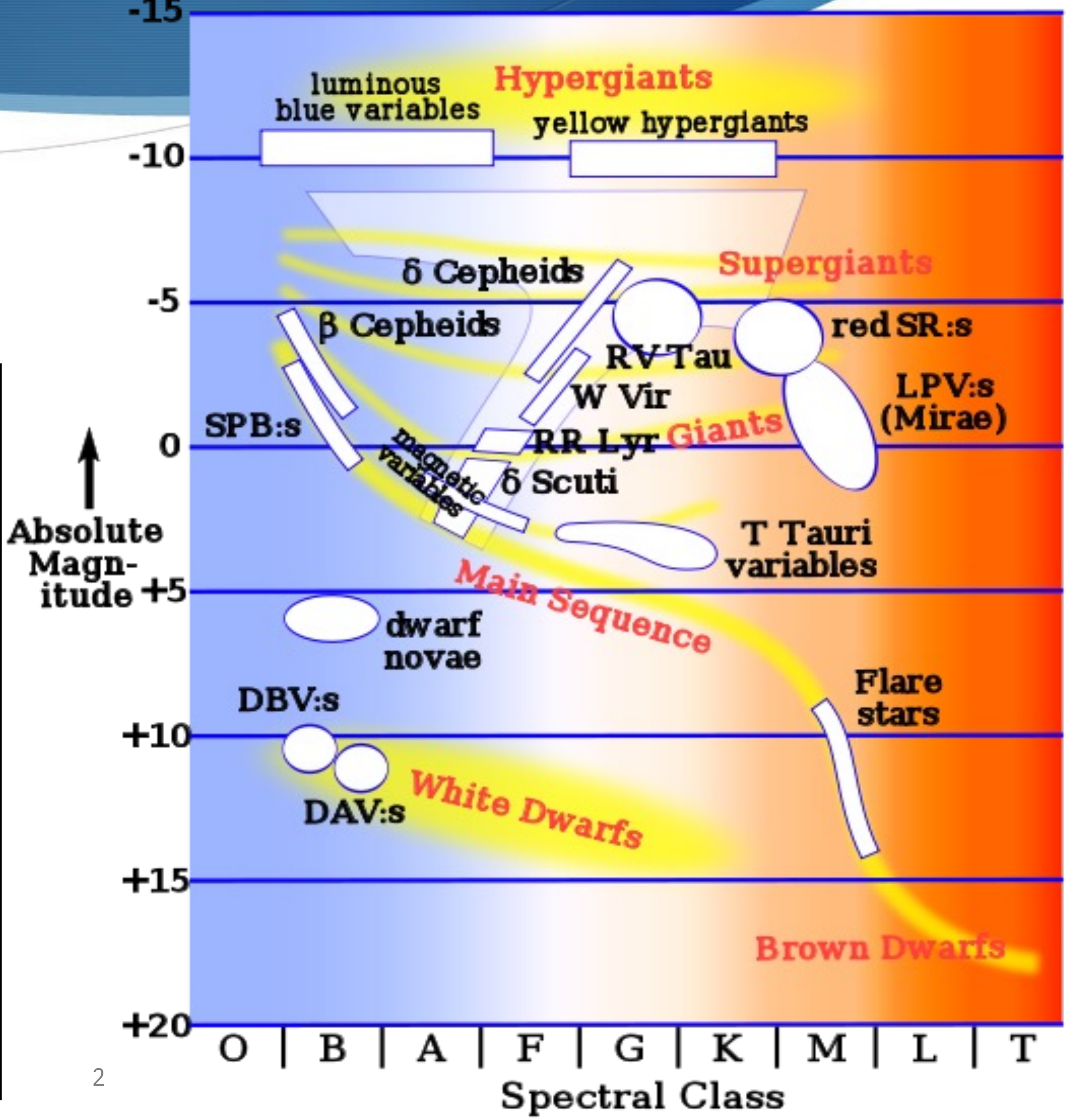
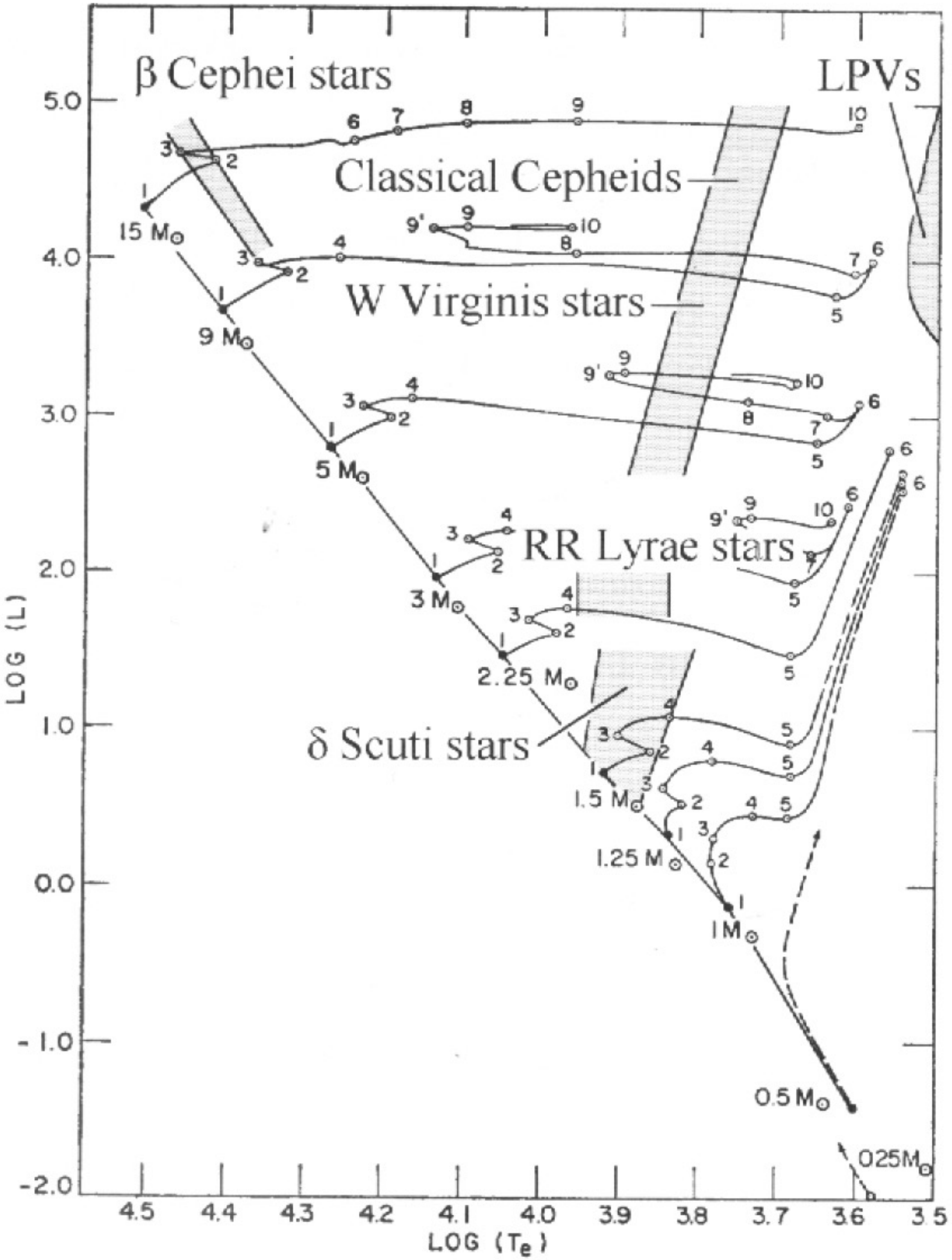
Pulsating Stars

a) Classes of pulsating stars

Many stars
Intrinsically variable
Subset: regular pulsation

Type	Period (d)
RR Lyrae	0.3-0.9
Cepheids	1-50
W Virginis	2-50
RV Tauri	60-200
β Cepheids	0.2
δ Scuti	0.2
LPV	100-700
Semi-regulars	100-200
Mira’s	150-700




β Cephei stars
LPVs
Classical Cepheids
W Virginis stars
RR Lyrae stars
δ Scuti stars
15 M⊙
9 M⊙
5 M⊙
3 M⊙
2.25 M⊙
1.5 M⊙
1.25 M⊙
1 M⊙
0.5 M⊙
0.25 M⊙
LOG (L)
5.0
4.0
3.0
2.0
1.0
0.0
- 1.0
-2.0
LOG (Te)
4.5
4.4
4.3
4.2
4.1
4.0
3.9
3.8
3.7
3.6
3.5

Le stelle variabili si dividono in:

- **W Virginis**- Sono Cefeidi dei Popolazione II (pochi metalli) sono 4 volte meno luminose delle Cefeidi classiche con lo stesso periodo. Posseggono una relazione luminosita'-periodo ma piu' bassa e parallela a quella delle Cefeidi classiche. Il periodo delle W Virginis e’ = 2-45 giorni
- **RR- Lyrae**- Sono stelle di Popolazione II e si trovano nel braccio orizzontale che si osserva negli ammassi globulari. (Periodo = 1.5-24 ore).
- **δ-Scuti**- Stelle evolute di tipo F vicine alla Sequenza Principale. Hanno sia oscillazioni radiali che non-radiali. (P= 1-3 ore).
- **ZZ Ceti**- Sono nane bianche pulsanti (P= 100-1000 sec). Hanno oscillazioni non-radiali.
- **Mira** o **variabili a lungo periodo**, LPVs- Insieme alle stelle β -Cephei si trovano al di fuori della striscia di instabilita' delle Cefeidi classiche e delle RR-Lyrae (P = 100-700 giorni). Sia LPV che le β –Cephei hanno oscillazioni radiali e non-radiali.
- **Cefeidi-** Variabili di Popolazione I con P= 1-50 giorni.

Pulsating stars lie in well-defined regions of the HRD

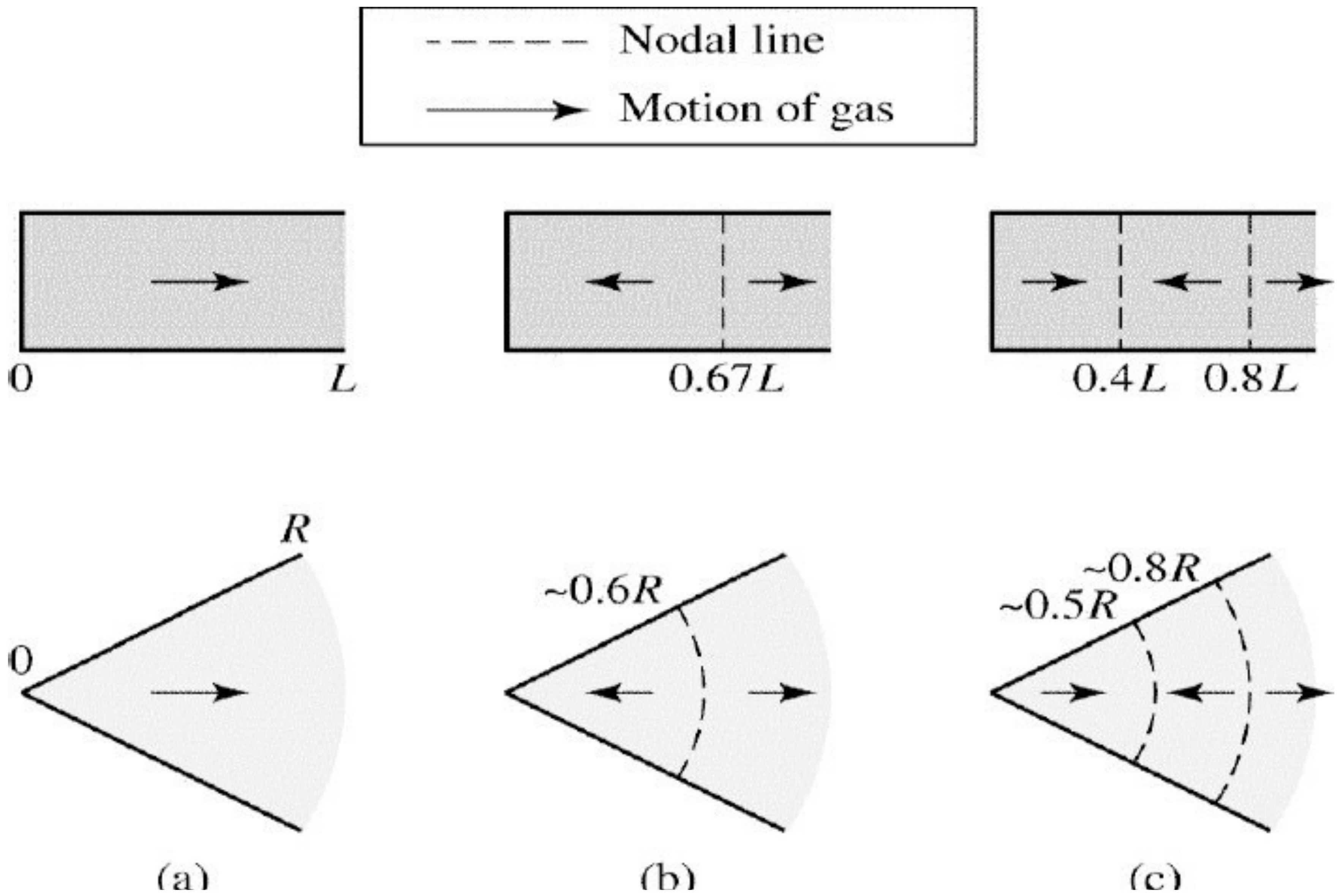
- Instability strip in HR diagram: narrow range in Teff spanning a wide range of luminosities

- Whenever stars pass through this instability strip in their evolution, they become pulsational unstable

- Stellar pulsations are due to sound waves resonating in their interiors

- Stellar pulsations depend on stellar structure

b) Period-mean density relation



Radial pulsations: standing waves; fundamental & overtones; nodes

Pulsation density perturbation with wavelength equal to
stellar diameter regular pulsation standing wave

First order: $\Pi \approx \frac{2R}{\overline{v}_s}$ with $\overline{v}_s$ mean sound speed $v_s = \sqrt{\Gamma_1 \frac{P}{\rho}}$

Assume: star oscillates around hydrostatic equilibrium

use virial theorem to estimate sound speed:

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = -\int_0^R \frac{4\pi r^3 G m(r) \rho(r)}{r^2} dr \qquad \Omega = -\alpha G M^2/R$$

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = [P(r) 4\pi r^3]_0^R - 3\int_0^R P(r) 4\pi r^2 dr$$

$$-3\int_0^R P(r) 4\pi r^2 dr = -3\int \frac{P}{\rho} dm \qquad dm = 4\pi r^2 \rho(r) dr.$$

$$-E_g = 2E_i$$

$$q\frac{GM^2}{R} = 3\int \frac{P}{\rho}\,dm = 3\int \frac{\upsilon_s^2}{\Gamma_1}\,dm \equiv 3\,\overline{\left(\frac{\upsilon_s^2}{\Gamma_1}\right)}M \approx 3\frac{\overline{\upsilon_s^2}}{\Gamma_1}M$$

$$\Pi \approx \frac{2R}{\bar{\upsilon}_s} = 2\left(\frac{3}{q\Gamma_1}\right)^{1/2}\left(\frac{R^3}{GM}\right)^{1/2} = 2\left(\frac{3}{q\Gamma_1}\right)^{1/2}\frac{3^{1/2}}{(4\pi G\bar{\rho})^{1/2}} \propto \frac{1}{\sqrt{G\bar{\rho}}}$$

QUALE SARA’ IL VALORE PER UNA CEFEIDI DI 5Msun E RAGGIO 50 Rsun?

c) Physics of stellar pulsations: k-mechanism

Eddington’s valve
Pulsations are oscillations around an equilibrium position:

1) Consider a layer that loses temporarily hydrostatic support and collapses
2) The layer will compress and heat up
3) The opacity will increase blocking radiation from below
4) Temperature and pressure below it will build up
5) Eventually, the layer will be pushed out again, expand, cool, and become more transparent
6) Radiation from below can escape
7) Cycle starts over

Steam engine: Radiation = steam; layer =piston; opacity=valve

Key is that opacity increases with compression

Kramer opacity: Normally, opacity decreases with compression

$$\kappa \propto \frac{\rho}{T^{3.5}}$$

as temperature increase wins over density increase

Partial ionization zones: Upon compression, internal energy is stored into increased ionization and density increase leads to increased opacity.

Upon expansion, stored internal (ionization) energy is released and decreased density leads to decreased opacity

Three partial ionization zones:

1) HI, HeI: 10,000–15,000K
2) HeII: 40,000K
3) Iron: 200,000K

Approximate opacity as: $\kappa = \kappa_0 \rho^n T^{-s}{}_0$ with n , $s > 0$ (Kramers: n=1, s=3.5)
Driving a pulsation needs heat exchange: non-adiabatic process
Use adiabatic relations to get approximate results

Write:
$$\frac{dT}{T} = (\Gamma_3 - 1)\frac{d\rho}{\rho} \Rightarrow T \propto \rho^{\Gamma_3 - 1} \Rightarrow \kappa \approx \rho^{1-3.5(\Gamma_3-1)}$$

If $1 - 3.5(\Gamma_3 - 1) > 0$ increases during compression
and decreases during expansion pulsation can be driven when $\Gamma_3 < 1.28$
Implications:
Fully ionized monatomic ideal gas has $\Gamma_3 = 5/3$: no pulsation
driven by -mechanism
In ionization zone Γ_3 decreases to ~1.1

K -mechanism can operate: during compression energy is use
to increase degree of ionization, T rises only little

d) “Real estate” of pulsation

H-He ionization zone: thickness ~ 10^4K
He+ zone: thickness ~$2x10^4$K: largest heat capacity
Stable pulsation possible when H, He, He+ zones lie at such a depth that
Envelope not so massive that pulsation is damped
Zone has sufficient heat capacity.

These conditions are fulfilled in classical instability strip
Computed properties of Cepheids agree well (Iben 1991 ApJS, 76)

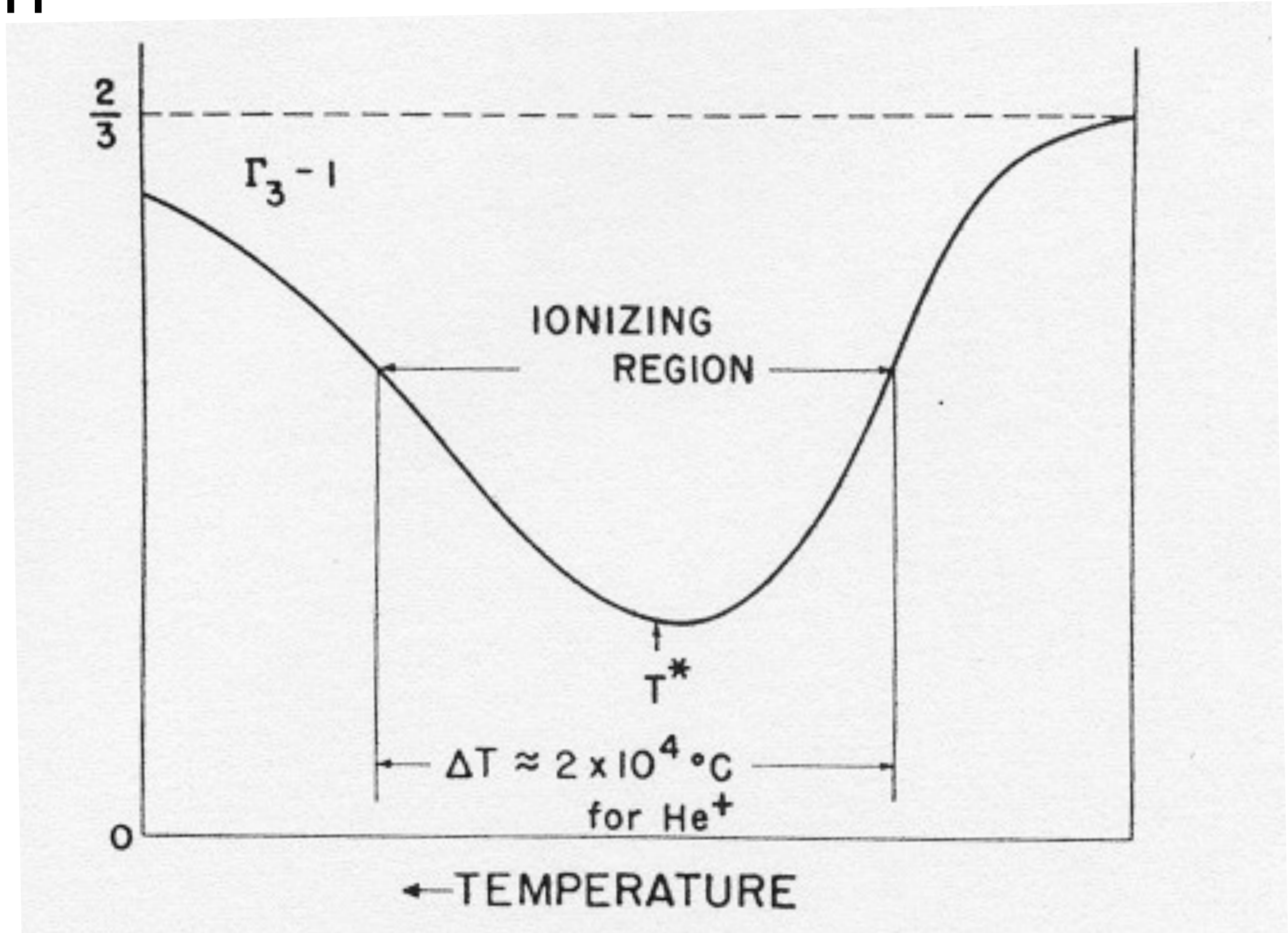


Fig. 27.4 $\Gamma_3 - 1$ vs. temperature (schematic) in the region of He^+ ionization in the equilibrium model of a stellar envelope.

1) Partial ionization zones (PIZ) are prone to pulsational instability

2) Real estate agents mantra: Location, location, location

1) PIZ at the surface, heat readily escapes

2) PIZ deeply inside, overlaying layers dampen pulsations

3) Location of PIZ:

Bottom of zone, radiative energy dammed upon compression.

Top of zone: energy can be radiated in a single period.

Work done during one cycle Cepheids, RR Lyrae

~60% energy He+ zone

~40% energy: H-He zone

Large dissipation below

Small dissipation above

Phase lag:

Effect of inertia of upper layers

5-10% of L temporarily stored in outer layers and released later

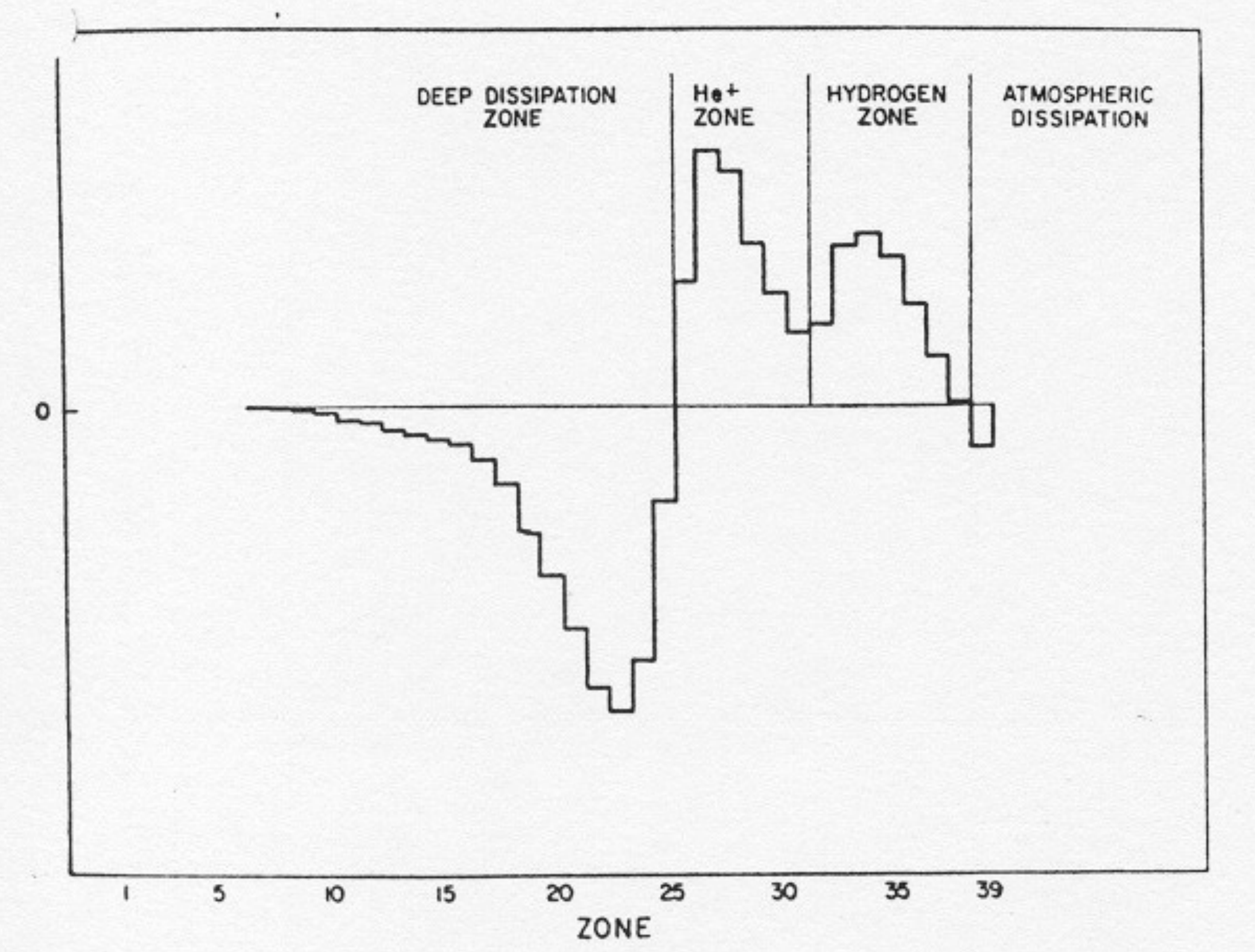
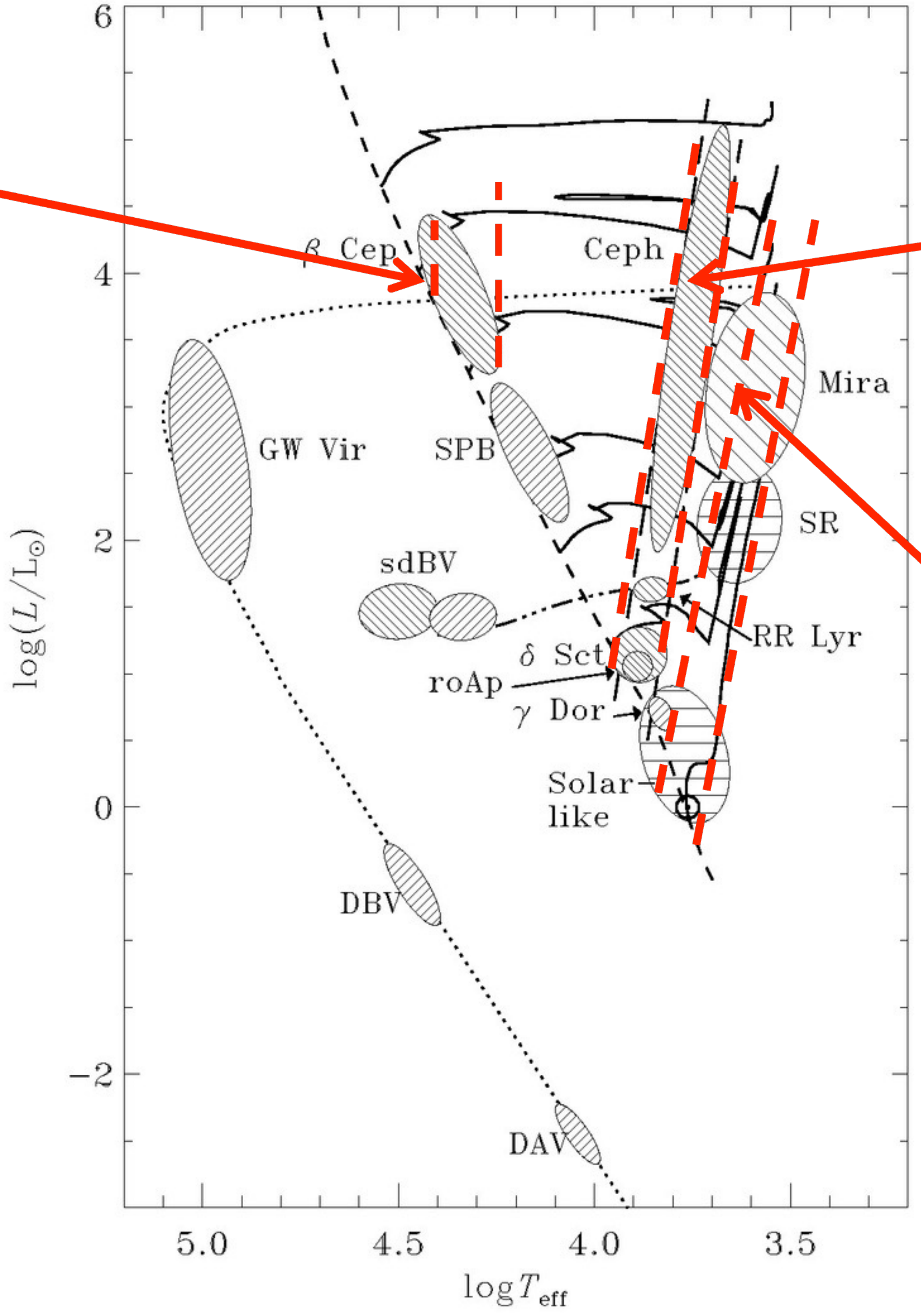


Fig. 27.15 The work W_i (on an arbitrary scale) done by the i^{th} mass shell on its surroundings around a complete period (see (27.194)) versus i for one of Christy's [Ch66a] unstable RR Lyrae models at limiting amplitude in the fundamental mode. (Parameters for this model are given in the text.) Positive values of W_i connote "driving" regions, negative values "damping" regions.

Pulsations & Partial Ionization Zones



iron ionization

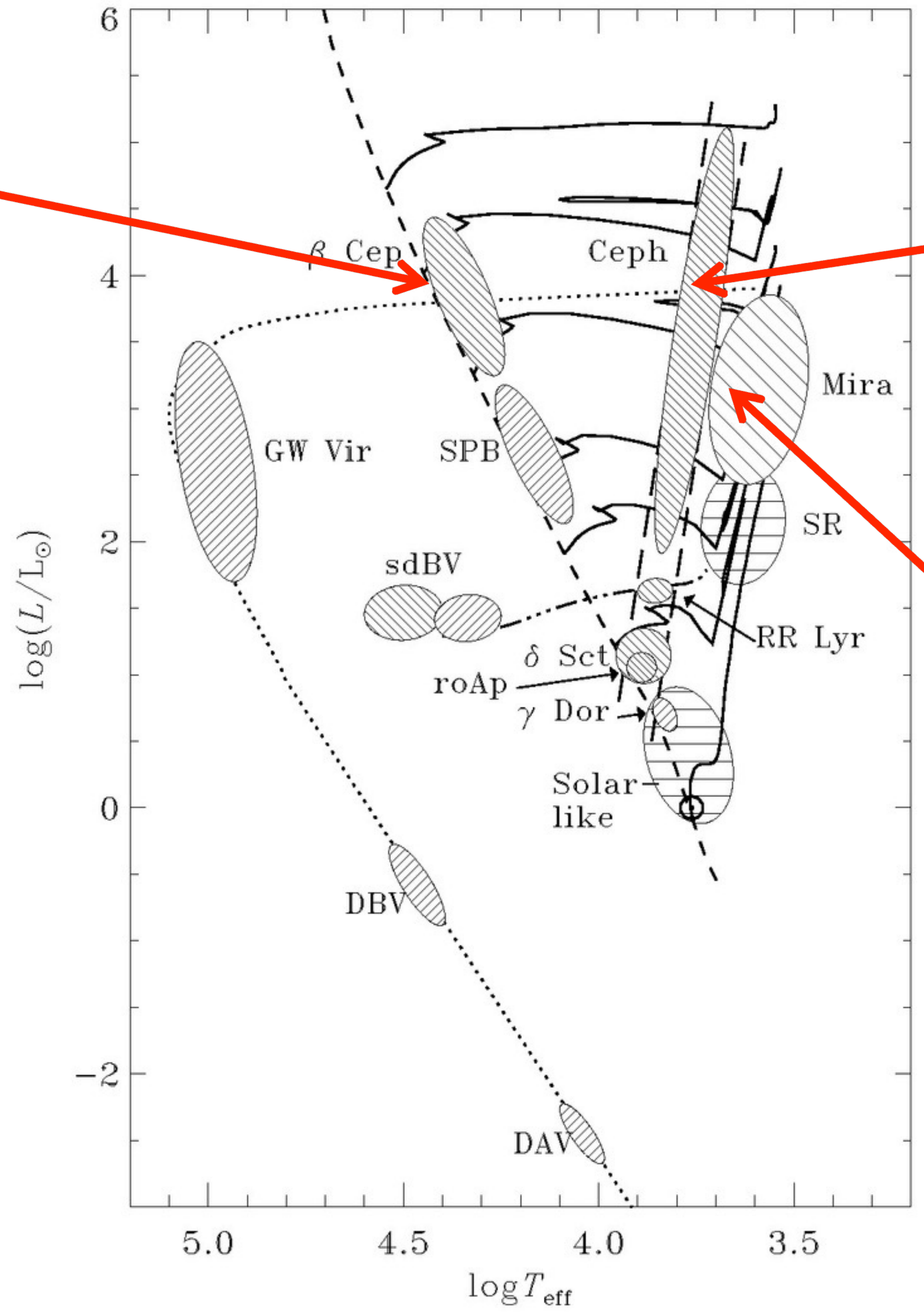
HeII ionization

HI & HeI ionization

Red edge due to location of partial ionization zone

Blue edge due to convection reaching this zone

Pulsations & Partial Ionization Zones



iron ionization

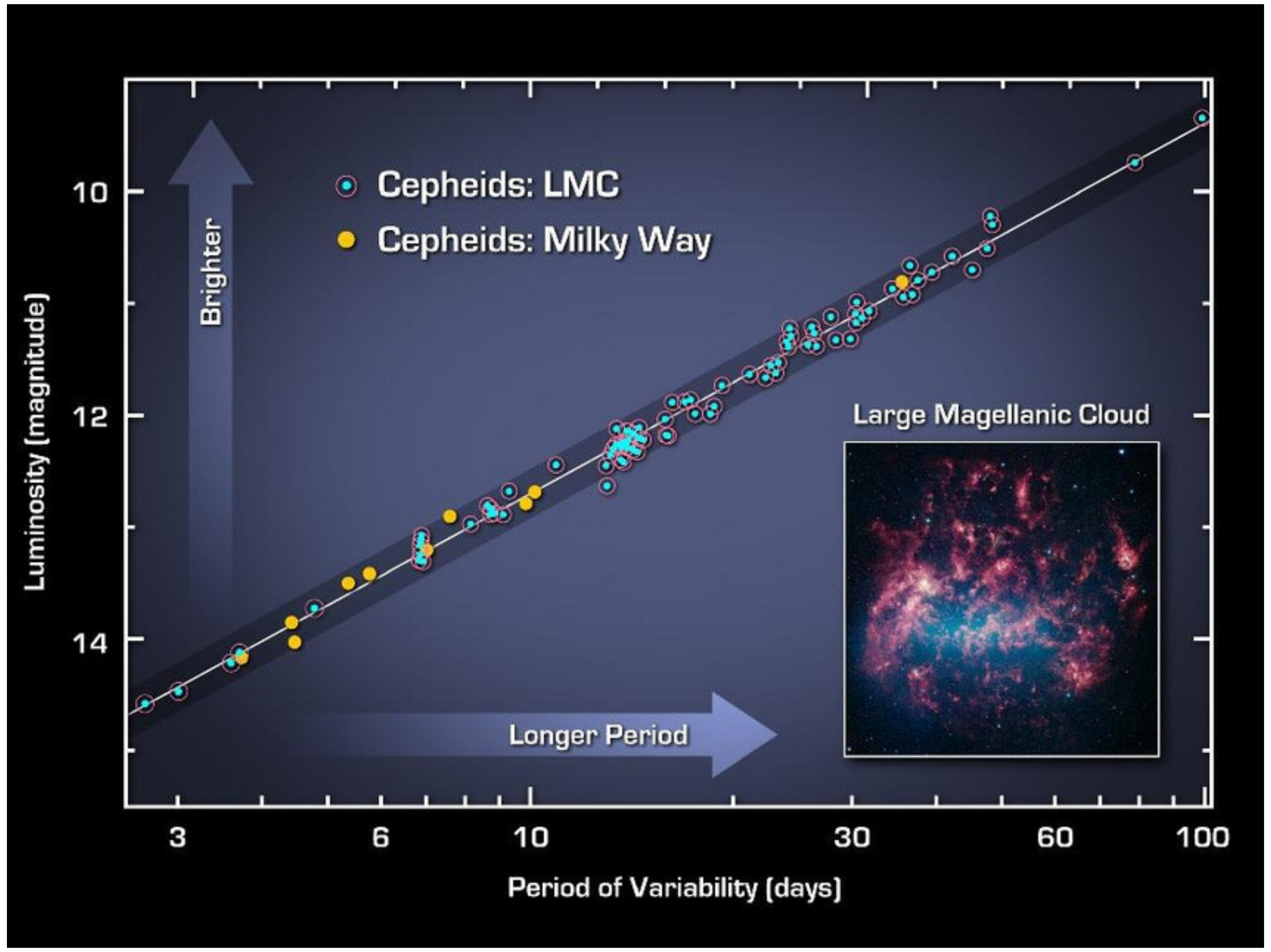
HeII ionization

HI & HeI ionization

e) Application

Period-luminosity relation

Leavitt discovered P-L relation by study of Cepheid variables in Magellanic Clouds



$$\Pi \propto 1/\sqrt{\rho}$$

$$L \propto R^{5/3}$$

$$L \propto M^{11/2} / R^{1/2}$$ (Radiative equilibrium & Kramers opacity)

$\Pi \propto L^{2/3}$ observed: $\Pi \propto L^{0.9}$