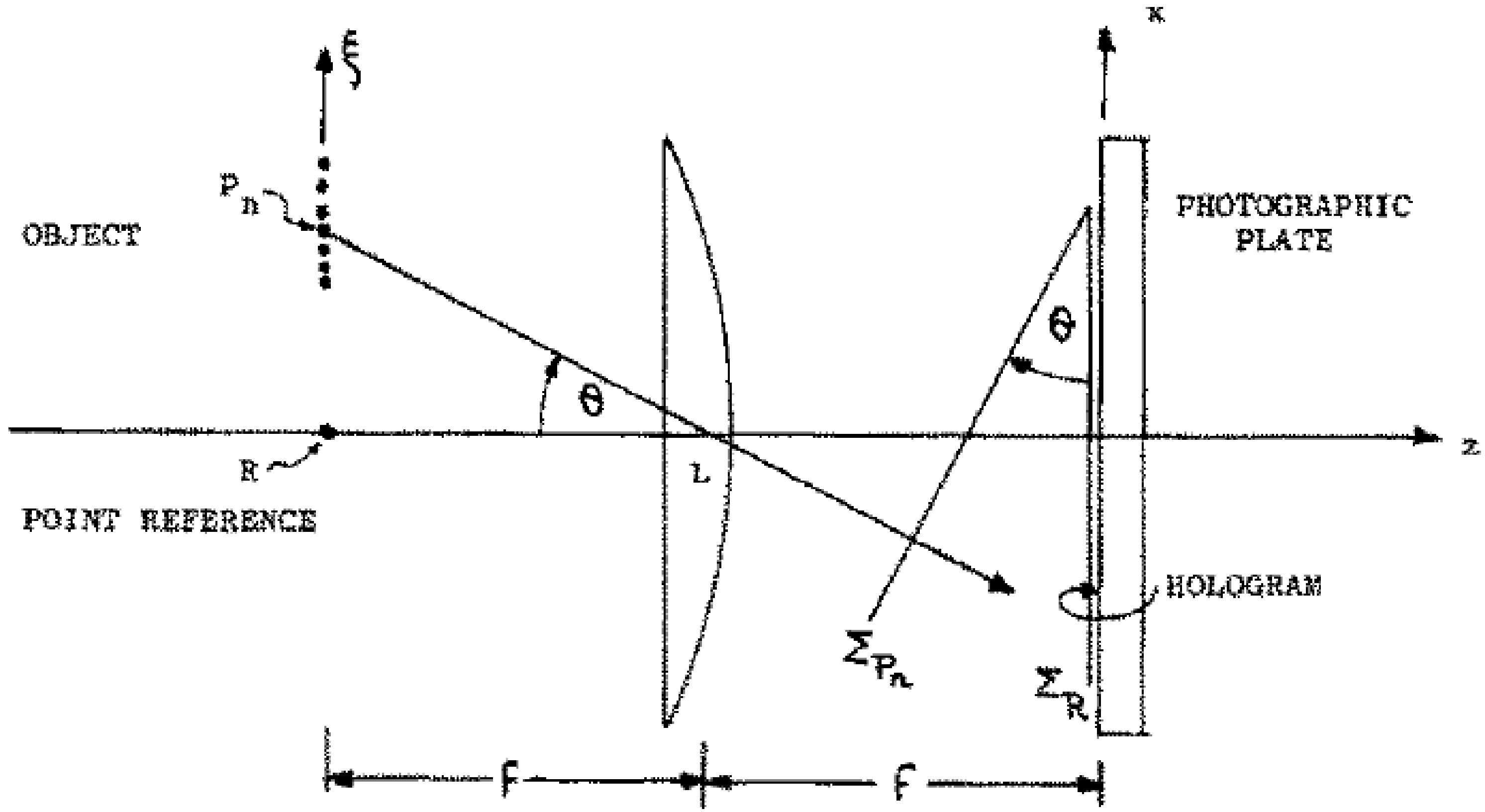
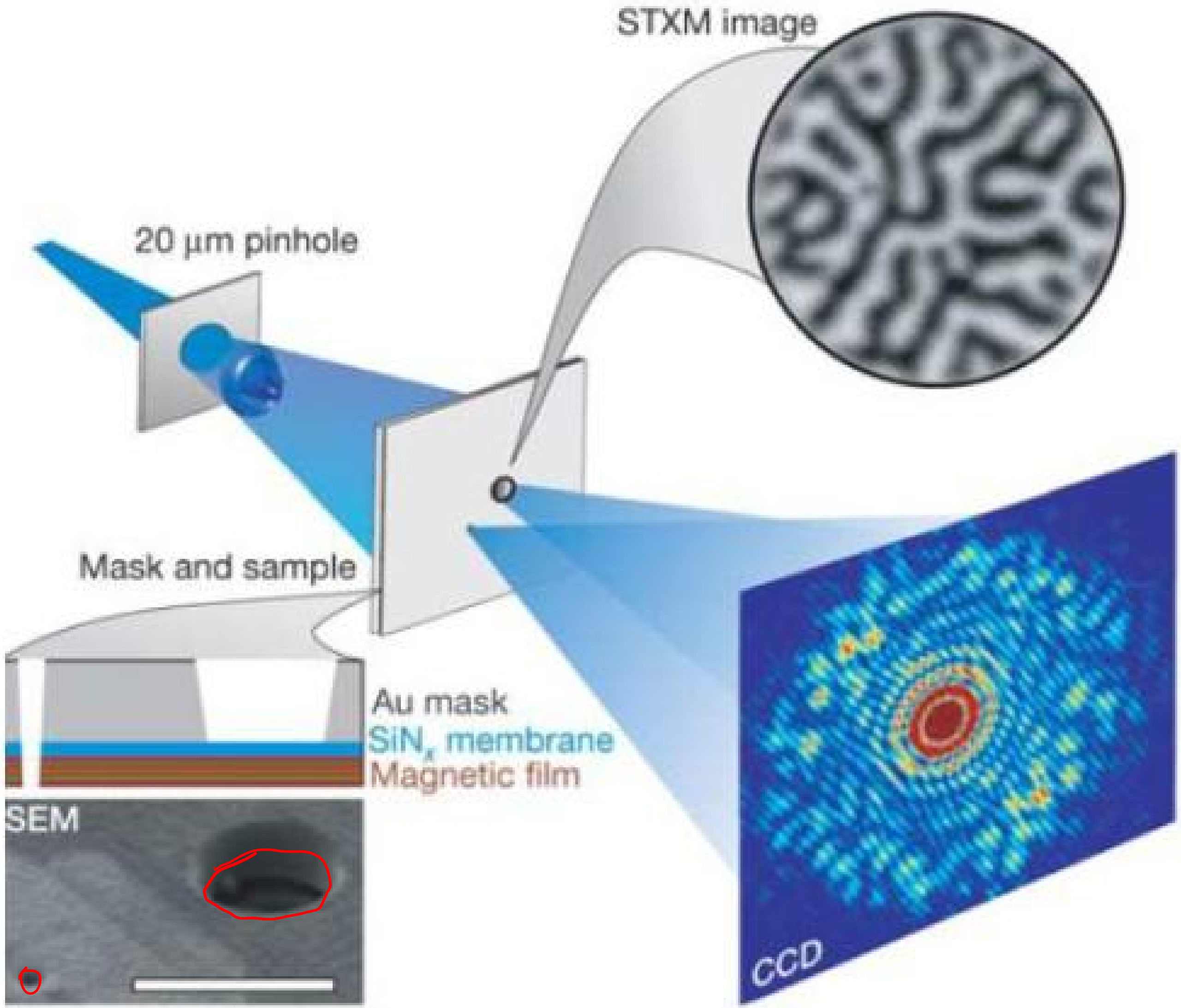


Fig. 1. Recording of a Fourier-transform hologram with a lens L. Σ_R = reference wavefront.

Source: G. Stroke, Appl. Phys. Lett. **6**, 201-203 (1965).

Fourier transform holography



Source: S. Eisebitt et al., Nature **432**, 885-888 (2004).

Fourier transform holography

$$\psi(\vec{r}) = \psi_r(\vec{r}) + \psi_o(\vec{r})$$

$$\Big\downarrow \text{F.T.}$$

$$\tilde{\psi}(\vec{u}) = \tilde{\psi}_r(\vec{u}) + \tilde{\psi}_o(\vec{u})$$

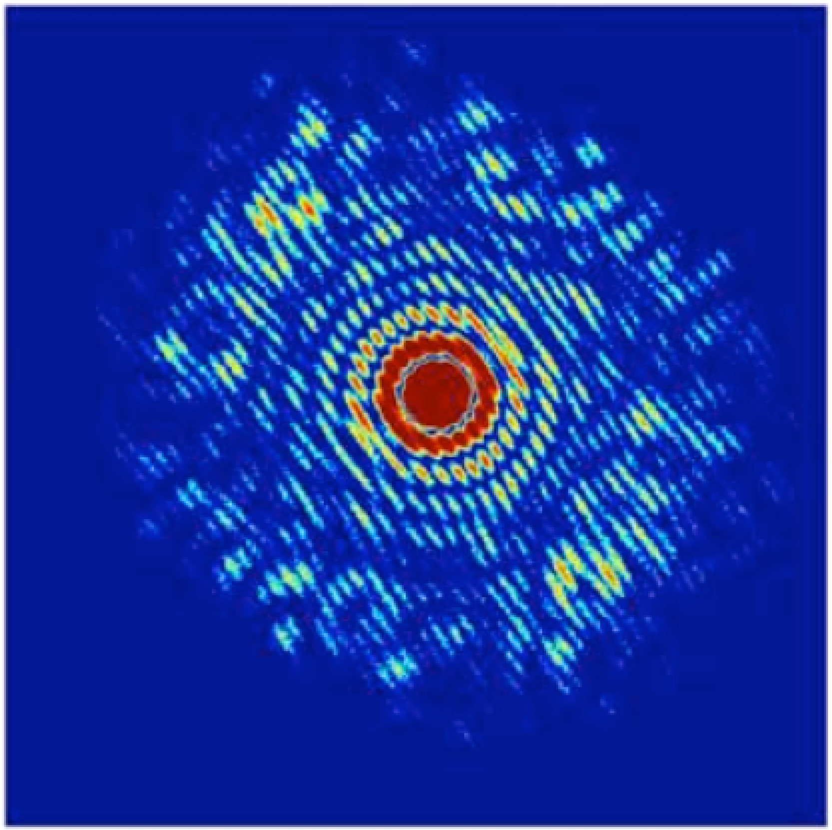
$$\Big\downarrow |\;|^2$$

$$I(\vec{u}) = |\tilde{\psi}_r(\vec{u})|^2 + |\tilde{\psi}_o(\vec{u})|^2 + \tilde{\psi}_r(\vec{u})\,\psi_o^*(\vec{u}) + \text{c.c.}$$

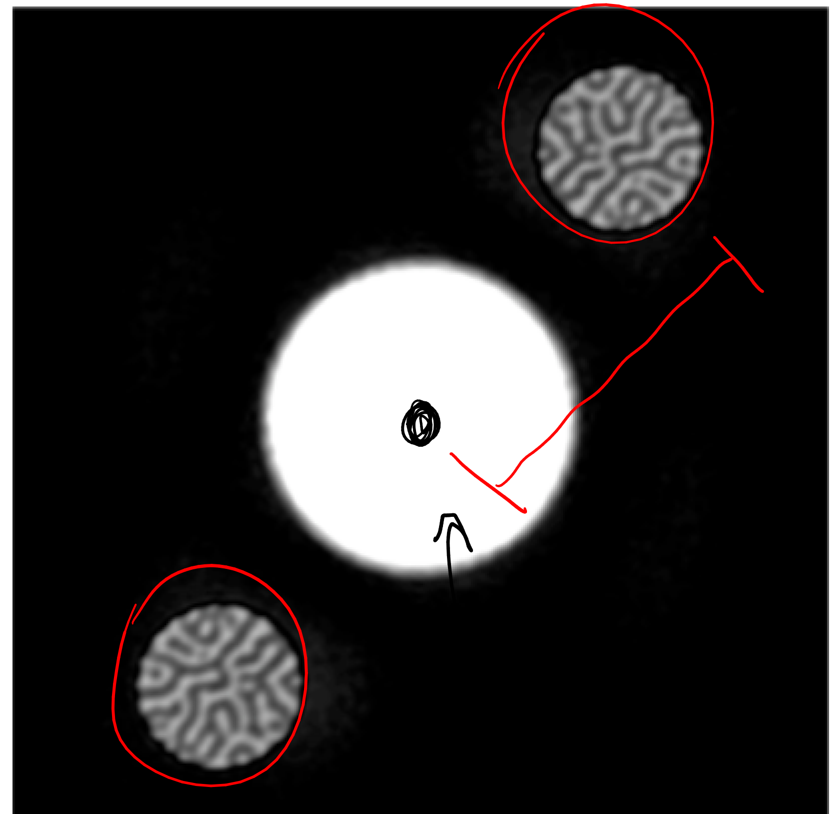
$$\Big\downarrow \mathcal{F}^{-1}$$

$$\mathcal{F}^{-1}\{I(\vec{u})\} = \psi_r \circledast \psi_r + \psi_o \circledast \psi_o + \underbrace{\psi_r \circledast \psi_o^*}_{\text{cross-correlations}} + \psi_r^* \circledast \psi_o$$

Fourier transform holography



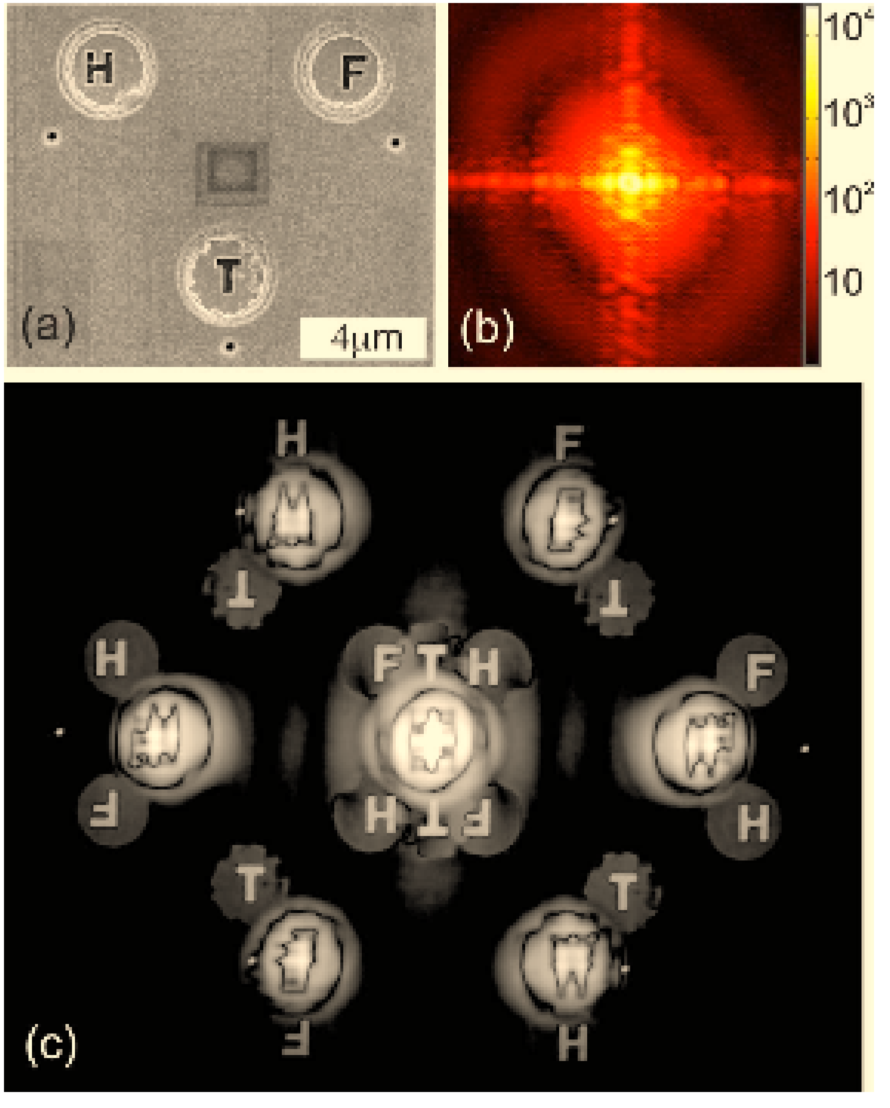
$\psi_r \circledast \psi_o$



$\psi_o \circledast \psi_o + \psi_r \circledast \psi_r$

Fourier transform holography

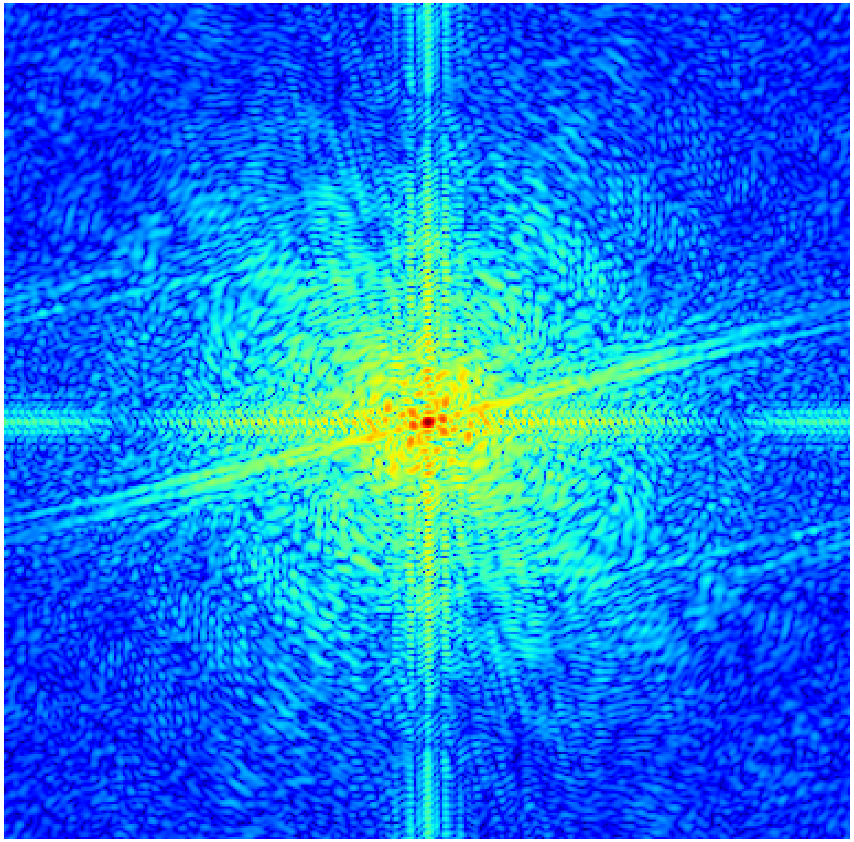
Multiple references



Source: W. Schlotter et al., Opt.. Lett. **21**, 3110-3112 (2006).

Coherent diffractive imaging

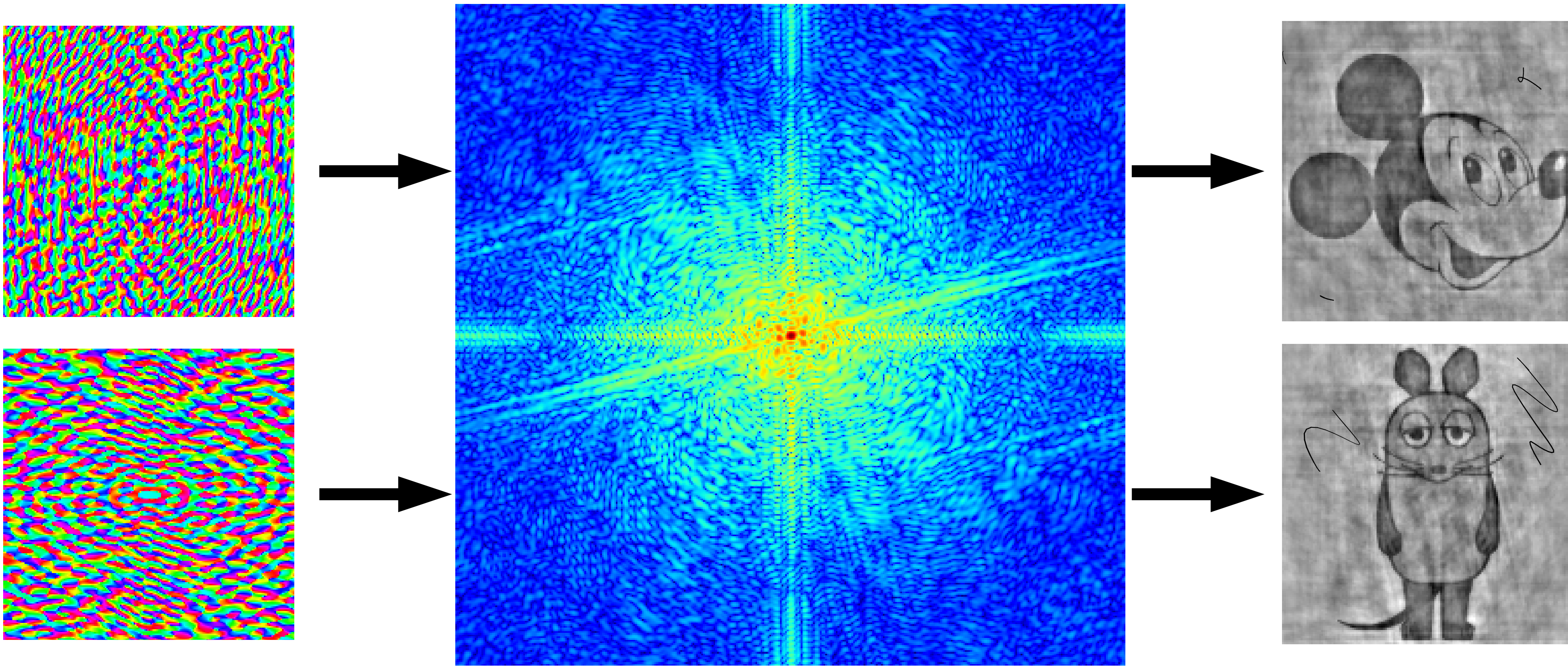
Diffraction pattern of an isolated sample



The phase problem

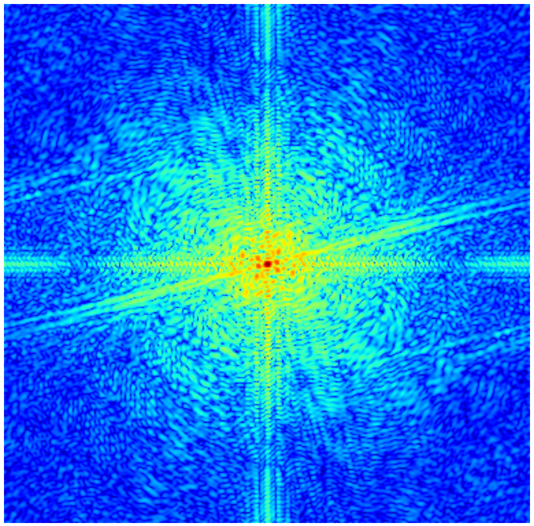
$e^{i\varphi(\vec{r})}$

$|\mathcal{F}\{\psi(\vec{r})\}|^2$



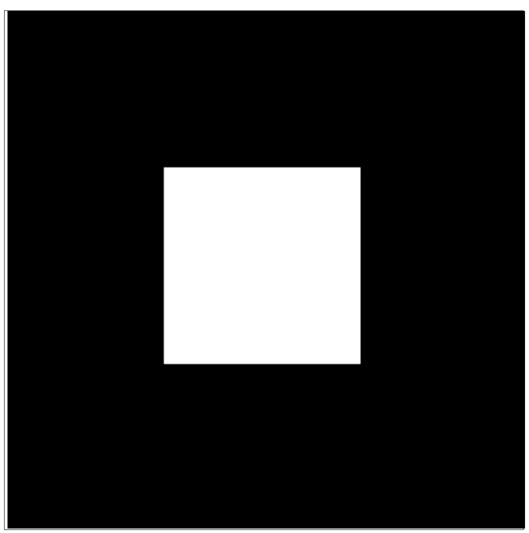
the phase part is very important to obtain the original image!

Coherent diffractive imaging



Two constraints

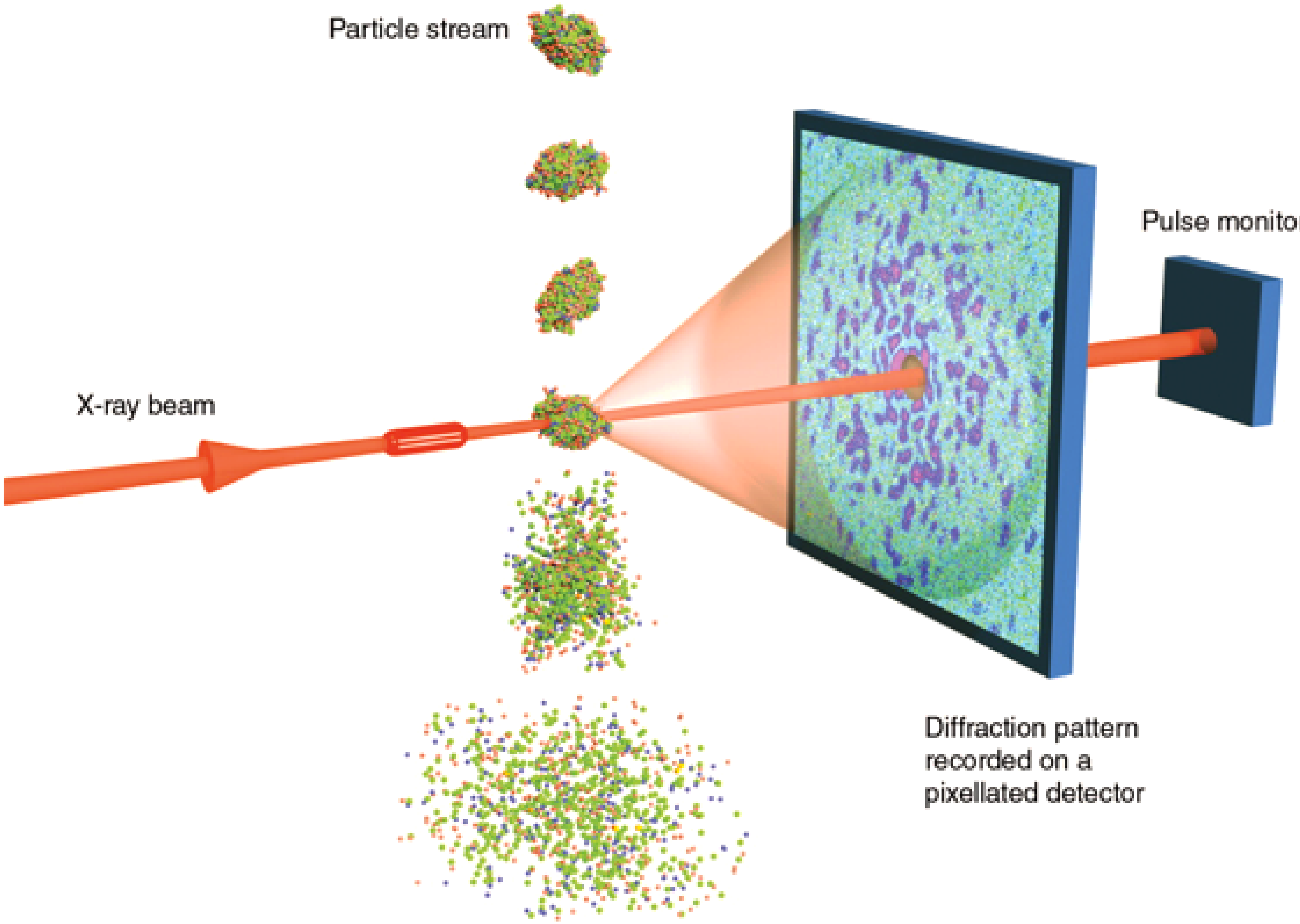
1. Solution has to be consistent with measured Fourier amplitudes



2. Solution is isolated

(anyway required to sample diffraction pattern sufficiently)

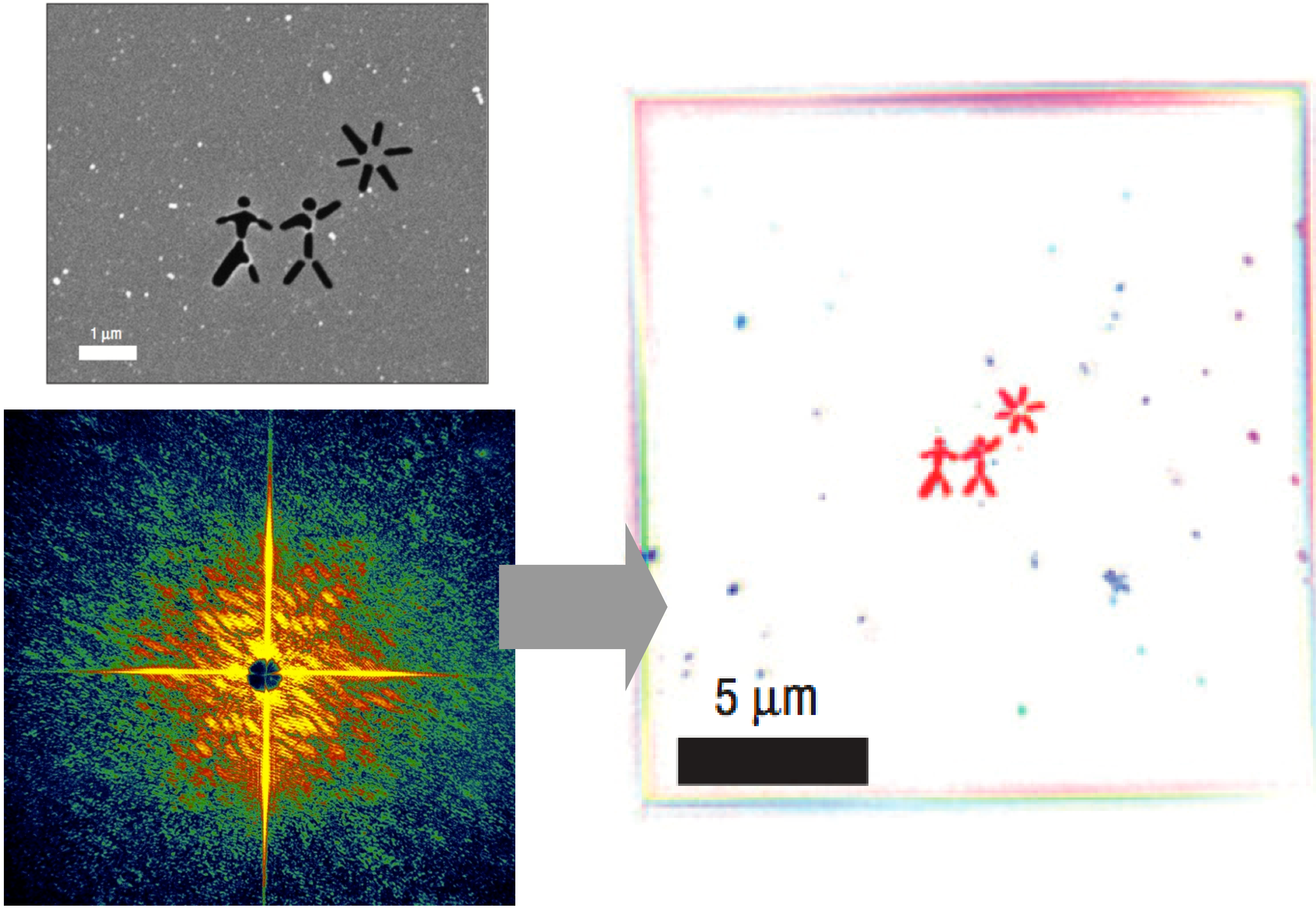
Radiation damage limits on radiation



R. Neutze *et al*, Nature **406**, 752 (2000)

K. J. Gaffney *et al*, Science **316**, 1444 (2007)

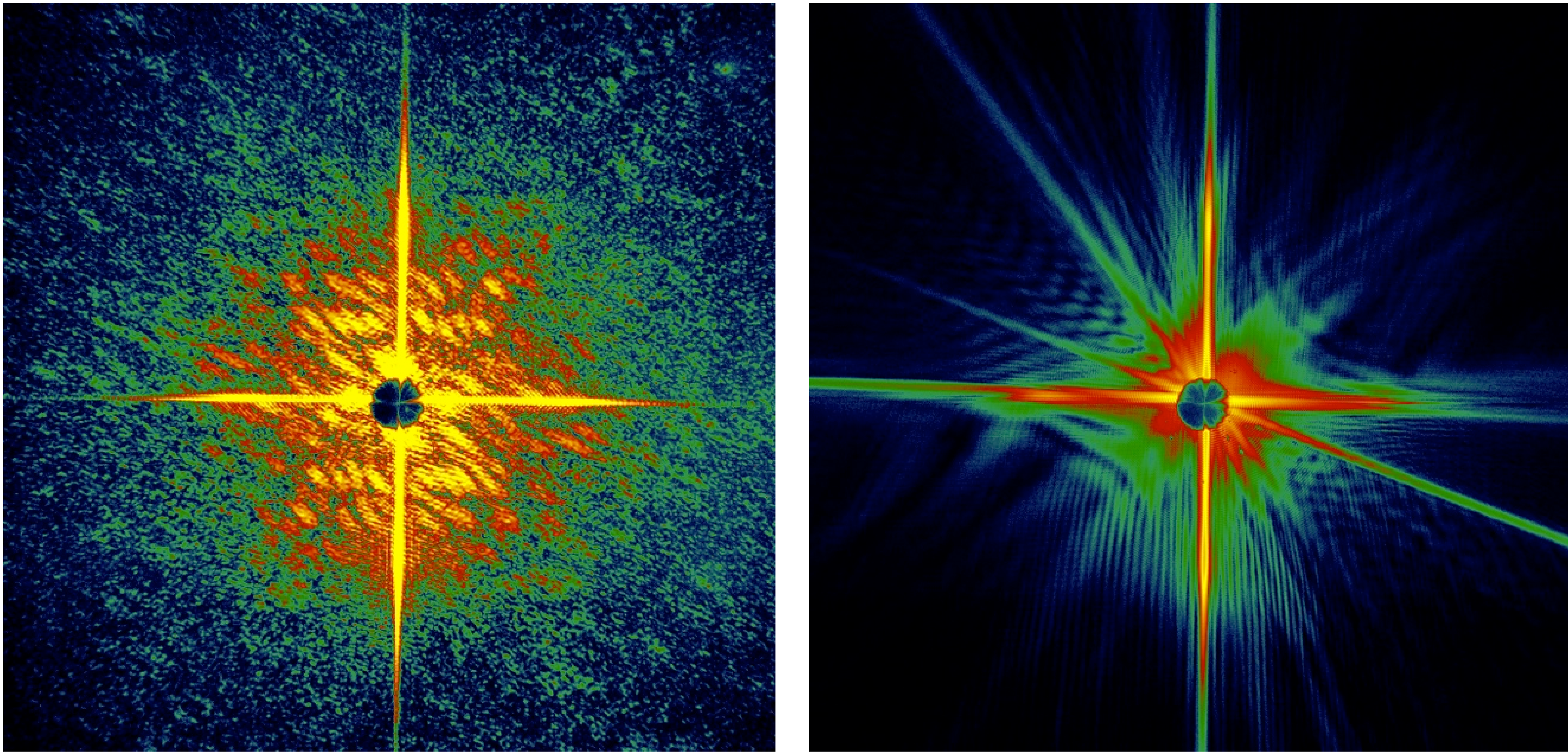
“Diffraction before destruction”



H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

“Diffraction before destruction”

The imaging pulse vaporized the sample



H. N. Chapman *et al*, Nat. Phys. **2**, 839 (2006)

Ptychography

- Scanning an isolated illumination on an extended specimen
- Measure full coherent diffraction pattern at each scan point
- Combine everything to get a reconstruction

Dynamische Theorie der Kristallstrukturanalyse durch Elektronenbeugung im inhomogenen Primärstrahlwellenfeld

1970

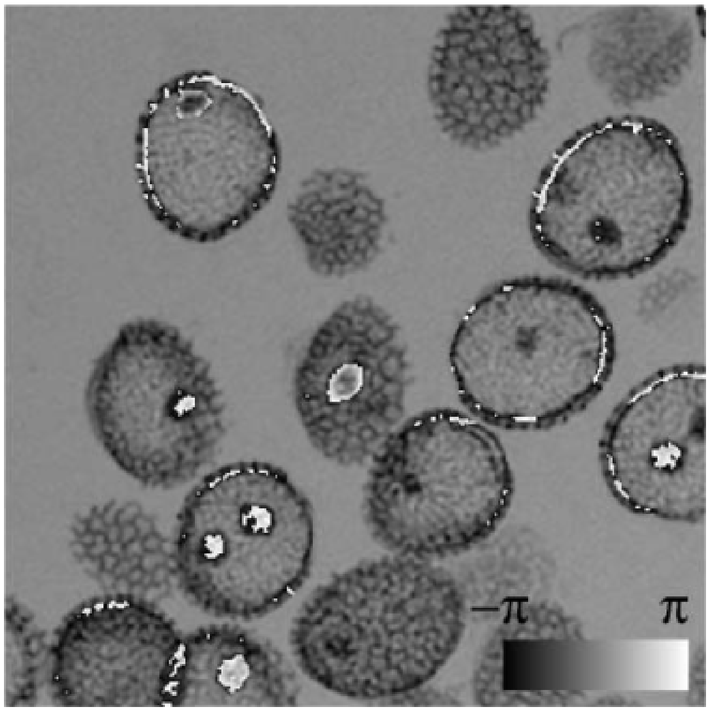
Von R. Hegerl und W. Hoppe

Some time ago a new principle was proposed for the registration of the complete information (amplitudes and phases) in a diffraction diagram, which does not – as does Holography – require the interference of the scattered waves with a single reference wave. The basis of the principle lies in the interference of neighbouring scattered waves which result when the object function $\varrho(x, y)$ is multiplied by a generalized primary wave function $p(x, y)$ in Fourier space (diffraction diagram) this is a convolution of the Fourier transforms of these functions. The above mentioned interferences necessary for the phase determination can be obtained by suitable choice of the shape of $p(x, y)$. To distinguish it from holography this procedure is designated "ptychography" ($\pi\tau\upsilon\xi$ = fold). The procedure is applicable to periodic and aperiodic structures. The relationships are simplest for plane lattices. In this paper the theory is extended to space lattices both with and without consideration of the dynamic theory. The resulting effects are demonstrated using a practical example.

Ptychography

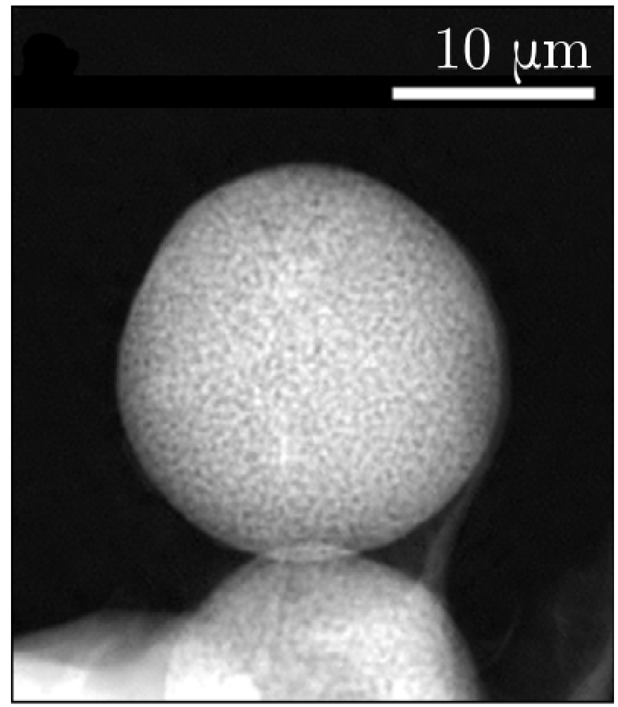
A few examples

Visible light



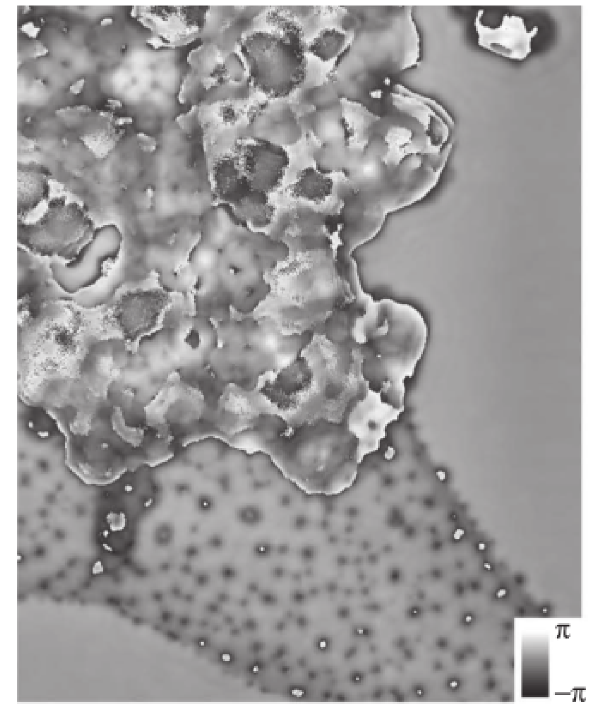
A. Maiden *et al.*, Opt. Lett. **35**, 2585-2587 (2010).

X-rays



P. Thibault *et al.*, New J. Phys **14**, 063004 (2012).

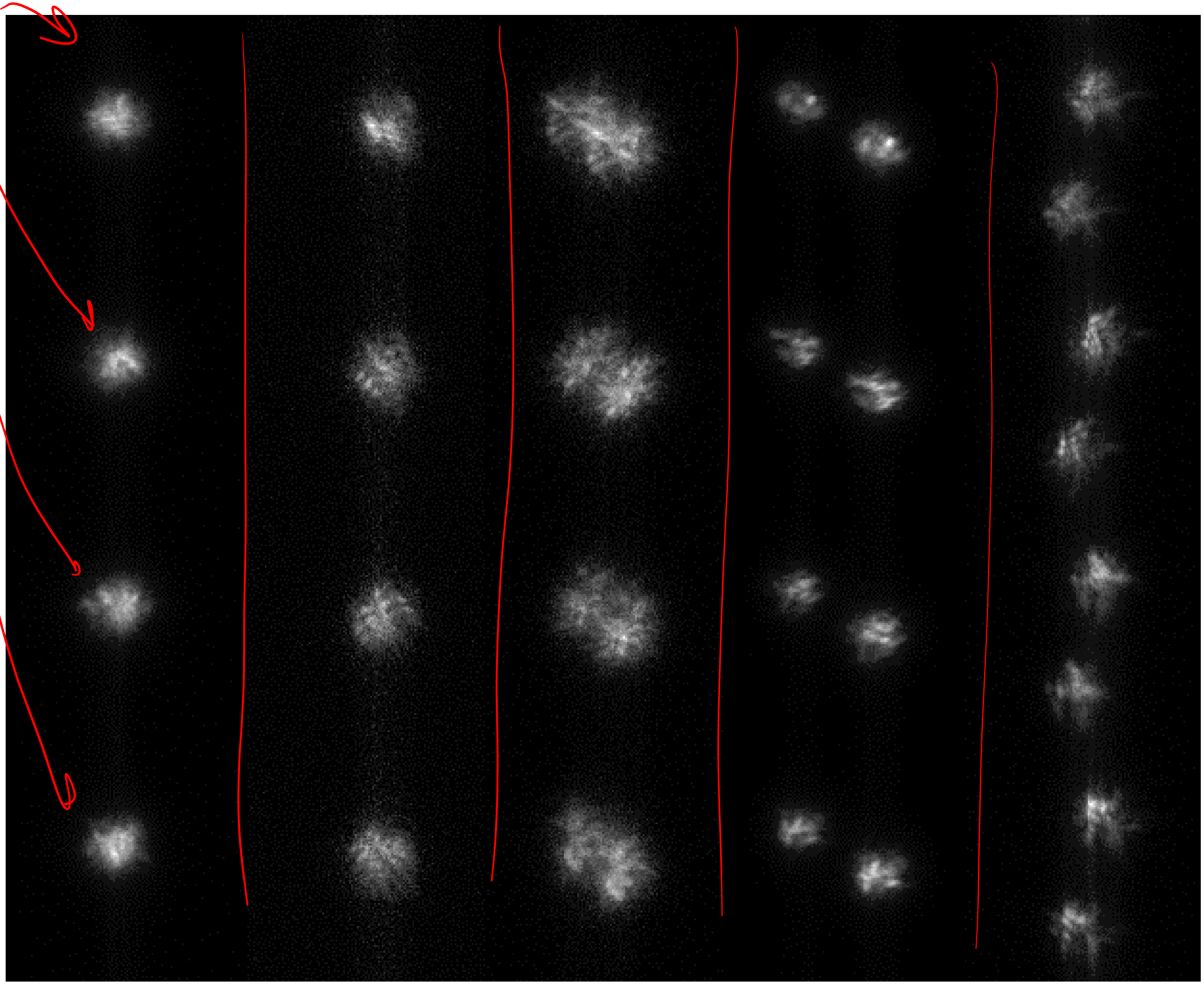
electrons



M. Humphry *et al.*, Nat. Comm. **3**, 730 (2012).

Speckle imaging in astronomy

a single object



result of air turbulence

Source:http://www.cis.rit.edu/research/thesis/bs/2000/hoffmann/thesis.html

Speckle imaging in astronomy

Model

incoherent imaging system

"instaneous PSF" changes with time because of turbulence

$$I(\vec{r}) = O * |P|^2$$

$$\tilde{I}(\vec{u}) = \tilde{O} \cdot P_A$$

autocorrelation of P

$$|\tilde{I}(\vec{u})|^2 = |\tilde{O}|^2 \, |P_A|^2$$

known quantity from fluid dynamics (well modelled)

average over multiple independent measurements

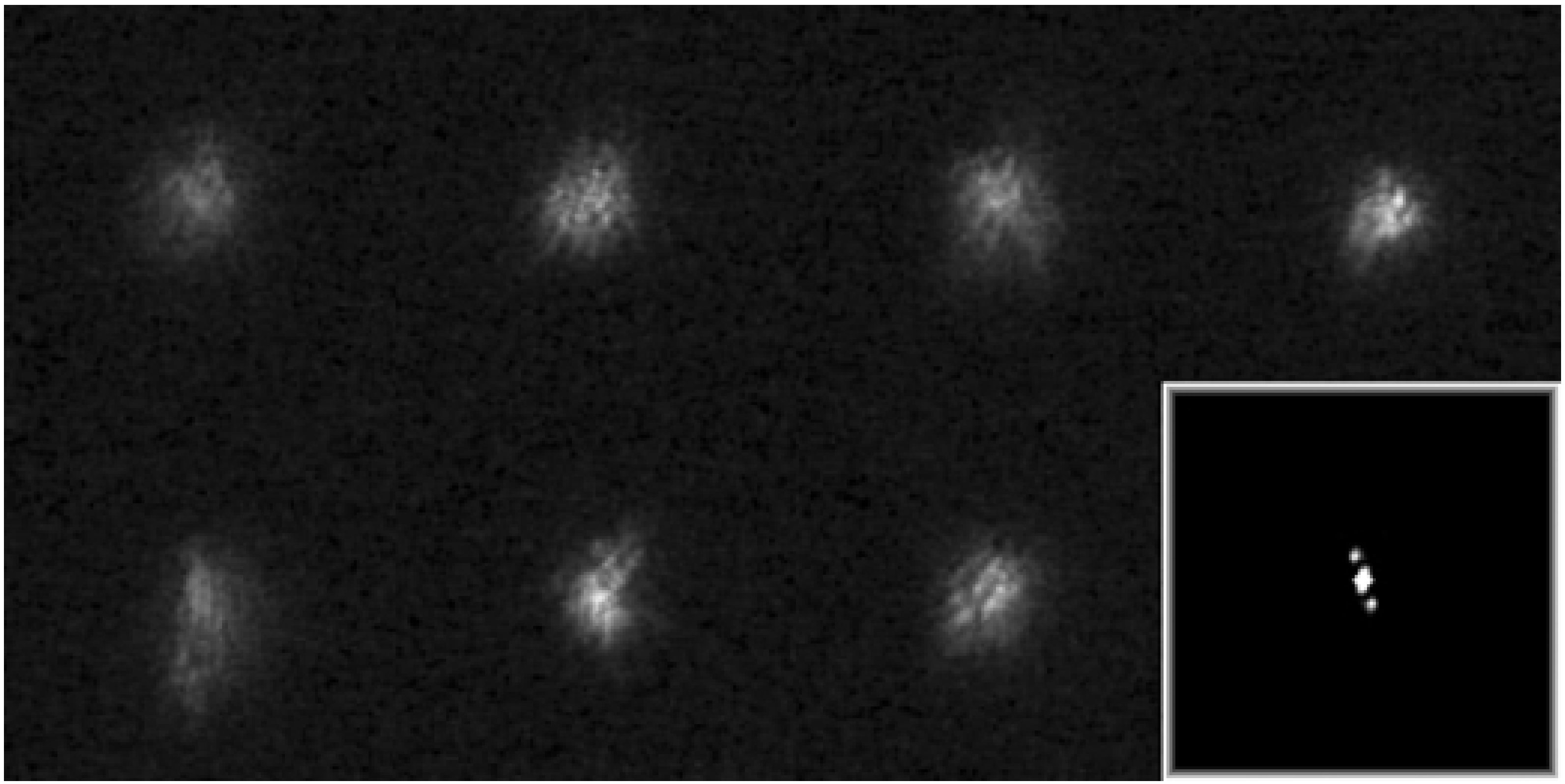
$$\langle |\tilde{I}|^2 \rangle = |\tilde{O}|^2 \langle |P_A|^2 \rangle$$

$$|\tilde{O}|^2 = \frac{\langle |\tilde{I}|^2 \rangle}{\langle |P_A|^2 \rangle}$$

model

recovering O from $|\tilde{O}|^2 \rightarrow$ same as CDI

Speckle imaging in astronomy
Retrieval of the autocorrelation

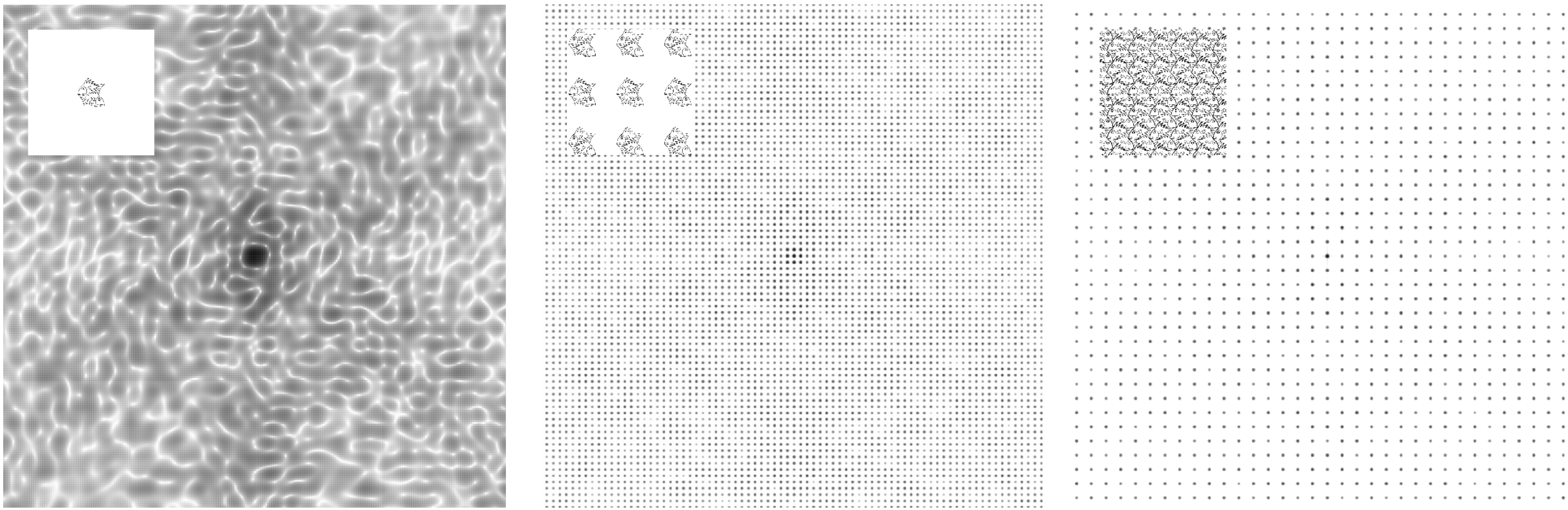


autocorrelation of O

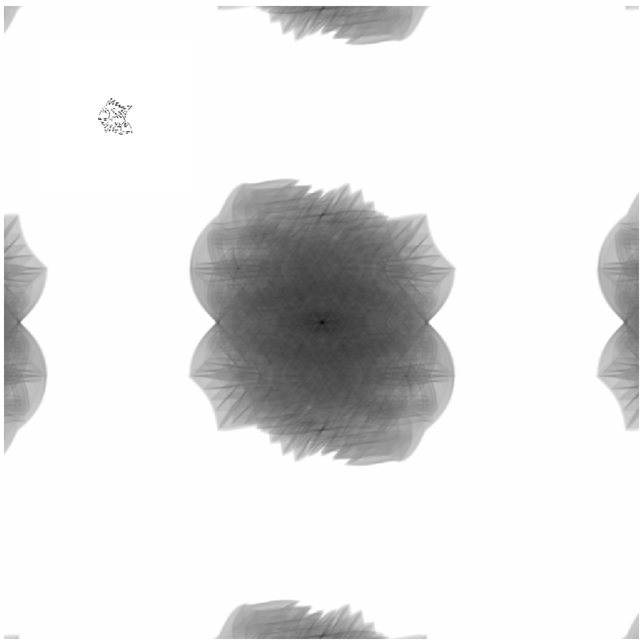
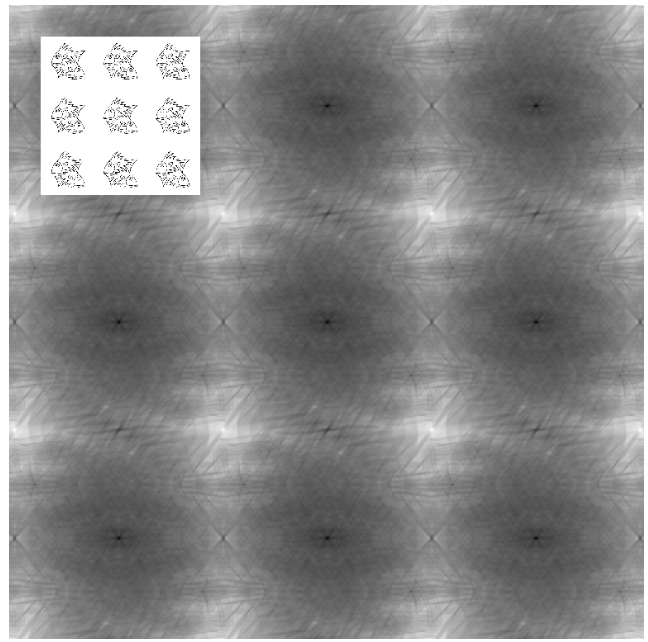
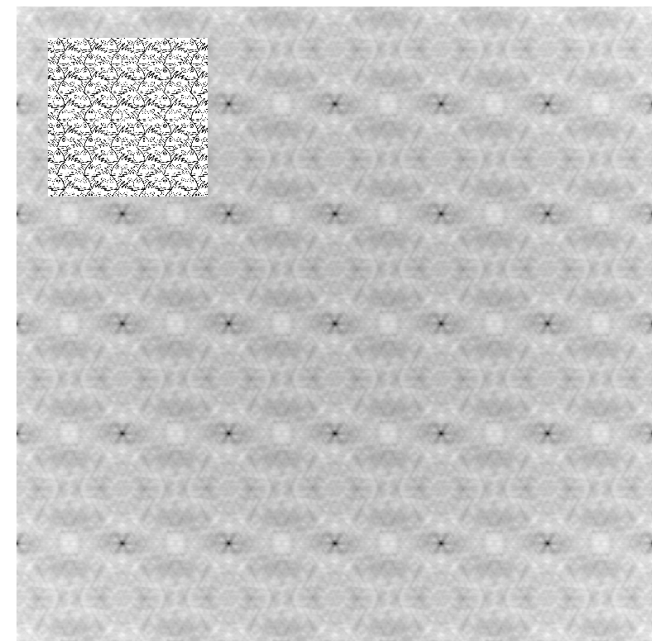
Source: http://www.astrosurf.com/hfosaf/uk/speckle10.htm

Crystallography

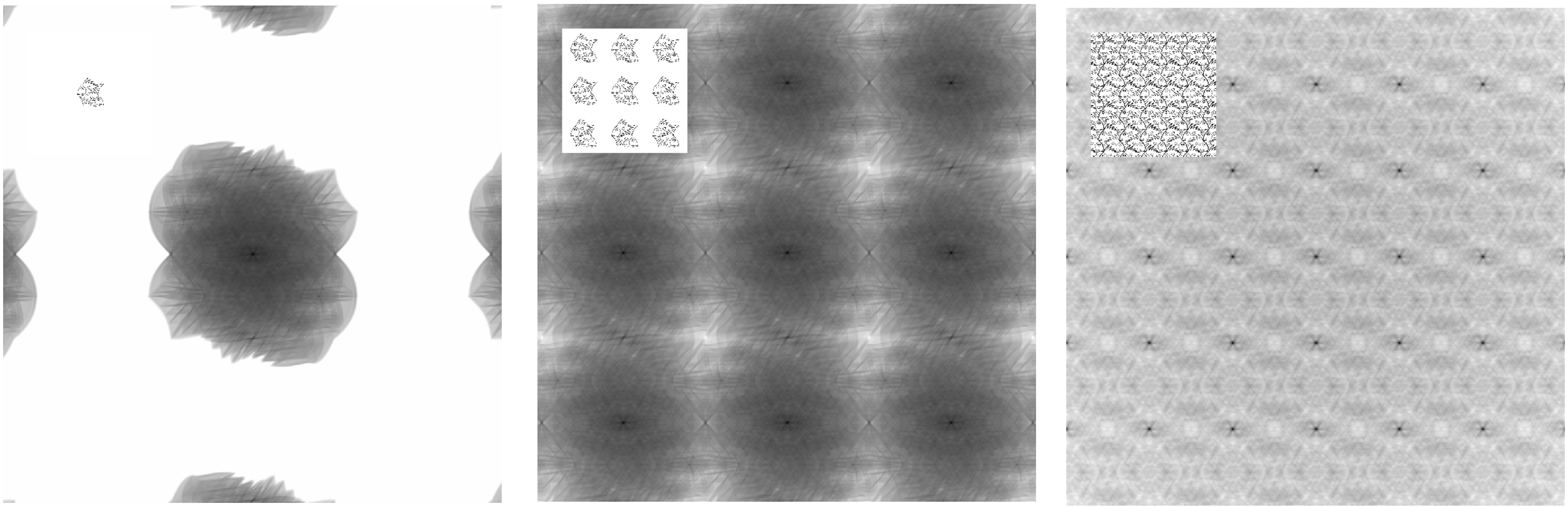
Diffraction by a crystal: Bragg peaks



Crystallography
Fourier transform of intensity: autocorrelation

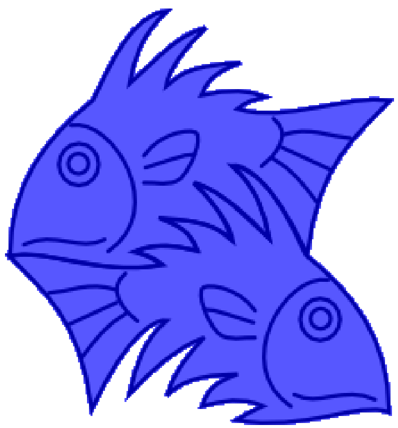




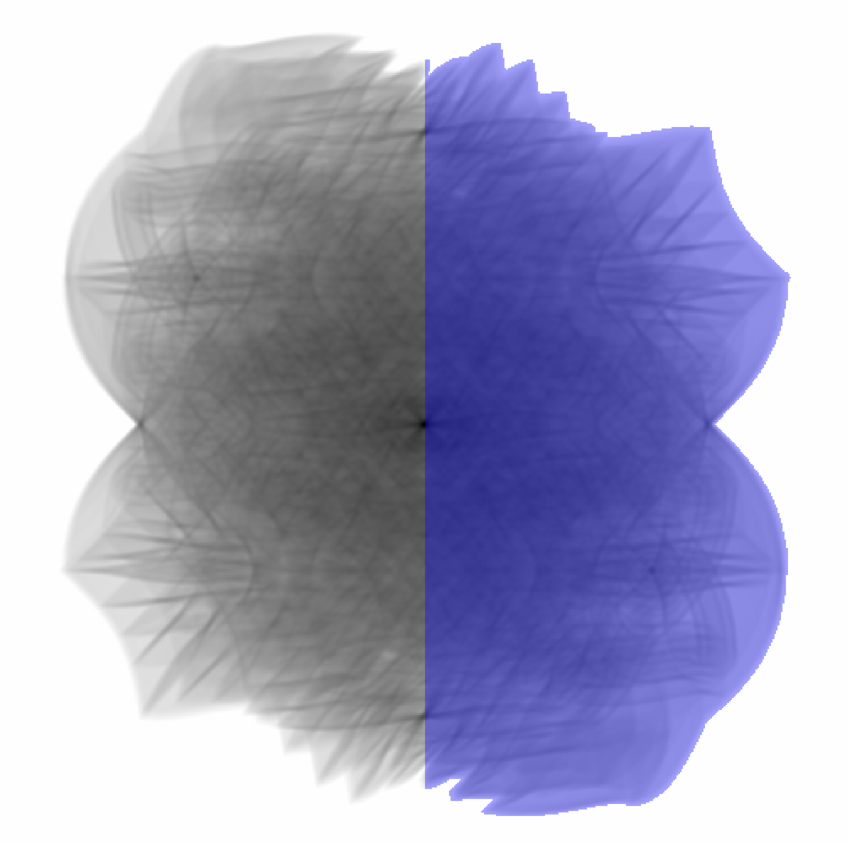
Crystallography



Crystallography

Problem is overconstrained with an isolated sample

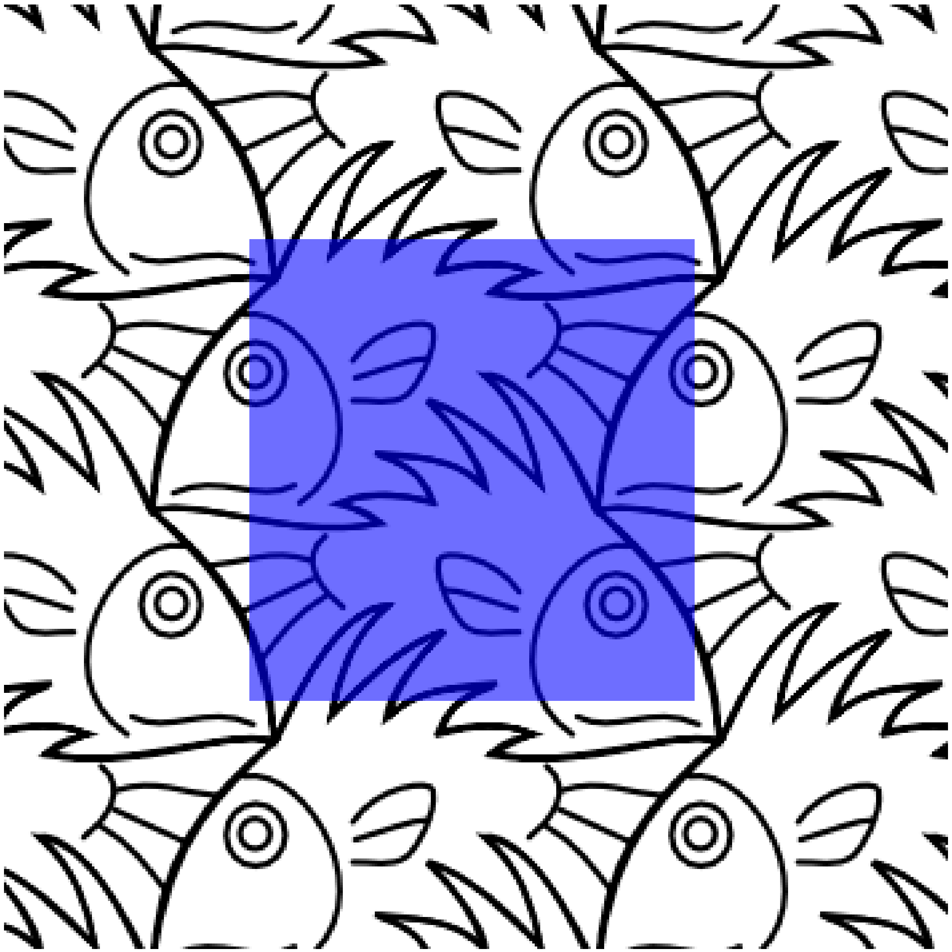




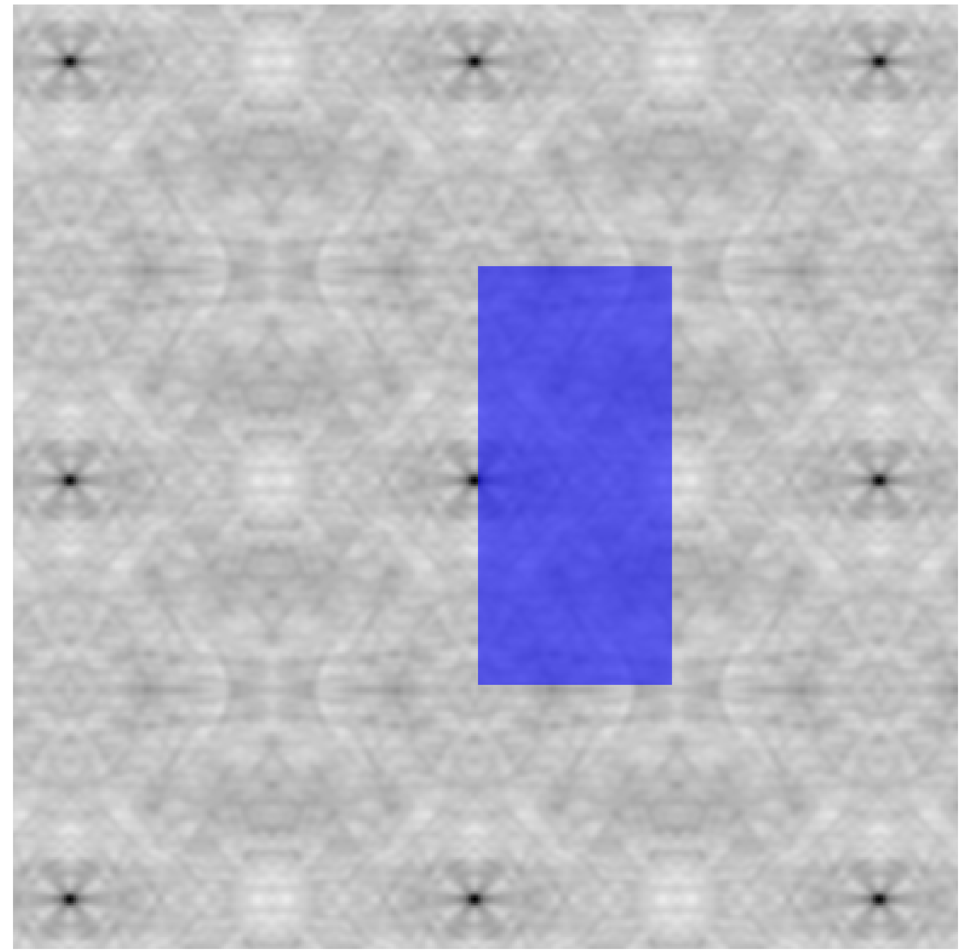
unknowns = N **constraints ≥ 2N**

Crystallography

Problem is **under**constrained with a crystal

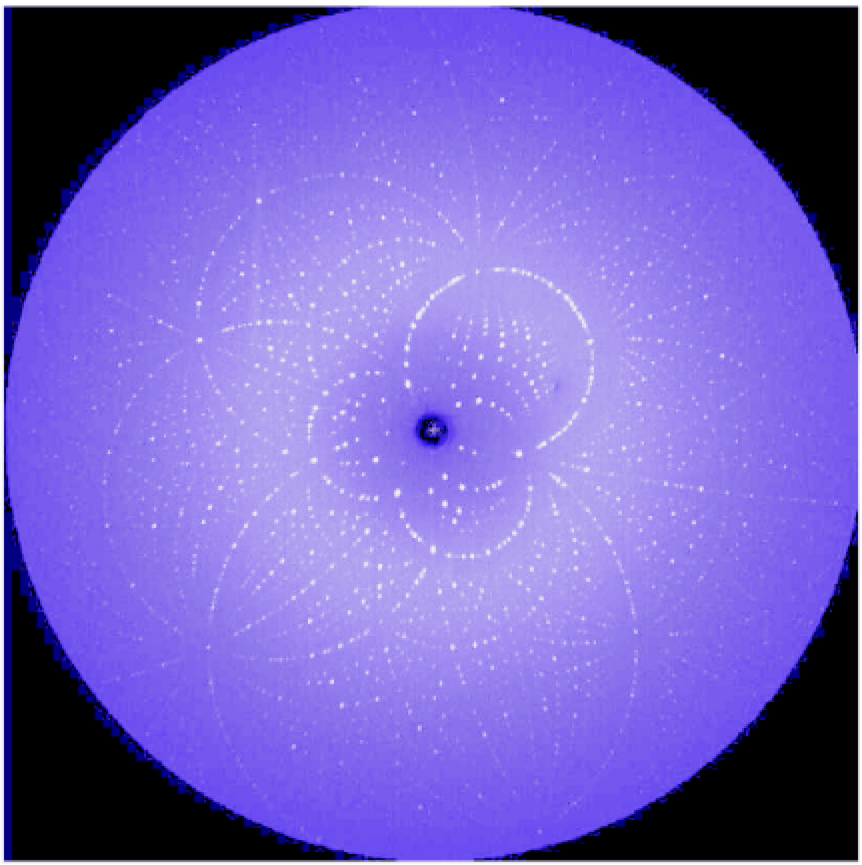


unknowns = N



constraints = N/2

Crystallography
Structure determination



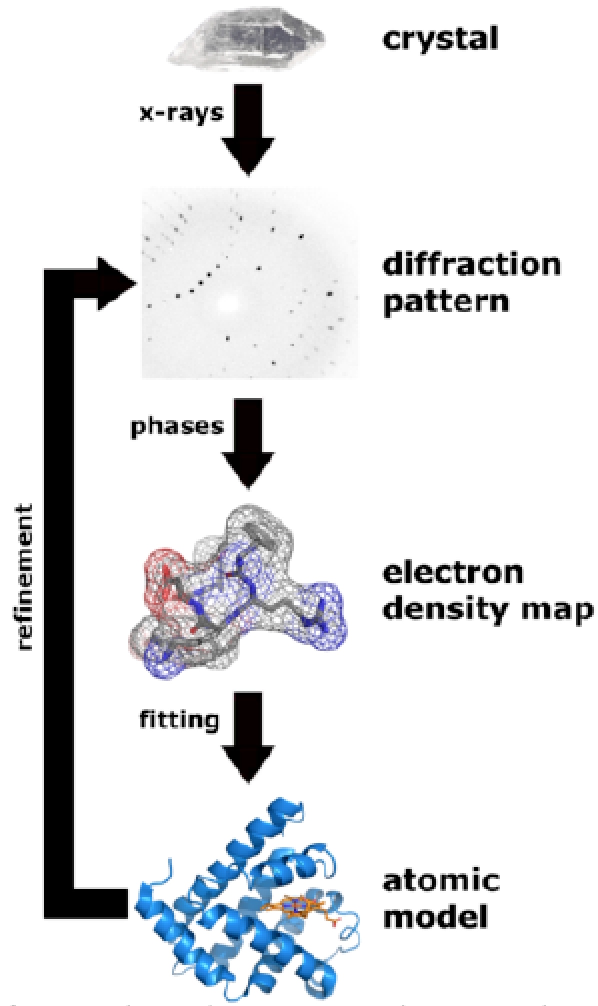


Image from Wikimedia courtesy Thomas Splettstoesser

Crystallography

Structure determination

- Hard problem: few measurements for the number of unknowns
- Luckily: crystals are made of atoms $\rightarrow$ strong constraint
- Also common: combining additional measurements (SAD, MAD, isomorphous replacement, ...)

Summary

Imaging from far-field amplitudes

- Used when image-forming lenses are unavailable (or unreliable) or to obtain more quantitative images.
- In general difficult because of the phase problem
- Solved with the help of additional information:
  - Strong *a priori* knowledge (e.g. CDI: support)
  - Multiple measurements (e.g. ptychography)