

# Cyber-Physical Systems

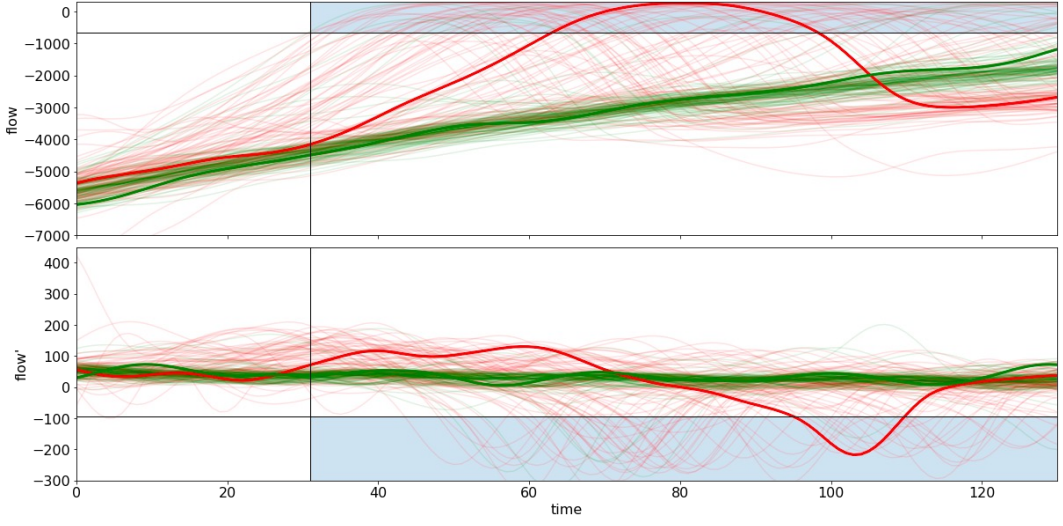
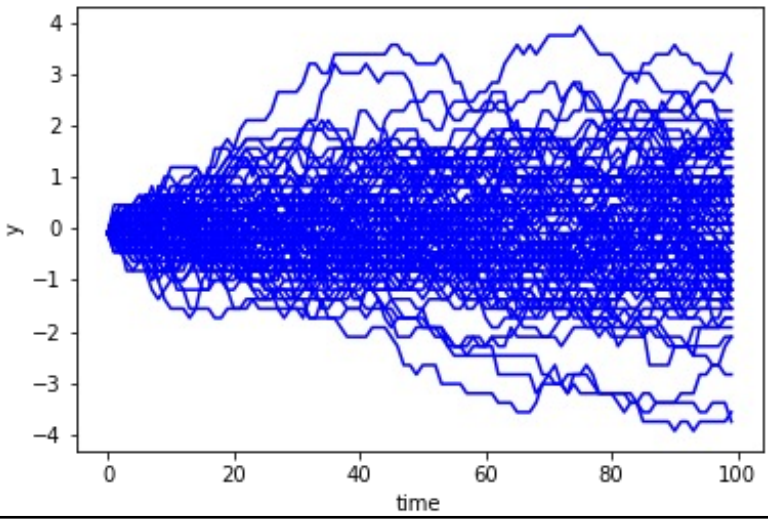
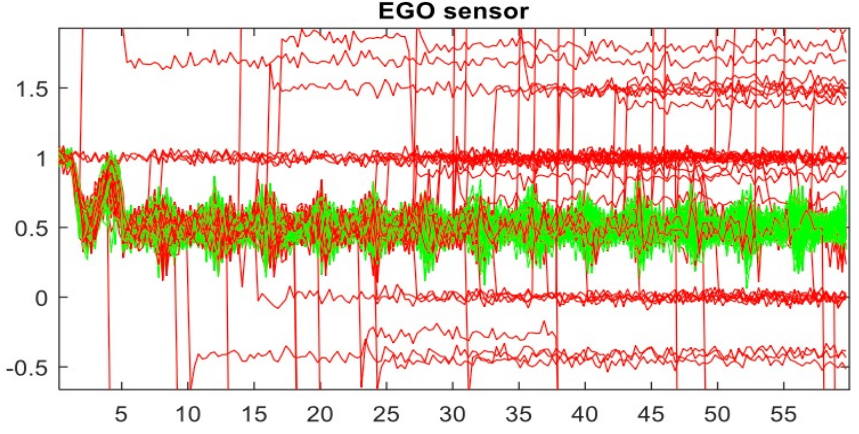
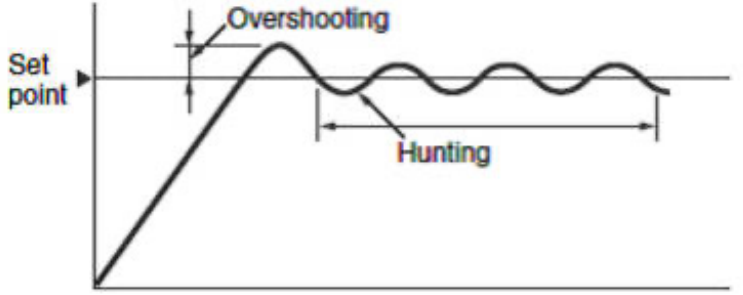
Laura Nenzi

Università degli Studi di Trieste  
I Semestre 2023

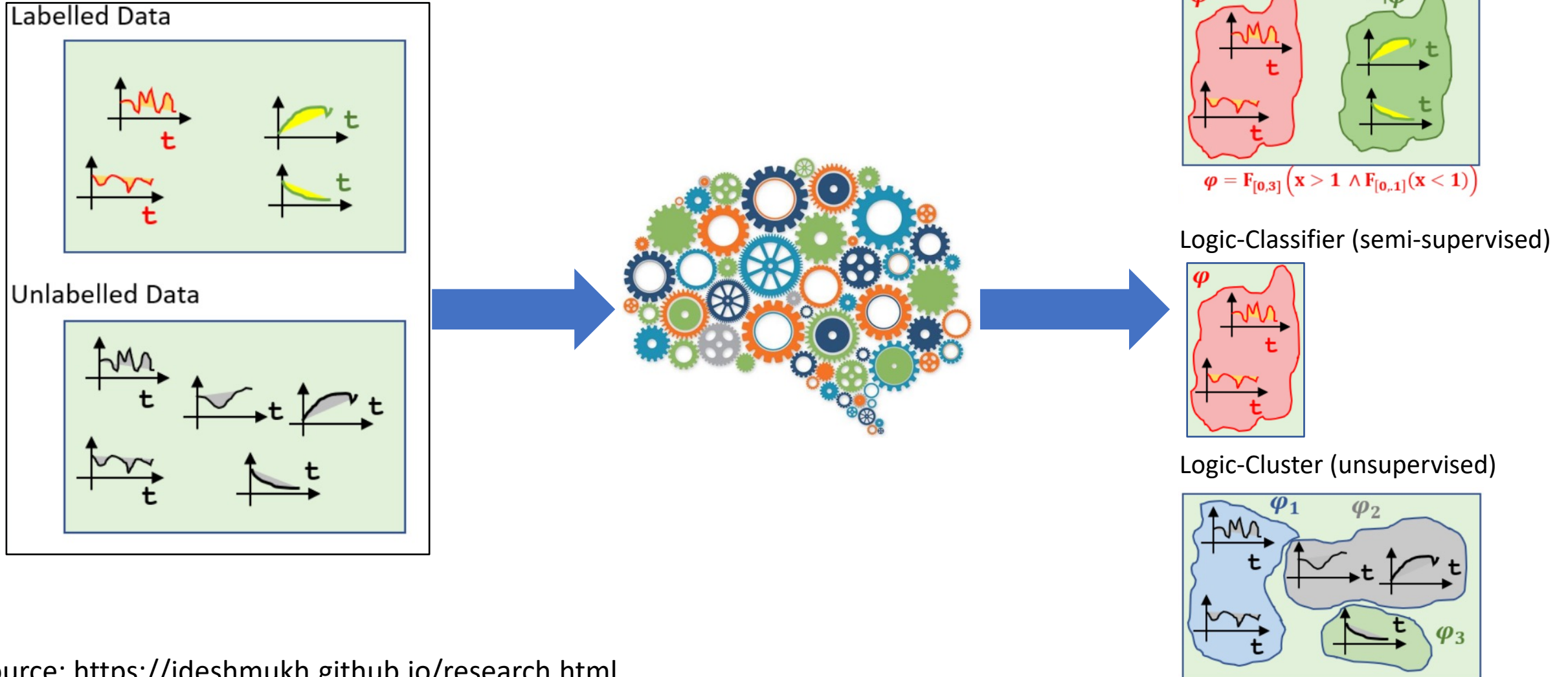
Lecture 21: STL learning

# Context and Problem

Data are trajectories



# Temporal Logic requirement mining



# STL Classifier: Problem Statement

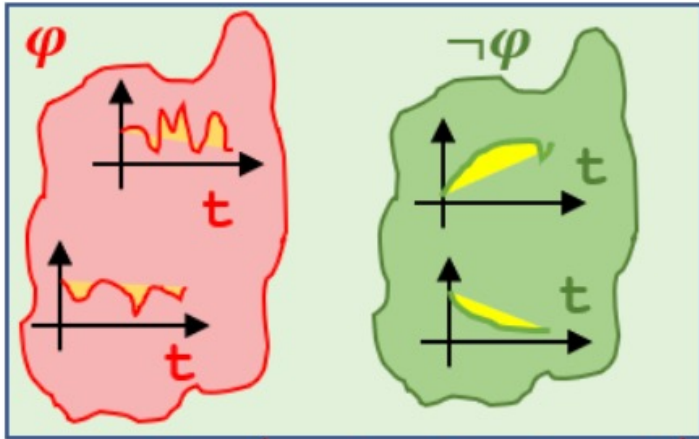
---

We want a way to search in the space of STL formulae considering training data  $X_{learn}$

## Supervised two-class classification problem

Training data set: two sets

- regular  $X_{learn}^+$
- anomalous  $X_{learn}^-$

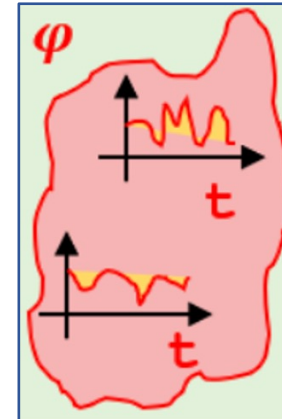


Find the best  $\phi$  that better separates the two sets.

## Semi-supervised one-class classification problem

Training data set: one set

- regular  $X_{learn}^+$



Find the “tight”  $\phi$  that is satisfied by the set

# Parametric Signal Temporal Logic

## Definition (PSTL syntax)

$$\phi := (x_i \bowtie \pi) \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \mathcal{U}_{[\tau_1, \tau_2]} \varphi_2$$

with  $\bowtie \in \{>, \leq\}$

- ▶  $\pi$  is **threshold** parameter
- ▶  $\tau_1, \tau_2$  are **temporal** parameters

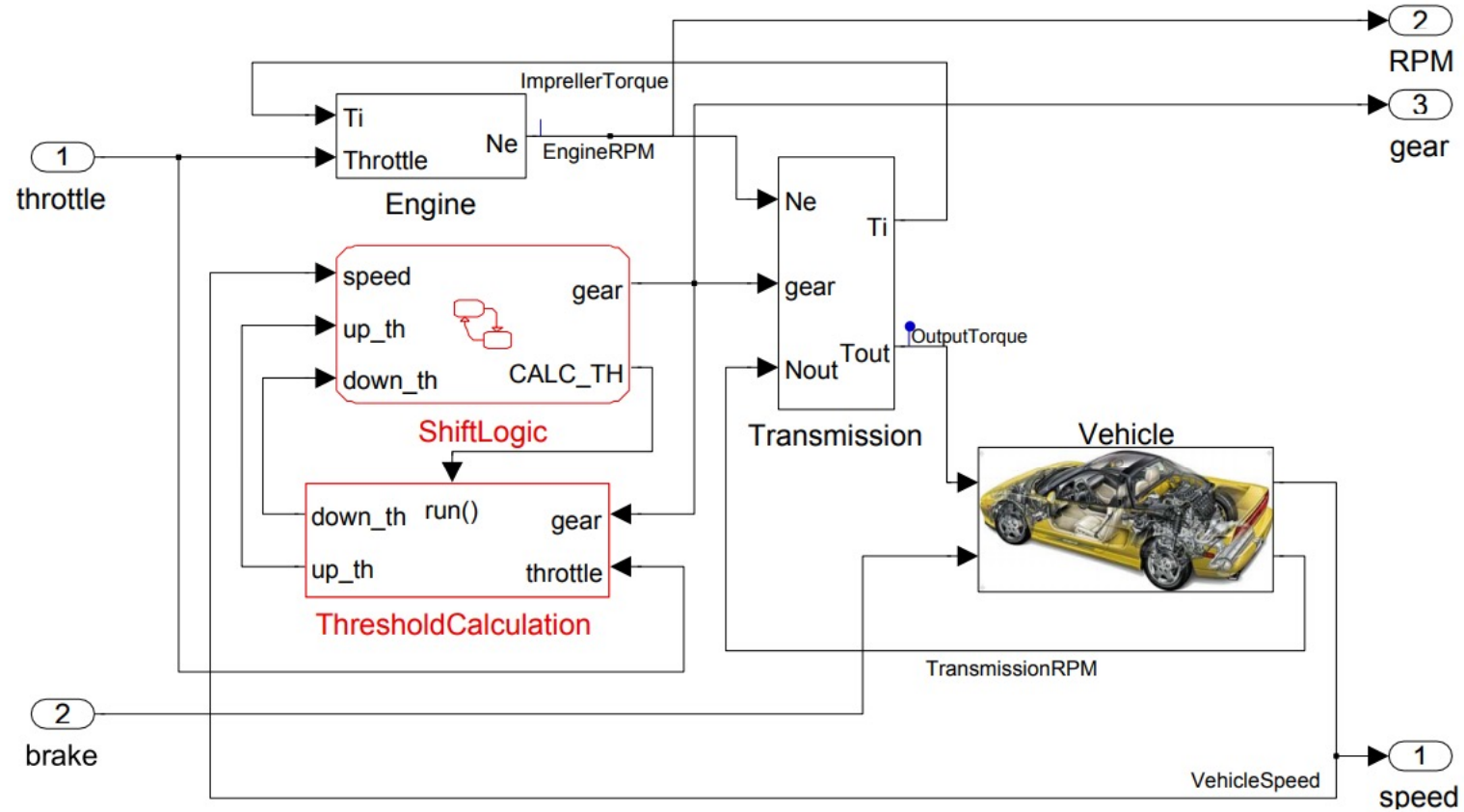
- ▶  $\mathbb{K} = (\mathcal{T} \times \mathcal{C})$  be the **parameter space**
- ▶  $\theta \in \mathbb{K}$  is a **parameter configuration**

e.g.,  $\phi = \mathcal{F}_{[a,b]}(x_i > k), \theta = (0, 2, 3.5)$  then  $\phi_\theta = \mathcal{F}_{[0,2]}(x_i > 3.5)$ .

# Specification Mining

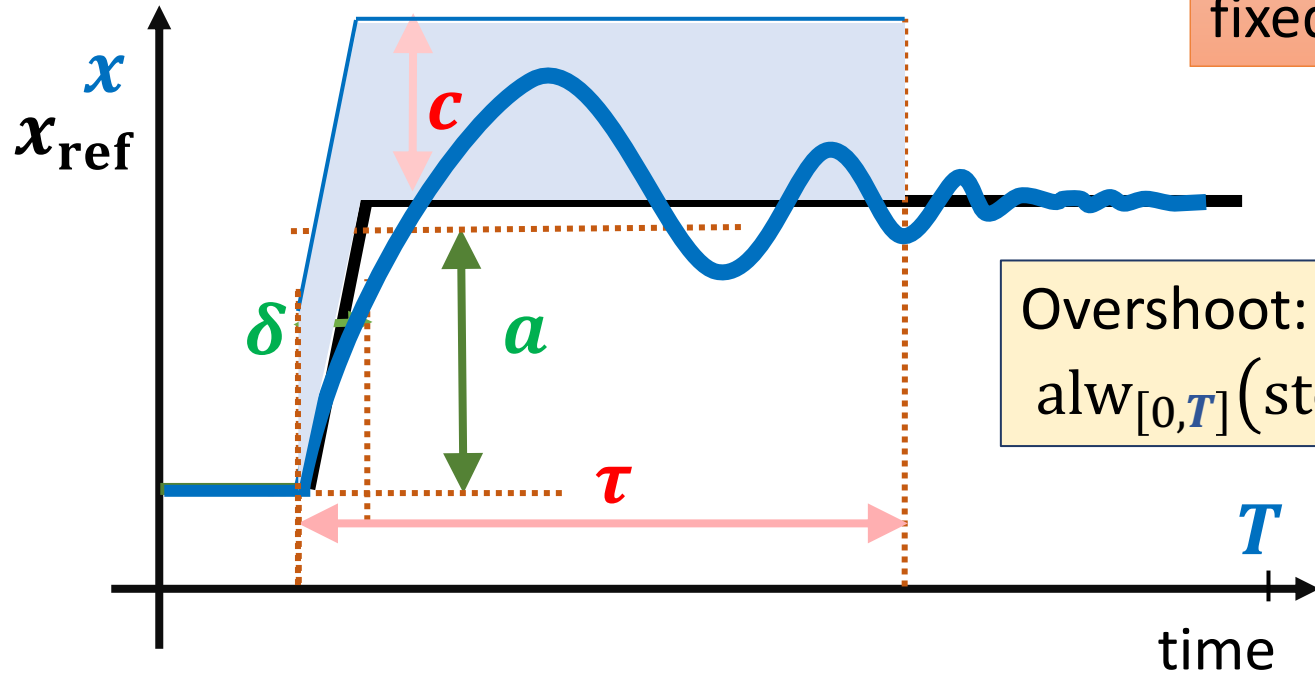
- ▶ Specification Mining: Try to find values of parameters of a PSTL formula from a given model
- ▶ Why?
  - ▶ Good to know “as-is” properties of the model
  - ▶ Finds worst-case behaviors of the model
  - ▶ Discriminates between regular and anomalous behaviours

# Specification Mining



- What is the maximum speed that the vehicle can reach ?
- What is the minimum dwell time in a given gear ?

# Specification Templates using PSTL



In previous lecture,  $a, c, T, \tau, \delta$  were some fixed values, here they represent parameters

Step:

$$\text{step}(y) := y(t + \delta) - y(t) > a$$

Overshoot:

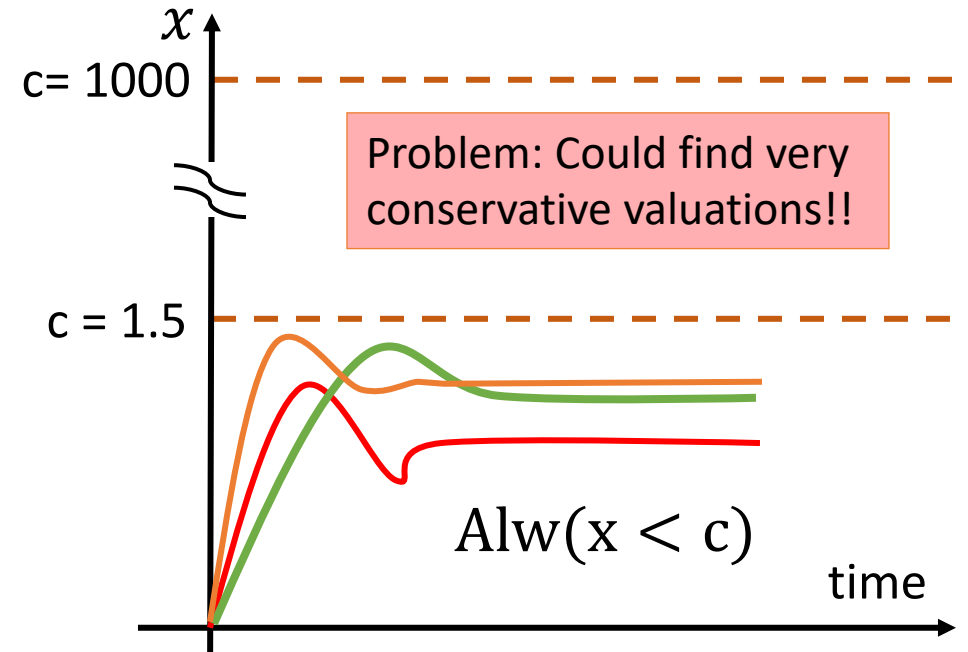
$$\text{alw}_{[0, T]}(\text{step}(x_{\text{ref}}) \Rightarrow \text{alw}_{[0, \tau]}(x(t) - x_{\text{ref}}(t) < c))$$



# Parameter inference for PSTL

- ▶ Given:
  - ▶ PSTL formula  $\varphi(\mathbf{p})$ , [ $\mathbf{p} = (p_1, p_2, \dots, p_m)$ ]
  - ▶ Traces  $x_1, \dots, x_n$
- ▶ Find:
  - ▶ ~~Valuation~~  $v(\mathbf{p})$  such that:  $\forall i : x_i \models \varphi(v(\mathbf{p}))$   
 *$\delta$ -tight valuation*
  - and  $\exists i : x_i \not\models \varphi(v(\mathbf{p}) \pm \delta)$  :  
i.e. small perturbation in  $v(\mathbf{p})$  makes some trace not satisfy formula

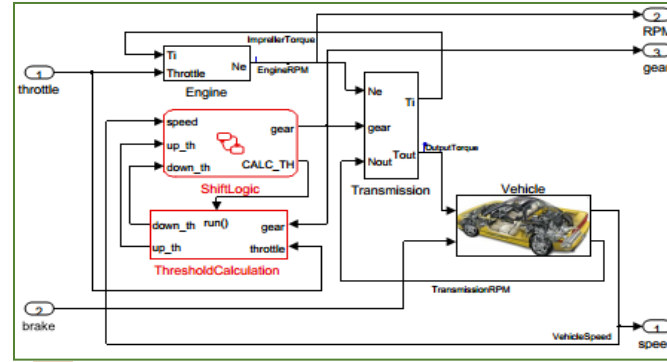
Finding  $\delta$ -tight valuations hard in general, but **efficient** for **Monotonic PSTL**



formula sat for given valuation  $\Rightarrow$   
 $\forall$  greater (or lesser) valuations sat

Binary search on parameter space

# Specification Mining



Falsification:  
 $\exists \text{ trace} \neq \text{Property?}$

Secret Sauce:

- Infer parameters for a given PSTL formula from traces
- Falsify given STL formula

Parameters  
Find "Tightest" Answers

Settling Time is ??  
Overshoot is ??  
Bounds on x are ??

Settling Time is 6 ms  
Overshoot is 5.5 KPa  
Upper Bound on x is 5

Settling Time is 6 ms  
Overshoot is 5.5 KPa  
Upper Bound on x is 5

NO

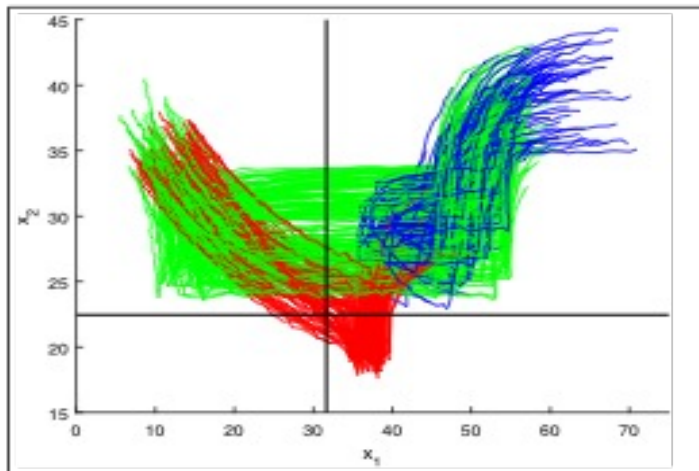
YES

# Learning STL classifiers

**Goal:** Given sets of good and bad trajectories (or generative models), learn STL properties that can separate the two behaviours (a STL classifier)

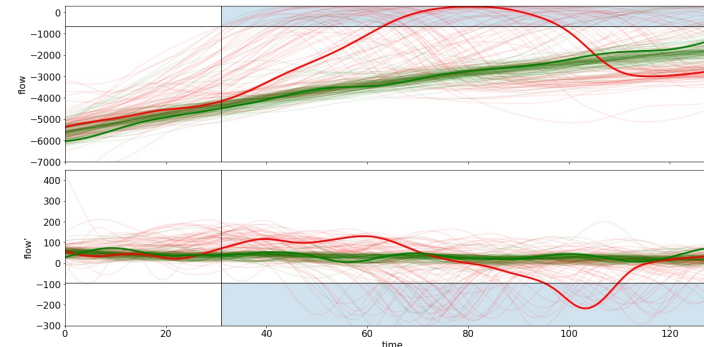
**Idea:** for a fixed template formula, learn optimally separating parameters by **Bayesian Optimisation**.

$$\phi = ((x_2 > 22.46) \mathcal{U}_{[49,287]}(x_1 \leq 31.65))$$



Maritime surveillance

$$\phi = \mathcal{F}_{[31,130]}((flow \geq -670) \vee (flow' \leq -94))$$



Light entrainment of biological oscillator

**Idea:** explore formula structure by **genetic programming** on syntactic trees

# Problem Formulation

## A supervised two-class classification problem

Given a training set of  $D_p$ (good) and  $D_n$ (bad) find the best  $\phi$  that better separates the two sets.

### Discrimination Function

$$G(\phi) = \frac{\mathbb{E}(R_\phi | \vec{X}_p) - \mathbb{E}(R_\phi | \vec{X}_n)}{\sigma(R_\phi | \vec{X}_p) + \sigma(R_\phi | \vec{X}_n)}.$$

**Observation:** only statistical and noisy evaluations of  $G(\phi)$

**Goal:** maximize  $G(\phi)$

# ROGE – RObustness GEnetic Algorithm

It is a bi-level optimization algorithm. A **GENetic algorithm** to learn the structure and a **Bayesian optimization algorithm** to learn the parameters.

**Require:**  $\mathcal{D}_p, \mathcal{D}_n, \mathbb{K}, Ne, Ng, \alpha, s$

- 1:  $gen \leftarrow \text{GENERATEINITIALFORMULAE}(Ne, s)$
- 2:  $gen_{\Theta} \leftarrow \text{LEARNINGPARAMETERS}(gen, G, \mathbb{K})$
- 3: **for**  $i = 1 \dots Ng$  **do**
- 4:    $subg_{\Theta} \leftarrow \text{SAMPLE}(gen_{\Theta}, F)$
- 5:    $newg \leftarrow \text{EVOLVE}(subg_{\Theta}, \alpha)$
- 6:    $newg_{\Theta} \leftarrow \text{LEARNINGPARAMETERS}(newg, G, \mathbb{K})$
- 7:    $gen_{\Theta} \leftarrow \text{SAMPLE}(newg_{\Theta} \cup gen_{\Theta}, F)$
- 8: **end for**
- 9: **return**  $gen_{\Theta}$

$$\phi_{best} = \operatorname{argmax}_{\phi_{\theta} \in gen_{\Theta}} (G(\phi_{\theta}))$$

# Learning the Parameters

## Problem

Given a PSTL formula  $\phi$ , a parameter space  $\mathbb{K}$ , find  $\Theta$  that maximises the discrimination function  $G(\phi_{\Theta})$ .



## Methodology

1. Sample  $\{(\theta_{(i)}, y_{(i)}), i = 1, \dots, n\}$
2. Emulate (**GP Regression**):  $G[R_{\phi}] \sim \text{GP}(\mu, k)$
3. Optimize the emulation via **GP-UCB algorithm**, new  $\theta_{(n+1)}$

$\exists \delta$  s.t.  $\mathbb{E}(R_{\phi_{\Theta^*}} | \vec{X}_p) > \delta$  and  $\mathbb{E}(R_{\phi_{\Theta^*}} | \vec{X}_n) \leq \delta$

**Translation.**  $(\vec{x} - \delta) \Rightarrow \mathbb{E}(R_{\phi_{\Theta^*}} | \vec{X}_p) > 0$  and  $\mathbb{E}(R_{\phi_{\Theta^*}} | \vec{X}_n) \leq 0$

# Learning the Structure

## Problem

Given a set of PSTL formulas  $gen$ , find the best  $\phi$  such that  $\phi_{\theta}$  maximises the discrimination function  $G(\phi_{\theta})$ .



## Methodology

- 1. GENERATEINITIALFORMULAE:**  $gen = \{\phi_1, \dots, \phi_{N_e}\}$
- 2. SAMPLE**( $gen_{\theta}, F$ ) =  $subg_{\theta}$ ,  $N_e/2$  formulae,  $F(\phi) = G(\phi) - S(\phi)$
- 3. EVOLVE**( $subg_{\theta}, \alpha$ ) =  $newg_{\theta}$ , based on two genetic operators, a **recombination** and a **mutation** operator.

## Regularization

Formula size penalty  $S(\phi)$  and complexity of initial population.

# Learning the Structure

## Problem

Given a set of PSTL formulas  $gen$ , find the best  $\phi$  such that  $\phi_{\theta}$  maximises the discrimination function  $G(\phi_{\theta})$ .



## Methodology

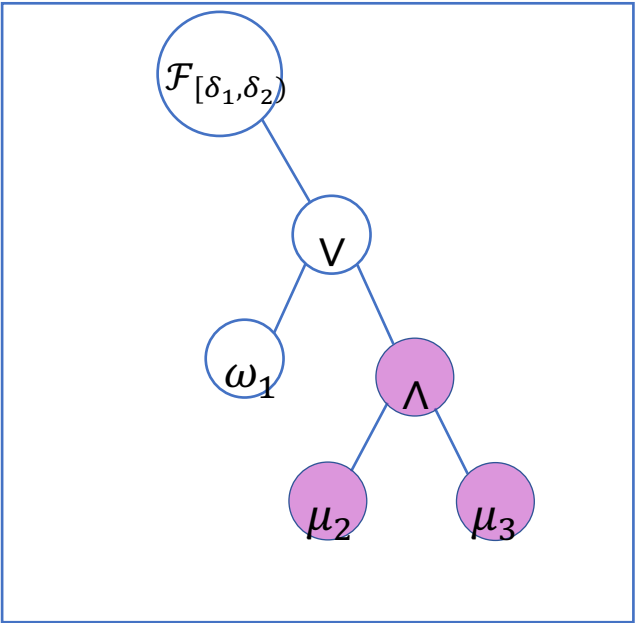
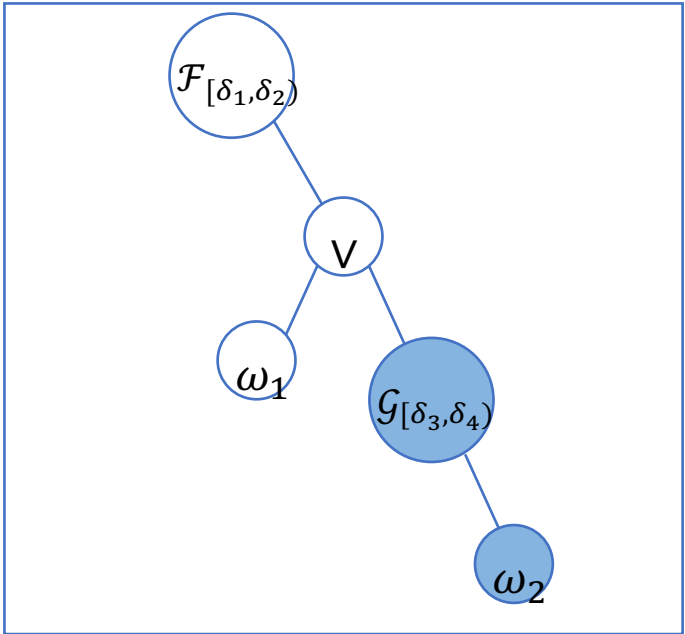
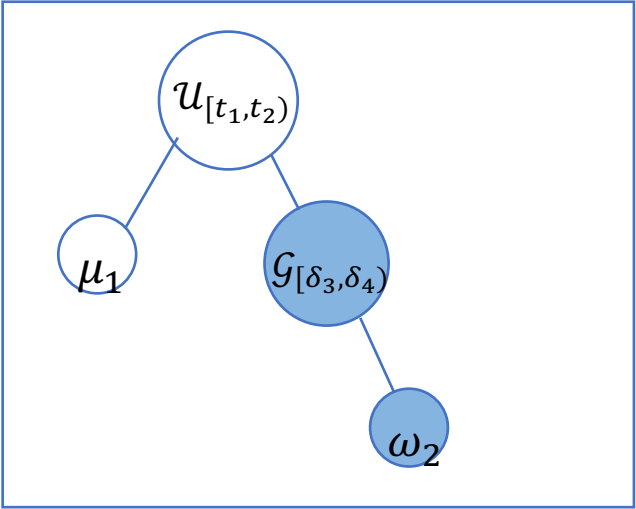
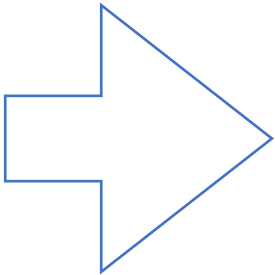
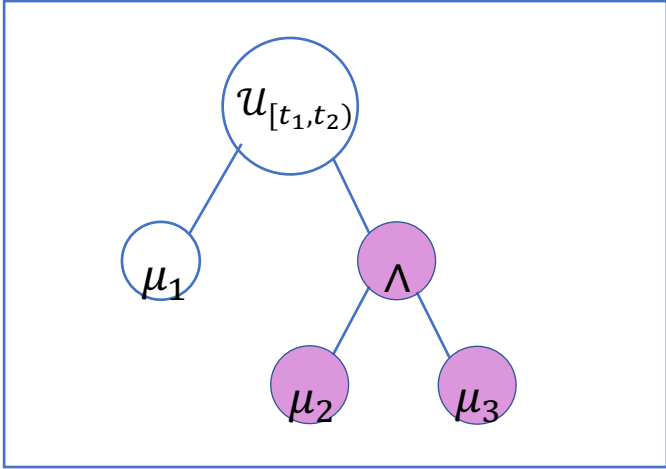
- 1. GENERATEINITIALFORMULAE:**  $gen = \{\phi_1, \dots, \phi_{N_e}\}$
- 2. SAMPLE**( $gen_{\theta}, F$ ) =  $subg_{\theta}$ ,  $N_e/2$  formulae,  $F(\phi) = G(\phi) - S(\phi)$
- 3. EVOLVE**( $subg_{\theta}, \alpha$ ) =  $newg_{\theta}$ , based on two genetic operators, a **recombination** and a **mutation** operator.

## Regularization

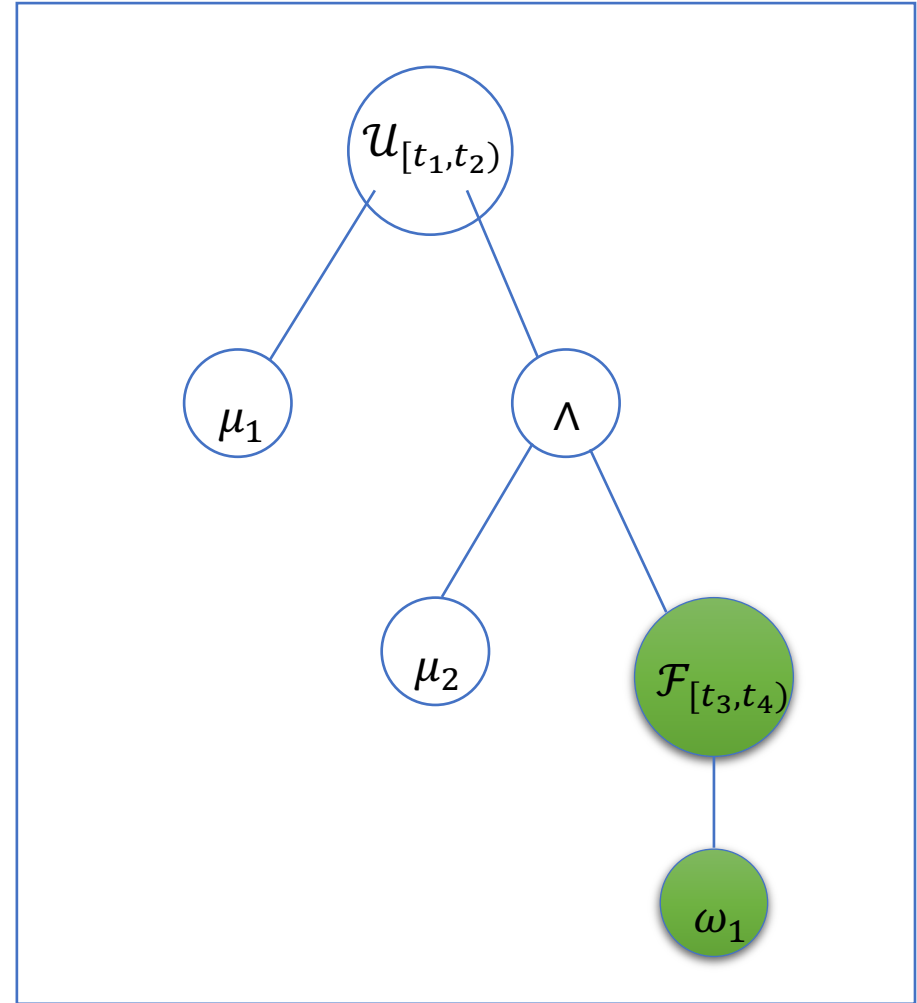
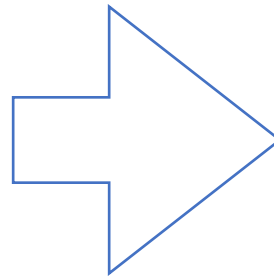
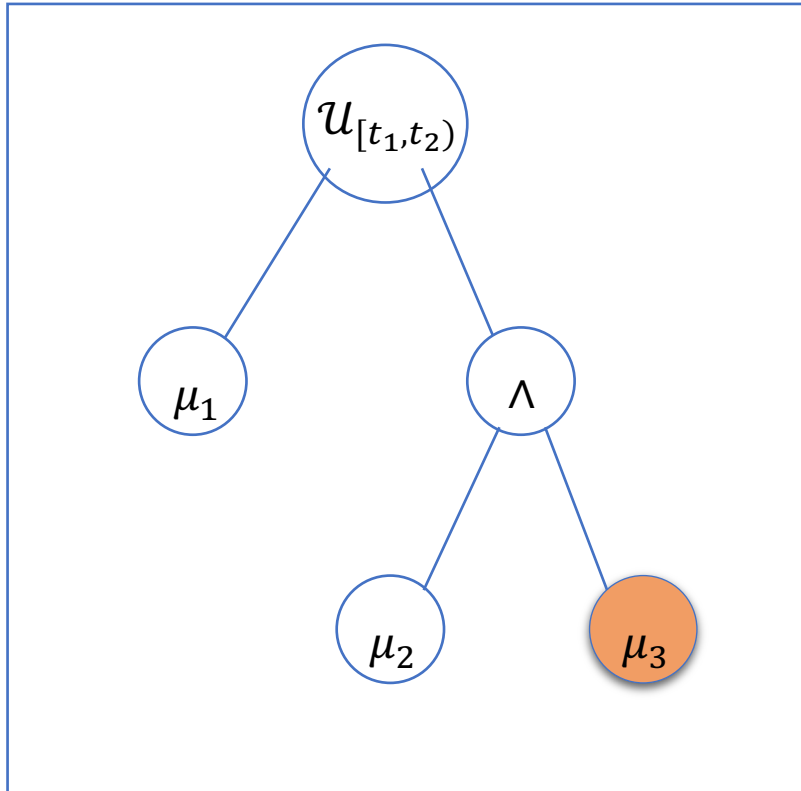
Formula size penalty  $S(\phi)$  and complexity of initial population.



# Recombination operator

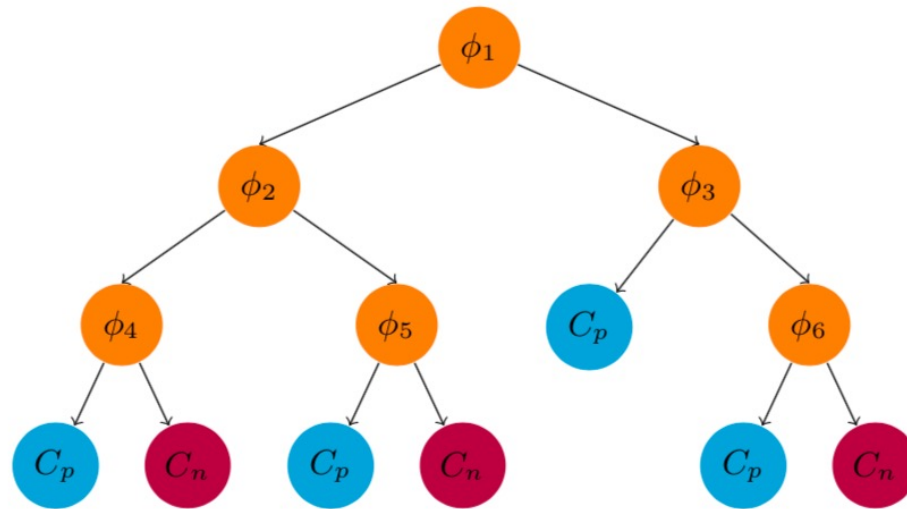


# Mutation operator



# A Decision Tree Approach to Data Classification

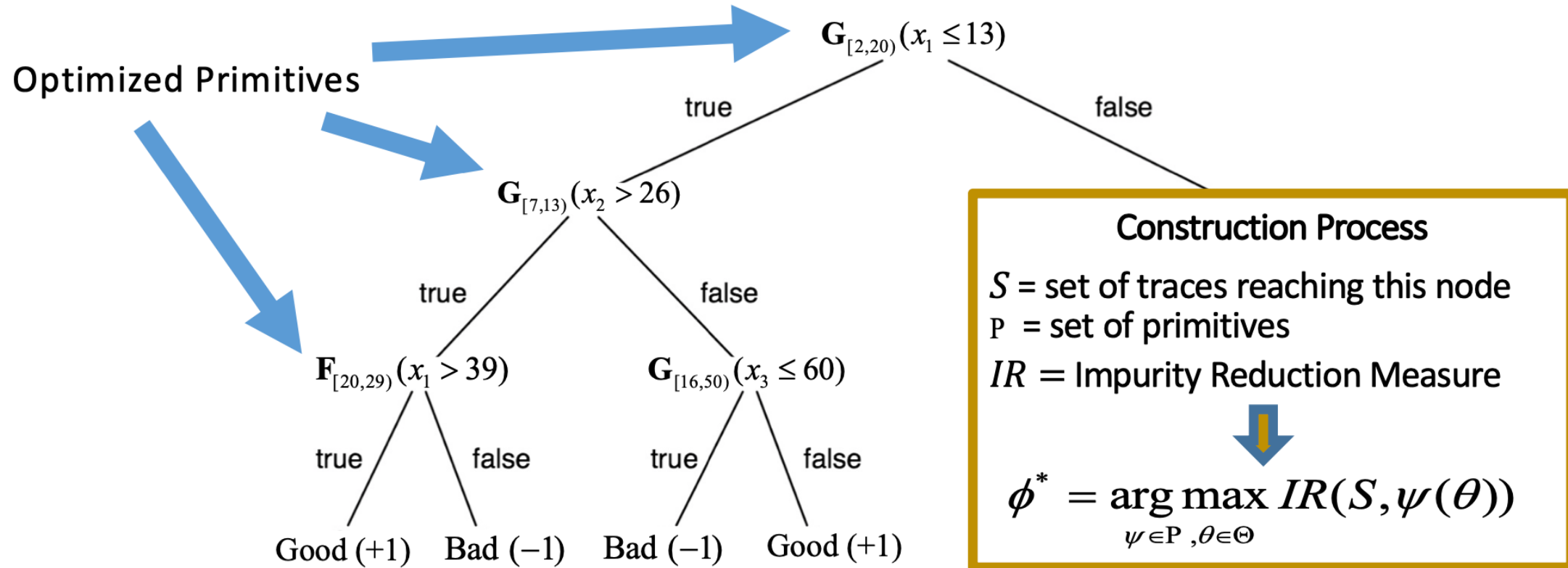
**DTL4STL** [2], that uses a decision tree algorithm for the structure of the formula and an heuristic impurity measure for parameter synthesis



$$\phi_{Tree} = (\phi_1 \wedge ((\phi_2 \wedge \phi_4) \vee (\neg\phi_2 \wedge \phi_5))) \vee (\neg\phi_1 \wedge (\phi_3 \vee (\neg\phi_3 \wedge \phi_6)))$$

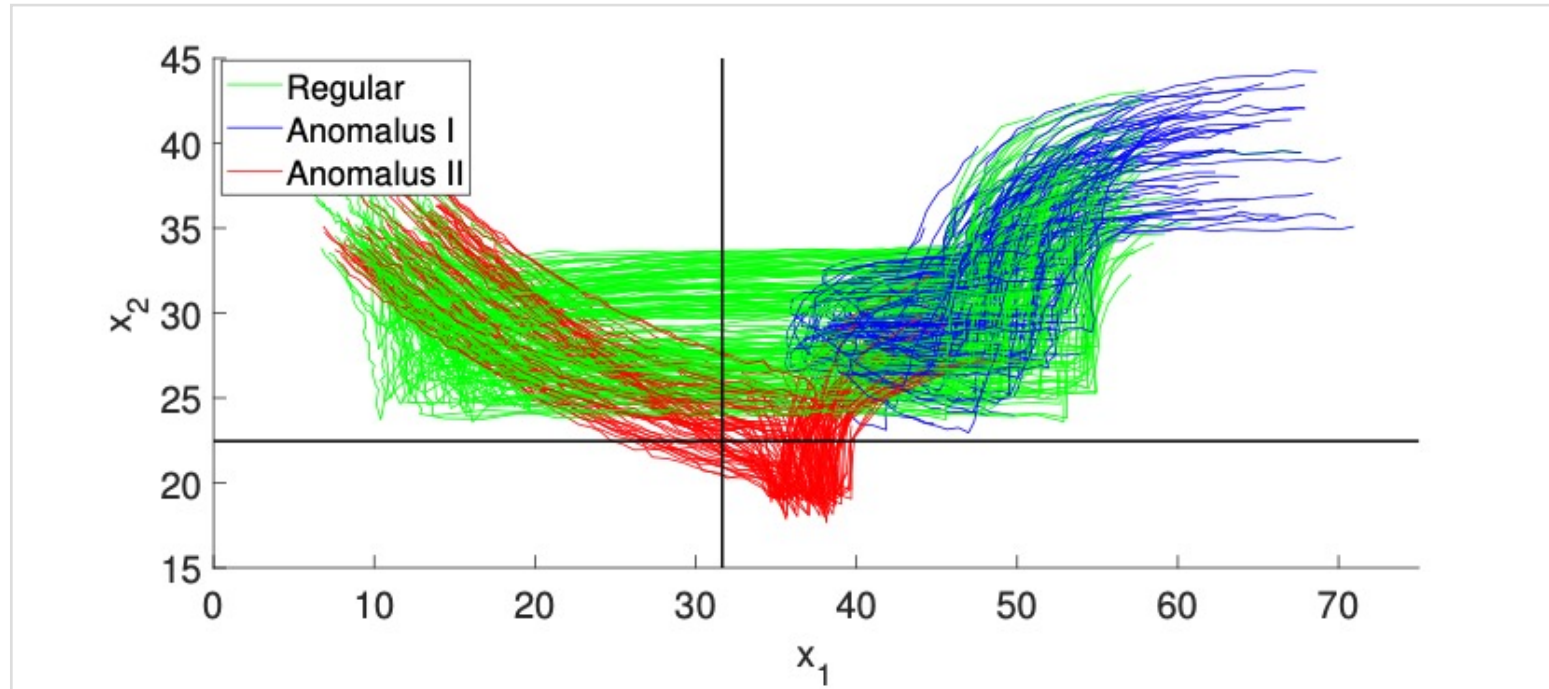
[2] Bombara, G et al, A Decision Tree Approach to Data Classification Using Signal Temporal Logic. In: Proc. of HSCC, 2016.

# A Decision Tree Approach to Data Classification



# Maritime Surveillance

Synthetic dataset of naval surveillance of 2-dimensional coordinates traces of vessels behaviours.



$$\phi_{ROGE} = ((x_2 > 22.46) \mathcal{U}_{[49,287]} (x_1 \leq 31.65))$$

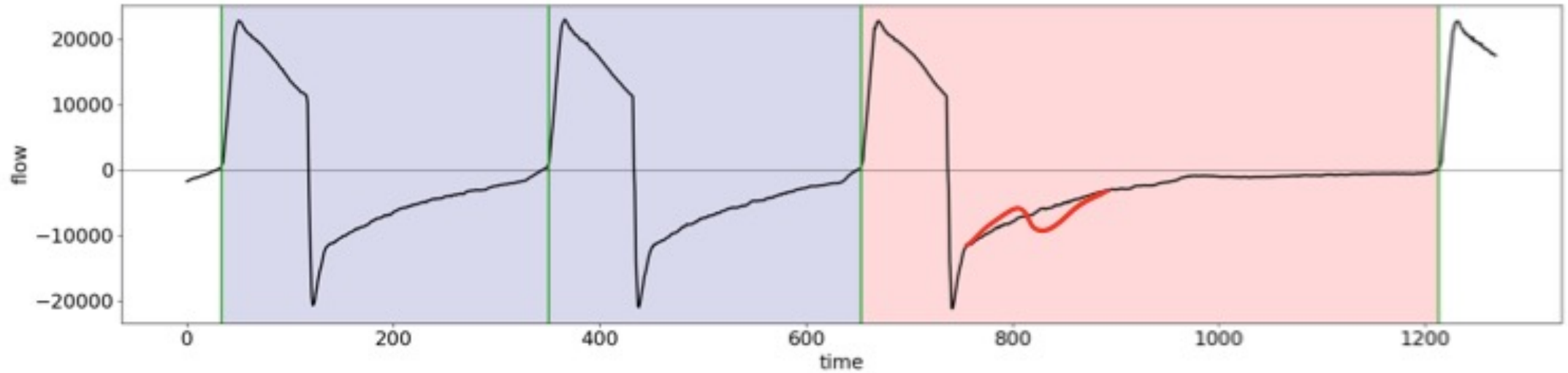
# Maritime Surveillance

$$\phi_{ROGE} = ((x_2 > 22.46) \mathcal{U}_{[49,287]} (x_1 \leq 31.65))$$

$$\begin{aligned} \psi_{DTL4STL} = & (((\mathcal{G}_{[187,196]} x_1 < 19.8) \wedge (\mathcal{F}_{[55.3,298]} x_1 > 40.8)) \vee ((\mathcal{F}_{[187,196]} x_1 > 19.8) \wedge \\ & ((\mathcal{G}_{[94.9,296]} x_2 < 32.2) \vee ((\mathcal{F}_{[94.9,296]} x_2 > 32.2) \wedge (((\mathcal{G}_{[50.2,274]} x_2 > 29.6) \wedge \\ & (\mathcal{G}_{[125,222]} x_1 < 47)) \vee ((\mathcal{F}_{[50.2,274]} x_2 < 29.6) \wedge (\mathcal{G}_{[206,233]} x_1 < 16.7)))))) \end{aligned}$$

	ROGE	DTL4STL	DTL4STL <sub>p</sub>
Mis. Rate	0	0.01 ± 0.013	0.007 ± 0.008
Comp. Time (sec.)	73 ± 18	144 ± 24	-

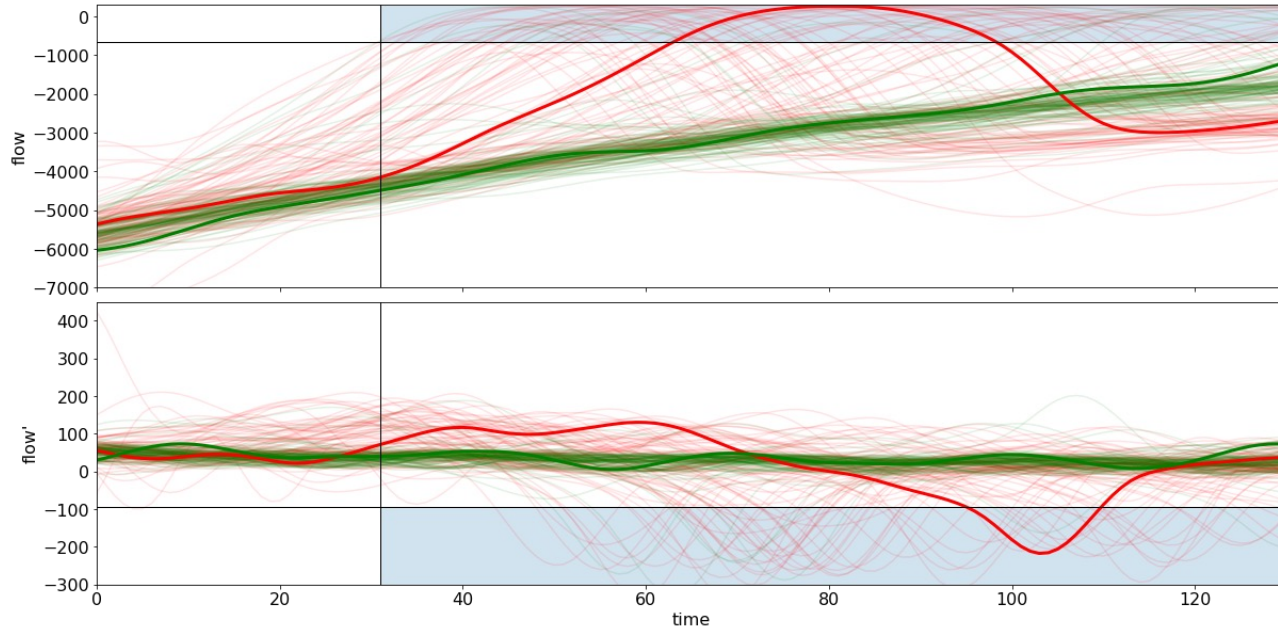
# Ineffective Inspiratory Effort (IIE)



- ▶ Regular breaths
- ▶ Irregular breaths (IIE)

# Ineffective Inspiratory Effort (IIE)

The dataset consists of 2-dim traces of flow and its derivative, flow'



	ROGE	DTL4STL
Mis. Rate	$0.17 \pm 0.01$	$0.23 \pm 0.07$
False. Pos	$0.20 \pm 0.02$	$0.23 \pm 0.07$
False. Neg	$0.14 \pm 0.02$	$0.20 \pm 0.15$
Comp. Time (sec.)	$65 \pm 7$	$201 \pm 7$

$$\phi = \mathcal{F}_{[31,130]}((\text{flow} \geq -670) \vee (\text{flow}' \leq -94))$$



# Bibliography

## Mining Requirements:

- ▶ Ezio Bartocci, Luca Bortolussi, Laura Nenzi, Guido Sanguinetti, System design of stochastic models using robustness of temporal properties. *Theor. Comput. Sci.* 587: 3-25 (2015)
- ▶ Jyo Deshmukh et al. Mining Requirements from Closed-loop Control Models (HSCC '13, IEEE Trans. On Computer Aided Design '15)
- ▶ Bartocci, E., Bortolussi, L., Sanguinetti, G.: Data-driven statistical learning of temporal logic properties, *FORMATS*, 2014
- ▶ Bufo, S., Bartocci, E., Sanguinetti, G., Borelli, M., Lucangelo, U., Bortolussi, L.i, Temporal logic based monitoring of assisted ventilation in intensive care patients, *ISoLA*, 2014.
- ▶ Nenzi L., Silveti S., Bartocci E., Bortolussi L. (2018) *A Robust Genetic Algorithm for Learning Temporal Specifications from Data*. *QEST 2018*. LNCS, vol 11024. Springer, Cham.
- ▶ Bombara, G et all, A Decision Tree Approach to Data Classification Using Signal Temporal Logic. In: *Proc. of HSCC*, 2016.