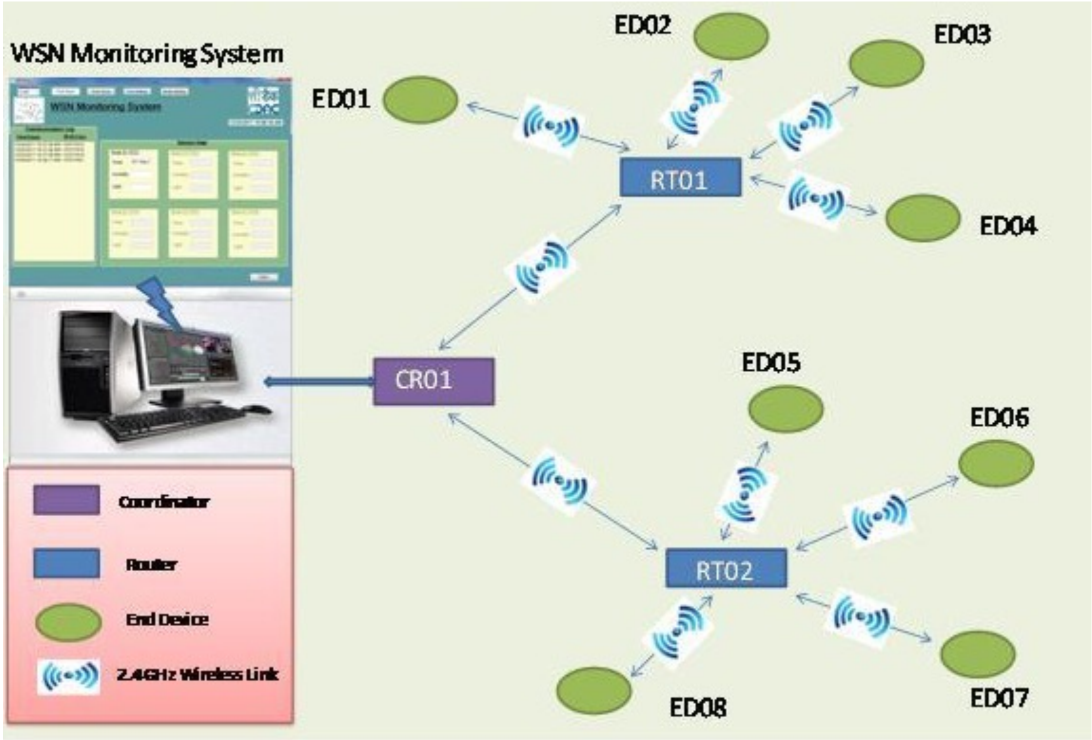
Cyber-Physical Systems

Laura Nenzi

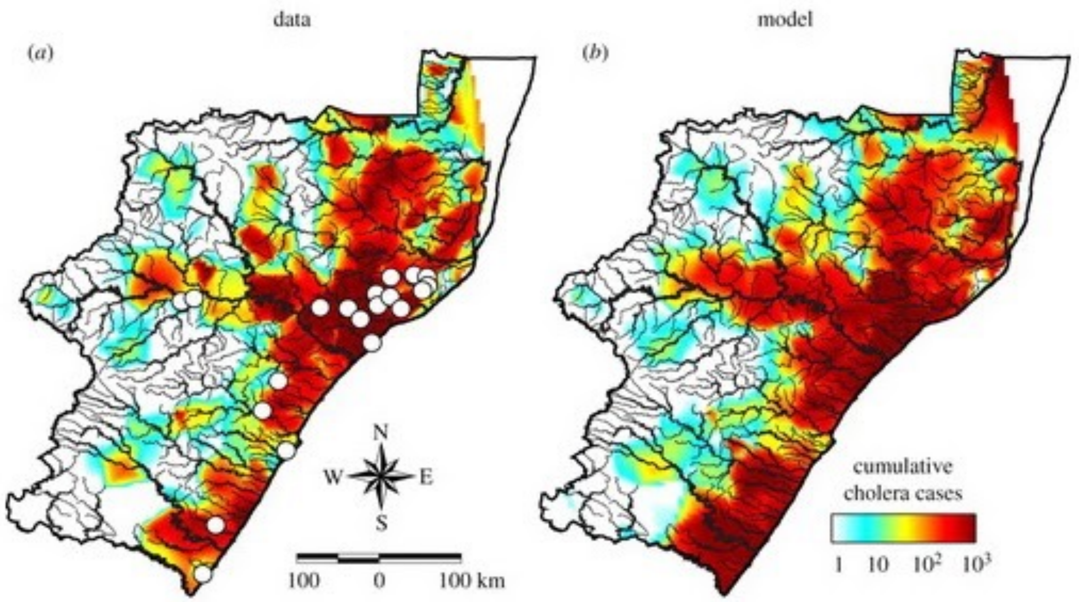
Università degli Studi di Trieste

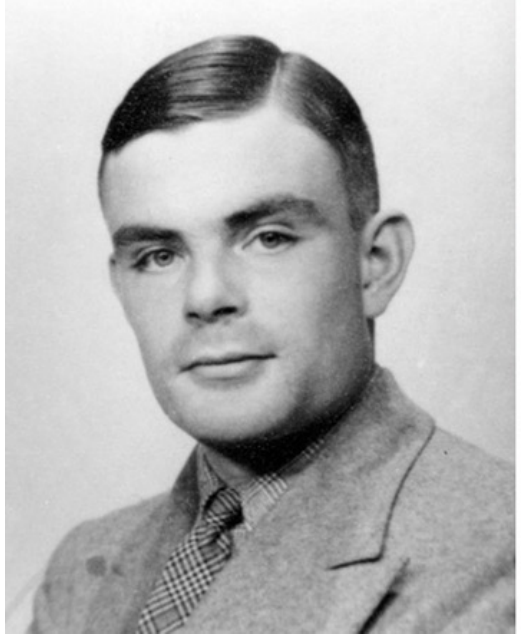
II Semestre 2023

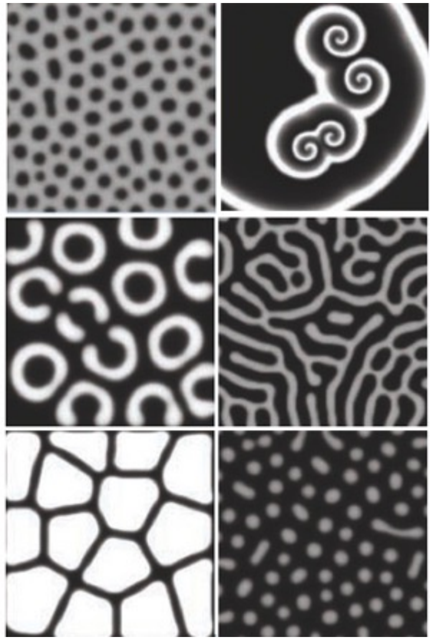
Lecture 16: Spatio-Temporal Reach and Escape Logic


WSN Monitoring System
ED01
ED02
ED03
ED04
ED05
ED06
ED07
ED08
RT01
RT02
CR01
Coordinator
Router
End Device
2.4GHz Wireless Link


MELBOURNE
$5 helmets are closer than you think
MELBOURNE
RACV

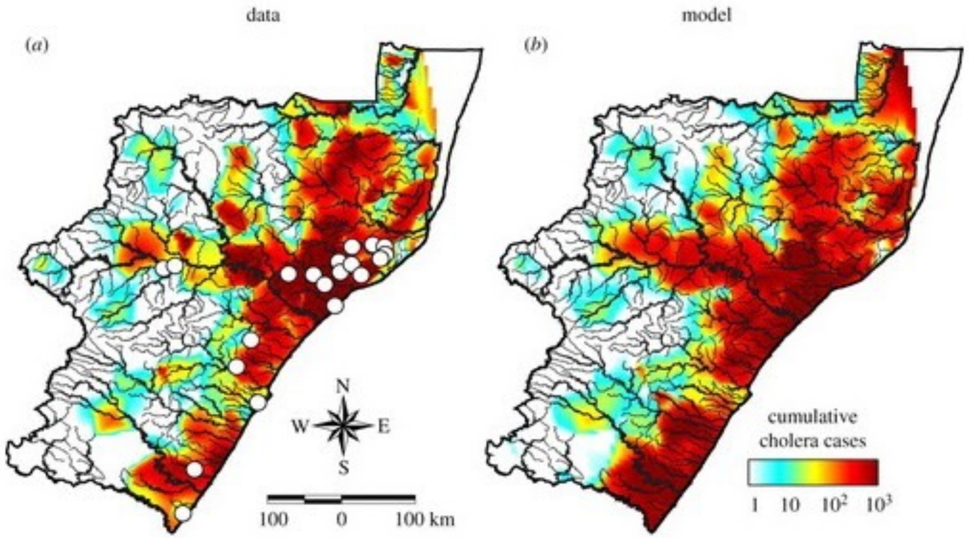

data
model
(a)
(b)
N
W
E
S
100
0
100 km
cumulative cholera cases
1
10
10^2
10^3

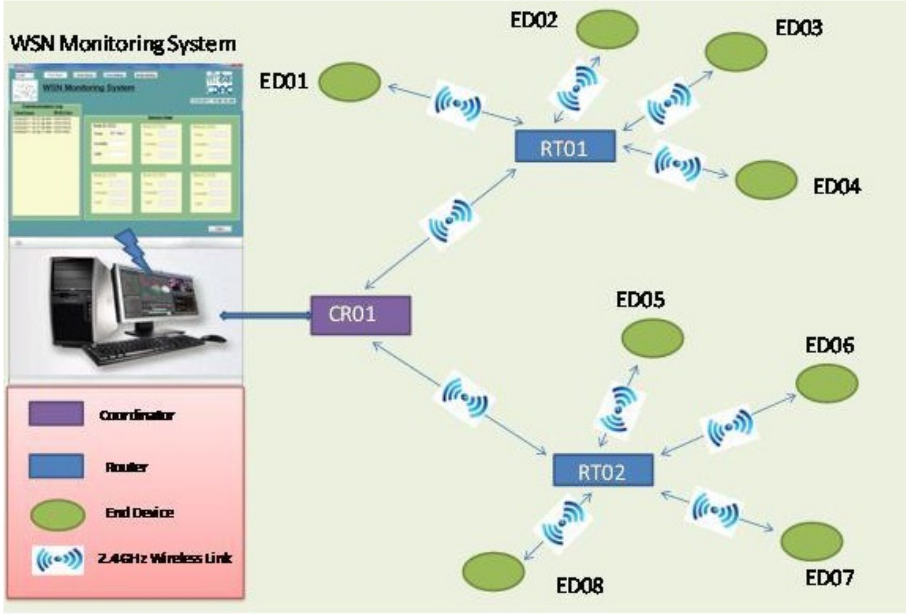




Availability: I can always find a station with at least one bike in a radius of 500 meters

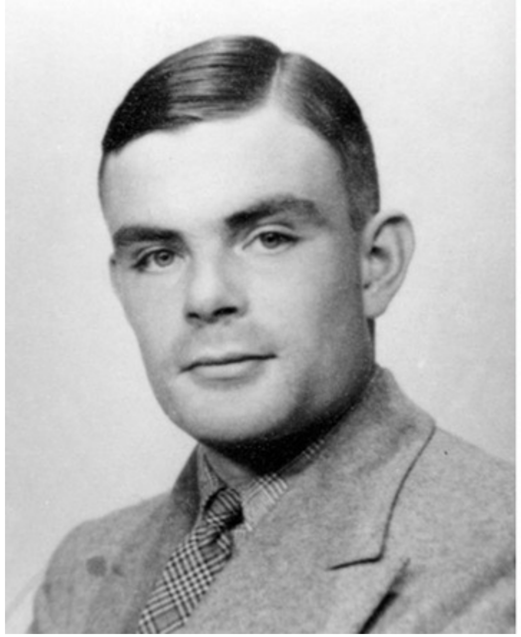
Spread: after 10 time units, there exists a location l′ at a certain distance from location l where the number of infected individuals is more than 50

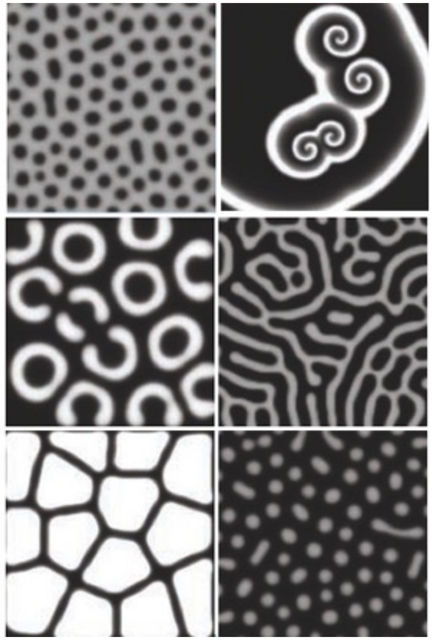




Reliability: we can always find a path of sensors such that all sensors have a battery level greater than 0.5

Spots: regions with low density of protein A are always surrounded by regions with high level of protein B





How to specify such spatio-temporal behaviours in a formal and human-understandable language ?

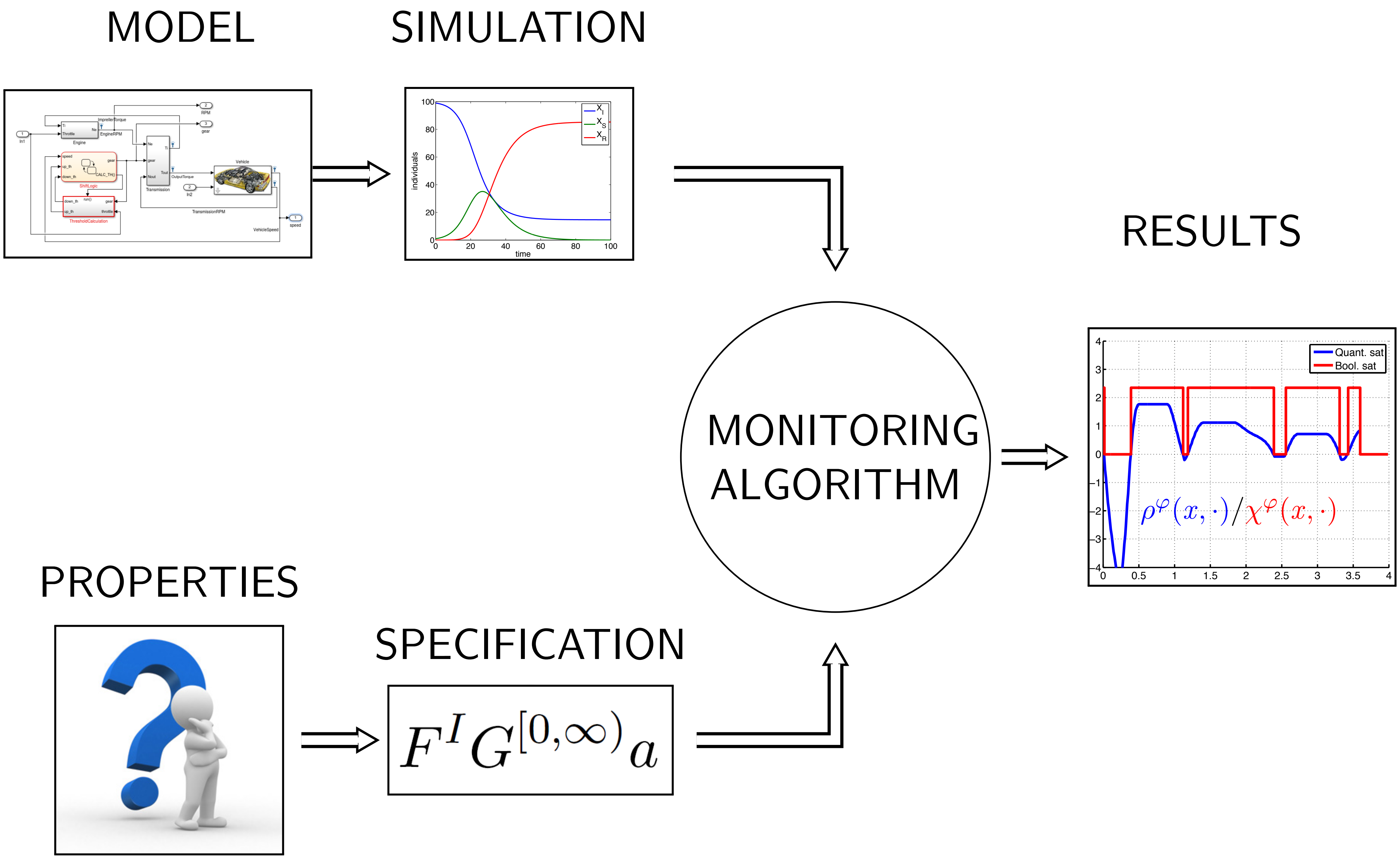
How to monitor their onset efficiently?

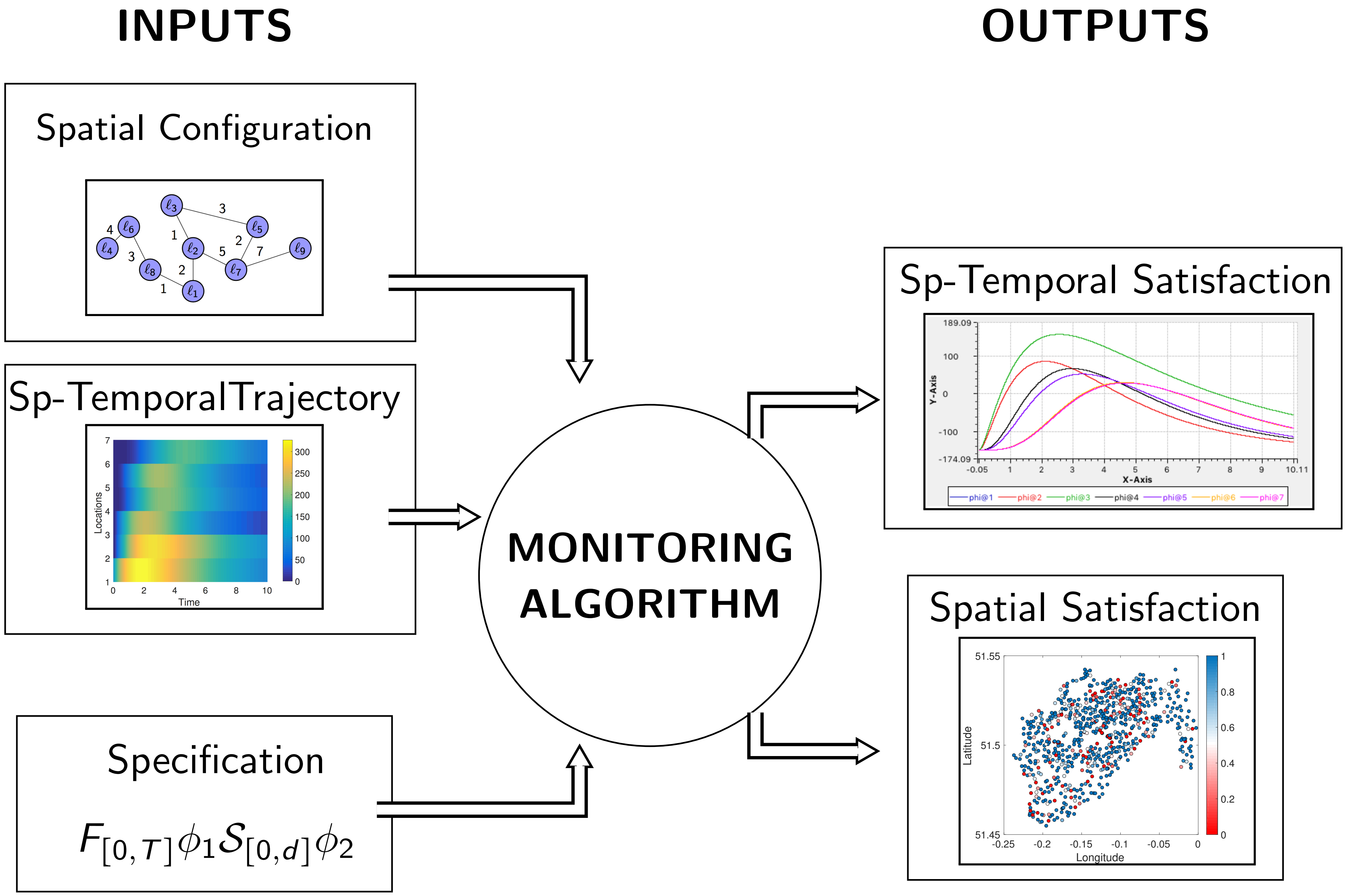
Part 1 :

- Space Model and traces
- Spatio- Temporal Reach and Escape Logic (STREL)

Part 2:

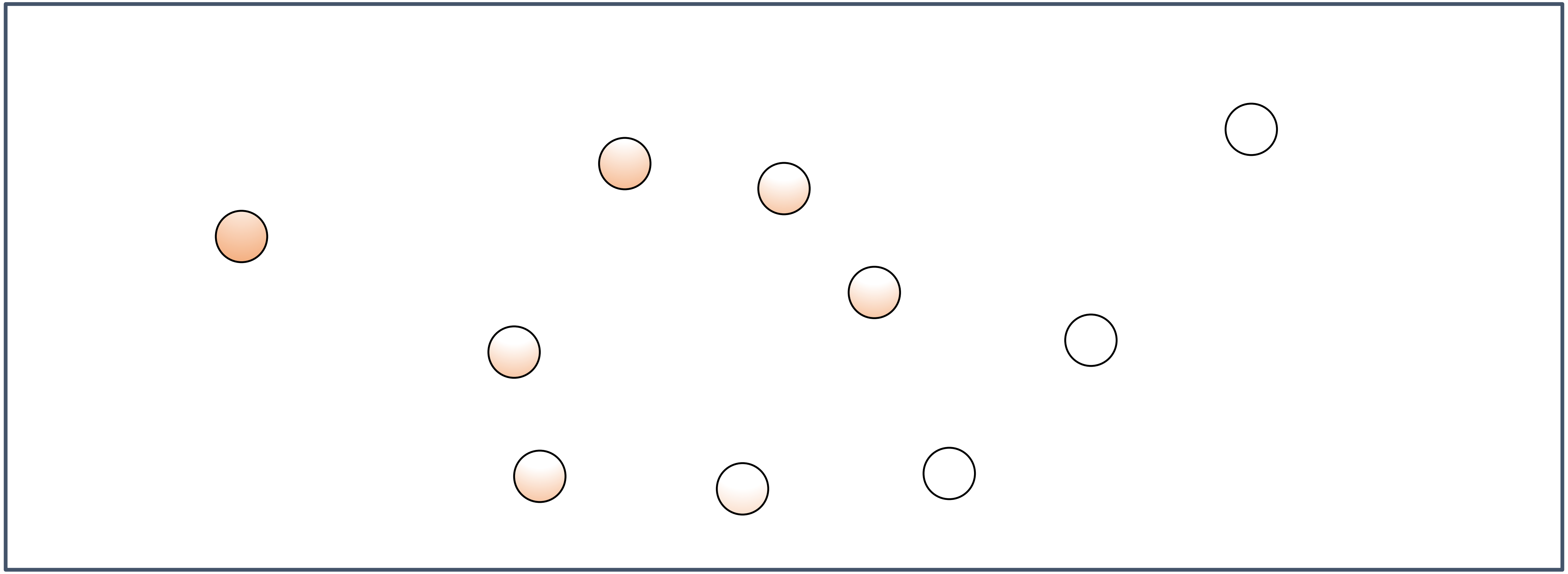
- Monitoring
- Applicability to different scenarios


MODEL
SIMULATION
individuals
time
X_I
X_S
X_R
RESULTS
MONITORING
ALGORITHM
Quant. sat
Bool. sat
$\rho^{\varphi}(x,\cdot)/\chi^{\varphi}(x,\cdot)$
PROPERTIES
SPECIFICATION
$F^I G^{[0,\infty)} a$



INPUTS
OUTPUTS
Spatial Configuration
Sp-TemporalTrajectory
Specification
$F_{[0,T]}\phi_1 \mathcal{S}_{[0,d]}\phi_2$
MONITORING ALGORITHM
Sp-Temporal Satisfaction
Spatial Satisfaction

Running Example: Wireless Sensor Network



Space Model, Signal and Traces

Spatial Configuration

We consider a discrete space described as a weighted (direct) graph

Reasons:

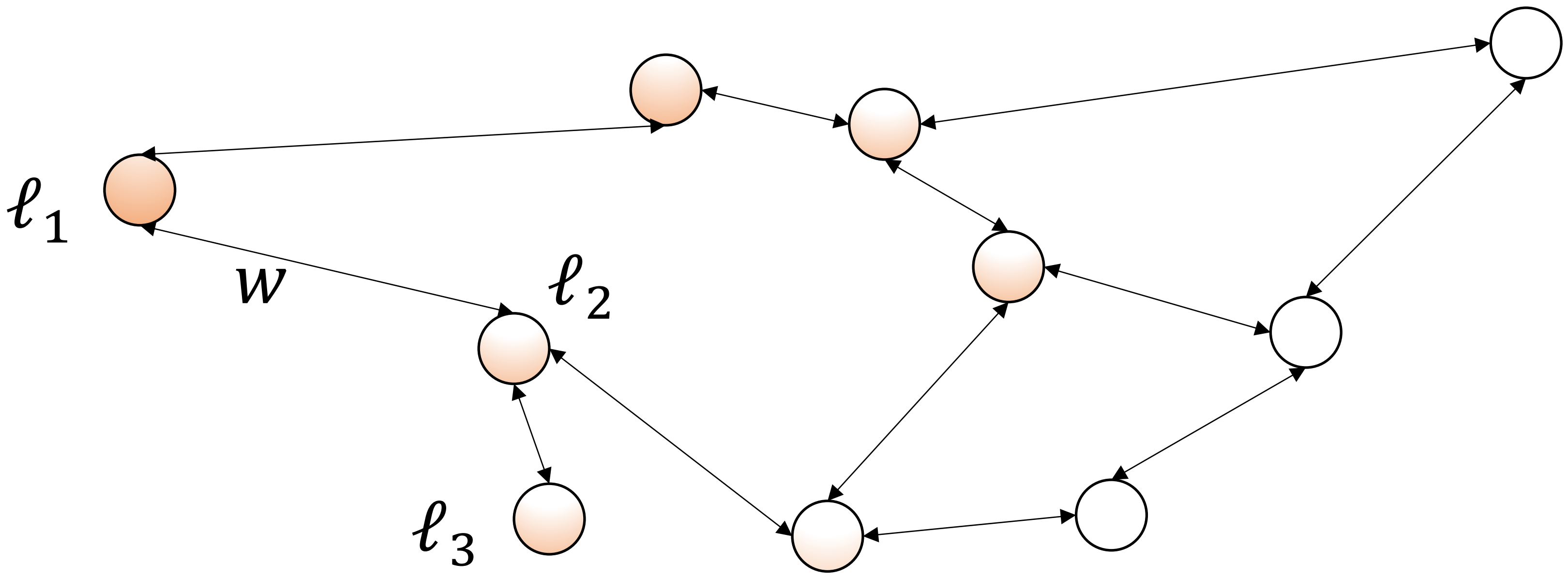
- many applications, like bike sharing systems, smart grid and sensor networks are naturally framed in a discrete spatial structure

- in many circumstances continuous space is abstracted as a grid or as a mesh, e.g. numerical integration of PDEs

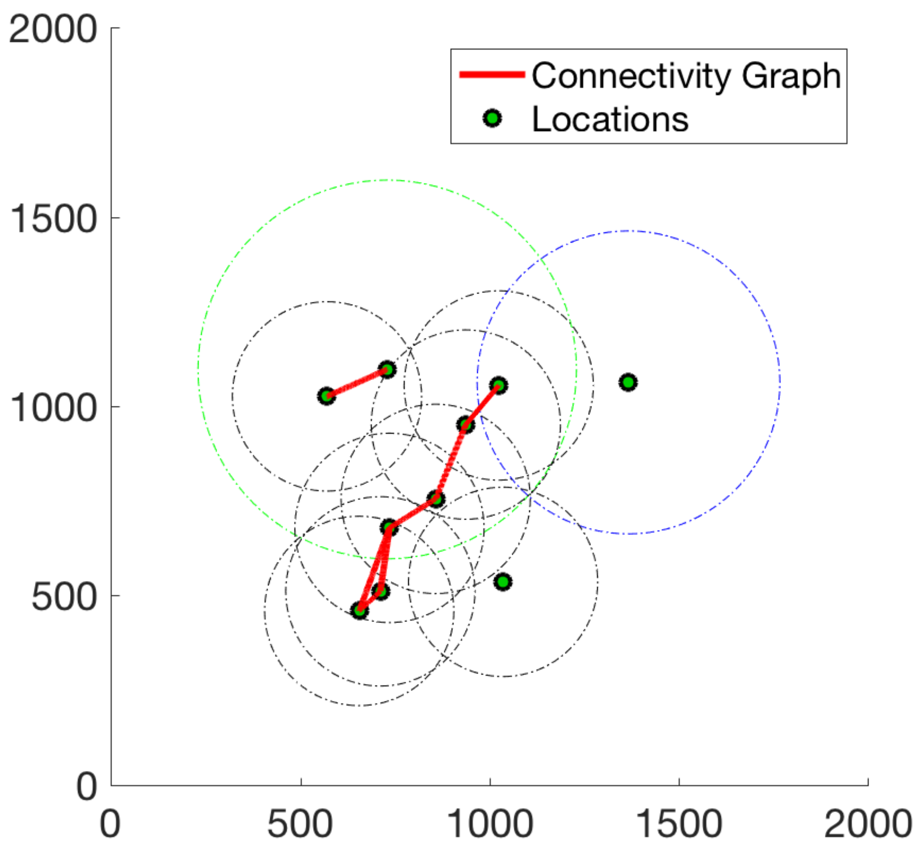
Space Model $S = \langle L, W \rangle$

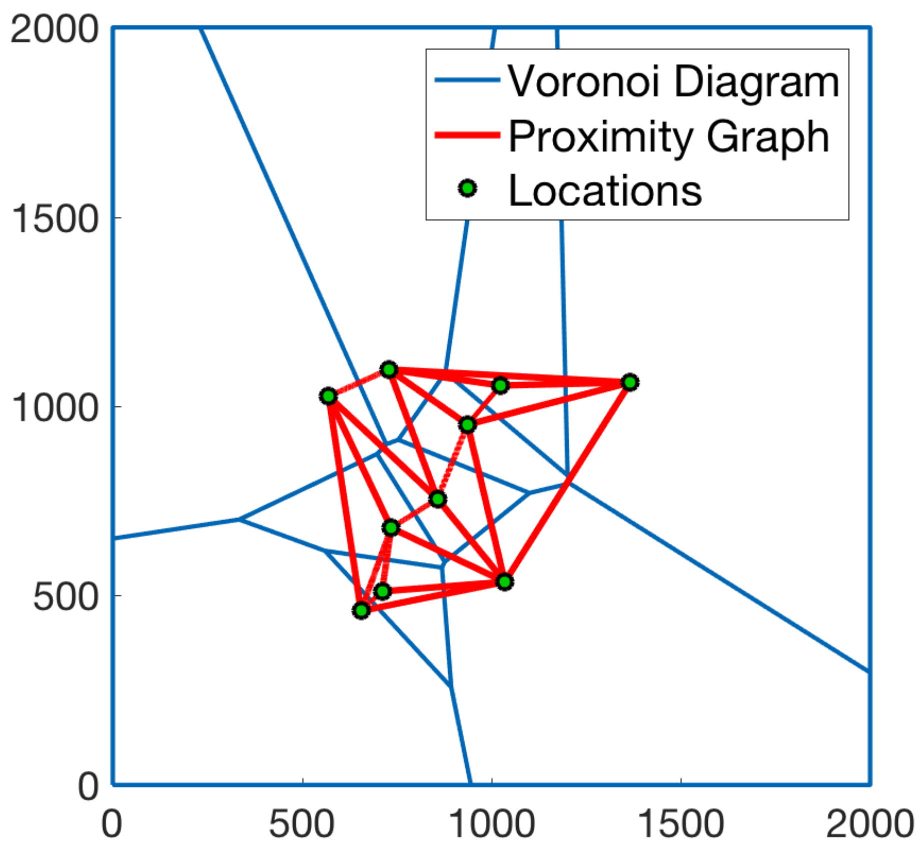
– L is a set of nodes that we call locations;

– $W \subseteq L \times \mathbb{R} \times L$ is a proximity function associating a label $w \in \mathbb{R}$ to distinct pair $\ell_1, \ell_2 \in L$. If $(\ell_1, w, \ell_2) \in W$, it means that there is an edge from ℓ_1 to ℓ_2 with weight $w \in \mathbb{R}$



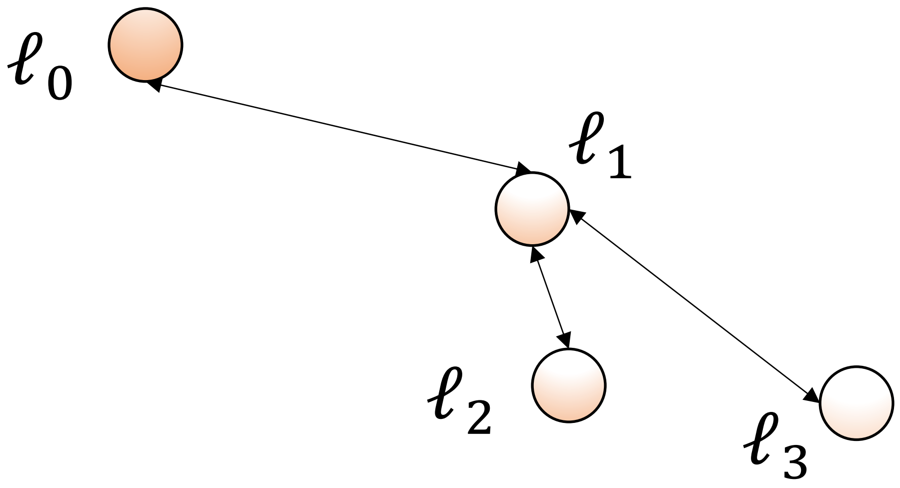
Example


Connectivity Graph
Locations
2000
1500
1000
500
0
0
500
1000
1500
2000


Voronoi Diagram
Proximity Graph
Locations
2000
1500
1000
500
0
0
500
1000
1500
2000

Route $\tau = \ell_0 \ell_1 \ell_2 \ldots$

It is a infinite sequence s.t. $\forall i \geq 0\ \exists\ w\ s.t. (\ell_i, w, \ell_{i+1}) \in W$



$\ell_0 \ell_1 \ell_2 \ell_1 \ldots$ is a route

$\ell_0 \ell_1 \ell_2 \ell_3 \ldots$ is a not route

$\tau[i]$ to denote the $i - th$ node τ

$\tau(\ell)$ to denote the first occurrence of $\ell \in \tau$

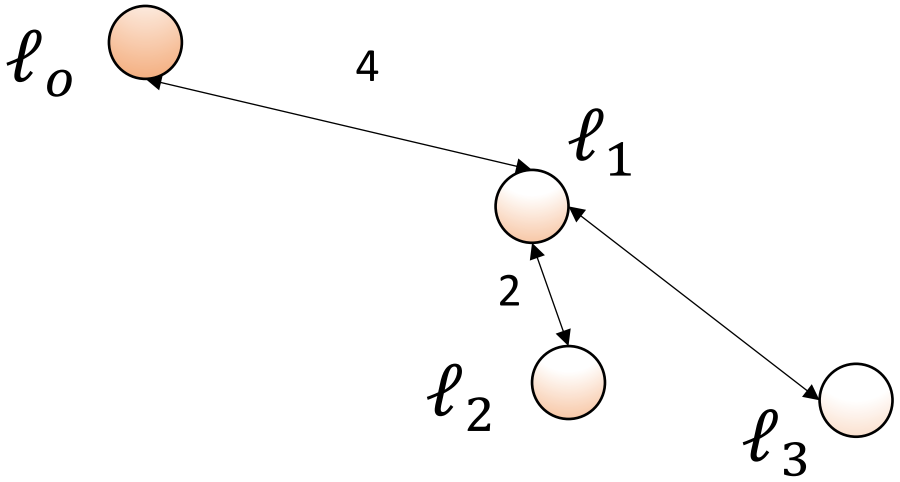
Route Distance $d_\tau^f[i]$

The distance $d_\tau^f[i]$ up to index i is:

$$d_\tau^f[i] = \begin{cases} 0 & i = 0 \\ f(d_{\tau[1..]}^f[i-1], w) & (i > 0)\ and\ \tau[0] \overset{w}{\mapsto} \tau[1] \end{cases}$$

$$d_\tau^f(\ell) = d_\tau^f[\tau(\ell)]$$

Route Distance $d_\tau^f[i]$



$$weight(x, y) = x + y$$

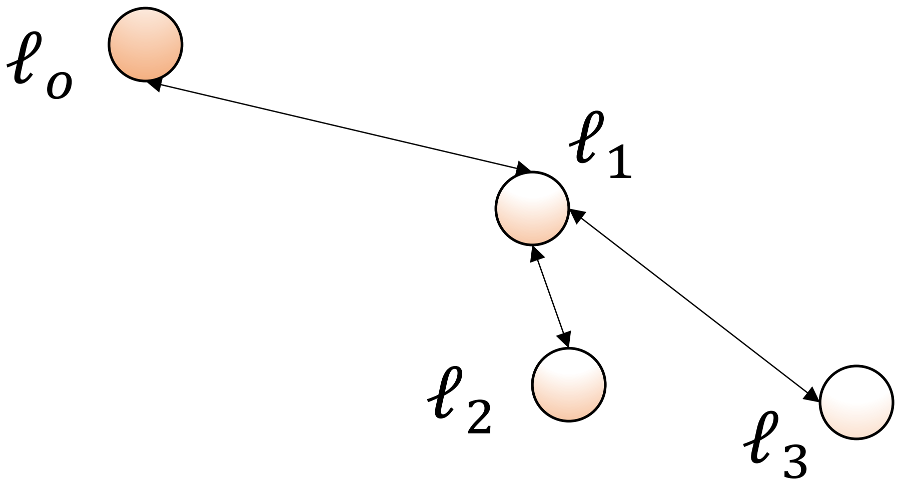
$$hops(x, y) = x + 1$$

$d_{\ell_0\ell_1\ell_2..}^{weight}[2] =$ weight($d_{\ell_1\ell_2..}^{weight}[1]$, 4) = $d_{\ell_1\ell_2}^{weight}[1]$ + 4 = …

= weight($d_{\ell_2..}^{weight}[0]$, 2) + 4 = 6

Location Distance $d_S^f[\ell_i, \ell_j]$

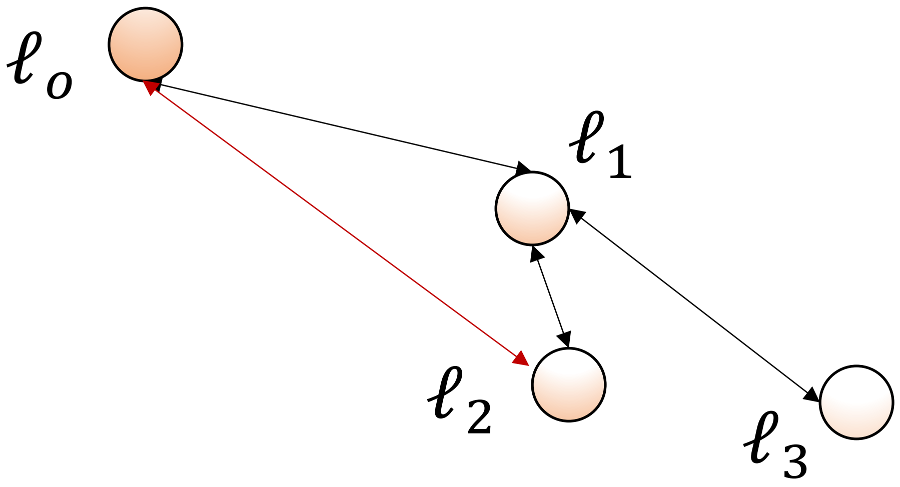
$$d_S^f[\ell_i, \ell_j] = \min\{d_\tau[\ell_j] \mid \tau \in Routes(S, \ell_i)\}$$



$d_S^{hops}[\ell_0, \ell_2]$ **= 2**

Location Distance

$$d_S^f[\ell_i, \ell_j] = \min\{d_\tau[\ell_j] | \tau \in Routes(S, \ell_i)\}$$



$d_S^{hops}[\ell_0, \ell_2]$ **= 1**

Signal and Trace

Spatio-Temporal Signals $\sigma: L \to \mathbb{T} \to D$

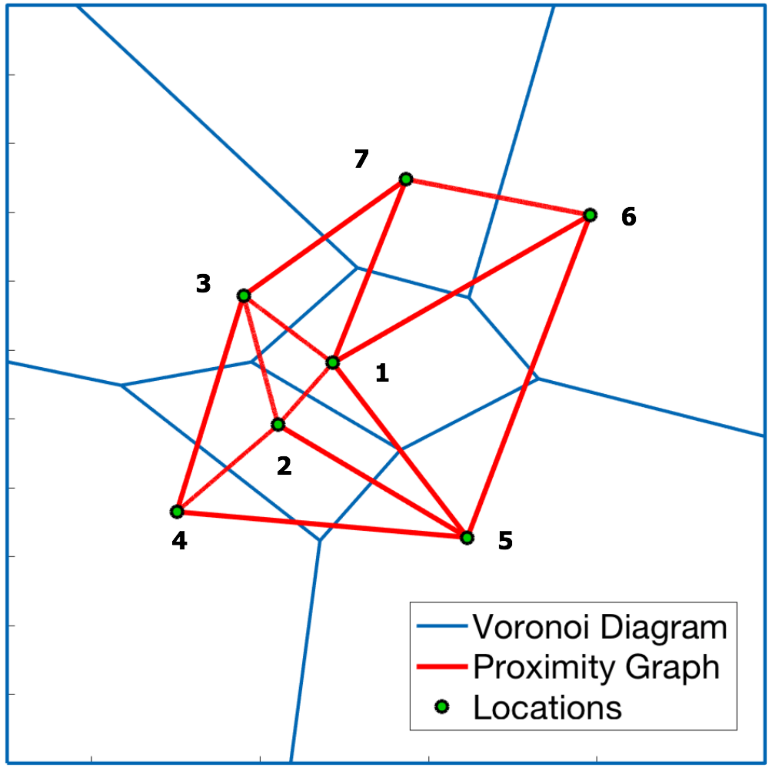
Spatio-Temporal Trace $\vec{\boldsymbol{x}}: L \to \mathbb{T} \to D^n$

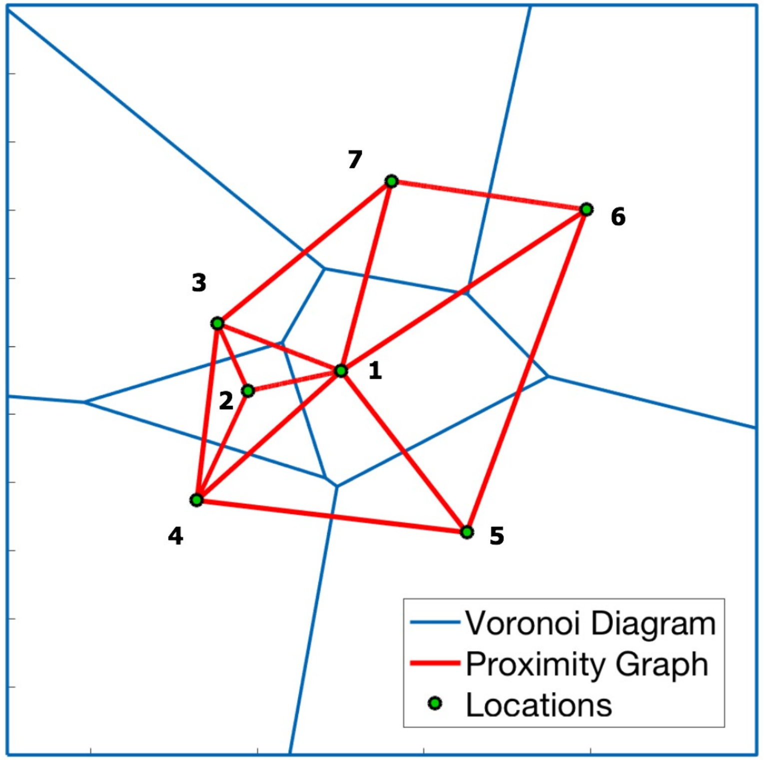
$$x(\ell) = (\nu_B, \nu_T)$$

$$x(\ell, t) = (\nu_B(t), \nu_T(t))$$

Dynamic Spatial Model

(t_i, S_i) for $i = 1, \ldots, n$ and $S(t) = S_i \forall t \in [t_i, t_{i+1})$





STREL

Spatio- Temporal Reach and Escape Logic (STREL)

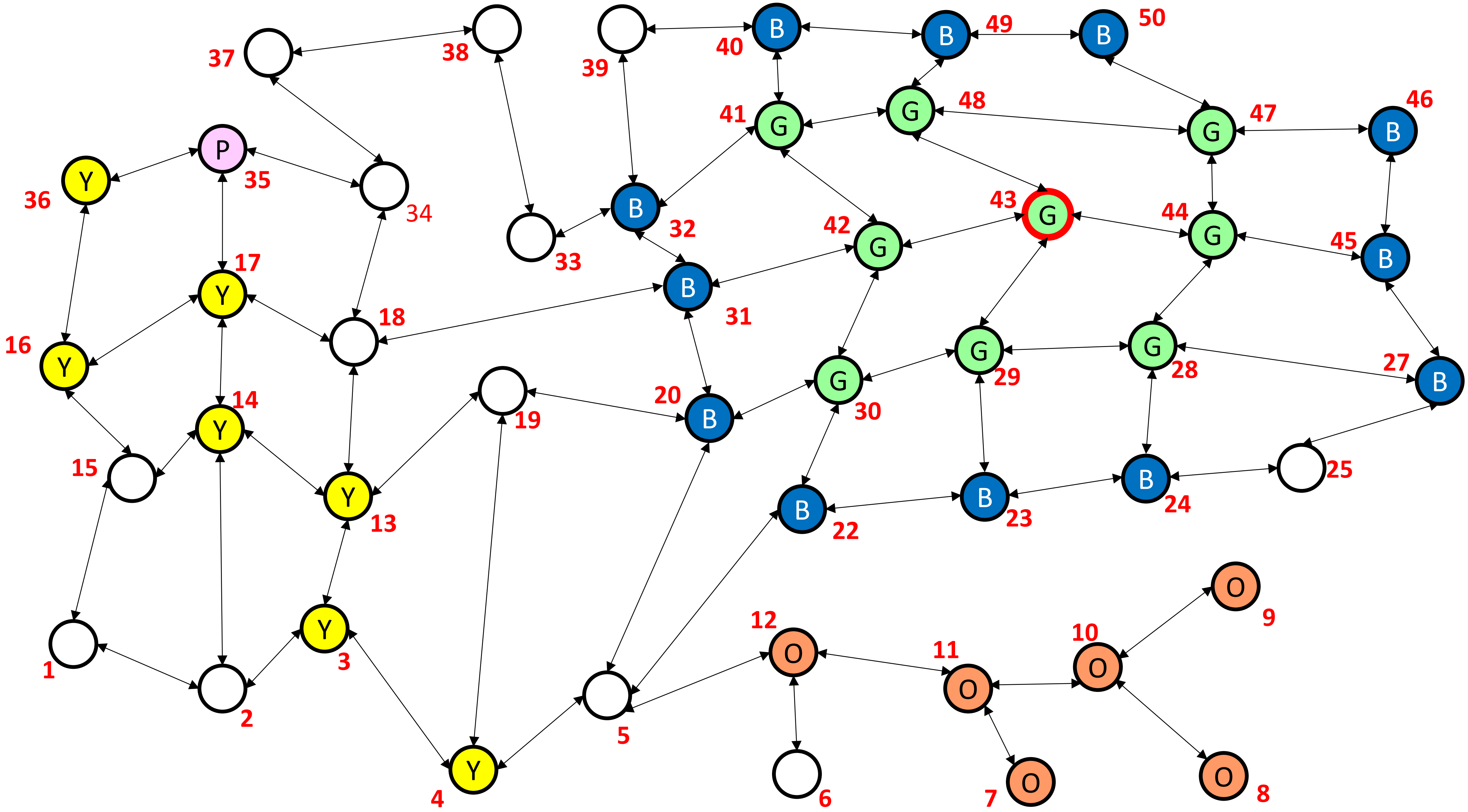
It is an extension of the Signal Temporal Logic with a number of spatial modal operators

STREL Syntax

$$\varphi := true \mid \mu \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \, \mathrm{U}_I \, \varphi_2 \mid \varphi_1 \, \mathrm{S}_I \, \varphi_2 \mid \varphi_1 \, \mathcal{R}_d^f \, \varphi_2 \mid \mathcal{E}_d^f \, \varphi$$

In addition, we can derive:

- The disjunction operator: $\vee$
- the temporal operators: $F_I, G_I, O_I, \mathrm{H_I}$
- the spatial operators: somewhere, everywhere and surround

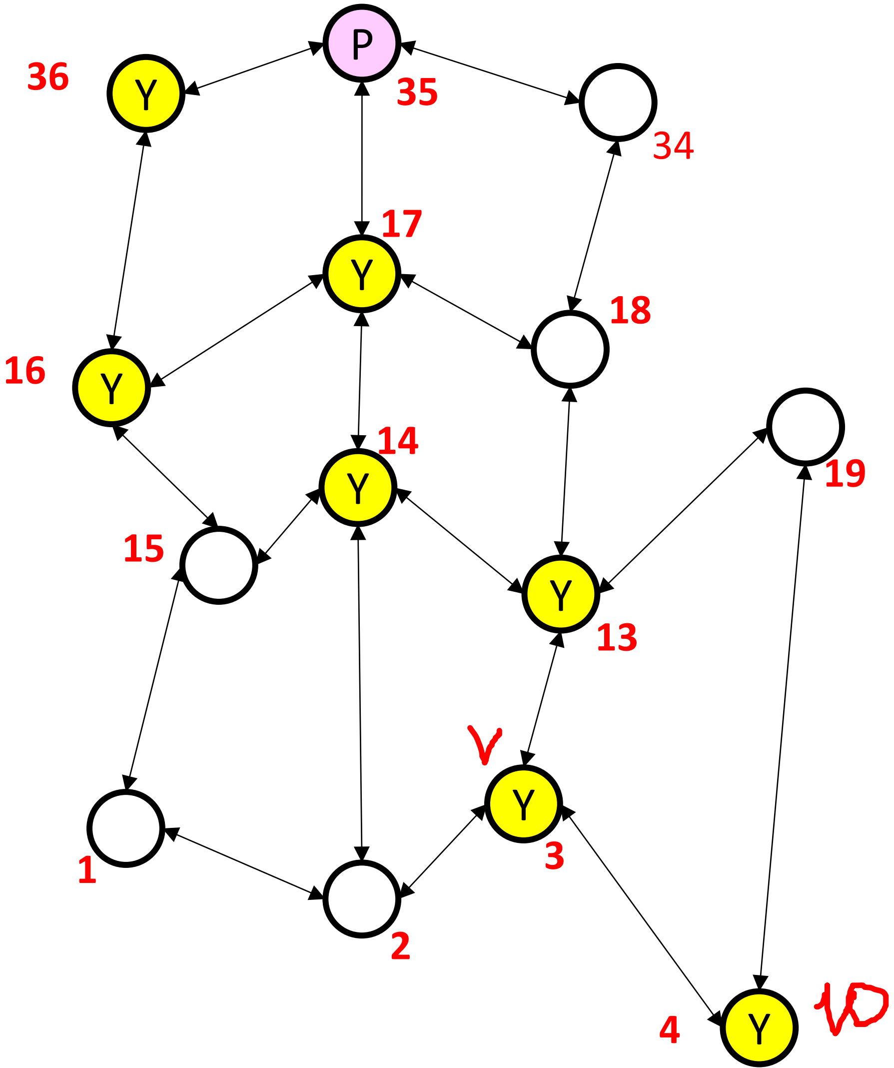


37
38
39
40
B
49
B
50
B
41
G
48
G
47
G
46
B
P
35
Y
36
34
43
G
44
G
45
B
32
B
33
42
G
17
Y
18
31
B
16
Y
29
G
28
G
27
B
14
Y
19
20
B
30
G
15
25
13
Y
B
24
B
23
B
22
O
9
12
O
11
O
10
O
Y
3
1
2
5
4
Y
6
7
O
8
O

Reach: $\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \varphi_2$

$(S, \vec{x}, \ell, t)$ satisfies $\varphi_1 \mathcal{R}^{f}_{[d_1,d_2]} \varphi_2$ iff it satisfies φ_2 in a location ℓ' reachable from ℓ through a route τ, with a length $d_\tau^f(\ell') \in [d_1, d_2]$ and such that $\tau[0] = \ell$ and all its elements with index less than $\tau(\ell')$ satisfy φ_1

Reach $yellow \mathcal{R}^{hops}_{[1,4]} pink$



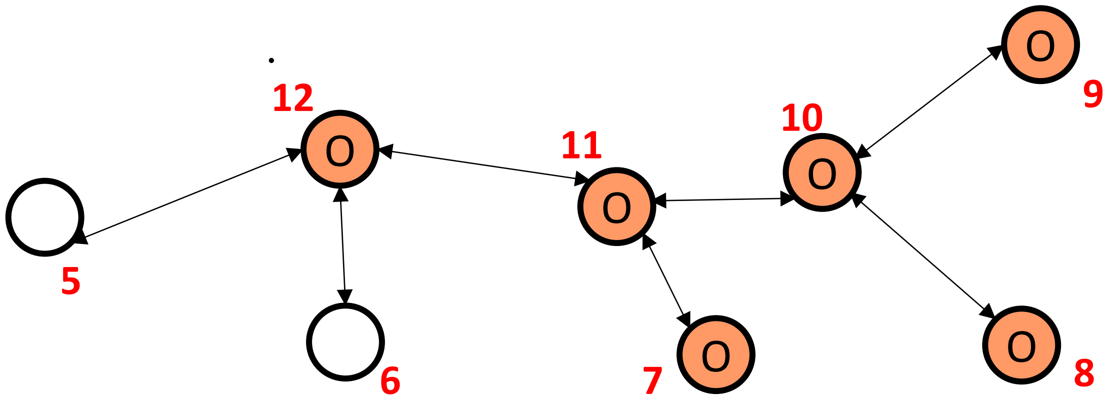
$$\tau = \ell_3 \ell_{13} \ell_{14} \ell_{17} \ell_{35}$$

$$d_{\tau}^{hops}(\ell_{35}) = 4, \text{ where } \tau[0] = \ell_3$$

Escape: $\mathcal{E}^{f}_{[d_1,d_2]}\varphi$

$(S, \vec{x}, \ell, t)$ satisfies $\mathcal{E}^{f}_{[d_1,d_2]}\varphi$ if and only there exists a route τ and a location $\ell' \in \tau$ such that $\tau[0] = \ell$, $d_S^f[\ell, \ell'] \in [d_1, d_2]$ and all elements $\tau[0]$, … $\tau[k]$ (with $\tau(l') = k$) satisfy φ

Escape: $\mathcal{E}^{hops}_{[3,\infty]} orange$

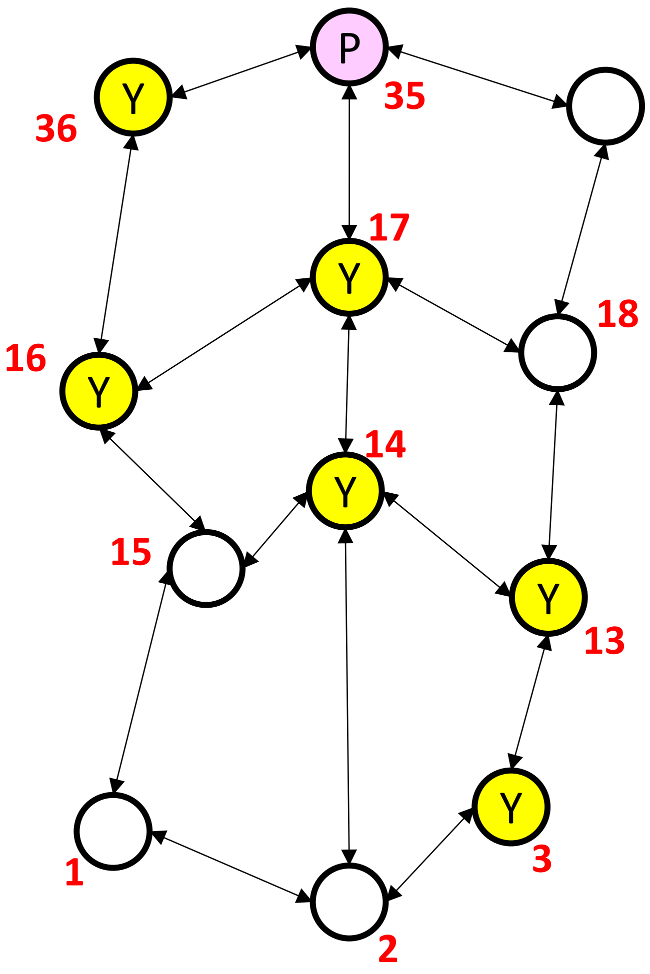


$$\tau = \ell_9 \ell_{10} \ell_{11} \ell_{12}$$

$$\tau[0] = \ell_9,\ \tau[3] = \ell_{12}$$

$$d_S^{hops}[\ell_9, \ell_{12}] = 3$$

Somewhere: $\Diamond^{f}_{[d_1,d_2]}\varphi$



$(S, \vec{x}, \ell, t)$ satisfies $\Diamond^{f}_{[d_1,d_2]}\varphi$ iff there exists a location ℓ' reachable from ℓ, and a s.t. $d^f_S[\ell, \ell'] \in [d_1, d_2]$, that satisfies φ

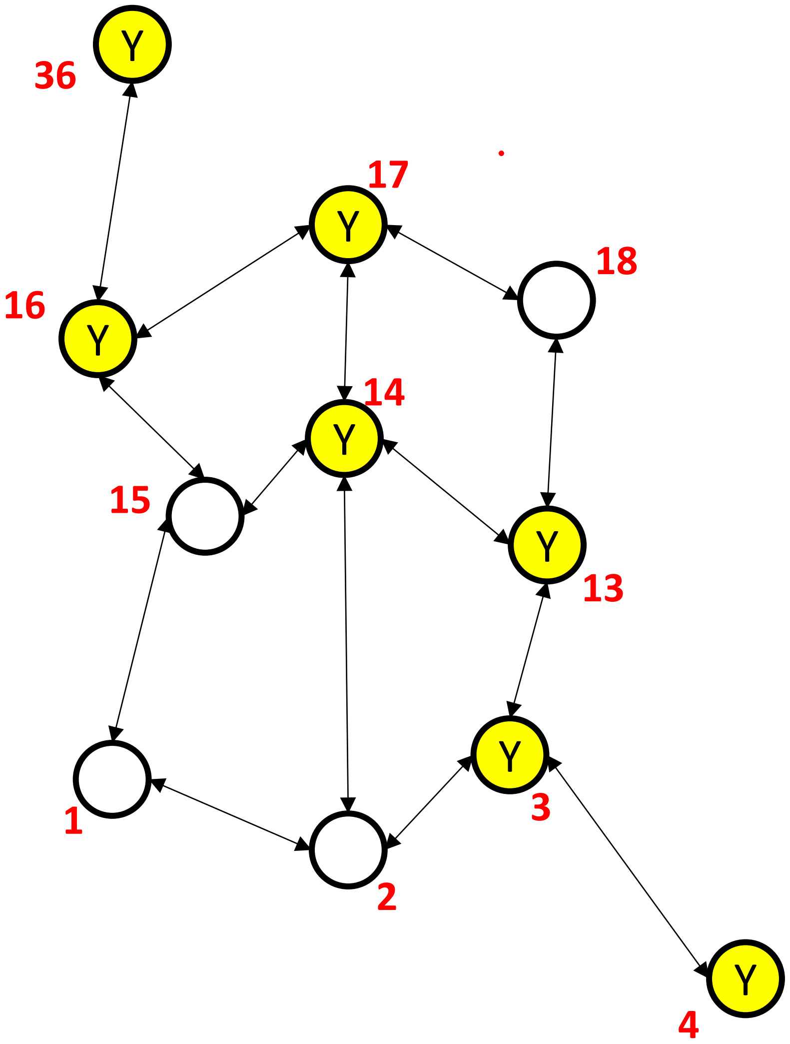
$$\Diamond^{hops}_{[3,5]} pink$$

$$\tau[0] = \ell_1,\ \tau[k] = \ell_{35}$$

$$\tau = \ell_1 \ldots \ell_{35}$$

$$d^{hops}_{\tau}(k) \in [3,5]$$

Everywhere: $\boxdot^{f}_{[d_1,d_2]}\varphi$



$(S, \vec{x}, \ell, t)$ satisfies $\boxdot^{f}_{[d_1,d_2]}\varphi$ iff all the locations ℓ' reachable from ℓ via a path s.t. $d_S^f[\ell,\ell'] \in [d_1, d_2]$, satisfy φ

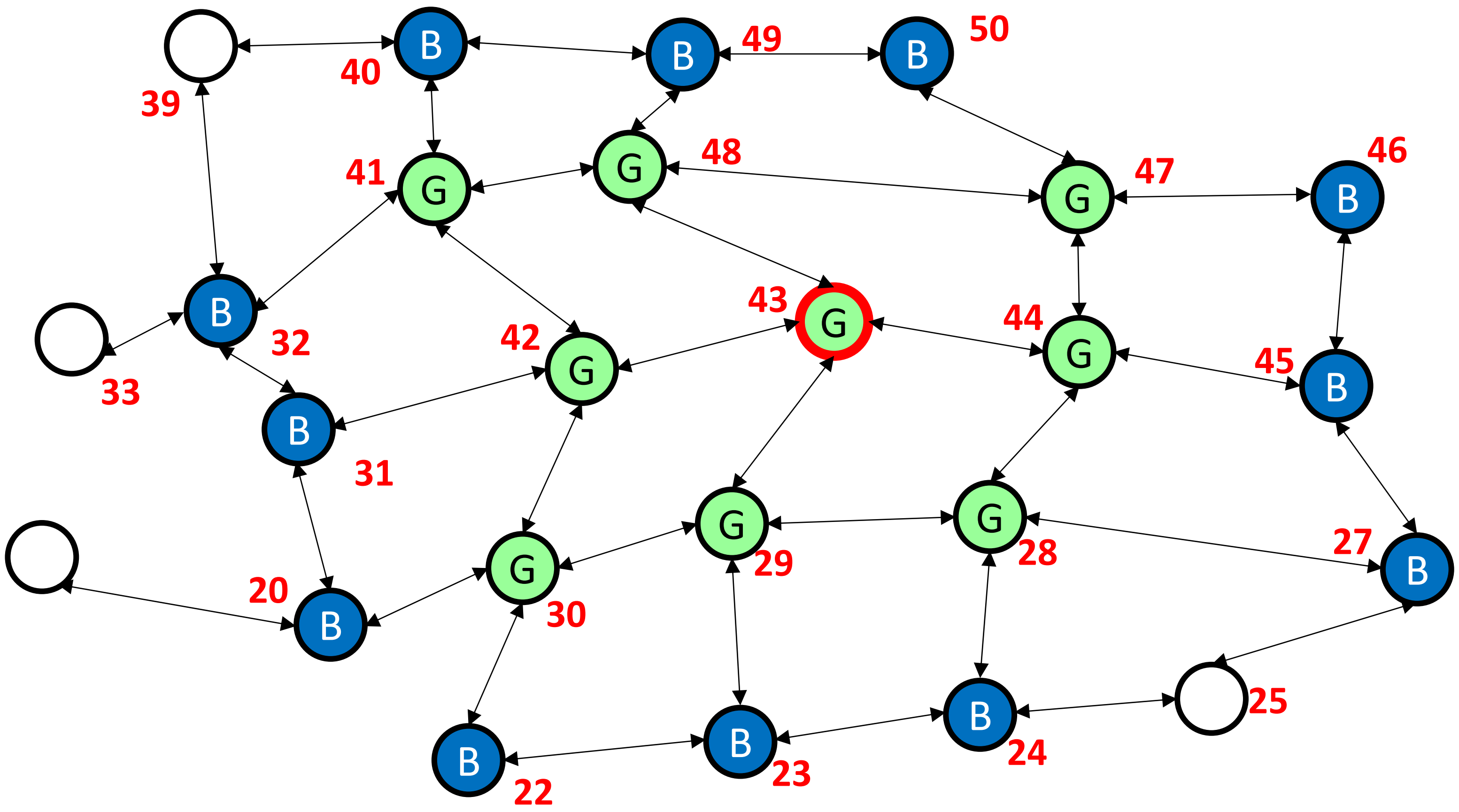
$$\boxdot^{hops}_{[2,3]} yellow$$

Surround: $\varphi_1 \circledcirc^{f}_{[d_1,d_2]} \varphi_2$

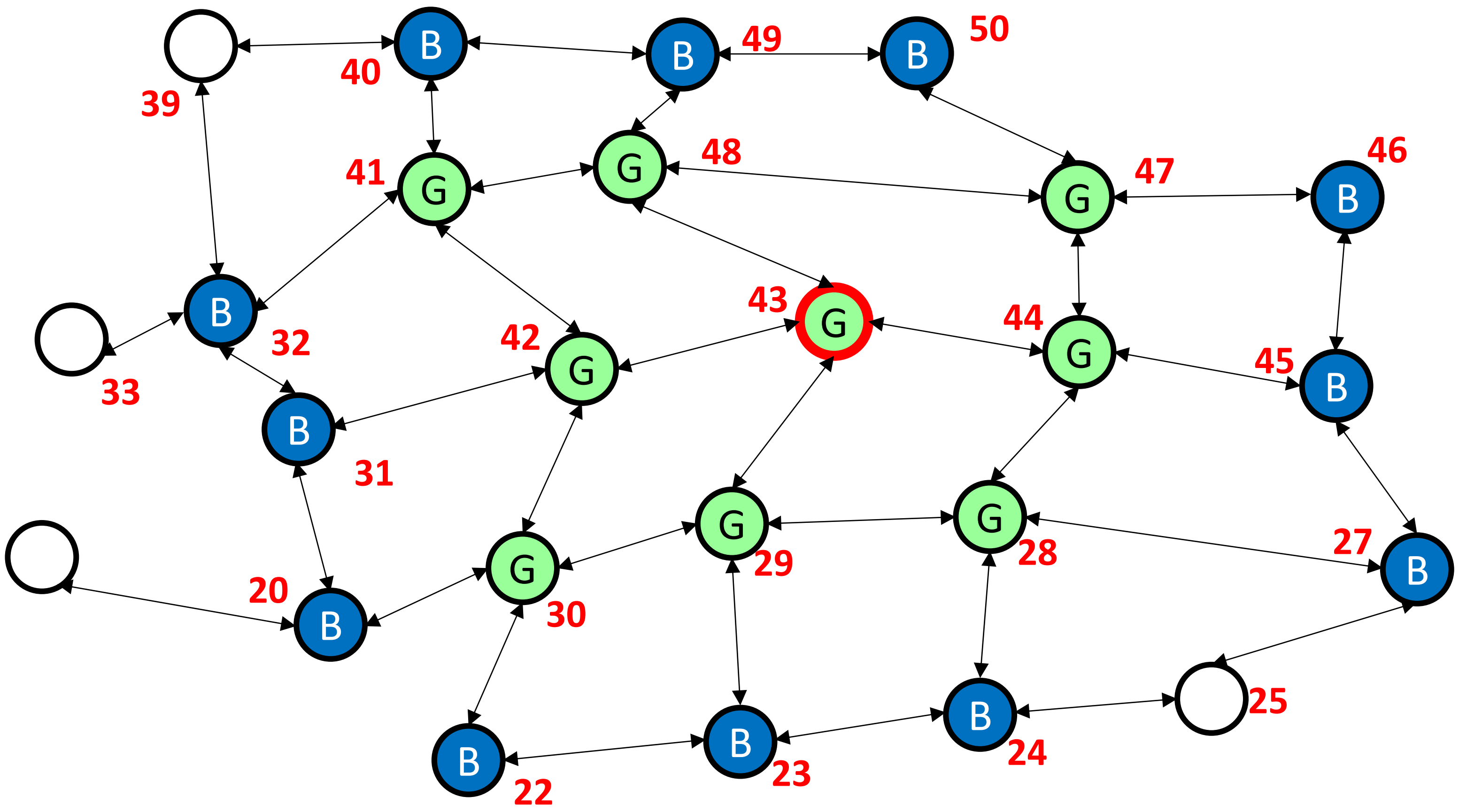
$(S, \vec{x}, \ell, t)$ iff there exists a φ_1-region that contains ℓ, all locations in that region satisfies φ_1 and are reachable from ℓ via a path with length less than d_2 .

All the locations that do not belong to the φ_1-region but are directly connected to a location of that region must satisfy φ_2 and be reached from ℓ via a path with length in the interval $[\mathrm{d}_1, d_2]$.

Surround: $green \circledcirc^{hops}_{[0,100]} blue$



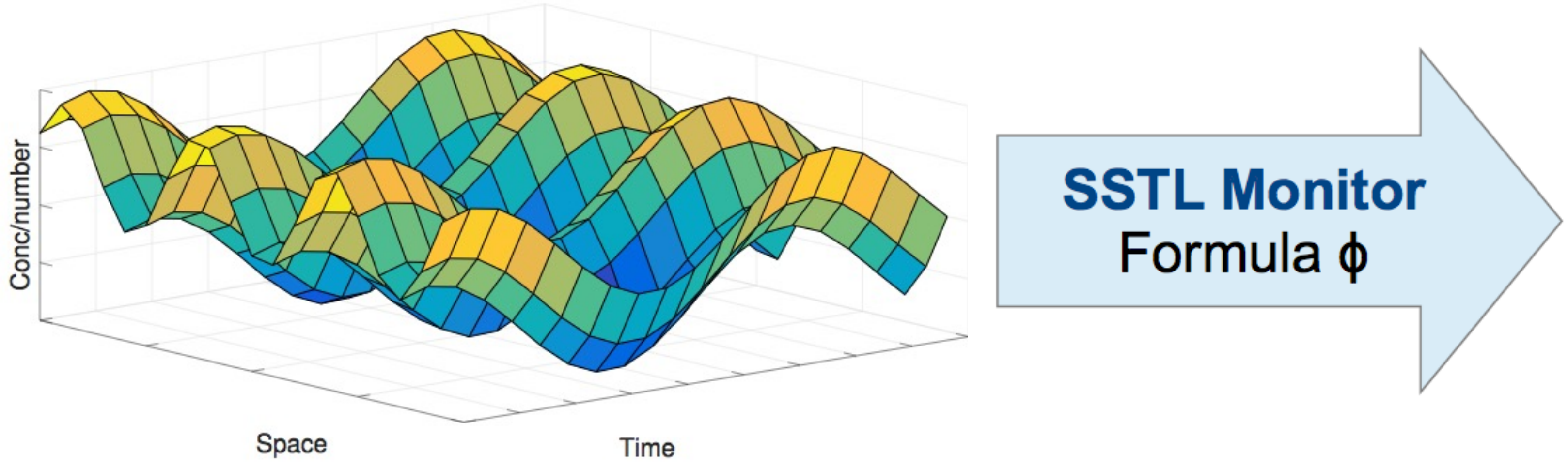
Surround: $green \circledcirc^{hops}_{[2,3]} blue$



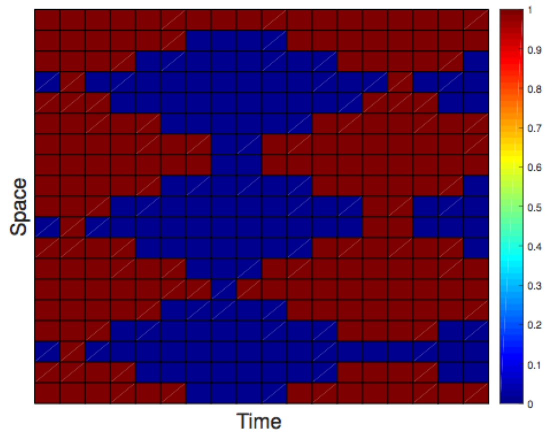
Offline Monitoring Algorithm

Spatial Boolean Signal

$s_\varphi : L \to [0, T] \to \{0, 1\}$ such that $s_\varphi(\ell, t) = 1 \Leftrightarrow (\mathcal{S}, \vec{x}, \ell, t) \models \varphi$



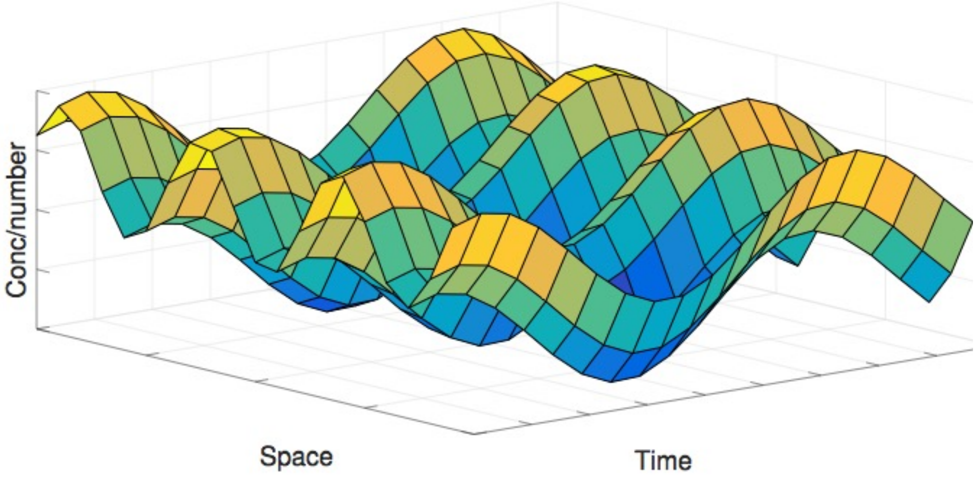
$s_\varphi(\ell, 0)$

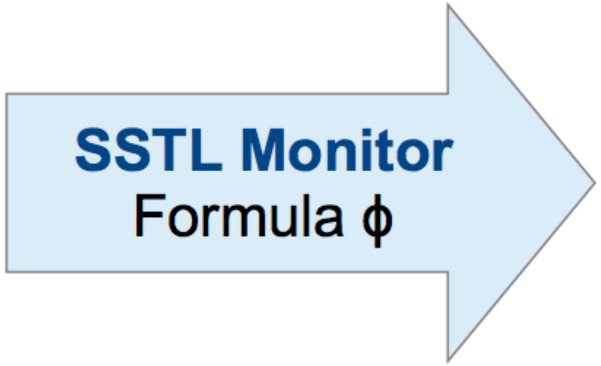


Offline Monitoring Algorithm

Spatial Quantitative Signal

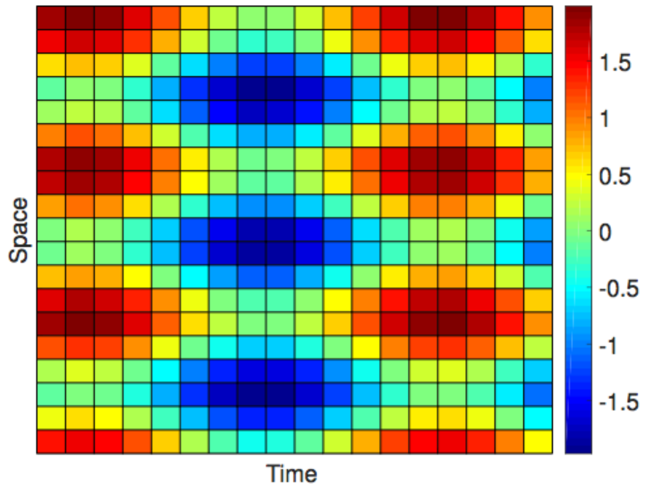
$\rho_\varphi : L \to [0, T] \to \mathbb{R} \cup \pm\infty$ such that $\rho_\varphi(\ell, t) = \rho(\mathcal{S}, \vec{x}, \ell, t)$

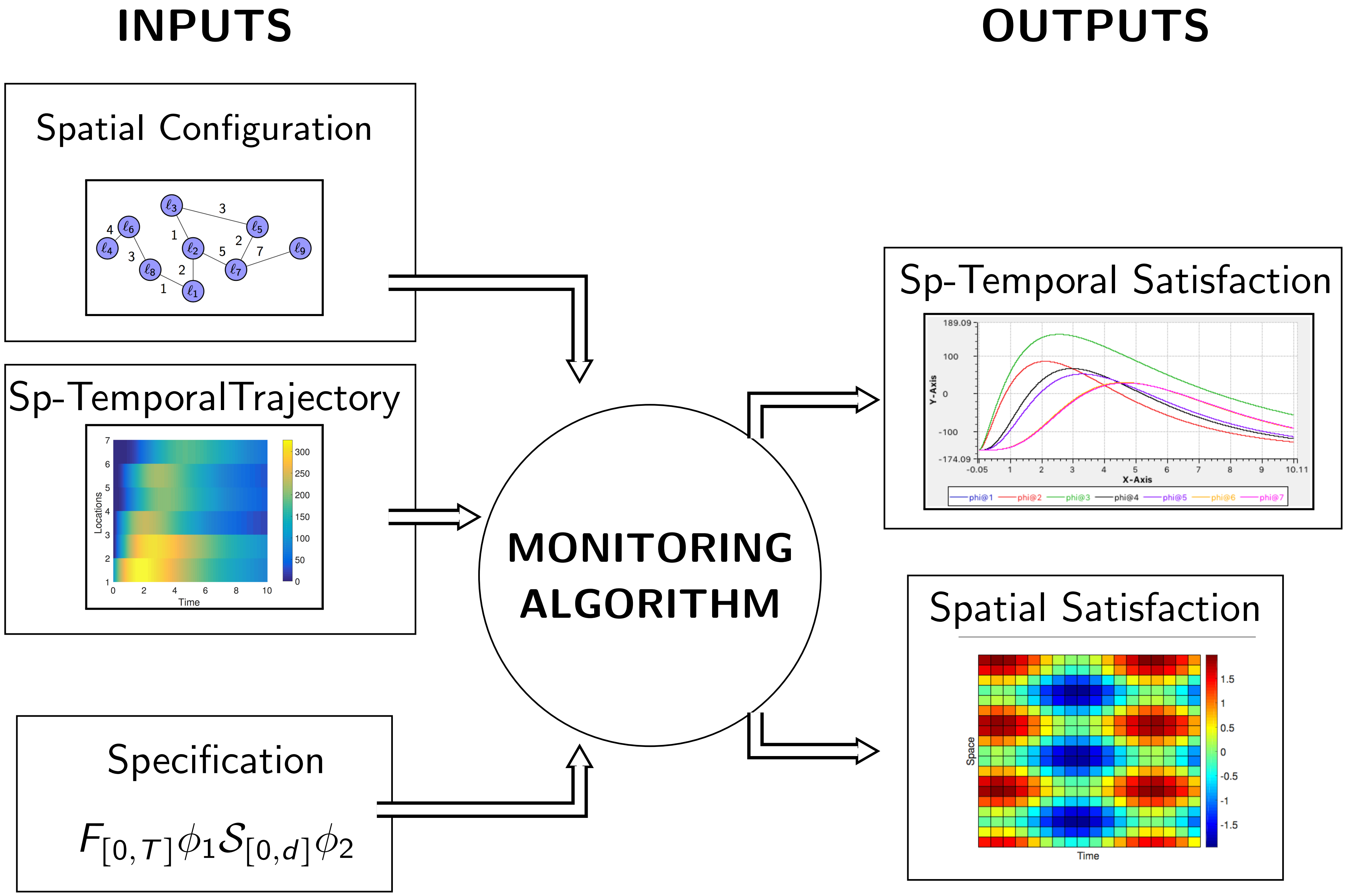




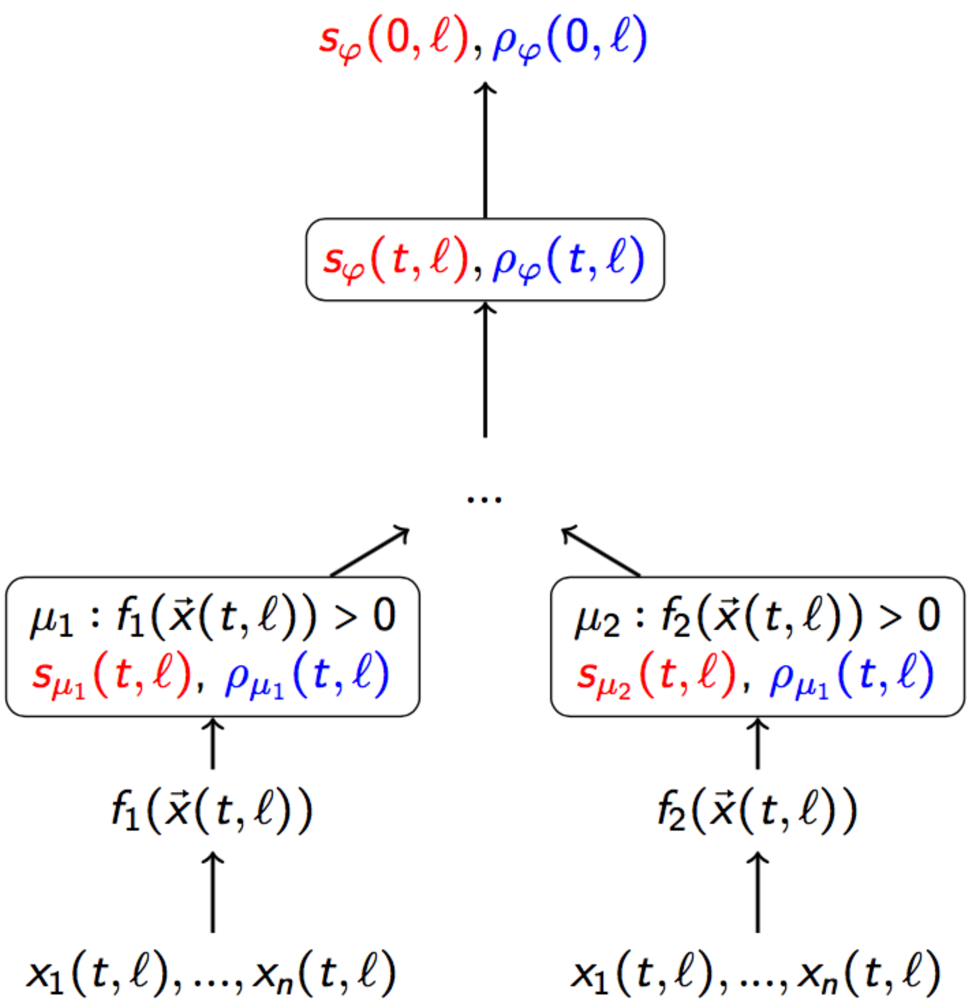
$\rho_\varphi(\ell, 0)$

Spatial Quantitative Satisfaction




INPUTS
OUTPUTS
Spatial Configuration
Sp-TemporalTrajectory
Locations
Time
Specification
$F_{[0,T]}\phi_1 \mathcal{S}_{[0,d]}\phi_2$
MONITORING
ALGORITHM
Sp-Temporal Satisfaction
X-Axis
Y-Axis
phi@1 phi@2 phi@3 phi@4 phi@5 phi@6 phi@7
Spatial Satisfaction
Space
Time

Offline Monitoring Algorithm



Spatial Boolean satisfaction
Spatial Quant. satisfaction

Spatial Boolean signals
Spatial Quant. signals

Secondary signals

Primary signals

Computational consideration

- Temporal operators: like in STL monitoring [1] is **linear** in the length of the signal times the number of locations in the spatial model.

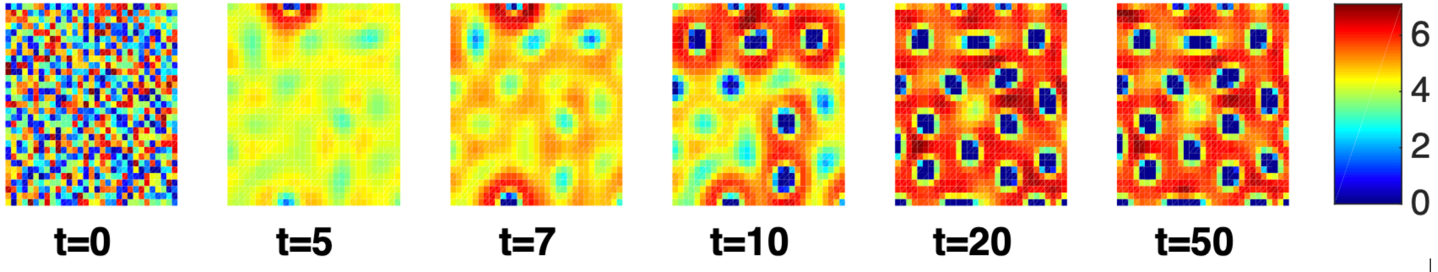
- Spatial properties are more expensive, they are based on a variations of the classical Floyd-Warshall algorithm.
  The number of operations to perform is **quadratic** for the reach operator and **cubic** for the escape

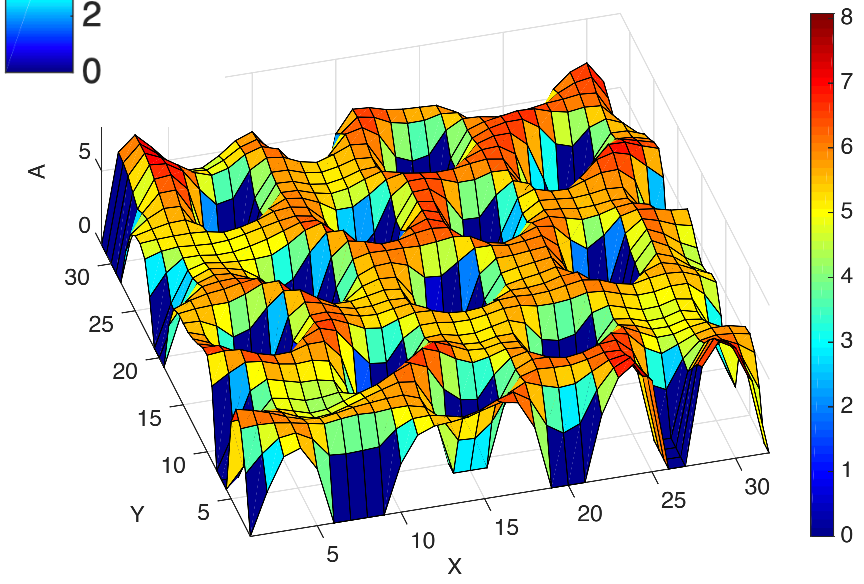
[1] O. Maler, T. Ferrére, and D. Nickovic. Efficient Robust Monitoring for STL. In Proc.CAV 2010

Static Space and Regular Grid

The formation of Patterns

The production of skin pigments that generate spots in animal furs:



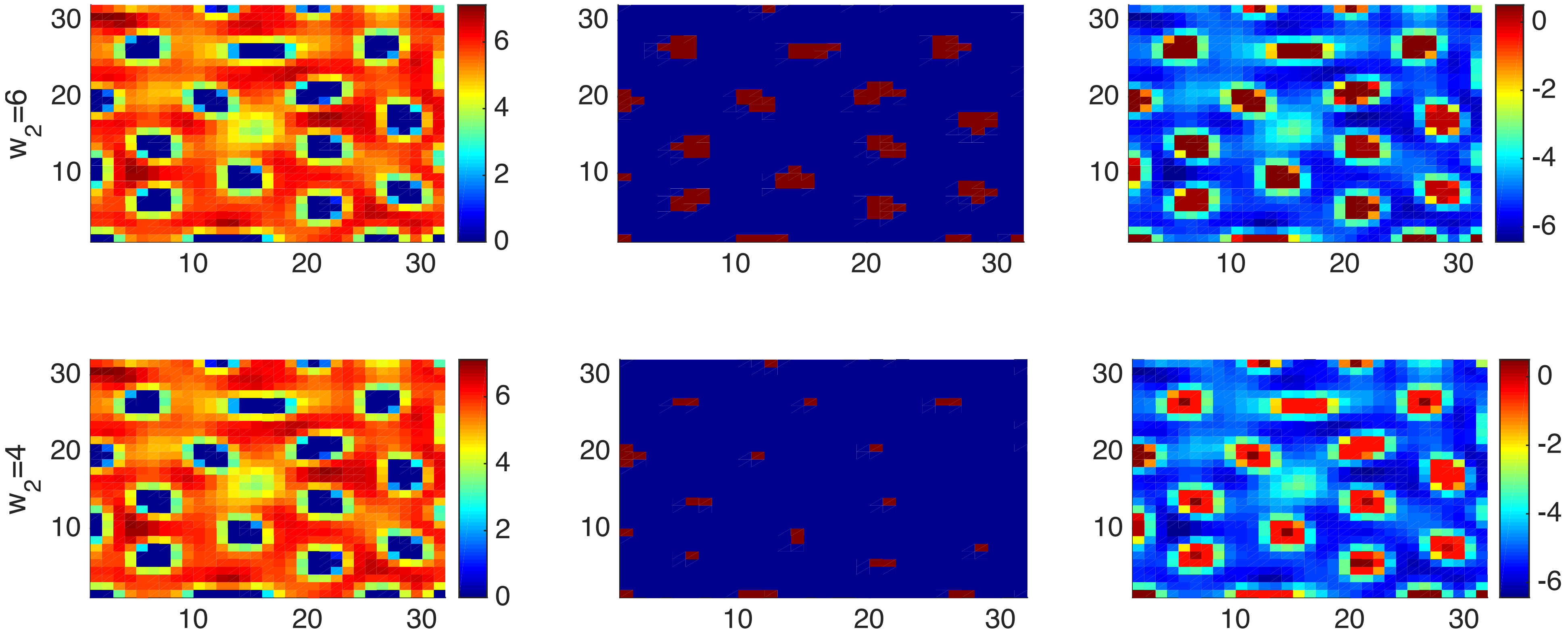


Space model: a K×K grid treated as a graph, cell$(i, j) \in L = \{1, \dots, K\} \times \{1, \dots, K\}$

Spatio-Temporal Trajectory: $x: L \rightarrow \mathbb{T} \rightarrow \mathbb{R}^2$ s.t. $x(\ell) = (x_A, x_B)$

Spot formation property

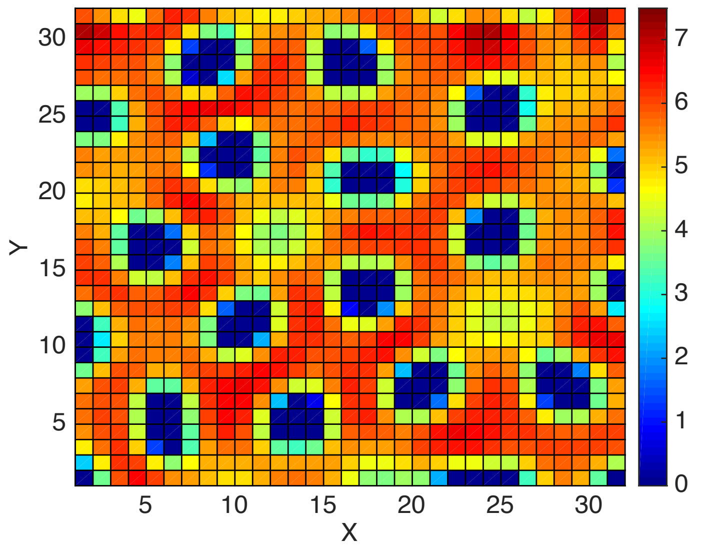
$$\phi_{spot_{form}} = F_{[19,20]} G((A \leq 0.5) \circledcirc^{hops}_{[1,w_2]} (A > 0.5))$$

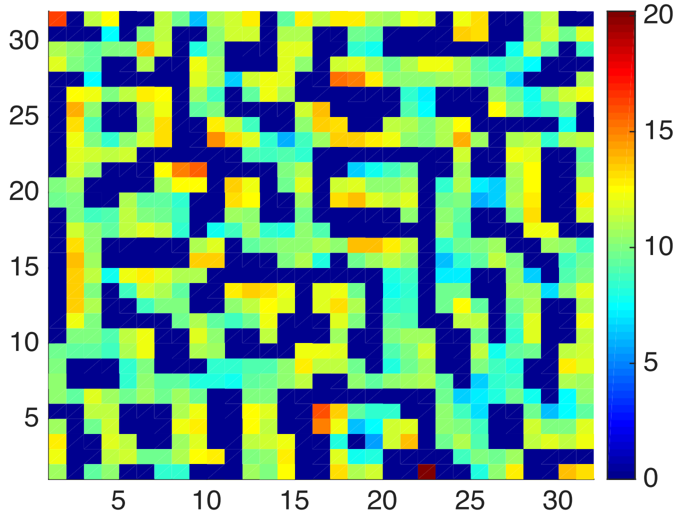


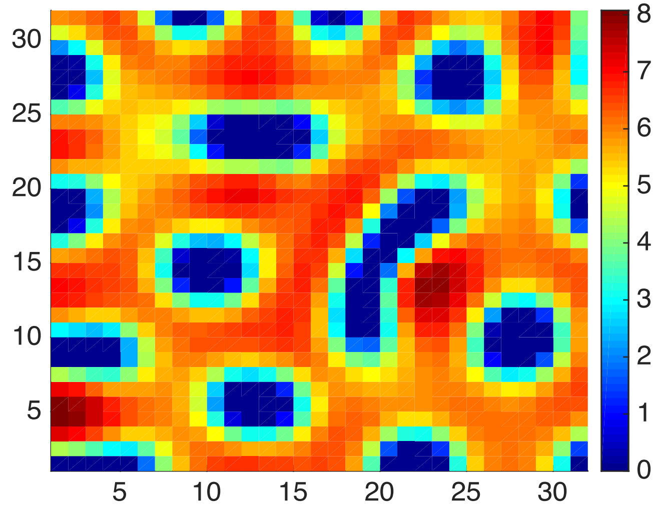
$\mathbf{x_A}(50, \ell)$ **Boolean sat.** **Quantitative sat.**

The formation of Patterns

$$\phi_{pattern} := \boxdot^{hops} \, \diamond^{hops}_{[0,15]} \, \phi_{spot_{form}}$$



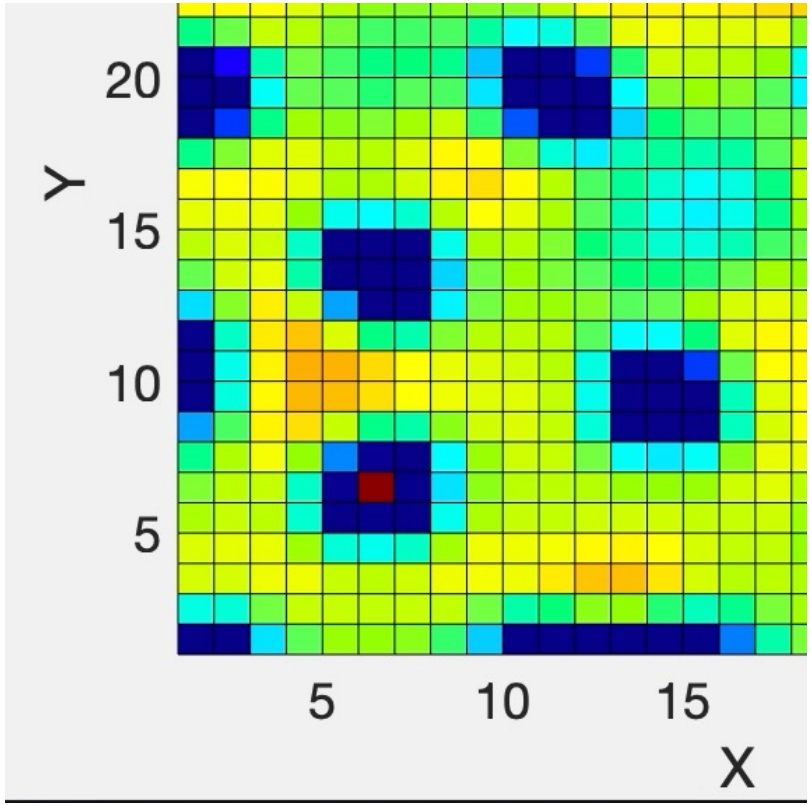




Perturbation Property

$$\phi_{\mathrm{pert}} := (x_A \geq 10) \wedge (\phi_{absorb} \circledcirc^{hops}_{[1,2]} \phi_{no\_effect})$$

- $\phi_{absorb} = F_{[0,1]} G_{[0,10]} (x_A < 3);$
- $\phi_{noeffect} := G_{[0,20]} (x_A < 3)$



Static Space and Stochastic Systems

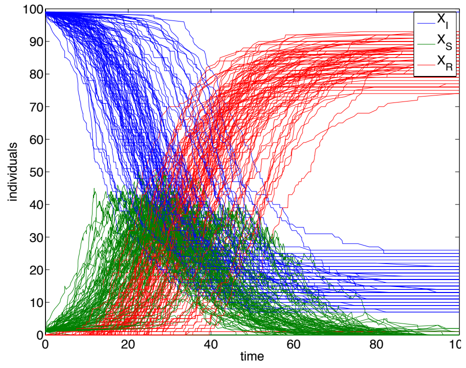
Application to Stochastic Systems

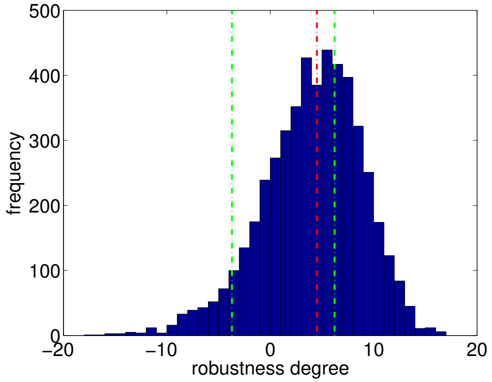
STREL can be applied on stochastic systems considering methodologies as Statistical Model Checking (SMC)

Stochastic process $\mathcal{M} = (\mathcal{T}, \mathcal{A}, \mu)$ where $\mathcal{T}$ is a trajectory space and $\boldsymbol{\mu}$ is a probability measure on a σ-algebra of $\mathcal{T}$.

We approximate the satisfaction probability $S(\varphi, t)$, i.e. the probability that a trajectory generated by the stochastic process $\mathcal{M}$ satisfies the formula ϕ.

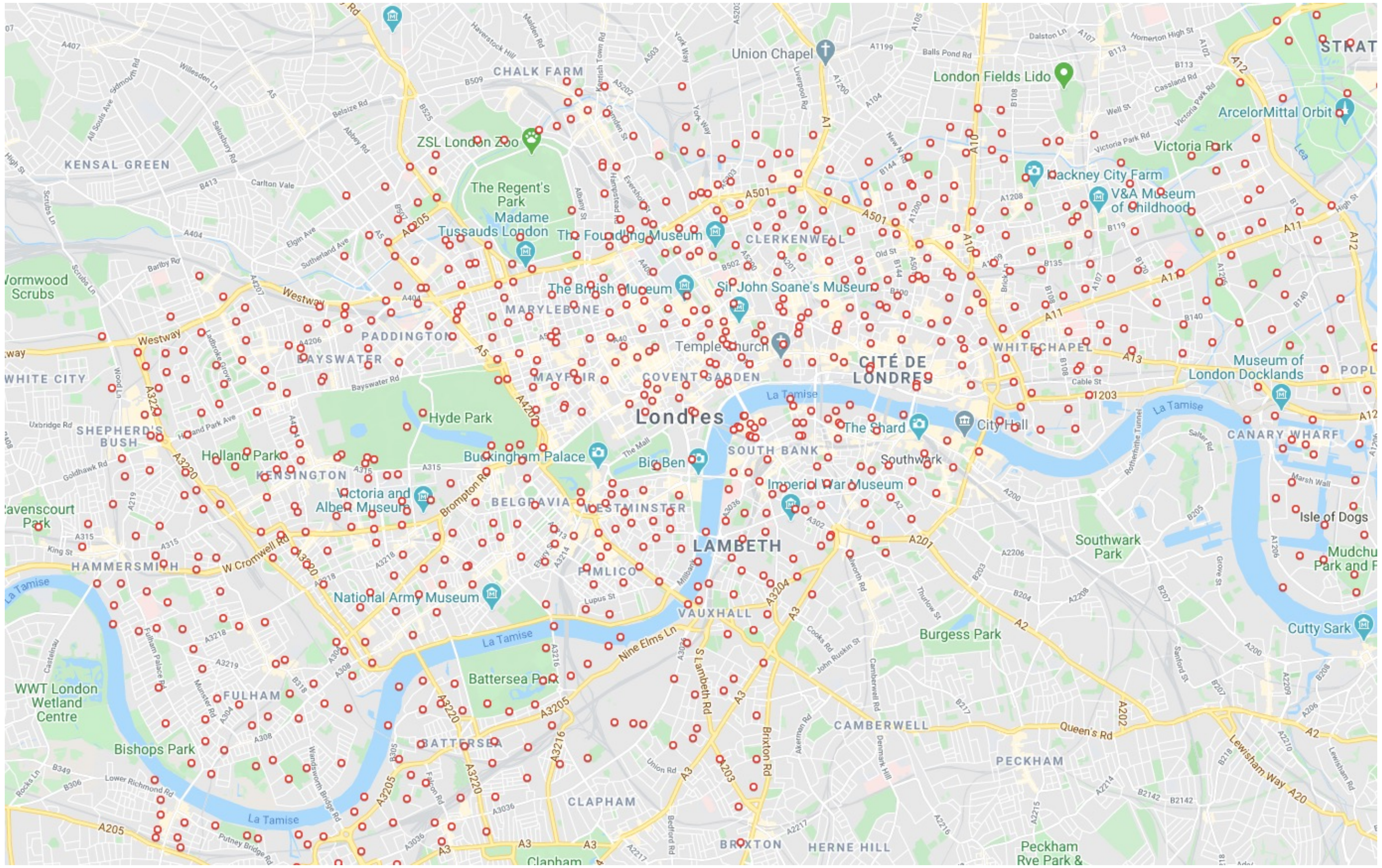
We can do something similar with the quantitative semantics computing the robustness distribution





Bike Sharing Systems (BSS)

London Santander Cycles Hire network



- 733 bike stations (each with 20-40 slots)
- a total population of 57,713 agents (users) picking up and returning bikes

We model it as a Population Continuous Time Markov Chain (PCTMC) with time-dependent rates, using historic journey and bike availability data.

Prediction for 40 minutes.

Bike Sharing Systems (BSS)

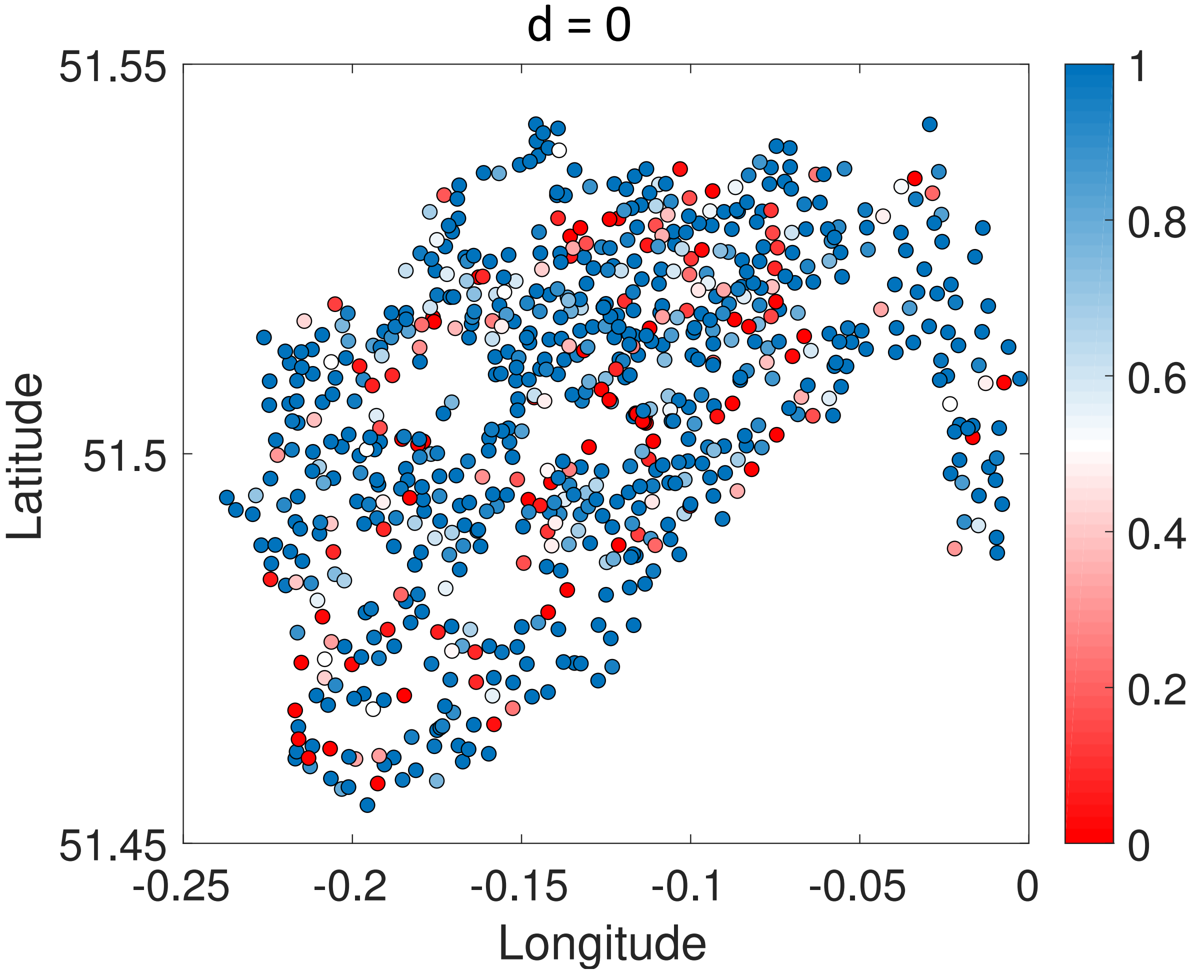
Spatio-Temporal Trajectory: $x: L -> \mathbb{T} \rightarrow \mathbb{Z}^2$ s.t. $x(i,t) = \left(B_i(t), S_i(t)\right)$

Space model

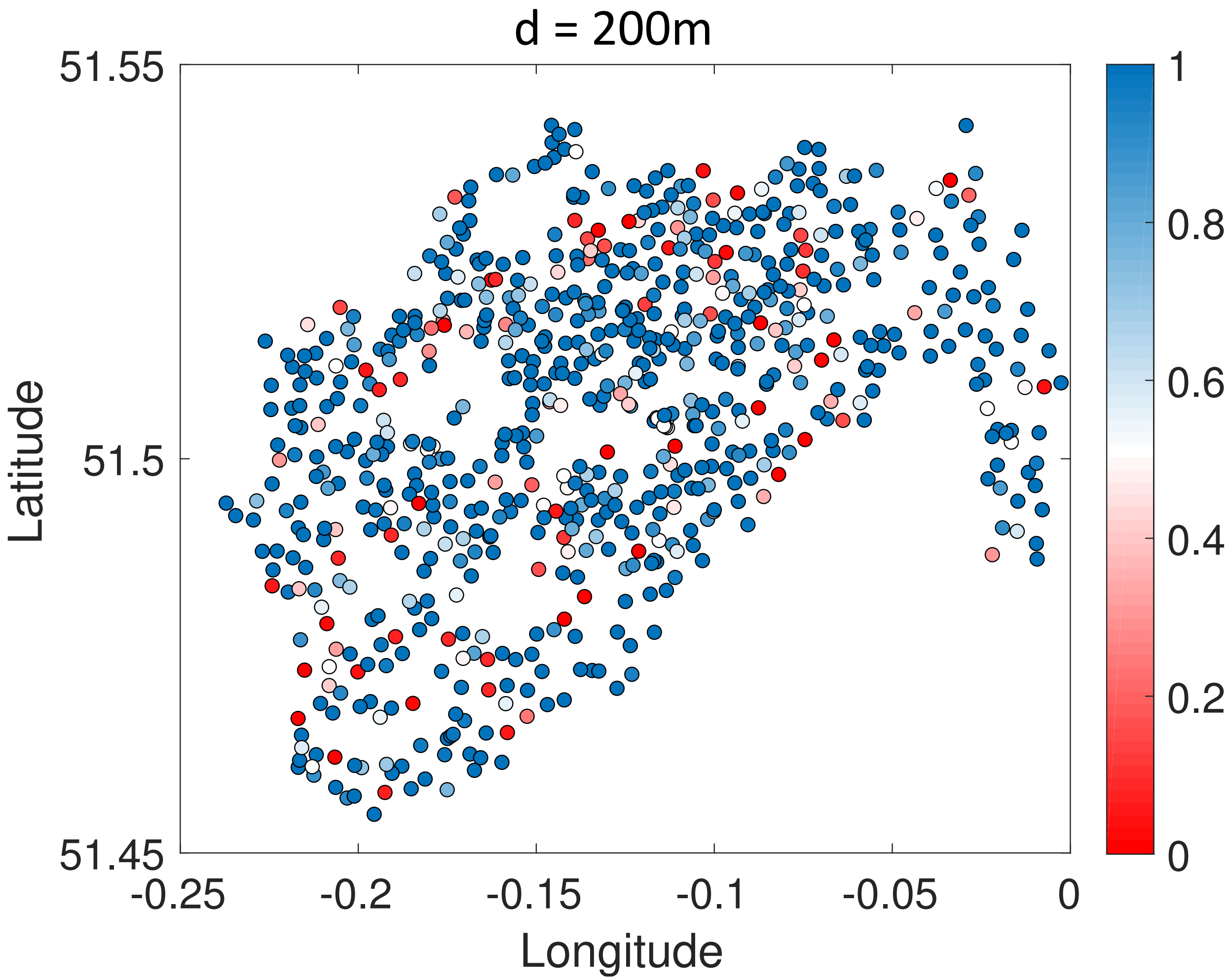
- Locations: $L = \{bike\ stations\}$,
- Edges: $\left(\ell_i, w, \ell_j\right) \in W$ iff $\mathrm{w} = \parallel \ell_i - \ell_j \parallel < 1\ kilometer$

Availability of Bikes

$$\phi_1 = \mathrm{G}\{\diamondsuit^{weight}_{[0,d]}(B > 0) \wedge \diamondsuit^{weight}_{[0,d]}(S > 0)\}$$



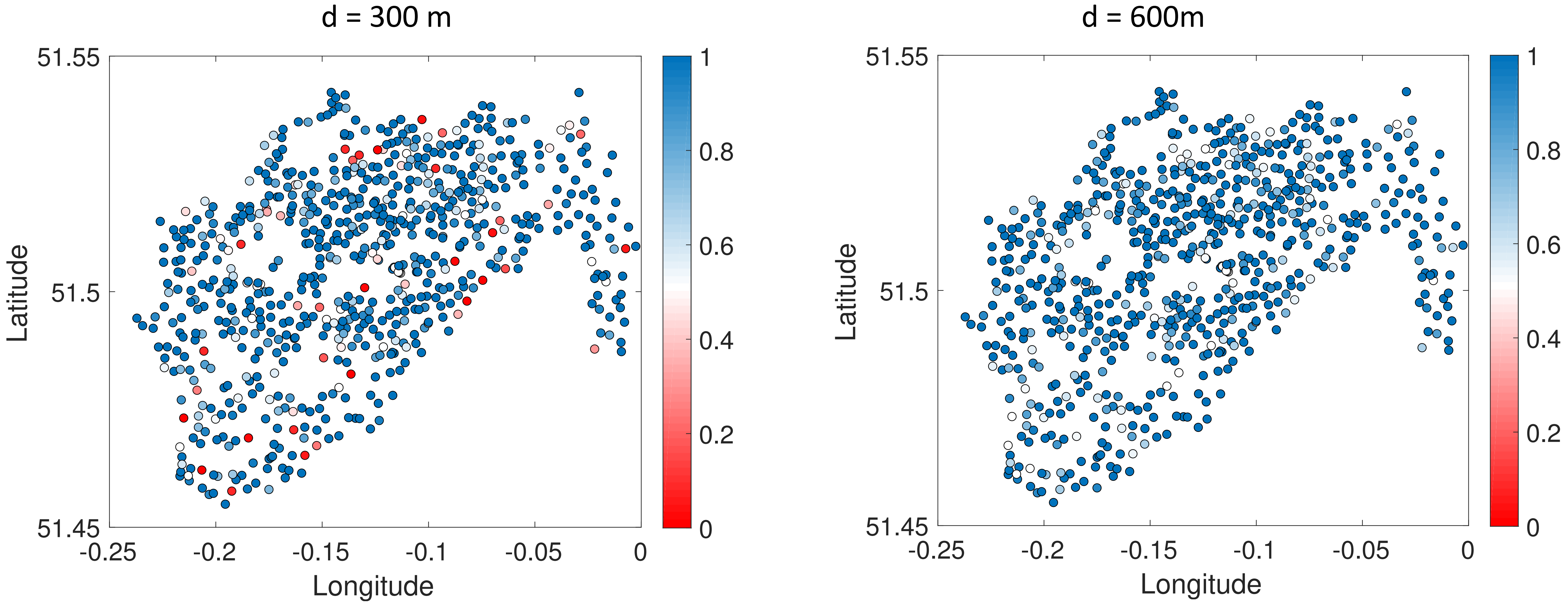
std in [0, 0.0158] , mean std = 0.0053.



std in [0, 0.0158] , mean std = 0.0039.

Availability of Bikes

$$\phi_1 = G\{\diamondsuit^{weight}_{[0,d]}(B > 0) \wedge \diamondsuit^{weight}_{[0,d]}(S > 0)\}$$



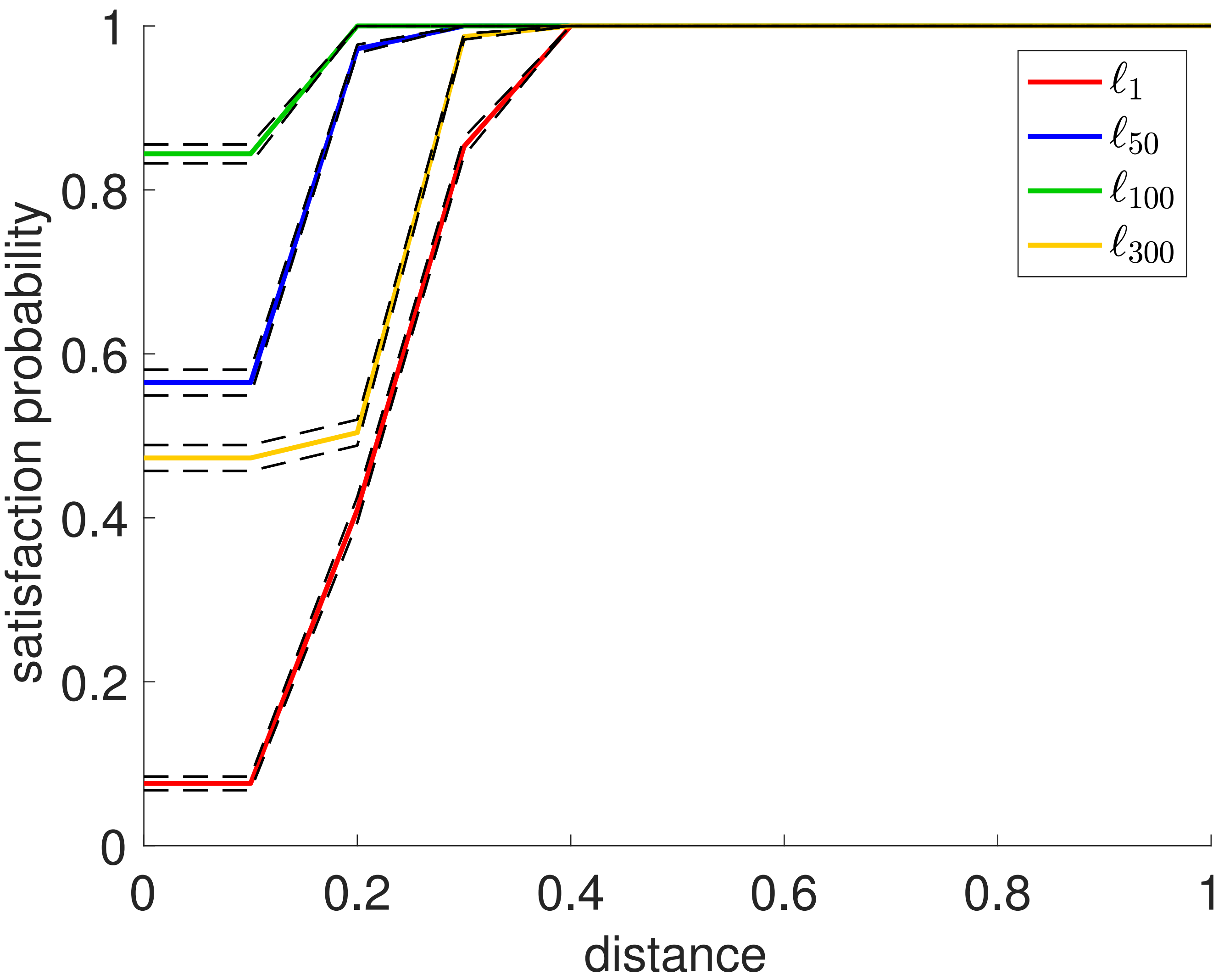
std in [0, 0.0151] , mean std = 0.0015.

std in [0, 0.0142] , mean std = 0.0002.

Availability of Bikes

$$\phi_1 = \mathrm{G}\{ \diamond\!\!\!\diamond^{weight}_{[0,d]}(B > 0) \wedge \diamond\!\!\!\diamond^{weight}_{[0,d]}(S > 0)\}$$

Satisfaction probability of some BBS stations vs distance d=[0,1.0]



Bike Sharing Systems (BSS)

$$\psi_1 = \mathrm{G}\{\diamondsuit_{[0,d]}^{weight}(\mathrm{F}_{[t_w,t_w]}B > 0) \wedge \diamondsuit_{[0,d]}^{weight}(\mathrm{F}_{[t_w,t_w]}S > 0)\}$$

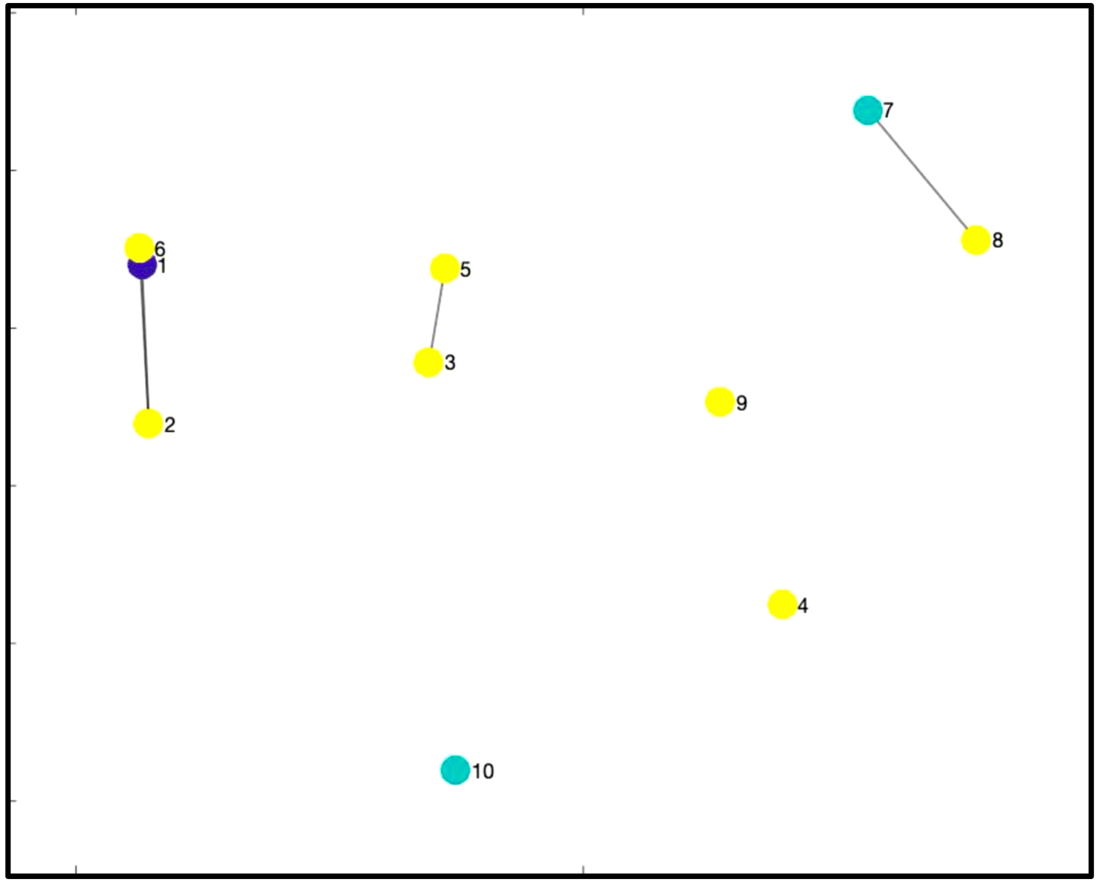
Average walking speed of 6.0 km/h, e.g. d = 0.5 km -> t_w = 6 minutes

The results similar to the results of previous property

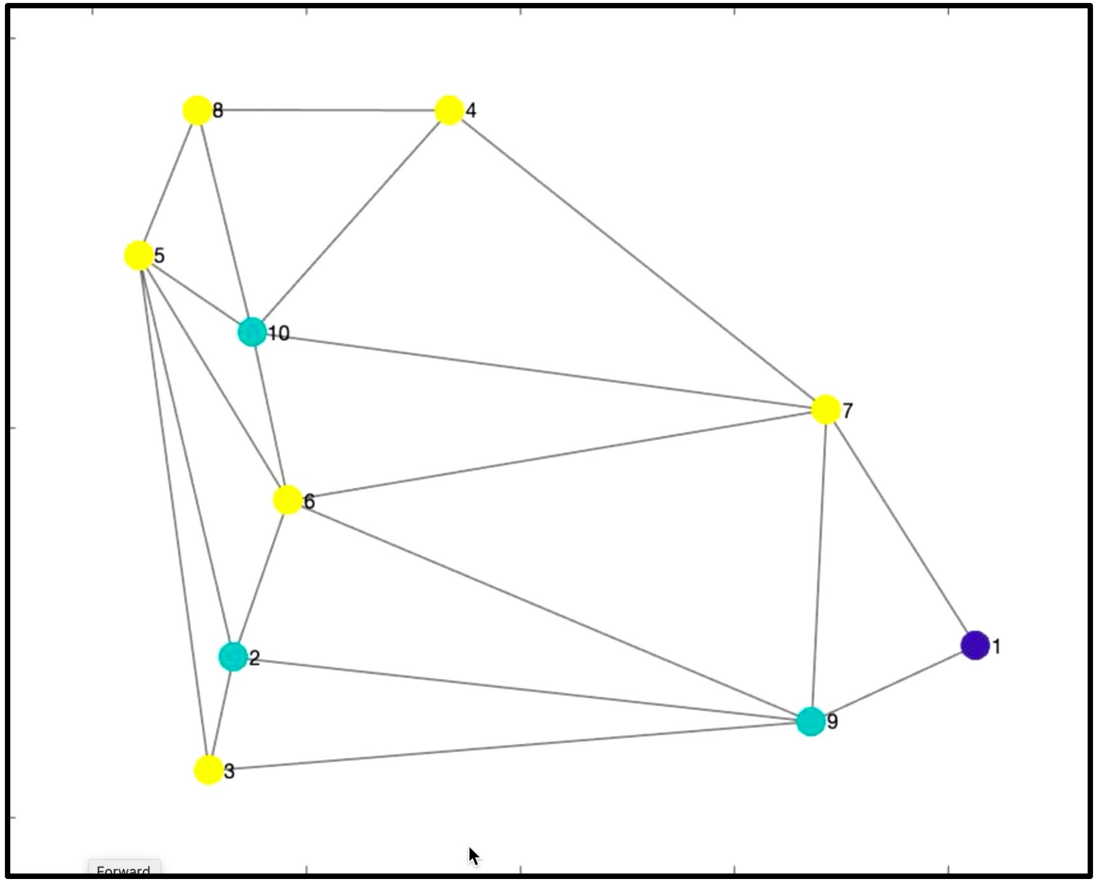
Dynamic Space

Mobile Ad-hoc sensor NETwork (MANET)

Coordinator Router End-devices



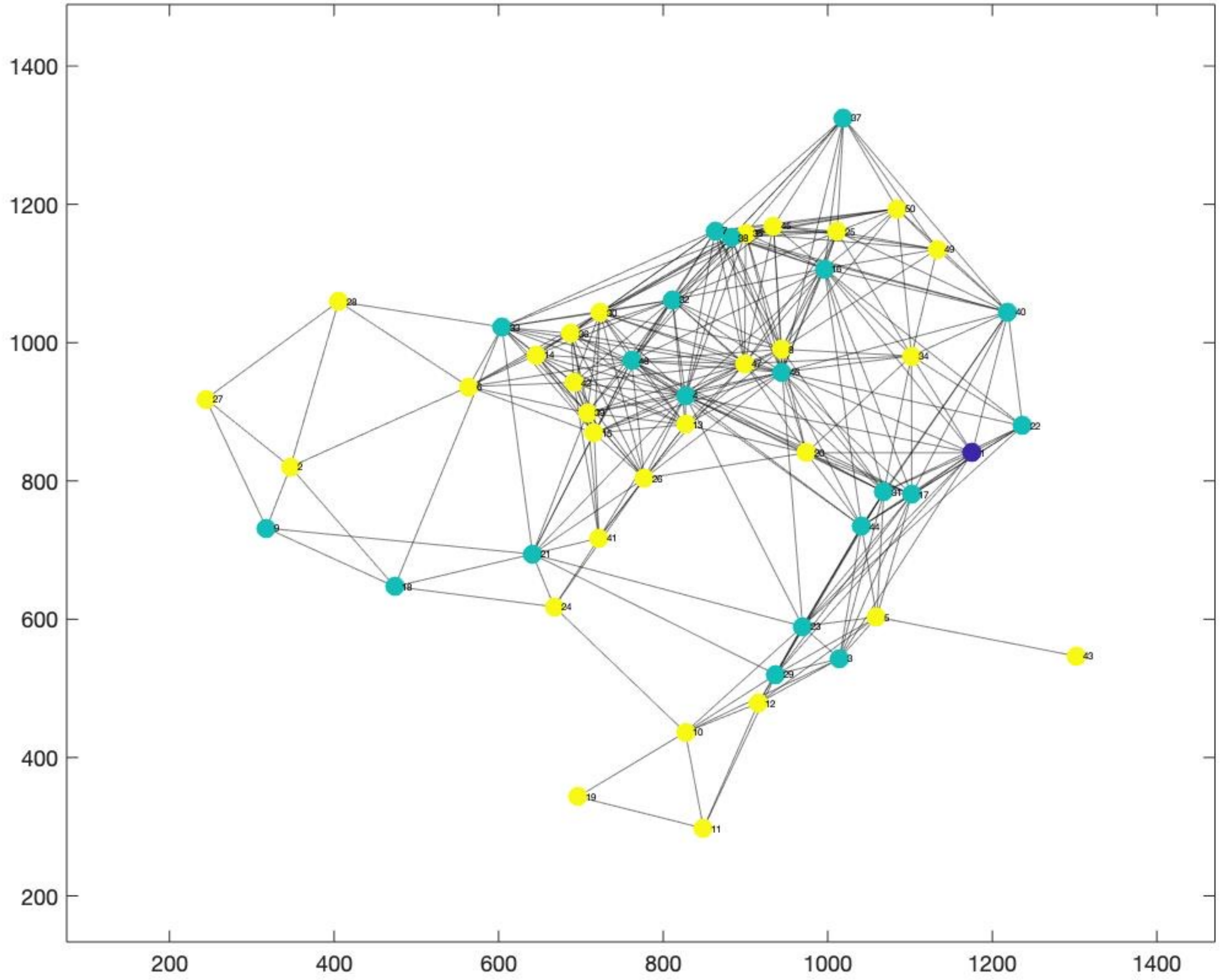
Connectivity Graph



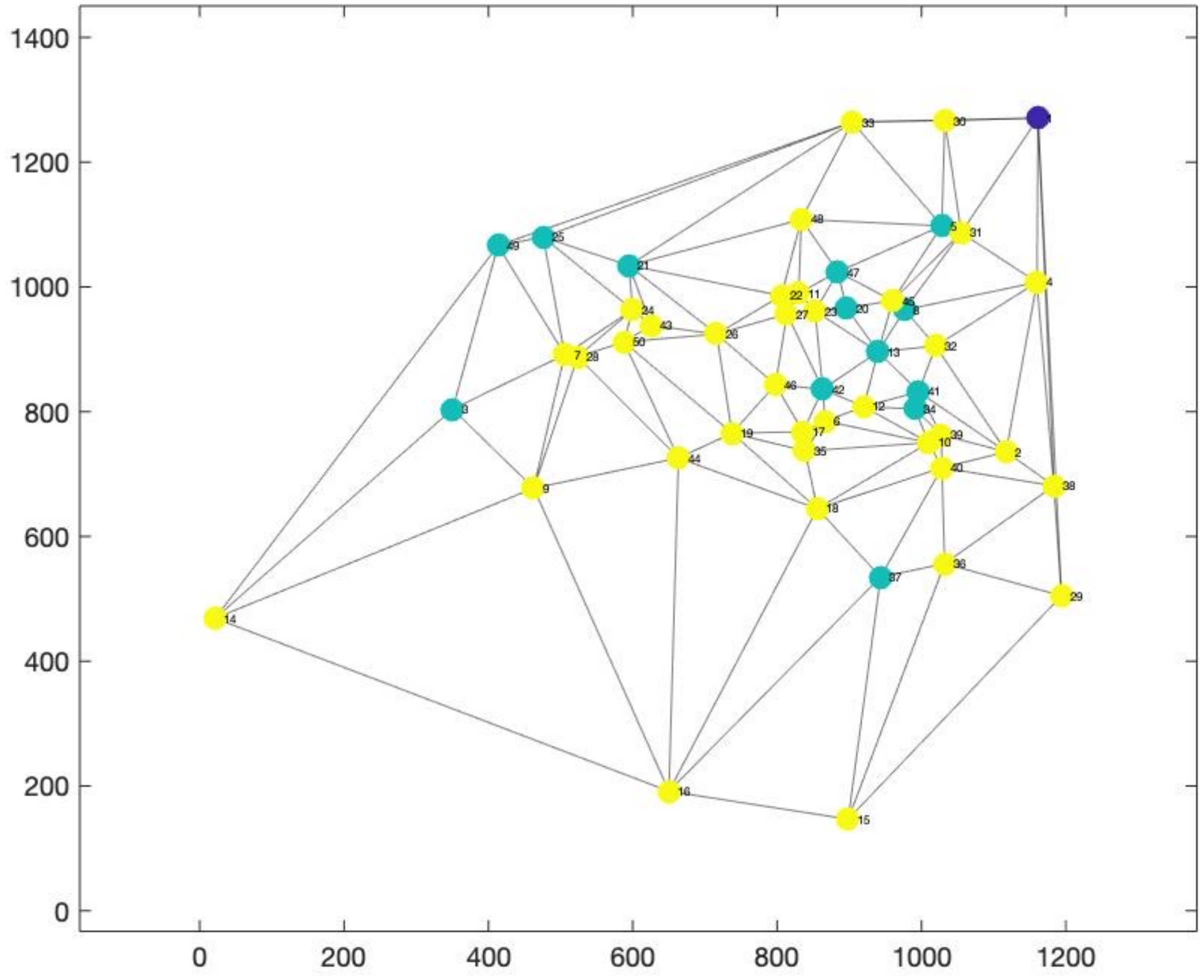
Proximity Graph

Mobile Ad-hoc sensor NETwork (MANET)

Coordinator ● Router ● End-devices ●



Connectivity Graph



Proximity Graph

Mobile Ad-hoc sensor NETwork (MANET)

Space model $S(t)$

- Locations: $L = \{devices\}$,
- Edges: $(\ell_i, w, \ell_j) \in W$ iff $\mathrm{w} = \| \ell_i - \ell_j \| < \min(r_i, r_j)$

Spatio-Temporal Trajectory: $x: L -> \mathbb{T} \to \mathbb{Z} \times \mathbb{R}^2$ s.t.

$x(i,t) = (nodeType, battery, temperature\ )$

$nodeType = 1, 2, 3$ for coordinator, rooter, and end_device

Connectivity in a MANET

“an end device is either connected to the coordinator or can reach it via a chain of routers”

“broken connection is restored within h time units”

Connectivity in a MANET

"an end device is either connected to the coordinator or can reach it via a chain of routers"

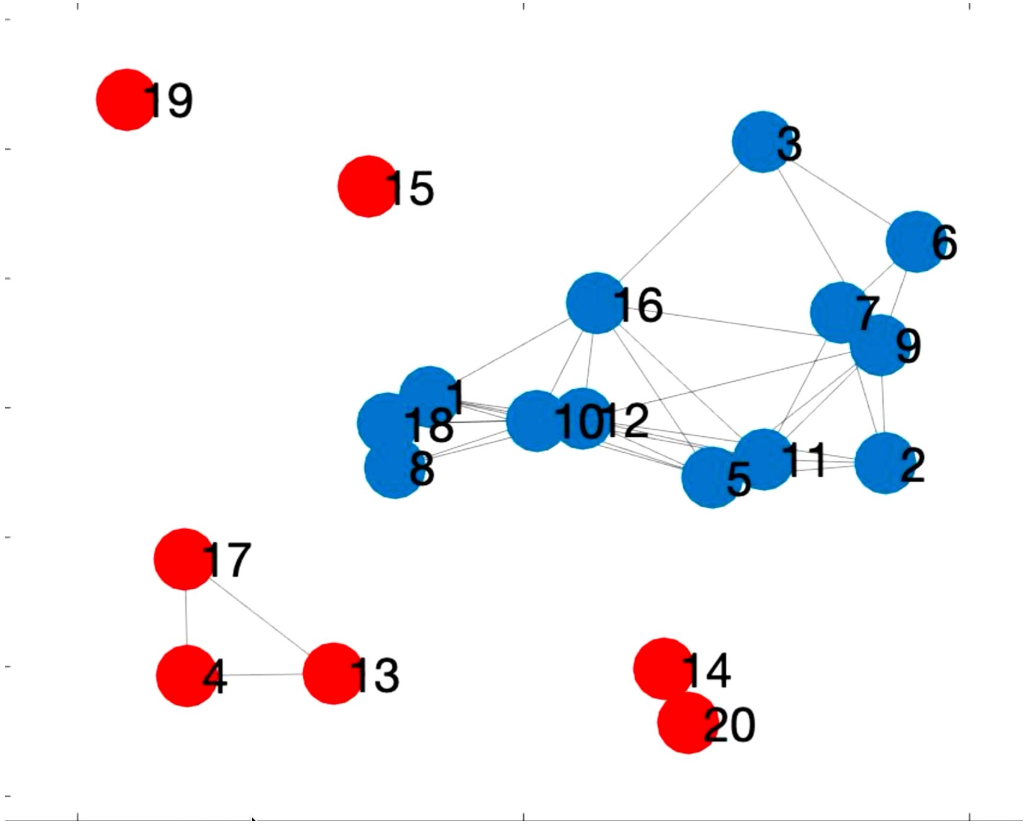
$$\phi_{connect} = device\mathcal{R}^{hop}_{[0,1]}(router\mathcal{R}^{hop}coord)$$

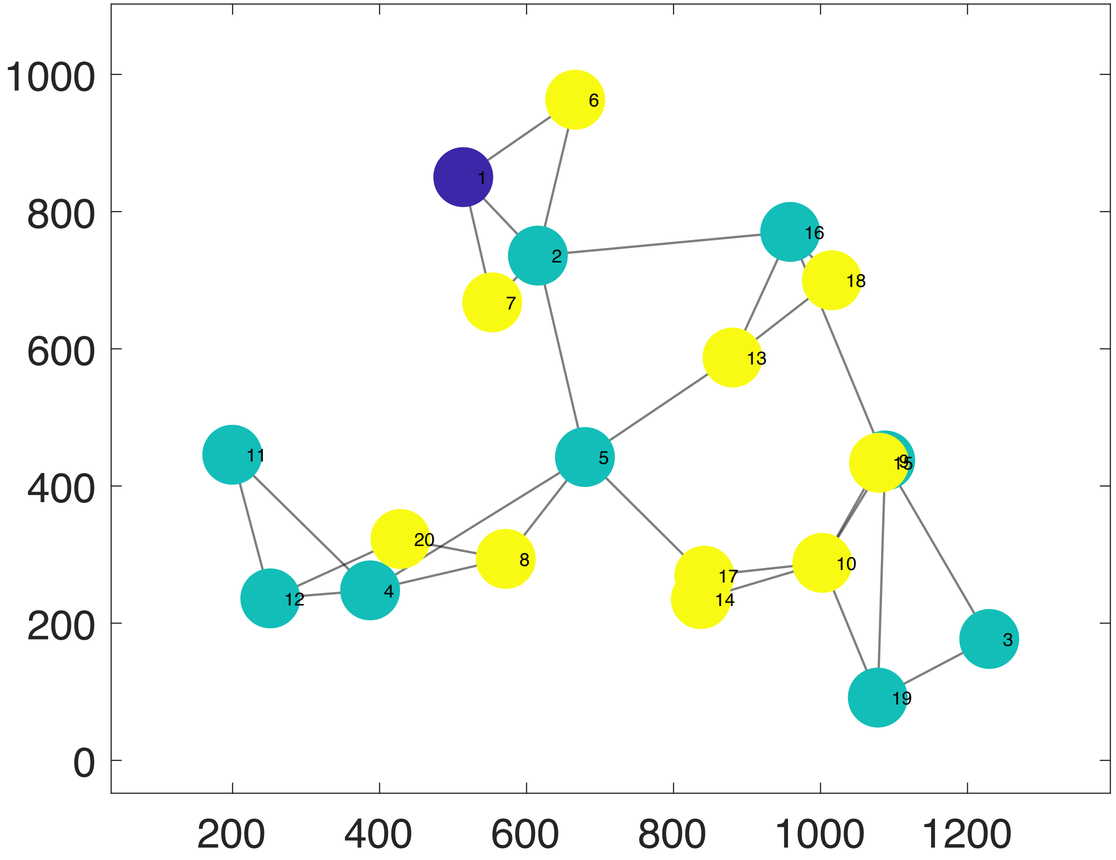
"broken connection is restored within h time units"

$$\phi_{connect\_restore} = \mathrm{G}(\neg\phi_{connect} \rightarrow \mathrm{F}_{[0,h]}\phi_{connect})$$

Boolean Satisfaction at each time step

$$\phi_{connect} = device\mathcal{R}^{hop}_{[0,1]}(router\mathcal{R}^{hop}coord)$$





Delivery in a MANET

"from a given location, we can find a path of (hops) length at least 5 such that all nodes along the path have a battery level greater than 0.5"

$$\psi_3 = \mathcal{E}^{hops}_{[5,\infty]}(battery > 0.5)$$

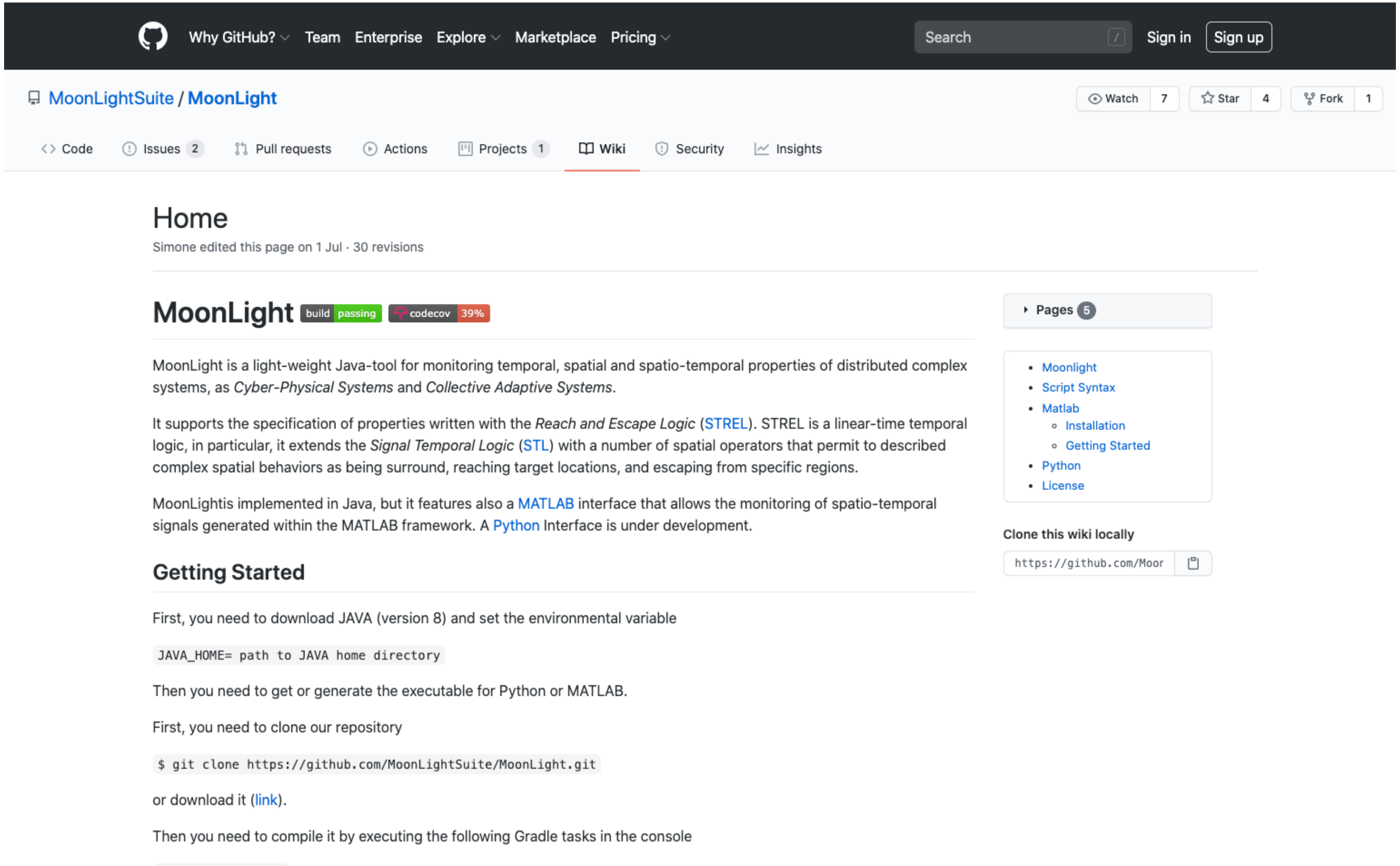
Reliability in a MANET

"reliability in terms of battery levels, e.g. battery level above 0.5

$$\phi_{reliable\_router} = ((battery > 0.5) \wedge router)\mathcal{R}^{hop} coord$$

$$\phi_{reliable\_connect} = device\mathcal{R}^{hop}_{[0,1]}(\phi_{reliable\_router})$$

Moonlight: https://github.com/MoonLightSuite/MoonLight/wiki



```
(atomicExpression)
| ! Formula
| Formula & Formula
| Formula | Formula
| Formula  -> Formula
| Formula until [a b] Formula
| Formula since [a b] Formula
| eventually [a b] Formula
| globally [a b] Formula
| once [a b] Formula
| historically [a b] Formula
| escape(distanceExpression)[a b] Formula
| Formula reach (distanceExpression)[a b] Formula
| somewhere(distanceExpression) [a b] Formula
| everywhere (distanceExpression) [a b] Formula
| {Formula}
```

Bibliography

Mining Requirements:

- Ezio Bartocci, Luca Bortolussi, Laura Nenzi, Guido Sanguinetti, System design of stochastic models using robustness of temporal properties. Theor. Comput. Sci. 587: 3-25 (2015)
- Jin, Deshmukh et al. Mining Requirements from Closed-loop Control Models (HSCC '13, IEEE Trans. On Computer Aided Design '15)
- Bartocci, E., Bortolussi, L., Sanguinetti, G.: Data-driven statistical learning of temporal logic properties, FORMATS, 2014
- Bufo, S., Bartocci, E., Sanguinetti, G., Borelli, M., Lucangelo, U., Bortolussi, L.i,Temporal logic based monitoring of assisted ventilation in intensive care patients, ISoLA, 2014.
- Nenzi L., Silvetti S., Bartocci E., Bortolussi L. (2018) *A Robust Genetic Algorithm for Learning Temporal Specifications from Data.* QEST 2018. LNCS, vol 11024. Springer, Cham.