$$\lim_{x \to 0^+} \sqrt{x}\, \lg(x) \sin\left(\frac{1}{x}\right) = 0$$

$$\left| \sin\left(\frac{1}{x}\right) \sqrt{x}\, \lg(x) \right| \le \sqrt{x}\, \lg \frac{1}{x} \to 0$$

$$\to 0$$

$$|\lg x| = -\lg x = \lg x^{-1}$$

$$\lim_{x \to 0^+} \frac{\lg\left(\frac{1}{x}\right)}{x^{-\frac{1}{2}}} = \lim_{y \to +\infty} \frac{\lg y}{y^{\frac{1}{2}}} = 0$$

$$y = x^{-1}$$

$$= \lim_{y \to +\infty} \frac{y^{-1}}{\frac{1}{2} y^{-\frac{1}{2}}} = \lim_{y \to +\infty}$$

$$= \lim_{y \to +\infty} 2 \cdot y^{-\frac{1}{2}} = 0$$

$$\lim_{x\to+\infty} \frac{\log(1+x+e^{2x}) - x^2\sin\left(\frac{1}{x}\right) - x}{\int_1^x t^{-2}\,dt - 1}$$

$$Den = -t^{-1}\Big]_1^x - 1 = -x^{-1} + 1 - 1 = -x^{-1}$$

$$\log\left(e^{2x}\left(1 + xe^{-2x} + e^{-2x}\right)\right) =$$

$$= 2x + \log(1 + xe^{-2x} + e^{-2x}) =$$

$\log(1+y) = y(1+o(1))$ per y vicino a 0

$$= 2x + (xe^{-2x} + e^{-2x})(1+o(1))$$

$$= 2x + o(x^{-10})$$

$$Num = \cancel{2}x + o(x^{-10}) - x^2\sin\left(\frac{1}{x}\right) - \cancel{x} =$$

$$-x^2\sin\left(\frac{1}{x}\right) = -x^2\left(\frac{1}{x} - \frac{1}{6x^3} + o(x^{-3})\right)$$

$$= -x + \frac{1}{6}x^{-1} + o(x^{-1})$$

$$= \cancel{x} - \cancel{x} + \frac{1}{6}x^{-1} + o(x^{-1})$$

$$Num = \frac{1}{6}x^{-1} + o(x^{-1}) = \frac{1}{6}x^{-1}(1+o(1))$$

$$\lim_{x\to+\infty} \frac{\frac{1}{6}x^{-1}}{-x^{-1}} = -\frac{1}{6}$$

$$f(x) = \left(\lg(x+x^2)\right)^{\tan x} =$$

$$= e^{\lg f(x)} =$$

$$= e^{\tan x \lg \lg(x+x^2)}$$

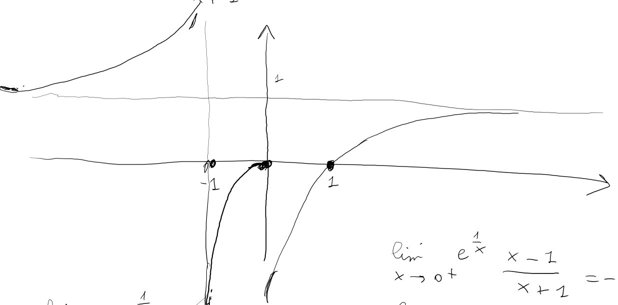
$$f(x) = e^{\tan x \lg \lg(x+x^2)}$$

$$f'(x) = f(x) \left(\tan x \lg \lg(x+x^2)\right)'$$

$$= f(x) \left((1+\tan^2 x) \lg\left(\lg(x+x^2)\right) + \right.$$

$$\left. + \tan x \frac{1}{\lg(x+x^2)} \frac{1}{x+x^2} (1+2x) \right)$$

$f(x) = e^{\frac{1}{x}} \frac{x-1}{x+1}$ $\qquad x \neq 0, -1$



$\lim_{x \to \infty} e^{\frac{1}{x}} \cancel{\frac{x}{x}} = 1$

$\lim_{x \to 0^+} e^{\frac{1}{x}} \frac{x-1}{x+1} = -\infty$

$\lim_{x \to 0^-} \qquad = 0$

$\lim_{x \to -1^+} e^{\frac{1}{x}} \frac{x-1}{x+1} = -\infty$

$\lim_{x \to -1^-} e^{\frac{1}{x}} \frac{x-1}{x+1} = +\infty$

$$f'(x) = e^{\frac{1}{x}} \left[-\frac{1}{x^2} \frac{x-1}{x+1} + \frac{\cancel{x}+1-(\cancel{x}-1)}{(x+1)^2} \right]$$

$$= \frac{e^{\frac{1}{x}}}{x+1} \left[\frac{-x+1}{x^2} + \frac{2}{x+1} \right]$$

$$= \frac{e^{\frac{1}{x}}}{(x+1)^2 x^2} \left(1 - x^2 + 2x^2 \right) = \frac{e^{\frac{1}{x}}}{(x+1)^2 x^2} \left(1 + x^2\right) > 0$$

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} \frac{e^{\frac{1}{x}}}{x^2} \qquad y = -\frac{1}{x}$$

$$= \lim_{y \to +\infty} \frac{e^{-y}}{\frac{1}{y^2}} = \lim_{y \to +\infty} \frac{y^2}{e^y} = 0$$

$$\int_1^2 \frac{1}{\sqrt{x}\,\sqrt{3-x}}\,dx$$

$$y = \sqrt{x}$$

$$dy = \frac{1}{2}\frac{1}{\sqrt{x}}\,dx$$

$$= 2\int_1^{\sqrt{2}} \frac{1}{\sqrt{3-y^2}}\,dy$$

$$y = \sqrt{3}\sin t$$

$$dy = \sqrt{3}\cos t\,dt$$

$$= 2\cancel{\sqrt{3}}\int_{\arcsin\left(\frac{1}{\sqrt{3}}\right)}^{\arcsin\left(\sqrt{\frac{2}{3}}\right)} \frac{\cos t}{\sqrt{\cancel{3}-\cancel{3}\sin^2 t}}\,dt = 2\left(\arcsin\left(\sqrt{\frac{2}{3}}\right) - \arcsin\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$\int \frac{1}{\sqrt{2+x^2}}\, dx$$

$$x = \sqrt{2}\ \mathrm{sh}\ t$$

$$dx = \sqrt{2}\ \mathrm{ch}\ t\ dt$$

$$\frac{x}{\sqrt{2}} = \mathrm{sh}\ t$$

$$= \int \frac{\cancel{\mathrm{ch}}\ t}{\cancel{\mathrm{ch}}\ t}\, dt = t + C$$

$$= \lg\left(\frac{x}{\sqrt{2}} + \sqrt{1+\frac{x^2}{2}}\right) + C$$

$$\int_0^1 \sqrt{1+4x^2}\, dx$$

$$\sin(x)\sin\left(\frac{1}{x}\right) \overset{?}{\in} L[1,+\infty)$$

$$\sin(x)\left(\frac{1}{x} - \frac{1}{6x^3} + o(x^{-3})\right) =$$

$$= \frac{\sin x}{x} - \sin x \; o(x^{-2})$$

ass. int.

$$L[1,+\infty)$$

$$\int_0^{2\pi} |\cos x| \, dx = \int_0^{\pi} |\cos x| \, dx + \int_{\pi}^{2\pi} |\cos x| \, dx$$

$$= 2 \int_0^{\pi} |\cos x| \, dx =$$

$$\int_{\pi}^{2\pi} |\cos(x)| \, dx = \qquad x = y + \pi, \quad dx = dy$$

$$= \int_0^{\pi} |\cos(y+\pi)| \, dy$$

$$\forall a$$

$$= 2 \int_a^{\pi+a} |\cos(x)| \, dx = 2 \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx}_{2}$$