STELLAR SPECTROSCOPY I.

MECHANICAL STATISTICS

Lezione VI- Fisica delle Galassie
Cap 8-9 Carrol & Ostlie

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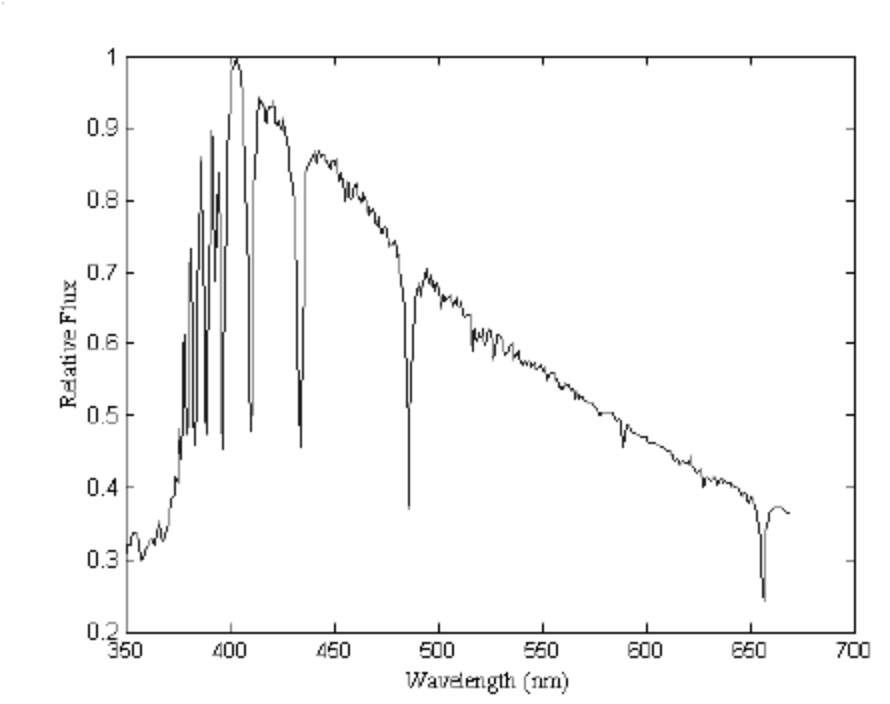
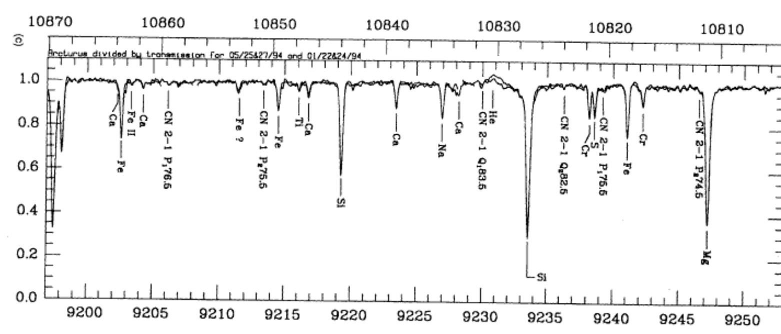
OUTLINE of the lecture:

The aim of this lecture is to learn how we can obtain information about the stellar parameters (temperature, surface gravity, global metallicity and individual element abundances) from a stellar spectrum

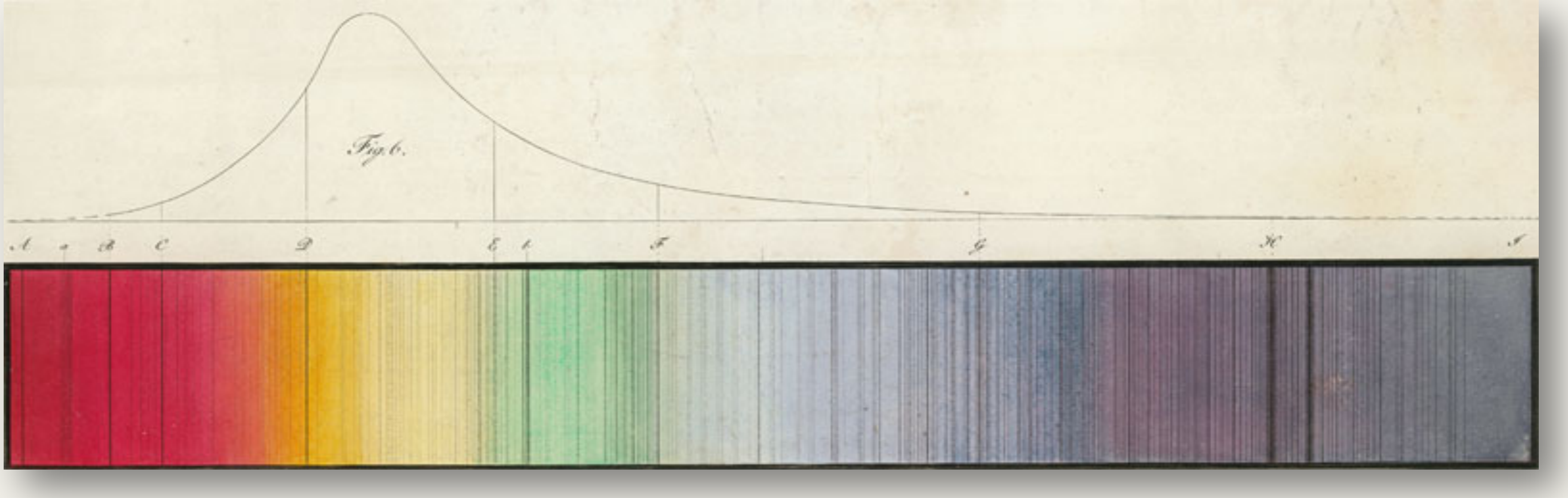




Globally it is a blackbody, but each small portion of the spectrum contains a large quantity of information

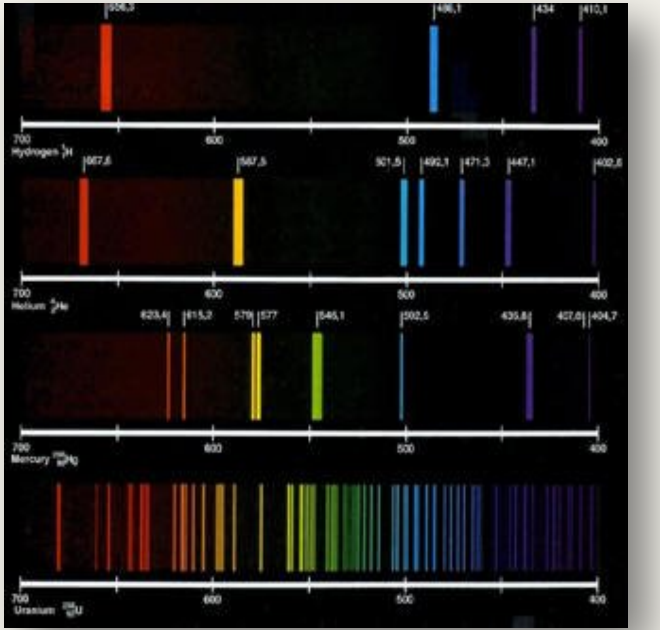
Stellar spectroscopy: a historical overview

- **In 1835,** Auguste Comte, a prominent French philosopher, stated that humans would **never be able to understand the chemical composition of stars.**
- He was soon proved wrong. In the latter half of the 19th century, astronomers began to embrace two new techniques—**spectroscopy** and photography.
- **Joseph Fraunhofer** mounted a prism in front of the objective lens of a small telescope, making for **the first time spectroscopy of the Sun and of bright stars.**
- He found that there were characteristic absorption lines present in the stellar spectra.



Spectra of stars: a historical overview

- A major advance was made in 1859 by Gustav Kirchhoff and Robert Bunsen
- In 1859, Bunsen reported to a colleague that Kirchhoff had made "a totally unexpected discovery." He had identified the cause of the dark lines seen in the solar spectra by Fraunhofer.
- When certain chemicals were heated characteristic bright lines appeared.
- In some cases these were at exactly the same positions in the spectrum as Fraunhofer's dark lines.
- The bright lines came from a hot gas, whereas the dark lines showed absorption of light in the cooler gas above the Sun's surface.



Spectra of stars: a historical overview

- The two scientists found that every chemical element produces a unique spectrum.

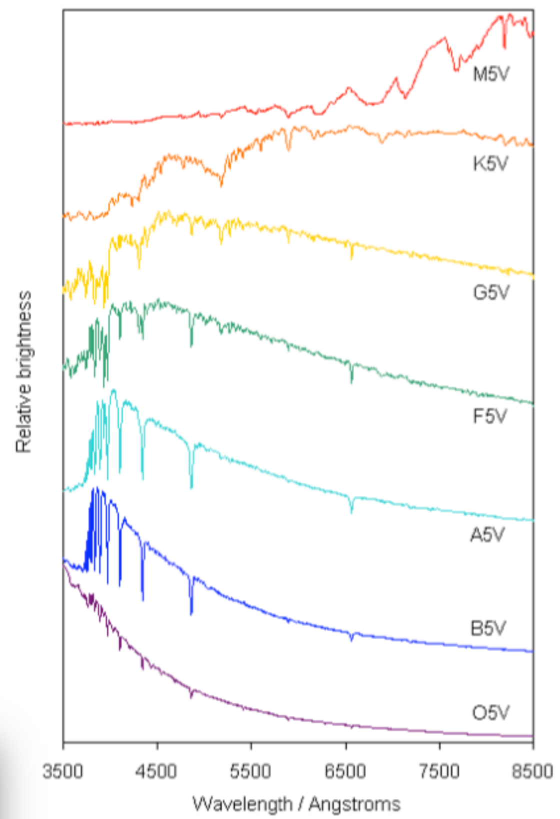
- **Kirchhoff and Bunsen recognized that this could be a powerful tool for "the determination of the chemical composition of the Sun and the fixed stars"**

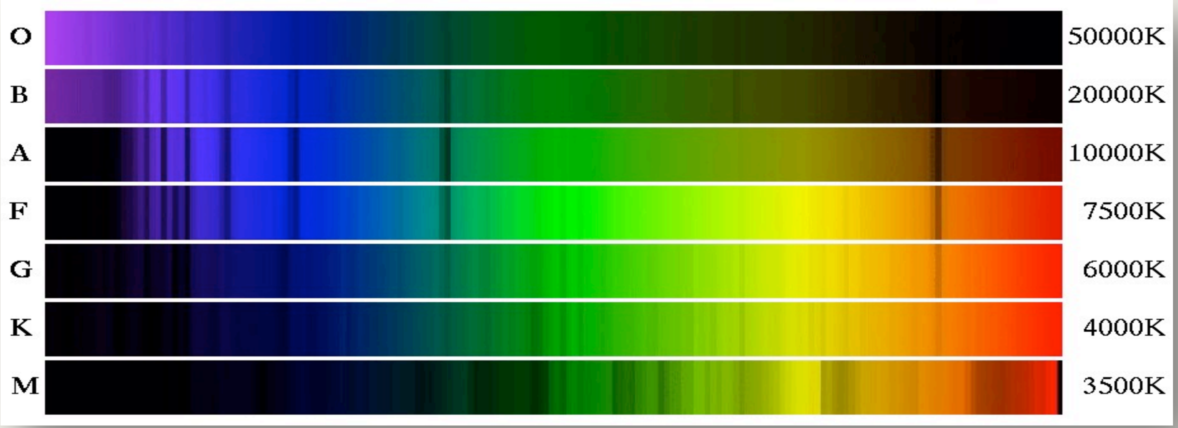
- Kirchhoff identified some **16 different chemical elements among the hundreds of lines observed in the Solar spectrum.**

Spectra of stars: an historical overview

The basis of the modern spectral classifications are funded on the work of Annie J. Cannon

- Cannon expanded the catalog to nine volumes with ~250,000 stars by 1924
- She developed a system of ten spectral types - **O, B, A, F, G, K, M, R, N, S** - that astronomers accepted in 1922.





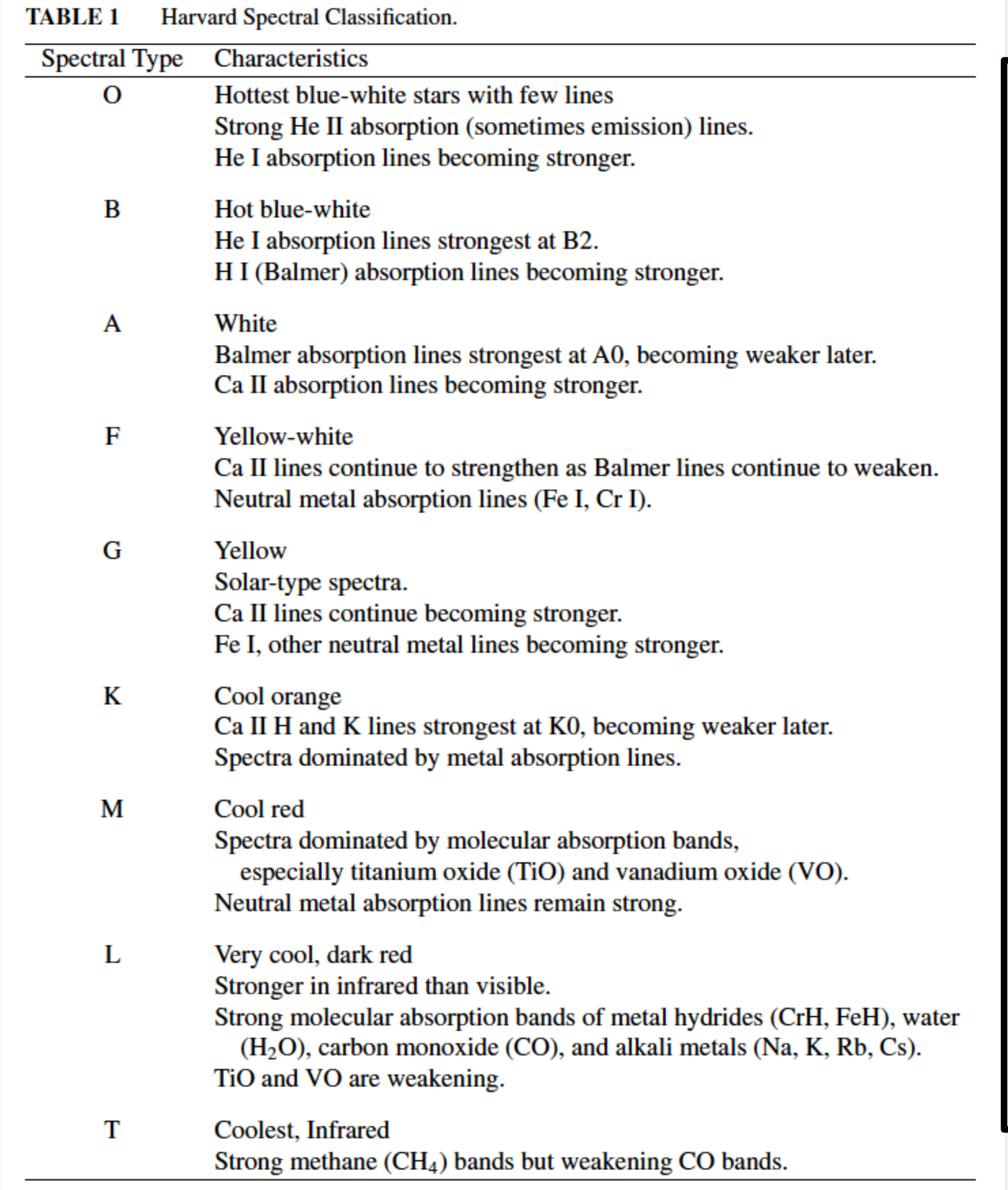


TABLE 1 Harvard Spectral Classification.

Spectral Type	Characteristics
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).
G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.
K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.
M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.
L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.
T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.

- Are the differences due to different composition?
- Or are they driven by different conditions of line formation?

Statistical mechanics:

Statistical properties of a system composed of many members.

For example, a gas that contains a huge number of particles with a large range of speeds and energies, as the gas that composes the stellar photospheres, can be studied through its statistical properties.

SPECTRA OF STARS: A HISTORICAL OVERVIEW

The composition of the Sun

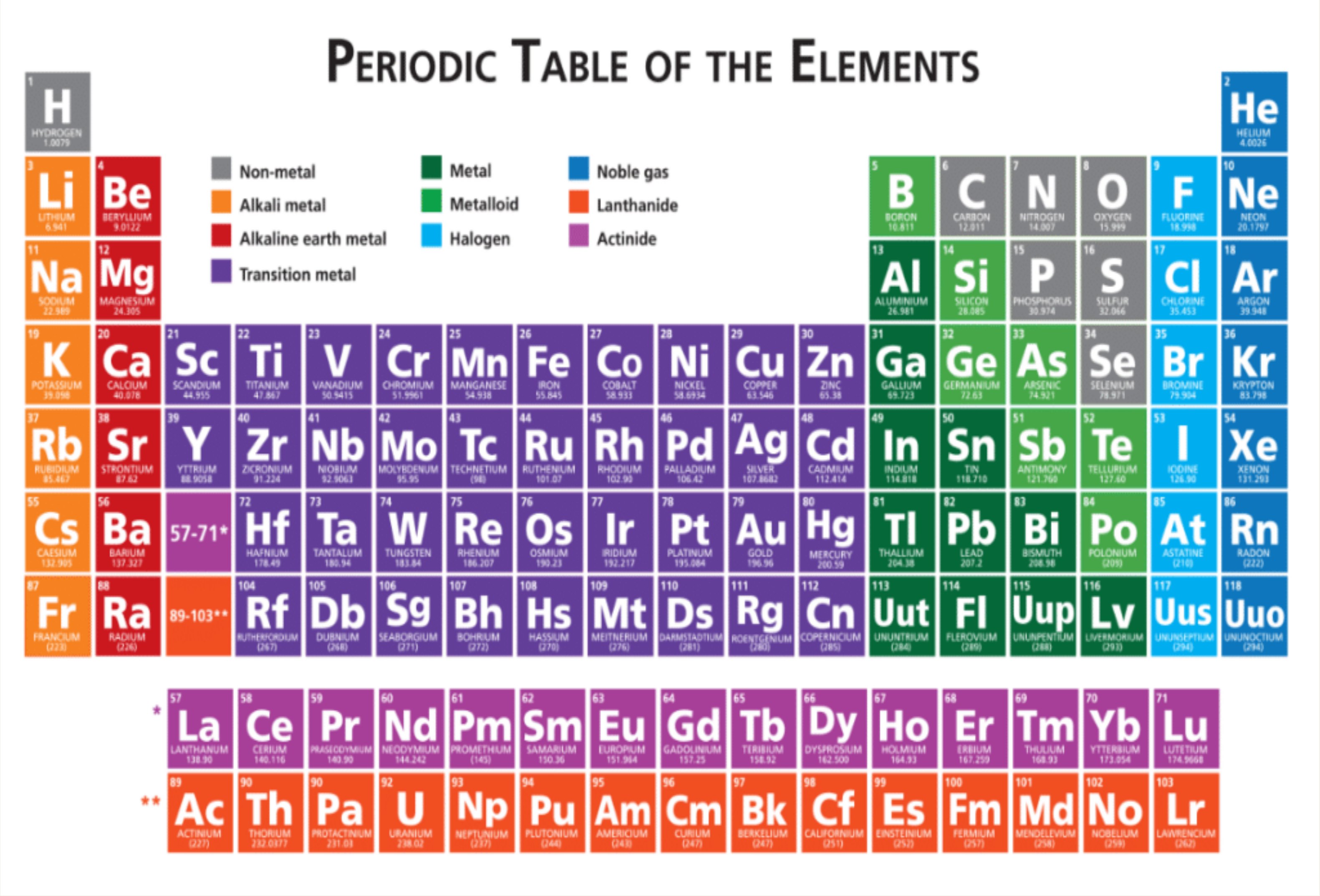
Cecilia Payne-Gaposchkin wrote her PhD Thesis in 1925.

Her major findings are:

- definitively establishing that the spectral sequence did correspond to quantifiable stellar temperatures.
- discovery that stars are made mainly of hydrogen and helium, contrary to the Earth composition

She was dissuaded from this conclusion by astronomer Henry Russel, who thought that stars would have the same composition as Earth. Russell conceded in 1929 that Payne was correct.

How is composed the Universe?



PERIODIC TABLE OF THE ELEMENTS

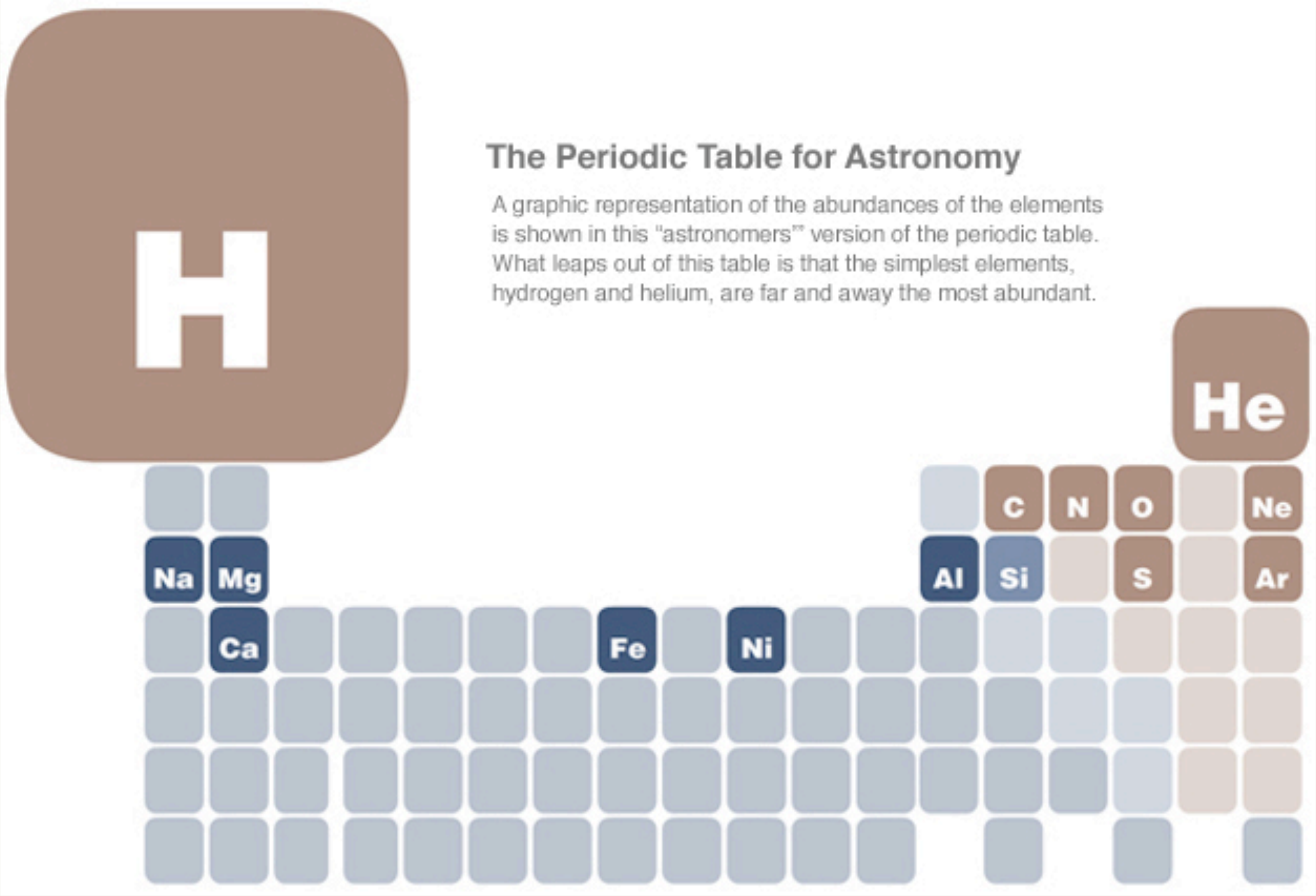
■ Non-metal
■ Alkali metal
■ Alkaline earth metal
■ Transition metal

■ Metal
■ Metalloid
■ Halogen

■ Noble gas
■ Lanthanide
■ Actinide

1 H HYDROGEN 1.0079																	2 He HELIUM 4.0026
3 Li LITHIUM 6.941	4 Be BERYLLIUM 9.0122											5 B BORON 10.811	6 C CARBON 12.011	7 N NITROGEN 14.007	8 O OXYGEN 15.999	9 F FLUORINE 18.998	10 Ne NEON 20.1797
11 Na SODIUM 22.989	12 Mg MAGNESIUM 24.305											13 Al ALUMINIUM 26.981	14 Si SILICON 28.085	15 P PHOSPHORUS 30.974	16 S SULFUR 32.066	17 Cl CHLORINE 35.453	18 Ar ARGON 39.948
19 K POTASSIUM 39.098	20 Ca CALCIUM 40.078	21 Sc SCANDIUM 44.955	22 Ti TITANIUM 47.867	23 V VANADIUM 50.9415	24 Cr CHROMIUM 51.9961	25 Mn MANGANESE 54.938	26 Fe IRON 55.845	27 Co COBALT 58.933	28 Ni NICKEL 58.6934	29 Cu COPPER 63.546	30 Zn ZINC 65.38	31 Ga GALLIUM 69.723	32 Ge GERMANIUM 72.63	33 As ARSENIC 74.921	34 Se SELENIUM 78.971	35 Br BROMINE 79.904	36 Kr KRYPTON 83.798
37 Rb RUBIDIUM 85.467	38 Sr STRONTIUM 87.62	39 Y YTTRIUM 88.9058	40 Zr ZICRONIUM 91.224	41 Nb NIOBIUM 92.9063	42 Mo MOLYBDENUM 95.95	43 Tc TECHNETIUM (98)	44 Ru RUTHENIUM 101.07	45 Rh RHODIUM 102.90	46 Pd PALLADIUM 106.42	47 Ag SILVER 107.8682	48 Cd CADMIUM 112.414	49 In INDIUM 114.818	50 Sn TIN 118.710	51 Sb ANTIMONY 121.760	52 Te TELLURIUM 127.60	53 I IODINE 126.90	54 Xe XENON 131.293
55 Cs CAESIUM 132.905	56 Ba BARIUM 137.327	57-71*	72 Hf HAFNIUM 178.49	73 Ta TANTALUM 180.94	74 W TUNGSTEN 183.84	75 Re RHENIUM 186.207	76 Os OSMIUM 190.23	77 Ir IRIDIUM 192.217	78 Pt PLATINUM 195.084	79 Au GOLD 196.96	80 Hg MERCURY 200.59	81 Tl THALLIUM 204.38	82 Pb LEAD 207.2	83 Bi BISMUTH 208.98	84 Po POLONIUM (209)	85 At ASTATINE (210)	86 Rn RADON (222)
87 Fr FRANCIUM (223)	88 Ra RADIUM (226)	89-103**	104 Rf RUTHERFORDIUM (267)	105 Db DUBNIUM (268)	106 Sg SEABORGIUM (271)	107 Bh BOHRIUM (272)	108 Hs HASSIUM (270)	109 Mt MEITNERIUM (276)	110 Ds DARMSTADTIUM (281)	111 Rg ROENTGENIUM (280)	112 Cn COPERNICIUM (285)	113 Uut UNUNTRIUM (284)	114 Fl FLEROVIUM (289)	115 Uup UNUNPENTIUM (288)	116 Lv LIVERMORIUM (293)	117 Uus UNUNSEPTIUM (294)	118 Uuo UNUNOCTIUM (294)
	*	57 La LANTHANUM 138.90	58 Ce CERIUM 140.116	59 Pr PRASEODYMIUM 140.90	60 Nd NEODYMIUM 144.242	61 Pm PROMETHIUM (145)	62 Sm SAMARIUM 150.36	63 Eu EUROPIUM 151.964	64 Gd GADOLINIUM 157.25	65 Tb TERIBIUM 158.92	66 Dy DYSPROSIUM 162.500	67 Ho HOLMIUM 164.93	68 Er ERBIUM 167.259	69 Tm THULIUM 168.93	70 Yb YTTERBIUM 173.054	71 Lu LUTETIUM 174.9668	
	**	89 Ac ACTINIUM (227)	90 Th THORIUM 232.0377	90 Pa PROTACTINIUM 231.03	92 U URANIUM 238.02	93 Np NEPTUNIUM (237)	94 Pu PLUTONIUM (244)	95 Am AMERICIUM (243)	96 Cm CURIUM (247)	97 Bk BERKELIUM (247)	98 Cf CALIFORNIUM (251)	99 Es EINSTEINIUM (252)	100 Fm FERMIUM (257)	101 Md MENDELEVIUM (258)	102 No NOBELIUM (259)	103 Lr LAWRENCIUM (262)	

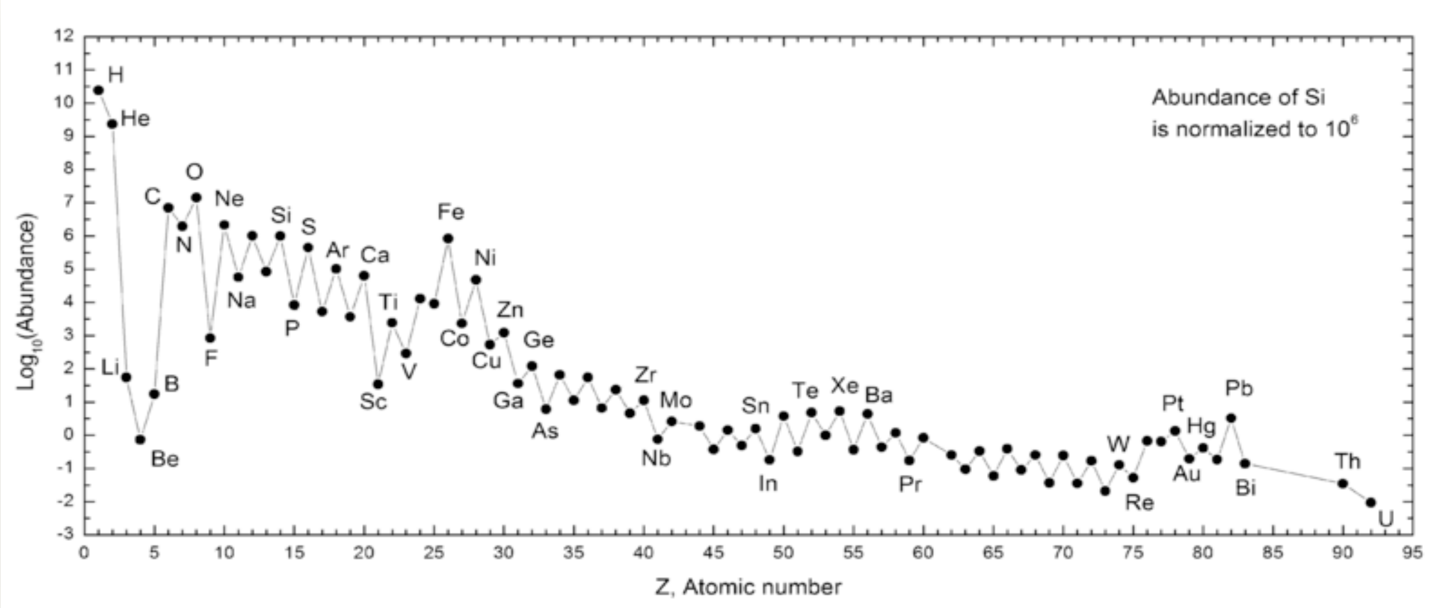
How is composed the Universe?



The Periodic Table for Astronomy

A graphic representation of the abundances of the elements is shown in this "astronomers'" version of the periodic table. What leaps out of this table is that the simplest elements, hydrogen and helium, are far and away the most abundant.

How is composed the Universe?



Elements in the cosmos:

→ Decline in abundances with atomic number

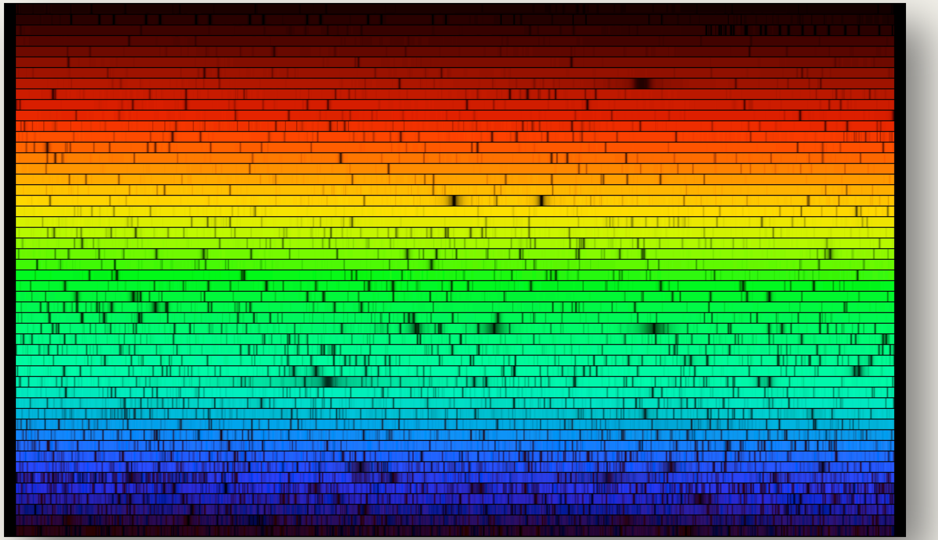
→ "odd-even effect” → elements that are even multiples of a He nucleus are enhanced → result of synthesis by alpha particle capture

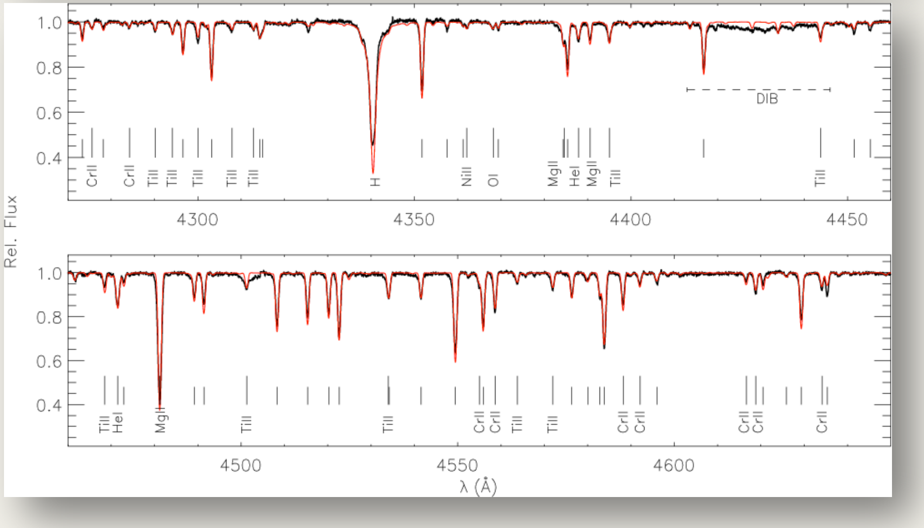
→ Drop in abundance for the light nuclei Li, Be, and B → instability of nuclei of mass 5, making the early creation of these elements in the Universe rare, as well as the easy destruction of these elements in stars,

→ Elements around iron (V, Cr, Mn, Fe, Co, Ni) show enhanced abundance, forming an "iron peak". These elements have the highest binding energy, which is the energy required to remove a nucleon.

From ASTR 5610 (Majewski) Lecture Notes

Stellar parameters and abundances





The abundances of most of these elements are present in stellar photospheres.

From the analysis of the stellar spectrum, we can derive:

- Effective temperature
- Surface gravity (pressure)
- Chemical composition of the stellar atmosphere

For details Gray (chapters 5–14) or Carroll & Ostlie (1996, chapters 9– 10)

Statistical Mechanics

This branch of physics studies the statistical properties of a system composed of many members. For example, a gas can contain a huge number of particles with a large range of speeds and energies.

Although in practice it would be impossible to calculate the detailed behavior of any single particle, the gas as a whole does have certain well-defined properties, such as its temperature, pressure, and density.

Maxwell–Boltzmann velocity distribution function

$$\boxed{n_v\,dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\,4\pi v^2\,dv,}$$

→ Gives the number of gas particles per unit volume having speeds between v and v + dv

→ Valid when the system of particles is assumed to have reached thermodynamical equilibrium

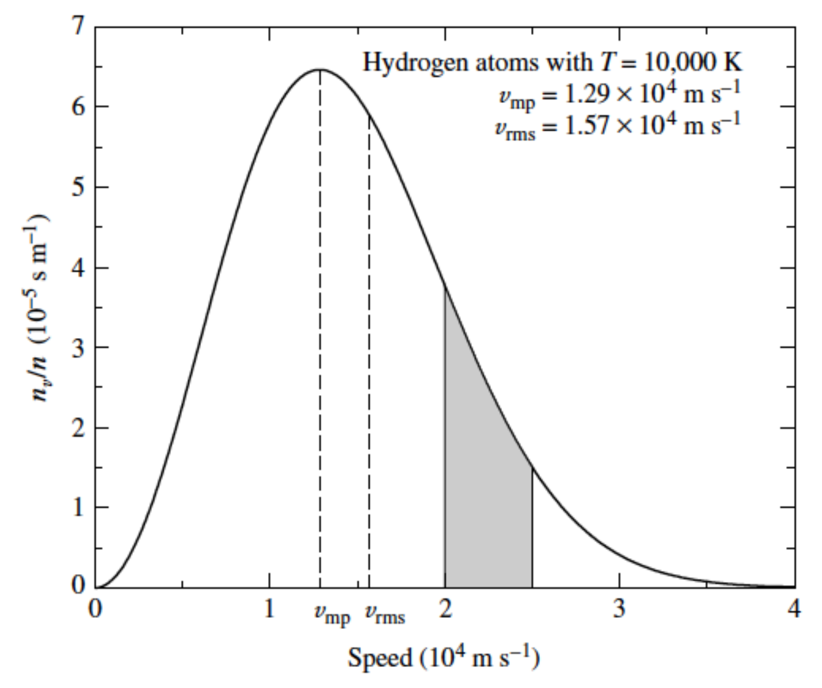
Shape of the distribution and most probable speed

The exponent of the distribution function is the ratio of a gas particle's kinetic energy, ½ m v^2 , to the characteristic thermal energy, kT .

$$v_{\mathrm{mp}} = \sqrt{\frac{2kT}{m}}.$$

the distribution peaks when these energies are equal

Maxwell–Boltzmann distribution function for hydrogen atoms at a temperature of 10,000 K.



→ Integrating the distribution in a velocity interval we have the number of atoms as a function of the temperature of the system

The Boltzmann equation

The atoms of a gas gain and lose energy as they collide.

The distribution in the speeds of the impacting atoms produces a definite distribution of the electrons among the atomic orbitals.

This distribution of electrons is governed by a fundamental result of statistical mechanics:

Orbitals of higher energy are less likely occupied by electrons.

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = e^{-(E_b-E_a)/kT}$$

→ P probability of the system to have energy E, and quantum numbers s (n, l, m_l, m_s).

→ $e^{-E/kT}$ is called the Boltzmann factor

→ Energy are expressed in eV

The Boltzmann factor

The Boltzmann factor plays such a fundamental role in the study of statistical mechanics

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = e^{-(E_b-E_a)/kT}$$

→ If $E_b > E_a$ and $T \rightarrow 0$

→ $-(E_b - E_a)/kT \rightarrow -\infty$

→ $P(s_b)/P(s_a) \rightarrow 0$

→ If we lower the temperature, there isn't any thermal energy available to raise the energy of an atom to a higher level.

→ If $E_b > E_a$ and $T \rightarrow \infty$

→ $-(E_b - E_a)/kT \rightarrow 0$

→ $P(s_b)/P(s_a) \rightarrow 1$

→ If we increase the temperature, any energy level can be reached → but we do not under-populate the lower energy level

Degenerate states and statistical weights

Energy levels of the system may be degenerate, with more than one quantum state having the same energy.

To account properly for the number of states that have a given energy, define **g to be the number of states with energy E**

→ g is called statistical weight

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}$$

The ratio of the probability $P(E_b)$ that the system will be found in any of the g_b degenerate states with energy E_b to the probability $P(E_a)$ that the system is in any of the g_a degenerate states with energy E_a

The Boltzmann equation

Stellar atmospheres contain a vast number of atoms, so the ratio of probabilities is indistinguishable from the ratio of the number of atoms.

$$\boxed{\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}.}$$

The equation of Boltzmann gives us the ratio between the numbers **of atom of a given element** in a given state of ionization (neutral, single ionized, etc.) **in two specific energy levels E_a and E_b, as a function of the system temperature.**

The Boltzmann equation: all levels

- Boltzmann’s Law **[EXCITED STATES]:**

This equation tells us **the probability of an atom to be in a given excited state.**

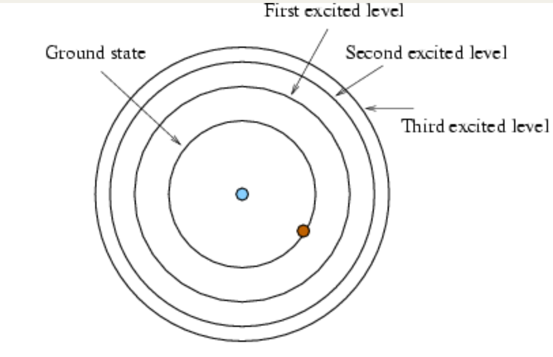
The relative population of excited states in a gas in thermodynamic equilibrium is given by the Boltzmann Excitation Distribution. The number of atoms of energy level n per unit volume N_n is proportional to the total number of atoms (N) of the same species:

$$\frac{N_n}{N} = \frac{g_n}{\mathcal{U}_n(T)} \exp\left(-\frac{E_n}{kT}\right)$$

where g_n is the statistical weight of the nth level, χ_n is the excitation potential of the nth level and $U_n(T)$ is the partition function of the particle in a gas of temperature T

Example: Balmer lines

For a gas of neutral hydrogen atoms, at what temperature will equal numbers of atoms have electrons in the ground state (n = 1) and in the first excited state (n = 2)?



Hydrogen atom, with the four lowest energy levels marked

$N_1/N_2=1$ → same number

$g_n=2\,n^2$ → $g_1=2$ for n=1 and $g_2=8$ for n=2 → number of degenerate states for Hydrogen

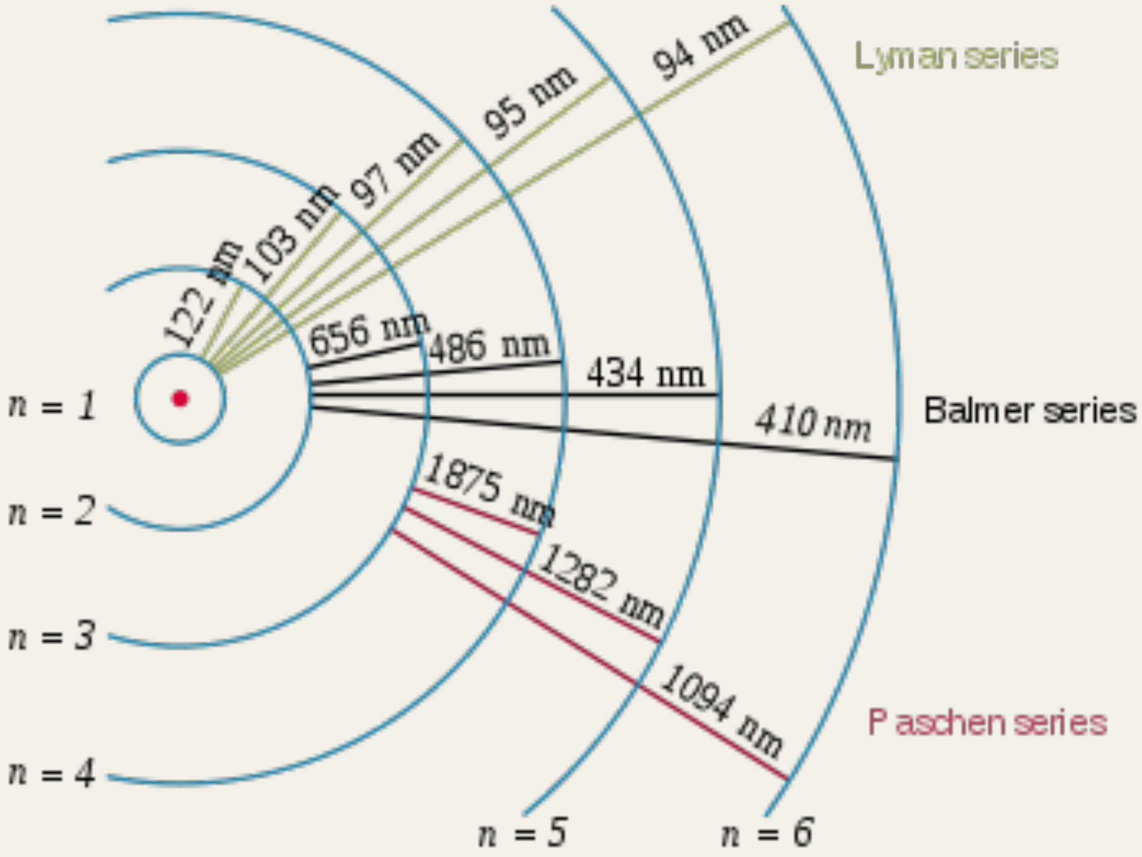
$1=N_1/N_2=4 \times e^{-[(-13.6\text{ eV}/4)-(-13.6\text{ eV}/1))]/kT}$ → Boltzmann's equation

(considering the energy levels of the Hydrogen atom $E = -E_0/n^2$, where $E_0 = 13.6$ eV)

$T= 10.2\text{ eV}/(k\ln(4)) = 8.5 \times 10^4$ K (where $k = 8.617\text{x}10{-}5\text{ eV K}^{-1}$).

Very high temperatures are required for a significant number of hydrogen atoms to have electrons in the first excited state.

Example: Balmer lines



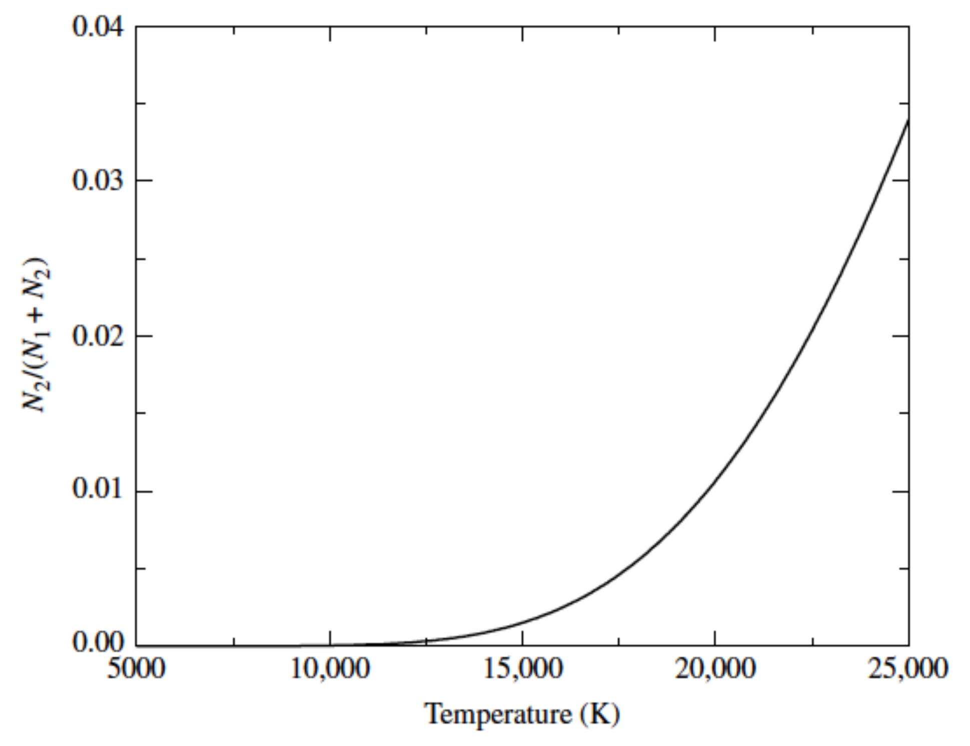
Example: Balmer lines

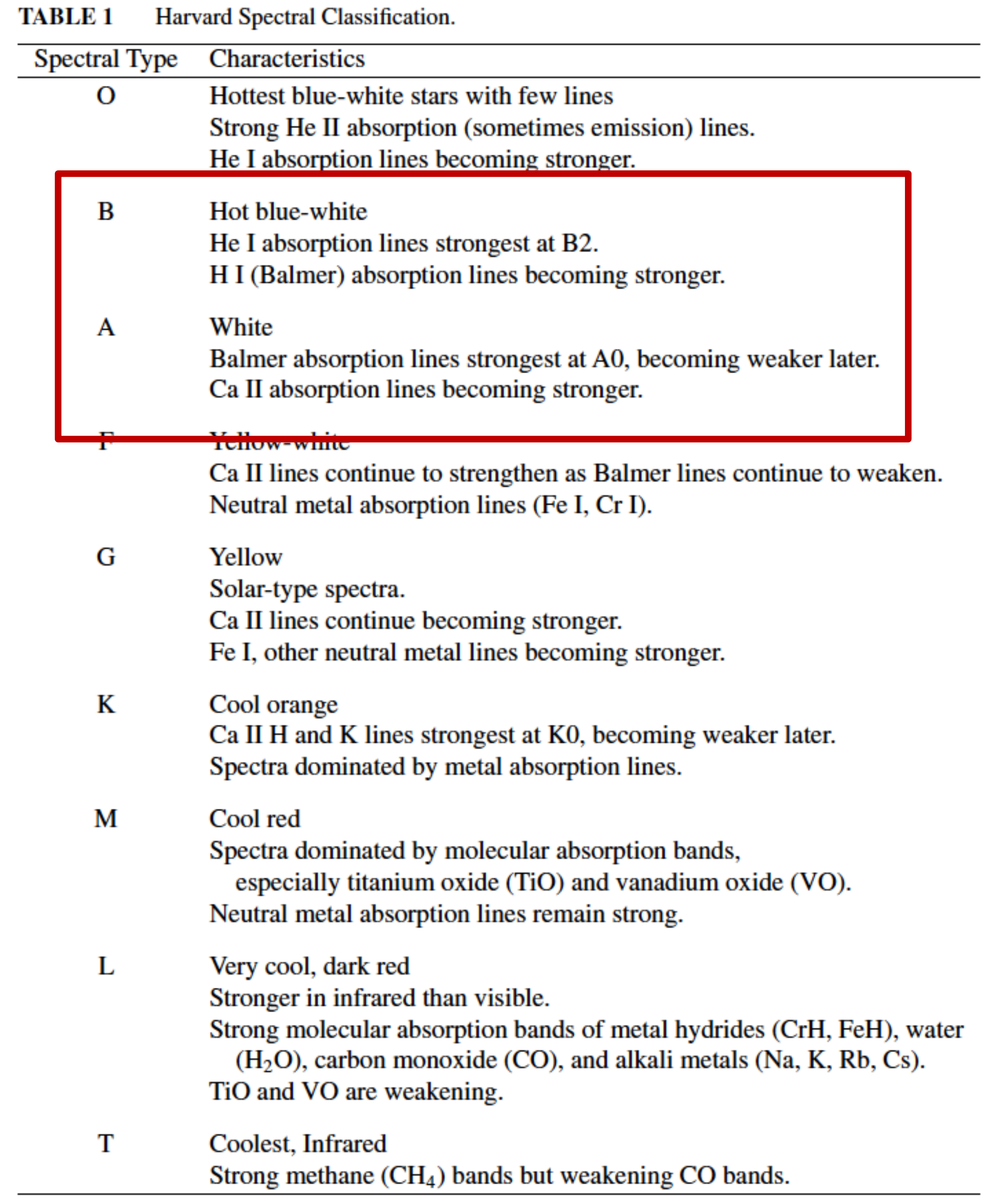
The Balmer absorption lines are produced by electrons in hydrogen atoms making an upward transition from the n= 2 orbital.

However, why do the Balmer lines reach their maximum intensity at a much lower temperature of ~9500 K and they are not present in the hottest stars?

Occupancy of the ground and first excited states as a function of temperature

→ Increasing the temperature does not increase the intensity of the Balmer lines





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T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.

The Saha equation: the partition function

The answer is related to the relative number of atoms in **different stages of ionization**

→ **If we increase the temperature, we give to the atom enough energy to ionize and to lose its electron**

→ For instance, the energy necessary to ionize H is 13.6 eV (from the ground state)

If the atom and ion are not in the ground state, we have to take an average over the orbital energies to allow for the possible partitioning of the atom's electrons among its orbitals.

E_J excited states

E_1 Ground state

$$Z = \sum_{j=1}^{\infty} g_j \, e^{-(E_j - E_1)/kT} .$$

Degenercy

The **partition function is the weighted sum of the number of ways the atom can arrange its electrons with the same energy**, with more energetic (and therefore less likely) configurations receiving less weight from the Boltzmann factor when the sum is taken.