CAUCHY

$$f_X(x) = \frac{1}{\pi(1+x^2)} \qquad x \in \mathbb{R}$$

$$E[X^+] = E[X^-] = +\infty$$

$$E[X^+] = E[\max\{X, 0\}] =$$

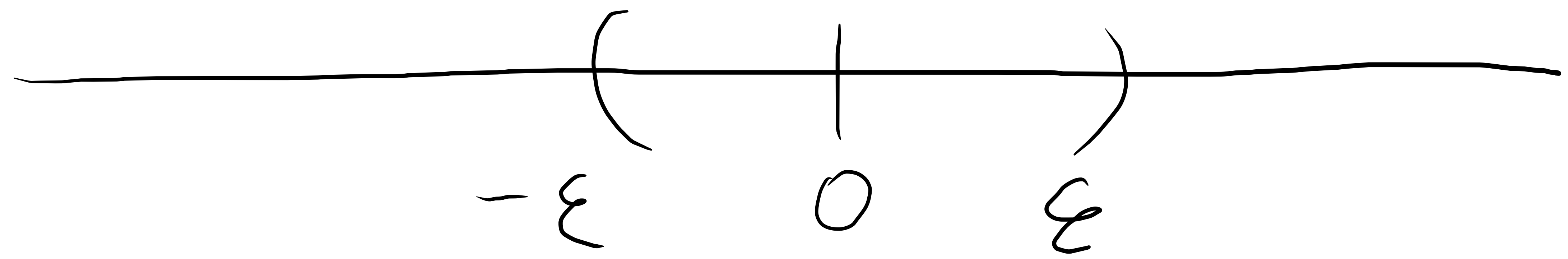
$$= \int_{-\infty}^{+\infty} \max\{x, 0\} \frac{1}{\pi(1+x^2)} dx =$$

$$= \int_0^{+\infty} \frac{x}{\pi(1+x^2)}\,dx$$

$$= \frac{1}{2\pi} \int_0^{+\infty} \frac{2x}{1+x^2}\,dx$$

$$= \frac{1}{2\pi} \underbrace{\Big[\log(1+x^2)\Big]_0^{+\infty}}_{+\infty \; - \; 0 \; = \; +\infty}$$

$$\{ E[X^p] \mid p \in \mathbb{N} \} \longrightarrow F_X \ ?$$



$$M_X(t) < +\infty \quad \forall\, t \in (-\varepsilon, \varepsilon)$$

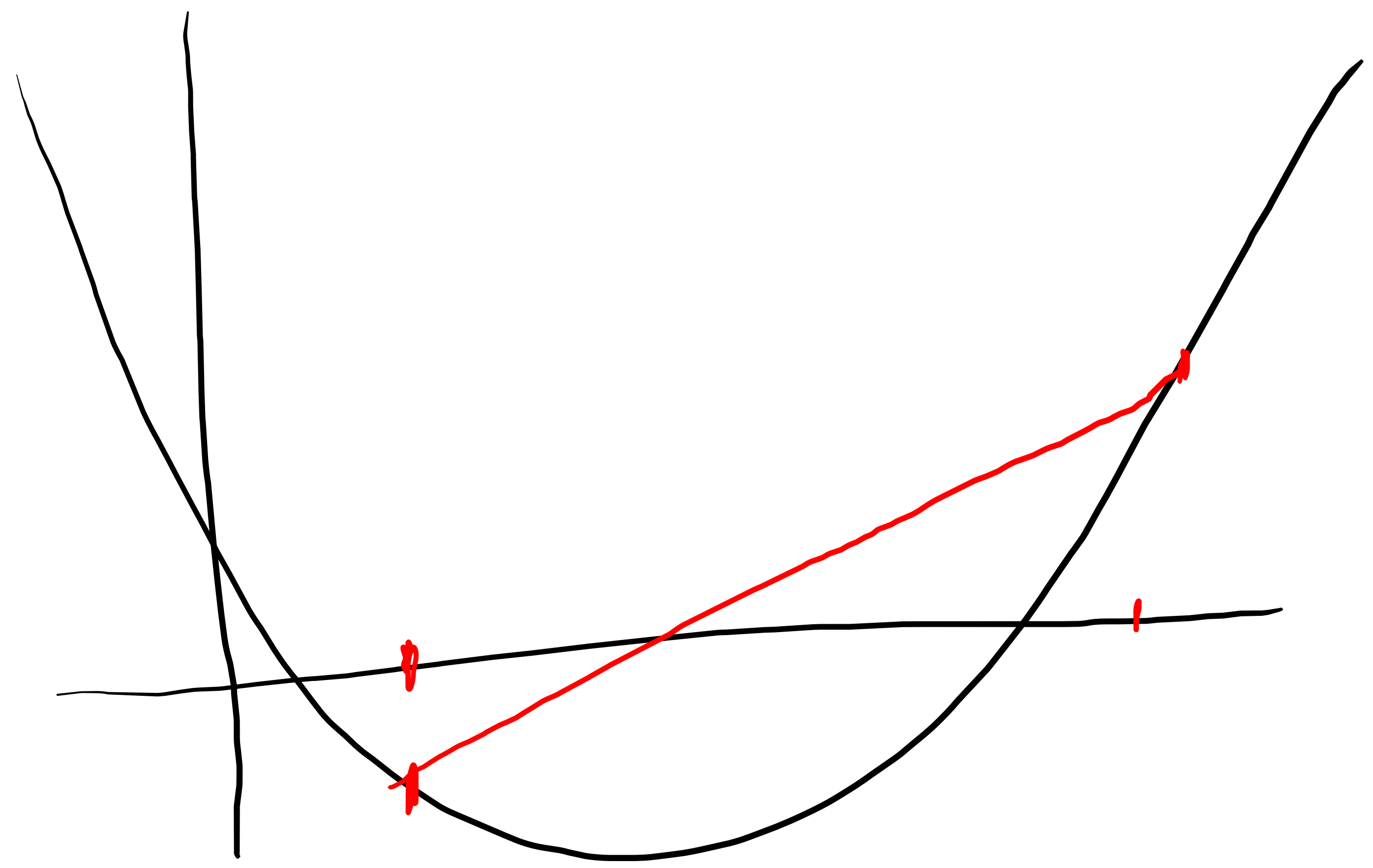
X LOGNORMALE

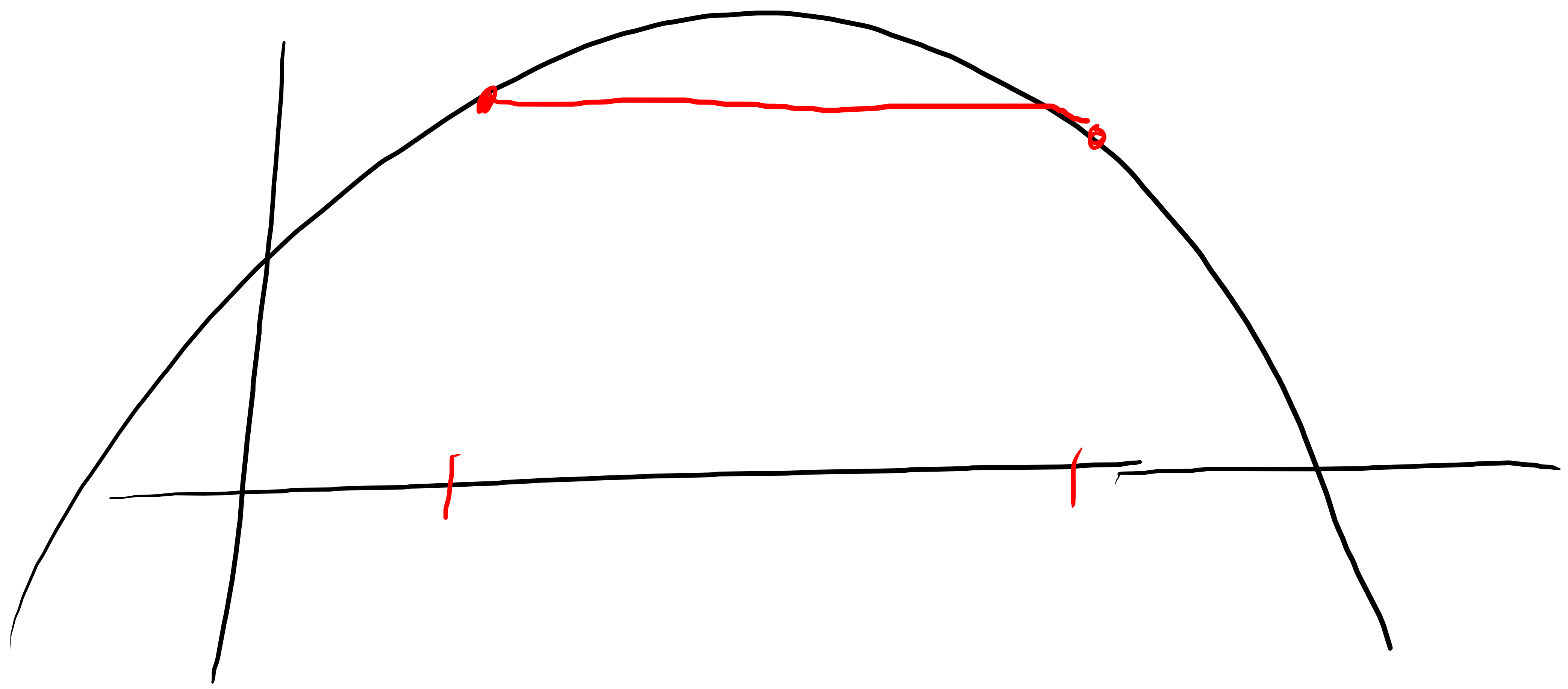
$M_X(t) < +\infty$ SOLO SE $t \le 0$

$$M_X(t) = E\left[e^{t \cdot X}\right]$$

$$e^{iz} = \cos z + i \cdot \operatorname{sen} z \qquad z \in \mathbb{R}$$

$$\underbrace{E\left[e^{itX}\right]}_{\varphi_X(t)\,:\,\mathbb{R} \to \mathbb{C}} = E\left[\cos(tX) + i \operatorname{sen}(tX)\right] \qquad < +\infty$$





$$f(\underbrace{\lambda x + (1-\lambda) y}) \leq \lambda f(x) + (1-\lambda) f(y) \quad \forall x, y \in I \text{ (INTERVALLO)}$$

$$\forall \; 0 \leq \lambda \leq 1$$

$$f(E[X]) \qquad E[f(X)]$$

$$X = \begin{cases} x & \lambda \\ y & 1-\lambda \end{cases}$$

$f(x) = |x|$ CONVESSA

$$|E[X]| \leq E[|X|]$$

$f(x) = x^2$ CONVESSA

$$E[X]^2 \leq E[X^2]$$

$f(x) = \log x$ CONCAVA

$$\log E(X) \geq E[\log X]$$

$X \sim \text{POISSON}(\lambda)$

$$E[X(X-1)(X-2)\cdot\ldots\cdot(X-k+1)] = \qquad k \geq 1$$

$$= \sum_{h=0}^{+\infty} h(h-1)(h-2)\cdot\ldots\cdot(h-k+1)\frac{\lambda^h}{h!}e^{-h}$$

$$= \sum_{h=k}^{+\infty} \cancel{h(h-1)\ldots(h-k+1)}\frac{\lambda^h}{\cancel{h\cdot(h-1)\ldots(h-k+1)}(h-k)(h-k-1)\ldots 3\cdot 2\cdot 1}e^{-h}$$

PRIMA
SONO
$=0$

$$= e^{-\lambda} \sum_{h=k}^{+\infty} \frac{\lambda^{h}}{(h-k)!} = e^{-\lambda}$$

$$h - k = i$$

$$i = 0, 1, 2, \ldots$$

$$= e^{-\lambda} \lambda^{k} \sum_{i=0}^{+\infty} \frac{\lambda^{i}}{i!} = 1 \;!$$

$$\lambda^{h} = \lambda^{i+k} = \lambda^{i} \cdot \lambda^{k}$$

$$= \lambda^{k}$$

$K = 2$

$$\underbrace{E[X(X-1)]}_{E[X^2] - \underbrace{E[X]}_{\lambda}} = \lambda^2 \quad \Big\} \Rightarrow E[X^2] = \lambda + \lambda^2$$

$$VAR[X] = \lambda + \lambda^2 - \lambda^2 = \lambda$$

$$M_X(t) = e^{\lambda(e^t - 1)}$$

$X \sim \text{POISSON}(\lambda)$

$$M'_X(0) = \lambda$$

$$M''_X(0) = \lambda + \lambda^2$$

$$E[X] = E[X \mathbb{1}_{\{X>\alpha\}} + X \mathbb{1}_{\{X \leq \alpha\}}]$$

$$X = X \cdot \mathbb{1}_{\Omega} = X \cdot \mathbb{1}_{\{X>\alpha\} \cup \{X \leq \alpha\}}$$

$$= E[\underbrace{\underbrace{X}_{>\alpha} \mathbb{1}_{\{X>\alpha\}}}_{\geq \alpha \mathbb{1}_{\{X>\alpha\}}}] + E[\underbrace{X \mathbb{1}_{\{X \leq \alpha\}}}_{\geq 0}]$$

$$\geq E[\alpha \mathbb{1}_{\{X>\alpha\}}] + 0$$

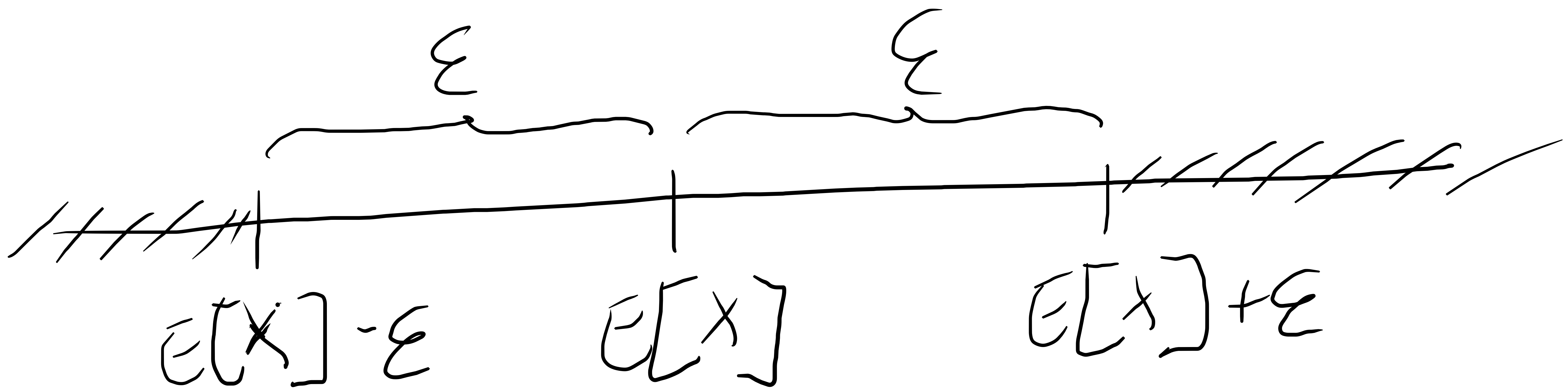
$$\alpha \cdot P(X > \alpha) \leq E[X]$$

$$P(\underbrace{|X - E(X)| > \varepsilon}) \leq \frac{\sigma_x^2}{\varepsilon^2} \qquad \varepsilon > 0$$

$$\Updownarrow$$

$$P(|X - E(X)|^2 > \varepsilon^2) \underset{\text{MARKOV}}{\leq} \frac{E[(X - E(X))^2]}{\varepsilon^2} = \frac{\sigma_x^2}{\varepsilon^2}$$

$$|X - E(X)| > \varepsilon \iff X < E[X] - \varepsilon \quad \vee \quad X > E[X] + \varepsilon$$



$$\varepsilon = k \cdot \sigma_X \qquad k = 1, 2, 3, \ldots$$

$$P\left(|X - E(X)| > k \cdot \sigma_X\right) \leq \frac{1}{k^2}$$

k-SIGMA

k	$1/k^2$
1	1
2	0.25
3	0.11
4	0.0625
5	0.04

$|X_n| \leq Y$

$-Y \leq X_n \leq Y$

$Y \in L^1$