MODEL ONLY OUTFLOW

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$$W(t) = \lambda(1-R)\psi$$

WIND PROP. TO SFR

$$\frac{dM_{TOT}}{dt} = -\lambda(1-R)\psi$$

$$\frac{dM_{gas}}{dt} = -\psi + E(t) - \lambda(1-R)\psi = -(1-R)\psi - \lambda(1-R)\psi$$

$$= -(1+\lambda)(1-R)\psi$$

$$\frac{dM_Z}{dt} = -\psi Z + E_Z(t) - \lambda Z(1-R)\psi$$

$$= -\psi Z + \psi Z R + (1-R)y_Z\psi - \lambda Z(1-R)\psi$$

$$\cancel{\frac{dM_{gas}}{dt}} Z + \frac{dZ}{dt} M_{gas} = \cancel{-\psi Z} + \cancel{\psi Z R} + (1-R)y_Z\psi - \lambda Z \cancel{(1-R)}\psi$$

$$\frac{dZ M_{gas}}{dM_{gas}} = \frac{-y_Z}{(1+\lambda)}$$

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$$dz = \frac{dM_{gs}}{M_{gs}}\left(\frac{-g_t}{1+\lambda}\right)$$

$$\boxed{M_{gs}(0) \neq M_{TOT}(t)}$$

IMPORTANT

$$z(t) = \int_{M_{gs}(0)}^{M_{gs}(t)} dM_{gs}$$

$$= \frac{g_t}{1+\lambda}\ln\left(\frac{M_{gs}(0)}{M_{gs}(t)}\right)$$

$$\boxed{M_{\cancel{gs}}\cancel{(0)} \cancel{=} M_{gs}(0)}$$

$$M_{TOT}(0) = M_{gs}(0)$$

$$\frac{dM_{TOT}}{dM_{gs}} = \frac{\lambda}{1+\lambda}$$

$$M_{TOT}(t) - M_{TOT}(0) = \left(M_{gs}(t) - M_{gs}(0)\right)\left(\frac{\lambda}{1+\lambda}\right)$$

$$M_{gs}(0)\left(1 - \frac{\lambda}{1+\lambda}\right) = M_{TOT}(t) - \frac{\lambda}{1+\lambda}M_{gs}(t)$$

$$M_{gs}(0) = (1+\lambda)M_{TOT}(t) - \lambda M_{gs}(t)$$

$$\boxed{z = \frac{g_t}{(1+\lambda)}\ln\left((1+\lambda)\mu^{-1} - \lambda\right)}$$

ONLY INFALL

$$M_{gas}(0) = M_0$$

SMALL MASS

$$\frac{dM_{TOT}}{dt} = \Lambda(1-R)\psi \qquad A(t) = \Lambda(1-R)\psi$$

$$\frac{dM_{gas}}{dt} = -(1-R)\psi + \Lambda(1-R)\psi = (\Lambda-1)(1-R)\psi$$

CONSTANT

$$\frac{dM_Z}{dt} = -\psi Z + R\psi Z + (1-R)y_Z\psi + \Lambda Z_A(1-R)\psi$$

$$= (-Z + y_Z + \Lambda Z_A)(1-R)\psi$$

$$\cancel{\frac{dM_{gas}}{dt}} Z + M_{gas}\frac{dZ}{dt} =$$

$$(Z_A\Lambda + y_Z - Z\Lambda)(1-R)\psi$$

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$$\frac{M_{gs}\, dZ}{dM_{gs}} = \frac{(Z_A \Omega + y_t - Z\Omega)}{\Omega - 1} \qquad x = -Z\Omega$$

$$\int_{M_0}^{M_{gs}} \frac{dM_{gs}}{M_{gs}} = \int_0^{Z} \frac{(\Omega - 1)\; dZ}{(Z_A \Omega + y_t - Z\Omega)}$$

$$\ln\left(\frac{M_{gs}}{M_0}\right) = -\frac{(\Omega - 1)}{\Omega} \ln\left(\frac{Z_A\Omega - Z\Omega + y_z}{Z_A\Omega + y_z}\right)$$

6 I WANT ~~VALU~~ QUANTITIES COMPUTED AT TIME t !!!

$$\frac{dM_{gs}}{dM_{TOT}} = \frac{\Omega - 1}{\Omega}$$

$$M_{gs}(t) - M_{gs}(0) = (M_{TOT}(t) - \underset{\equiv\, M_{gs0}}{M_{TOT}(0)})\, \frac{\Omega - 1}{\Omega}$$

$$M_{gs}(0)\left(1 - \frac{\Omega - 1}{\Omega}\right) = M_{gs}(t) - M_{TOT}(t)\, \frac{\Omega - 1}{\Omega}$$

$$M_{gs}(0)\left(\frac{1}{\Omega}\right) = M_{gs}(t) - \frac{(\Omega - 1)}{\Omega} M_{TOT}(t)$$

$$M_{gs}(t) = M_{gs}(t)\,\Omega - (\Omega - 1)\, M_{Tot}(t)$$

$$\ln\left(\frac{\Omega M_{gs}(t) - (\Omega - 1) M_{Tot}(t)}{M_{gs}(1)}\right) = \frac{\Omega - 1}{\Omega} \ln\left(\frac{z_A \Omega - z\Omega + y_t}{z_A \Omega + y_t}\right)$$

$$\left(\Omega - (\Omega - 1)\bar{\mu}^{-1}\right)^{\Omega/\Omega - 1} = \frac{z_A \Omega - z\Omega + y_t}{z_A \Omega + y_t}$$

$$(z_A \Omega + y_t)\left(\Omega - (\Omega - 1)\bar{\mu}^{-1}\right)^{\Omega/(\Omega - 1)} = z_A \Omega - z\Omega + y_t$$

$$z\Omega = (z_A \Omega + y_t)\left(1 - \left[\Omega - (\Omega - 1)\bar{\mu}^{-1}\right]^{\Omega/\Omega - 1}\right)$$

$$z = \frac{\dots}{\Omega}$$

EXTREME INFALL

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$\alpha = 1 \qquad \lambda = 0 \qquad Z_A = 0$

$M_{gas}(0) = M_0$

$$\frac{dM_{TOT}}{dt} = (1-R)\psi$$

$$\frac{dM_{gas}}{dt} = (1-R)\psi - (1-R)\psi = 0$$

$$\frac{dM_Z}{dt} = -\psi Z + \psi Z R + y_Z(1-R)\psi$$

$$\frac{dM_Z}{dt} = (-Z + y_Z)(1-R)\psi$$

$$\frac{dM_Z}{dt} = M_{gas}\frac{dZ}{dt} + \cancel{Z\frac{dM_{gas}}{dt}} =$$

I CAN~~T~~ NOT DIVIDE BY $M_{gas} = 0$

$$M_{gas}\frac{dZ}{dt} = (y_Z - Z)(1-R)\psi$$

$$\frac{M_{gas}\, dZ}{dM_{TOT}} = (y_Z - Z)$$

$$\frac{dZ}{y_Z - Z} = \frac{1}{M_{gas}}\int_{M_{gas}(0)}^{M_{TOT}} dM_{TOT}$$

$$-\ln\left(\frac{y_z - z}{y_z}\right) = M_{Tot} - \underset{\substack{\parallel \\ M_{ges}(t)}}{M_{ges}(0)} \qquad \frac{1}{M_{ges}(t)}$$

$$= -\ln\left(\frac{y_z - z}{y_z}\right) = \mu^{-1} - 1$$

$$y_z - z = y_z \, e^{-(\mu^{-1} - 1)}$$

$$z = y_z\left(1 - e^{-(\mu^{-1} - 1)}\right)$$

INFALL +OUTFLOW

$$\frac{dM_{TOT}}{dt} = (\Lambda - \lambda)(1-R)\psi$$

$$\frac{dM_{gas}}{dt} = (\Lambda - \lambda - 1)(1-R)\psi$$

$$\frac{dM_Z}{dt} = -\psi Z + R\psi Z + (1-R)\,y_Z\,\psi + \Lambda Z_A(1-R)\psi - \lambda Z(1-R)\psi$$

$$= \left(-Z + y_Z + \Lambda Z_A - \lambda Z\right)(1-R)\psi(t)$$

$$M_g(0) = M_T(0) = M_0$$

$$\frac{dM_g}{dM_{TOT}} = \frac{(\Lambda - \lambda - 1)}{(\Lambda - \lambda)}$$

$$M_g = \frac{(\Lambda - \lambda - 1)}{\Lambda - \lambda} M_T + K$$

$$K = M_g(0) - \frac{(\Lambda - \lambda - 1)}{\Lambda - \lambda} M_T(0)$$

$$K = \frac{M_0}{\Lambda - \lambda}$$

$$M_0 = (\Lambda - \lambda) M_{ges} - (\Lambda - \lambda - 1) M_T$$

$$\frac{dM_z}{dt} = \frac{d(zM_g)}{dt} = z\frac{dM_g}{dt} + M_g\frac{dz}{dt}$$

$$(z_A \Lambda - z - z\lambda + g_z)(1-R)\psi(t) = M_g\frac{dz}{dt} + z(\Lambda - \lambda - 1)(1-R)\psi(t)$$

$$M_g\frac{dz}{dt} = (z_A \Lambda - z - z\lambda + g_z - z\Lambda + z\lambda + z)(1-R)\psi(t)$$

$$M_g\frac{dz}{dt} = (z_A \Lambda + g_z - z\Lambda)(1-R)\psi$$

DIVIDO

$$\frac{M_g\, dz}{dM_g} = \frac{(z_A \Lambda + g_z - z\Lambda)}{\Lambda - \lambda - 1}$$

$$\int_{M_0}^{M_g} \frac{dM_g}{M_g} = \int_0^{Z} \frac{(\Omega-\lambda-1)}{Z_A\Omega - Z\Omega + y_Z}\, dZ$$

$\log(x^k) = k \log x$

$$\ln\left(\frac{M_g}{M_0}\right) = \frac{-(\Omega-\lambda-1)}{\Omega}\ \ln\left(\frac{Z_A\Omega - Z\Omega + y_Z}{Z_A\Omega + y_Z}\right)$$

$$\ln\left(\frac{(\Omega-\lambda)M_{ges} - (\Omega-\lambda-1)M_T}{M_{ges}}\right) = \frac{(\Omega-\lambda-1)}{\Omega}\ \ln\left(\frac{Z_A\Omega - Z\Omega + y_Z}{Z_A\Omega + y_Z}\right)$$

$$\left((\Omega-\lambda) - (\Omega-\lambda-1)\bar{\mu}^{-1}\right)^{\Omega/\Omega-\lambda-1} = \left(\frac{Z_A\Omega - Z\Omega + y_Z}{Z_A\Omega + y_Z}\right)$$

$$(Z_A\Omega + y_Z)\left((\Omega-\lambda) - (\Omega-\lambda-1)\bar{\mu}^{-1}\right)^{\Omega/\Omega-\lambda-1} = Z_A\Omega - Z\Omega + y_Z$$

$$Z\Omega = (Z_A\Omega + y_Z)\left(1 - \left[(\Omega-\lambda) - (\Omega-\lambda-1)\bar{\mu}^{-1}\right]^{\Omega/\Omega-\lambda-1}\right)$$

$$Z = \left(Z_A + \frac{y_Z}{\Omega}\right)$$