$X_n \geq 0$

$S_n = X_1 + X_2 + \dots + X_n$

$0 \leq S_n \leq S_{n+1}$

QUINDI PER THM CONVERGENZA MONOTONA

$$\underbrace{\lim_{n \to +\infty} \underbrace{E[S_n]}_{E[X_1] + \dots + E[X_n]}}_{\sum_{n=1}^{\infty} E[X_n]} = E\Big[\underbrace{\lim_{n \to +\infty} S_n}_{\sum_{n=1}^{\infty} X_n}\Big]$$

$$X_n \sim GAMMA(\underbrace{1}_{\alpha}, \underbrace{2^{n-1}}_{\lambda}) \sim EXP(2^{n-1}) \geq 0$$

$$\sum_{n=1}^{\infty} X_n \leq +\infty$$

$$E\left[\sum_{n=1}^{\infty} X_n\right] = \sum_{n=1}^{\infty} E[X_n] = \sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$

QUINDI

$$\sum_{n=1}^{\infty} X_n < +\infty \quad Q.C.$$

$$1 + \alpha + \alpha^2 + \dots + \alpha^n + \dots = \frac{1}{1-\alpha} \quad \alpha \neq 1$$

$$X_n = \begin{cases} -\frac{1}{n!} & 1-p \\ \frac{1}{n!} & p \end{cases} \qquad \sum \frac{\lambda^n}{n!} = e^{\lambda}$$

$$\sum_{n=1}^{\infty} E[\underbrace{|X_n|}_{=\frac{1}{n!}}] = \sum_{n=1}^{\infty} \frac{1}{n!} = e < +\infty$$

QUINDI $\sum |X_n| \in L^1$ (QUINDI $< +\infty$ q.c.)

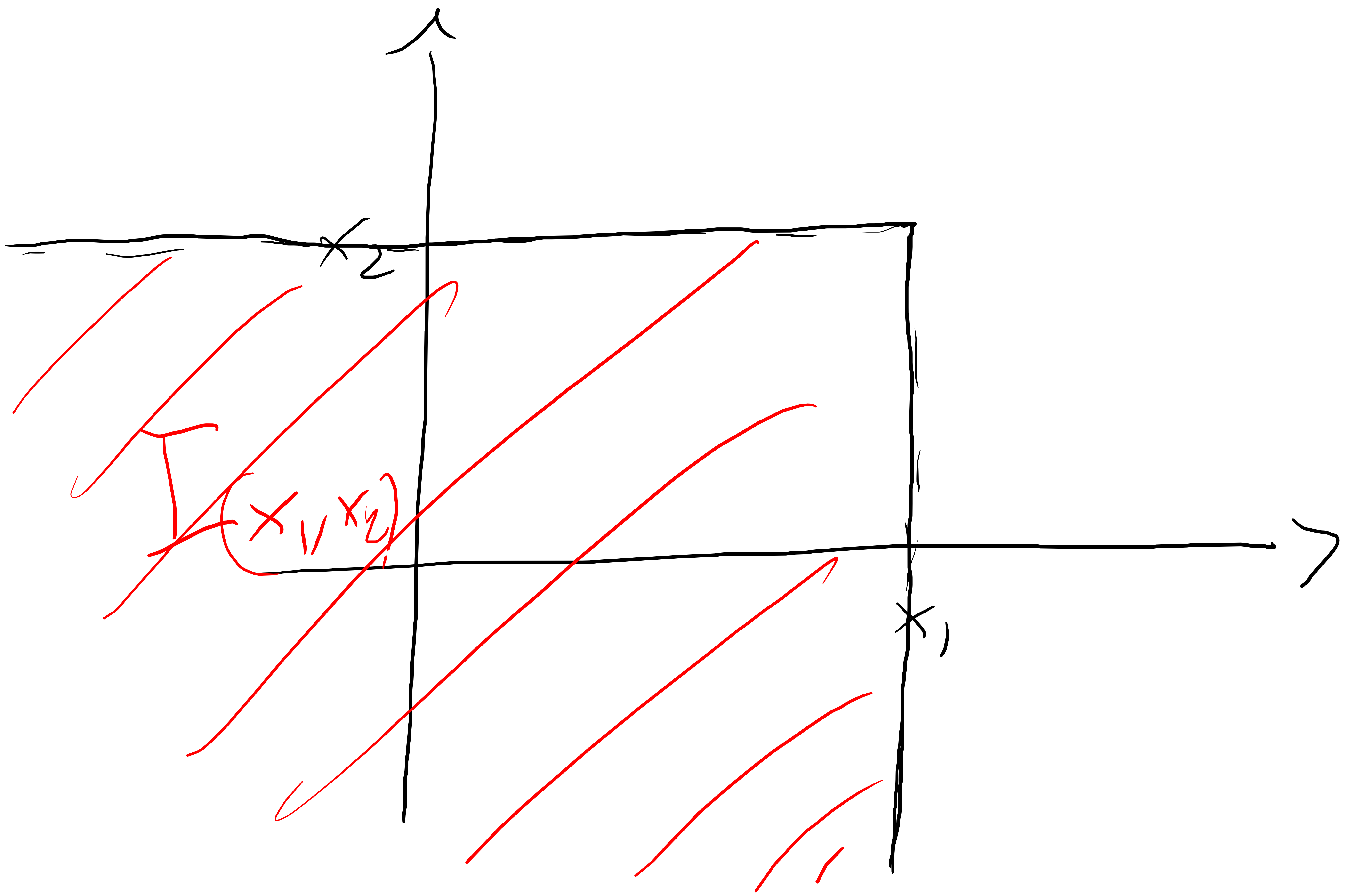
QUINDI $\sum X_n < +\infty$ q.c.

INOLYRE

$$E\left[\sum_{n=1}^{\infty} X_n\right] = \sum_{n=1}^{\infty} \underbrace{E\left[X_n\right]}_{(1-p)\left(-\frac{1}{n!}\right)+p\left(\frac{1}{n!}\right)} = \sum_{n=1}^{\infty} \frac{1}{n!}(2P-1)$$

$$= (2P-1)\cdot e$$

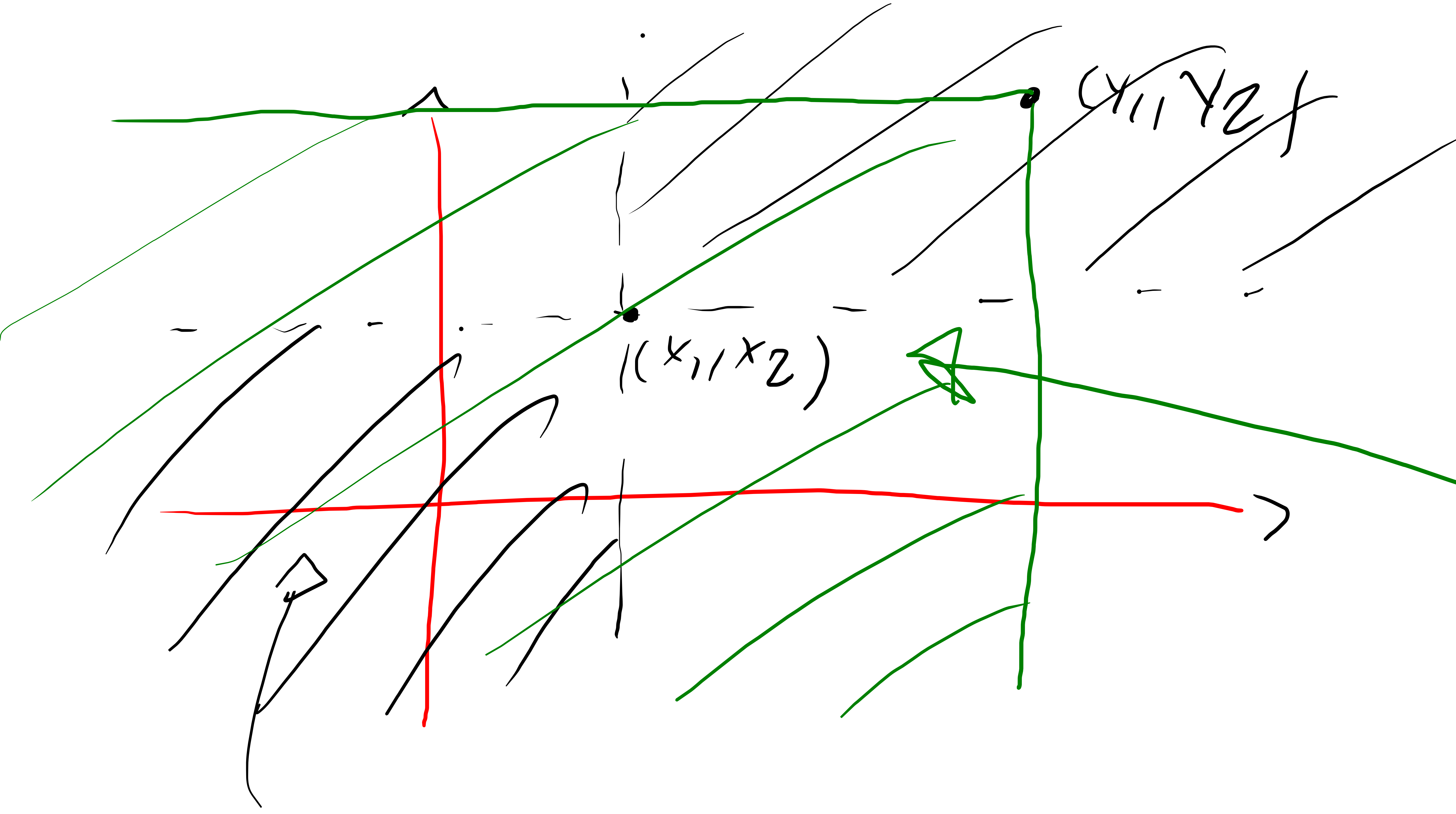
$$I_{(x_1, x_2)} = (-\infty, x_1] \times (-\infty, x_2]$$



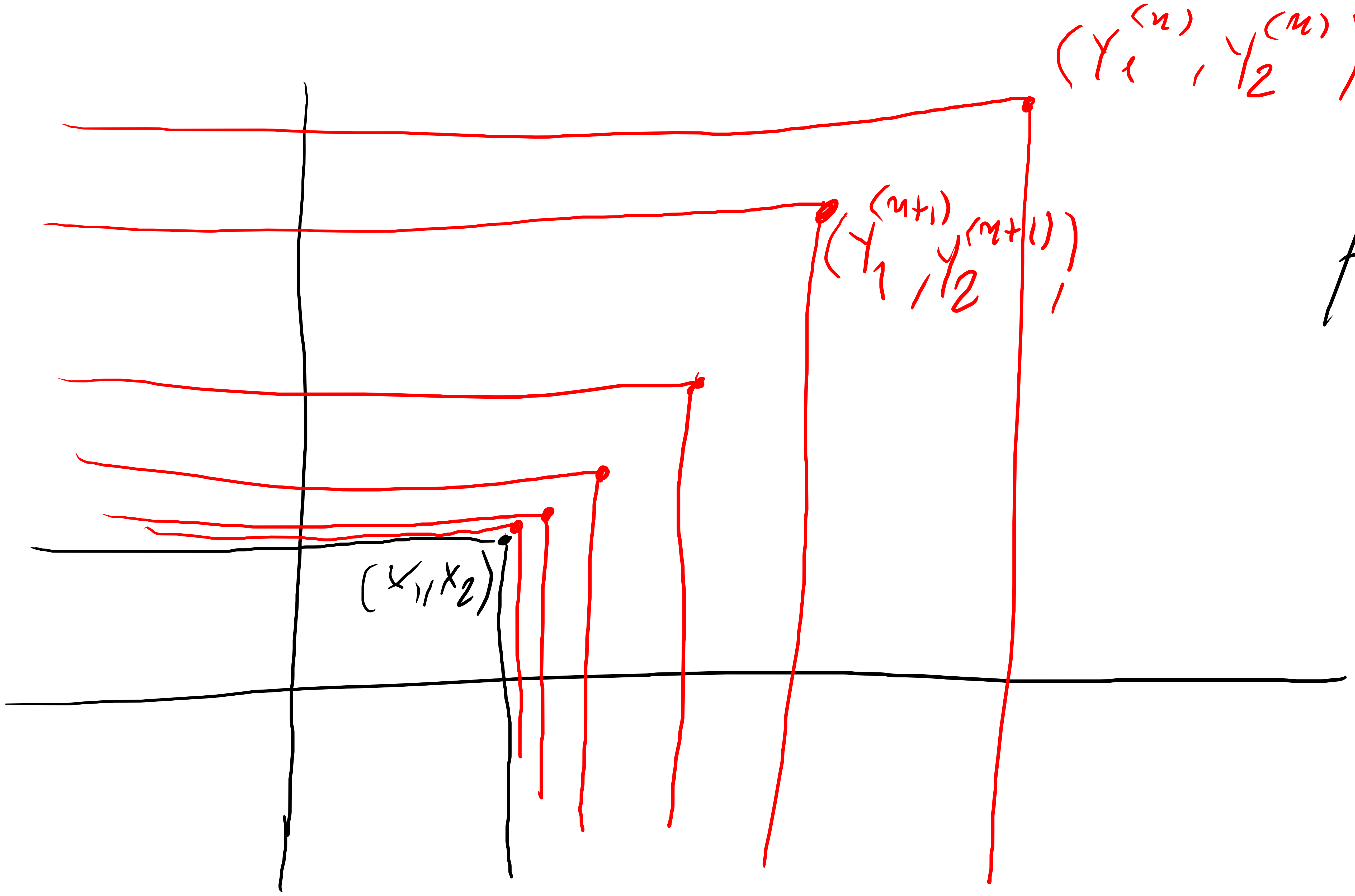
$$\{(X_1, X_2) \in I_{(x_1, x_2)}\}$$

$$=$$

$$\{X_1 \leq x_1,\ X_2 \leq x_2\}$$



$\{X_1 \leq x_1, X_2 \leq x_2\}$ $\{X_1 \leq y_1, X_2 \leq y_2\}$



$Y_1^{(n)} \downarrow X_1 \qquad Y_2^{(n)} \downarrow X_2$

$A_n = \{X_1 \le Y_1^{(n)}, X_2 \le Y_2^{(n)}\}$

$A_n \supset A_{n+1}$

DECRESCENTE PER INCLUSIONE

$\lim P(A_n) = P(\cap_n A_n)$

$F_{(X_1,X_2)}(Y_1^{(n)}, Y_2^{(n)})$

$$\lim A_n = \bigcap_{n=1}^{\infty} A_n = \bigcap_{n=1}^{\infty} \{ X_1 \le y_n^{(1)}, X_2 \le y_n^{(2)} \}$$

$$= \{ X_1 \le x_1, X_2 \le x_2 \}$$

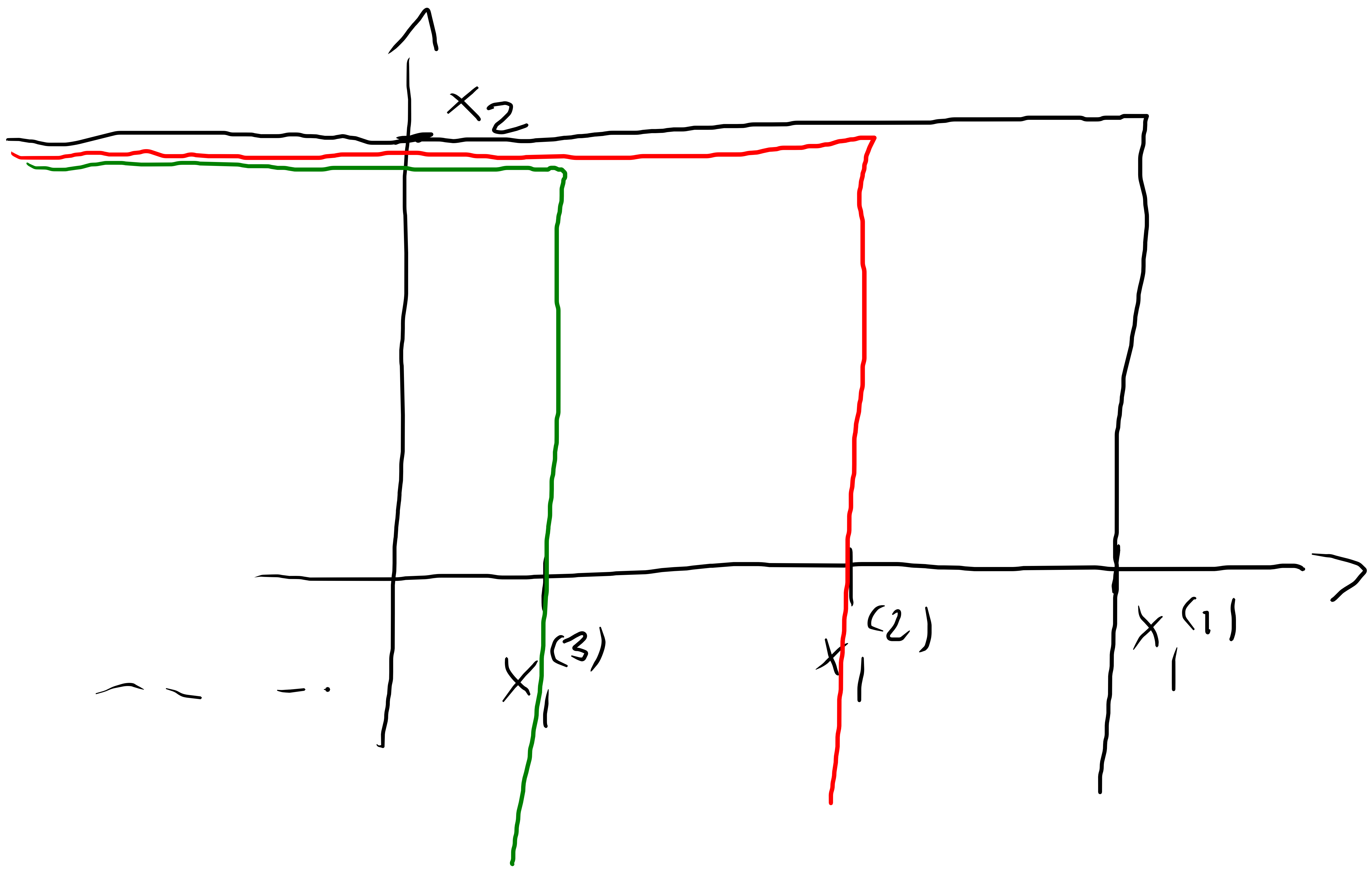
$$P(\lim A_n) = F_{(X_1, X_2)}(x_1, x_2)$$

$x_i^{(k)} \downarrow -\infty$

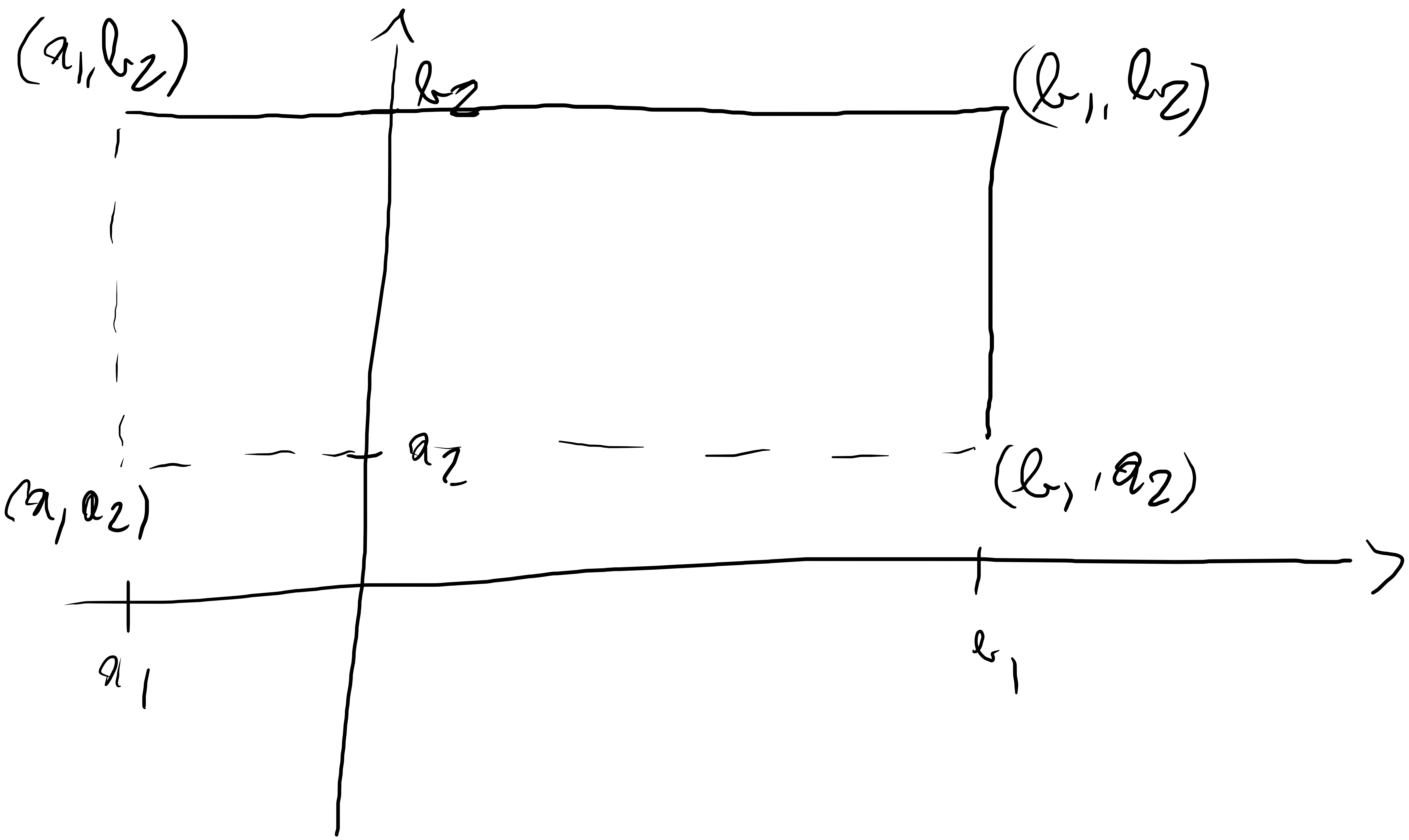
$$\{X_1 \le x_1, \ldots, X_i \le x_i^{(k)}, \ldots X_n \le x_n\} = A_k$$

DECRESCENTE PER INCLUSIONE

$$\lim A_k = \bigcap_{k=1}^{\infty} A_k = \emptyset$$



(X_1, X_2) $\qquad I = (a_1, b_1] \times (a_2, b_2]$

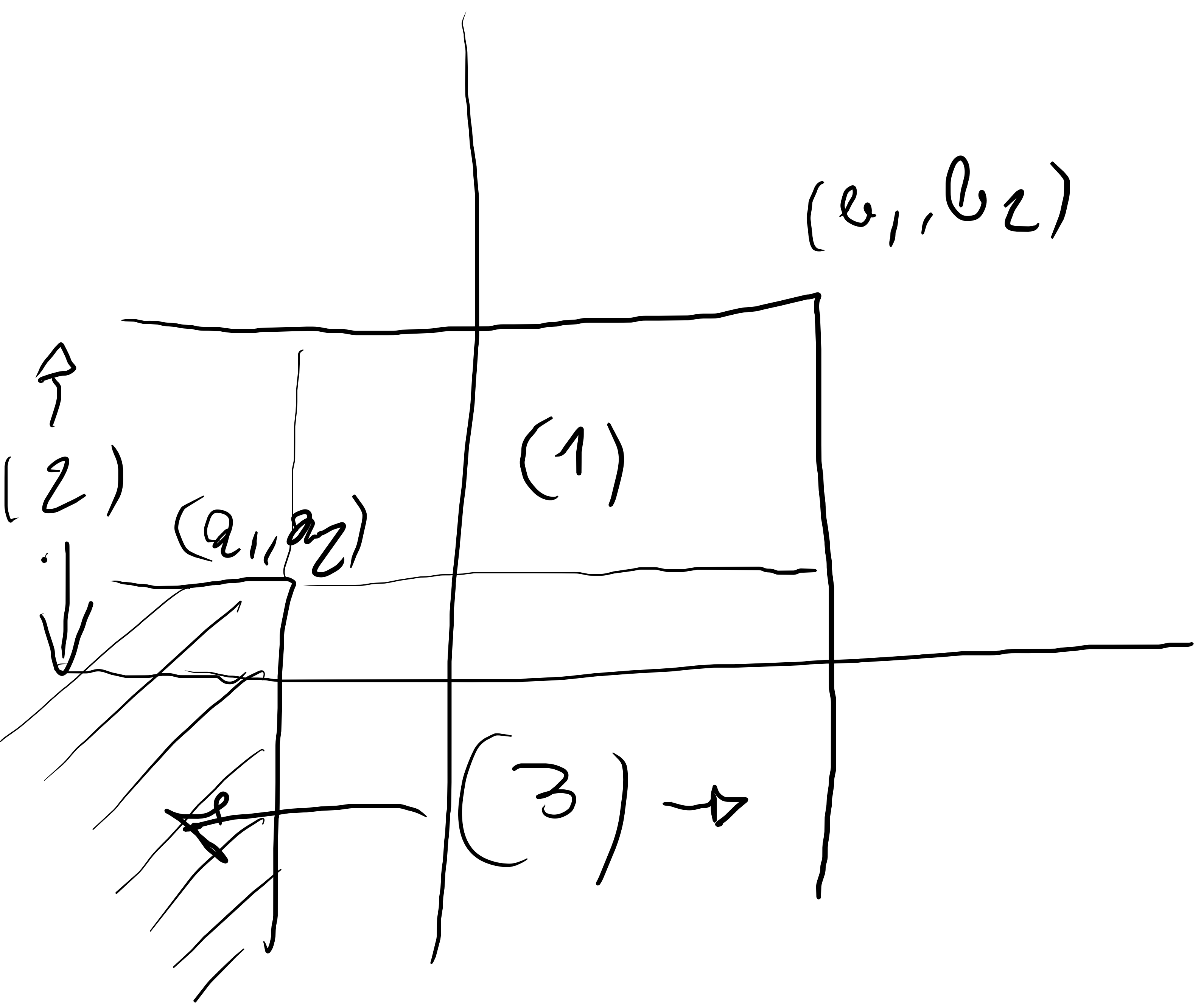


$P((X_1, X_2) \in I)$

$$\{X_1 \le b_1, X_2 \le b_2\} = \overset{(1)}{\{a_1 < X_1 \le b_1, a_2 < X_2 \le b_2\}} \cup$$

$$\cup \overset{(2)}{\{X_1 \le a_1, X_2 \le b_2\}}$$

$$\cup \overset{(3)}{\{X_1 \le b_1, X_2 \le a_2\}}$$



$$(1) \cap (2) = \emptyset$$

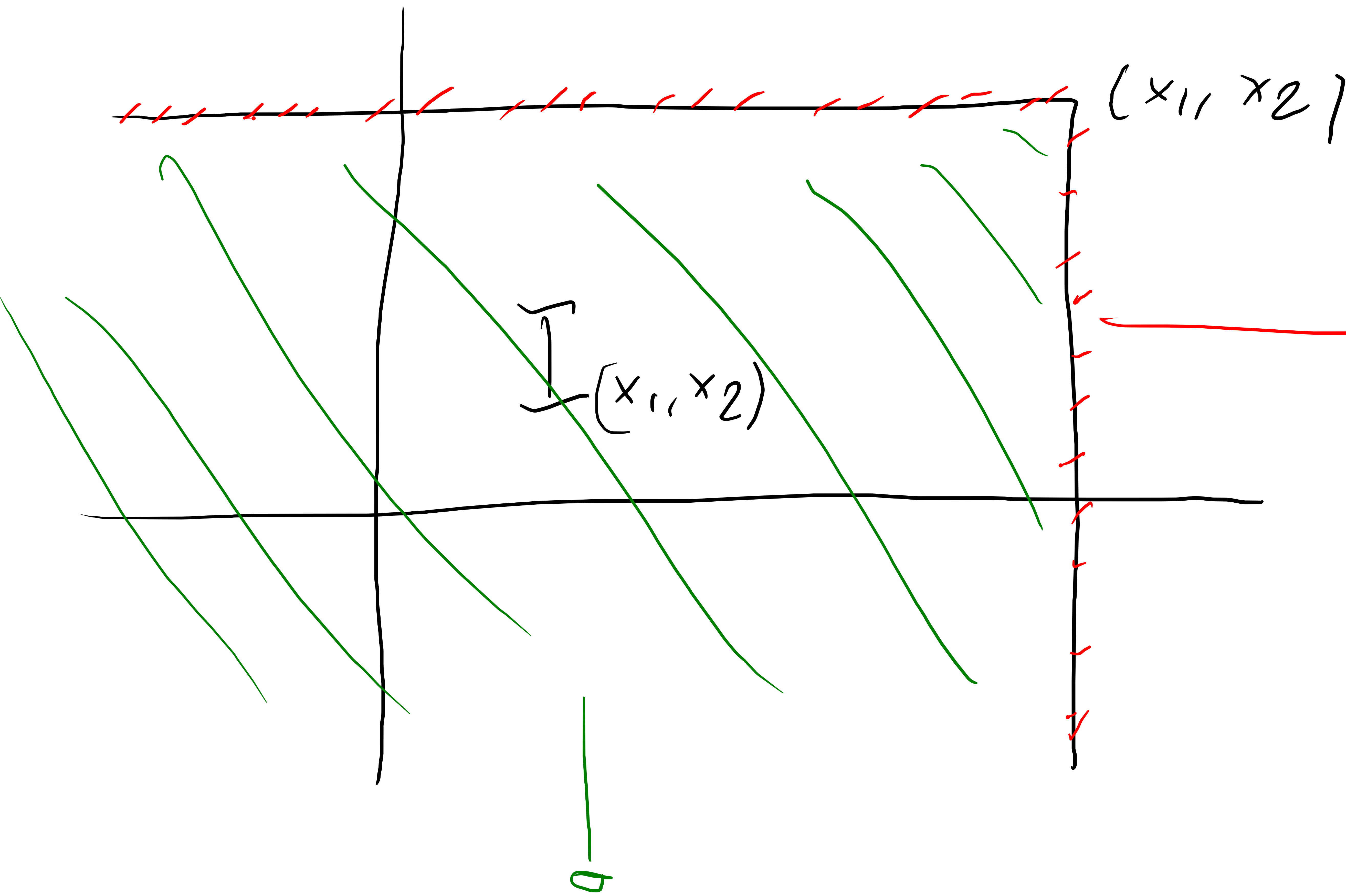
$$(1) \cap (3) = \emptyset$$

$$(2) \cap (3) = \{X_1 \le a_1, X_2 \le a_2\}$$

PASSANDO ALLE PROB.

$$F(b_1, b_2) = P\left((X_1, X_2) \in I\right) + F(a_1, b_2) + F(b_1, a_2)$$

$$- 0 - 0 - F(a_1, a_2) + 0$$



$$Fr(I_x) = \{(y_1, y_2) \mid y_1 = x_1,\ y_2 \le x_2 \ \cup\ y_1 \le x_1,\ y_2 = x_2\}$$

$$INT(I_x) = (-\infty, x_1) \times (-\infty, x_2)$$

$$\Delta^{(1)}_{a_1,b_1}\Big(\underbrace{\Delta^{(2)}_{a_2,b_2}\ F_{(X_1,X_2)}(x_1,x_2)}_{F_{(X_1,X_2)}(x_1,b_2)\,-\,F_{(X_1,X_2)}(x_1,a_2)}\Big) =$$

$$= F_{(X_1,X_2)}(b_1,b_2) - F_{(X_1,X_2)}(b_1,a_2) - \Big(F_{(X_1,X_2)}(a_1,b_2) - F_{(X_1,X_2)}(a_1,a_2)\Big)$$

$$= F_{X_1,X_2}(b_1,b_2) - F(b_1,a_2) - F(a_1,b_2) + F(a_1,a_2)$$

$$\geq 0$$

$$\lim_{x_2 \to +\infty} F_{(X_1,X_2)}(x_1,x_2) = F_{X_1}(x_1)$$

$$x_2^{(n)} \uparrow +\infty \qquad A_n = \{X_1 \leq x_1,\ X_2 \leq x_2^{(n)}\}$$

SUCCESSIONE NON DECRESCENTE

$$P(\underbrace{\lim_n A_n}) = \lim_n P(A_n)$$

$$\bigcup_n A_n = \{X_1 \leq x_1\}$$

$X_1 \quad x_1^{(1)}, x_2^{(1)}, \text{—} \text{—}, x_m^{(1)} \cdot \text{—}$

$X_2 \quad x_1^{(2)}, x_2^{(2)}, \text{—}, x_m^{(2)}, \text{—}$

$$P(X_1 = x_j^{(1)}) = P(\{X_1 = x_j^{(1)}\} \cap \Omega)$$

$$= P\left(\{X_1 = x_j^{(1)}\} \cap \bigcup_{e} \{X_2 = x_e^{(2)}\}\right)$$

$$= \sum_{e} \underbrace{P(X_1 = x_j^{(1)}, X_2 = x_e^{(2)})}_{p_{j,e}}$$

$$U \le V$$

$$1 \le i \le 6$$
$$1 \le j \le 6$$

$$P(U=i, V=j) = \begin{cases} 0 & \text{se } i>j \\ \dfrac{1}{36} & i=j \\ P(\{X_1=i, X_2=j\} \cup \{X_1=j, X_2=i\}) = \dfrac{1}{18} & i<j \end{cases}$$

V

U \ V	1	2	3	4	5	6	
1	$1/36$	$1/18$	$1/18$	$1/18$	$1/18$	$1/18$	$11/36$
2	0	$1/36$	$1/18$	$1/18$	$1/18$	$1/18$	$9/36$
3	0	0	$1/36$	$1/18$	$1/18$	$1/18$	$7/36$
4	0	0	0	$1/36$	$1/18$	$1/18$	$5/36$
5	0	0	0	0	$1/36$	$1/18$	$3/36$
6	0	0	0	0	0	$1/36$	$1/36$
	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	

LANCIO UN DADO n VOLTE

$X_1 \sim$ N. DI VOLTE IN CUI ESCE 1

X_2 " " " " " 2

⋮

X_6 " " " " " 6

$X_1 + X_2 + \ldots + X_6 = n$

$X \sim$ NORMALE

(X_1, X_2) $\quad X_1 = X \;,\; X_2 = X$

$$E\left[\frac{U}{V}\right] = \frac{1}{1}\cdot\frac{1}{36} + \frac{1}{2}\frac{1}{18} + \frac{1}{3}\frac{1}{18} + \text{— — —} + \frac{1}{6}\frac{1}{18} +$$

$$\frac{2}{1}\cdot 0 + \frac{2}{2}\frac{1}{36} + \frac{2}{3}\frac{1}{18} + \text{— —} \frac{2}{6}\frac{1}{18} +$$

$$0 + 0 + \frac{3}{3}\frac{1}{36} + \text{— — —}$$

$$\text{— — — —}$$