$$\lim_{x\to 0^+}\int_x^{3x} \lg\left(1+\frac{1}{\sqrt{t}}+\frac{1}{t}\right) dt = \int_0^0 \lg\left(1+\frac{1}{\sqrt{t}}+\frac{1}{t}\right) dt$$

$$\int_x^{3x} \lg\left(1+\frac{1}{\sqrt{t}}+\frac{1}{t}\right) dt = \int_x^{3x} \lg\left(\frac{1}{t}\left(1+\sqrt{t}+t\right)\right) dt$$

$$= \int_x^{3x} \lg\left(\frac{1}{t}\right) dt + \int_x^{3x} \lg(1+\sqrt{t}+t)\, dt$$

$t \to \lg(1+\sqrt{t}+t)$ è in $C^0([0,+\infty))$

$[0,+\infty)$ $[0,+\infty)$

$\Longrightarrow \lg(1+\sqrt{t}+t) \in L[0,1]$

$$\int_x^{3x} \lg(1+\sqrt{t}+t)\, dt = \int_0^{3x} \lg(1+\sqrt{t}+t)\, dt - \int_0^{x} \lg(1+\sqrt{t}+t)\, dt$$

$x\to 0^+$ $\to 0$ $\quad$ $C^0([0,1])$ $\quad$ $x\to 0^+$ $\to 0$

$$\int_x^{3x} \lg\left(\frac{1}{t}\right) dt \overset{\text{se volessi}}{=} \int_0^{3x} \lg\left(\frac{1}{t}\right) dt - \int_0^{x} \lg\left(\frac{1}{t}\right) dt$$

$x\to 0^+$ $\to 0$

$$\lg\left(\frac{1}{t}\right) = \lg\left(t^{-1}\right) = -\lg(t)$$

$$\int_x^{3x} \lg\left(\frac{1}{t}\right) dt = -\int_x^{3x} \lg(t)\, dt$$

$$0 < \int_x^{3x} \lg\left(\frac{1}{t}\right) dt < \int_x^{3x} \lg\left(\frac{1}{x}\right) dt = 2x \lg\left(\frac{1}{x}\right)$$

$x\to 0^+$ $\to 0$

$$f(x) = \left[x + \left(x^3 + (x^2 + x + 1)^{2x}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}\left[x + \left(x^3 + (x^2 + x + 1)^{2x}\right)^{\frac{1}{3}}\right]^{-\frac{1}{2}}$$

$$\cdot \left\{1 + \frac{1}{3}\left(x^3 + (x^2 + x + 1)^{2\cancel{x}}\right)^{-\frac{2}{3}} \left(3x^2 + \left((x^2 + x + 1)^{2x}\right)'\right)\right\}$$

$$\left((x^2 + x + 1)^{2x}\right)' = \left(e^{2x \lg(x^2 + x + 1)}\right)'$$

$$= e^{2x \lg(x^2 + x + 1)} \, 2\left(x \lg(x^2 + x + 1)\right)'$$

$$= 2(x^2 + x + 1)^{2x}\left(\lg(x^2 + x + 1) + x\,\frac{2x + 1}{x^2 + x + 1}\right)$$

$$\lim_{x\to 0^+}\int_x^{2x}\frac{1}{t}\,dt \neq 0$$

$$\int_x^{2x}\frac{1}{t}\,dt = \lg t\Big]_x^{2x} = \lg(2x)-\lg(x) =$$
$$= \lg\left(\frac{2\cancel{x}}{\cancel{x}}\right) = \lg 2$$

$$\lim_{x\to 0^+}\int_x^{2x}\frac{1}{\lg(1+t+t^2)}\,dt$$

$$\lg(1+y) = y-\frac{y^2}{2}+o(y^2)$$
$$\lg(1+\underbrace{t+t^2}_{y}) = t+t^2-\frac{(t+t^2)^2}{2}+o\left((t+t^2)^2\right)$$
$$= t+t^2-\frac{(t+t^2)^2}{2}+o(t^2)$$
$$= t+\frac{t^2}{2}+o(t^2)$$

$$\int_x^{2x}\frac{1}{t+\frac{t^2}{2}+o(t^2)}\,dt = \int_x^{2x}\frac{1}{t}\;\frac{1}{1+\frac{t}{2}+o(t)}\,dt$$
$$= \int_x^{2x}\frac{1}{t}\;\frac{1}{1+o(1)}\,dt$$
$$= \int_x^{2x}\frac{1}{t}\,(1+o(1))\,dt$$
$$= \underbrace{\int_x^{2x}\frac{1}{t}\,dt} + \int_x^{2x}\frac{o(1)}{t}\,dt$$
$$= \lg 2 + \int_x^{2x}\frac{o(1)}{t}\,dt \xrightarrow{x\to 0^+} 0$$

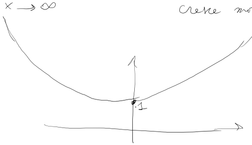
$$\int_x^{2x}\frac{o(1)}{t}\,dt = x\,\frac{o(1)}{t}\Big|_{t=c_x} \qquad x\le c_x\le 2x$$

$$0 \le \left|\frac{x}{c_x}\;o(1)\Big|_{t=c_x}\right| \le o(1)\Big|_{t=c_x} \xrightarrow{x\to 0^+} 0$$

$$f(x) = \frac{e^{x^2}}{1+x^2}$$

$\mathrm{dom}\, f = \mathbb{R}$

$\lim_{x\to\infty} f(x) = +\infty$ perchè l'esponenziale cresce molto più rapidamente.



$$f'(x) = \left(\frac{e^{x^2}}{1+x^2}\right) = e^{x^2}\,\frac{2x(\cancel{1}+x^2) - \cancel{2x}}{(1+x^2)^2} =$$

$$= \frac{2x^3 e^{x^2}}{(1+x^2)^2}$$

$f'(x) = 0 \iff x = 0$

$f'(x) > 0 \iff x > 0$

$$f''(x) = \frac{(2x^3 e^{x^2})'(1+x^2)^{\cancel{2}} - 2x^3 e^{x^2}\, 2\,\cancel{(1+x^2)}\,2x}{(1+x^2)^{\cancel{4}\,3}}$$

$$(x^3 e^{x^2})' = 3x^2 e^{x^2} + 2x^4 e^{x^2}$$

$$= 2\,\frac{(3x^2 e^{x^2} + 2x^4 e^{x^2})(1+x^2) - 4x^4 e^{x^2}}{(1+x^2)^3}$$

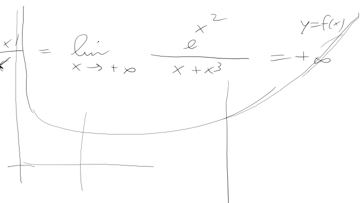
$$= \frac{2e^{x^2}x^2}{(1+x^2)^3}\left[(3+2x^2)(1+x^2) - 4x^2\right]$$

$$= \frac{2e^{x^2}x^2}{(1+x^2)^3}\left[\underbrace{2x^4 + \overset{x^2}{\cancel{5}x^2} + 3 \cancel{- 4x^2}}\right]$$

$$f''(x) = \frac{2e^{x^2}x^2}{(1+x^2)^3}\left[2x^4 + x^2 + 3\right] \geq 0$$

f è convessa

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{e^{x^2}}{x+x^3} = +\infty$$



$$\int_1^{+\infty} \frac{1}{x^3+x^2+x+1}\,dx$$

$$\frac{1}{(x^3+x^2)+(x+1)} = \frac{1}{x^2(x+1)+(x+1)} = \frac{1}{(x^2+1)(x+1)}$$

$$\frac{1}{x^3+x^2+x+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$A = \left.\frac{1}{x^2+1}\right|_{x=-1} = \frac{1}{2} \qquad B = -\frac{1}{2}$$

$$= \frac{\frac{1}{2}}{x+1} + \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{\frac{1}{2}(x^2+1) - \frac{1}{2}x(x+1) + C(x+1)}{(x+1)(x^2+1)}$$

$$-\frac{1}{2}+C=0$$

$$C=\frac{1}{2}$$

$$\frac{1}{2}\int_1^{+\infty}\left[\frac{1}{x+1} - \frac{x-1}{x^2+1}\right]dx$$

$$\frac{1}{2}\int_1^{R}\left[\frac{1}{x+1} - \frac{x-1}{x^2+1}\right]dx = \frac{1}{2}\left(\lg(R+1) - \lg 2\right)$$

$$- \frac{1}{4}\int_1^R \frac{2x}{x^2+1}\,dx + \frac{1}{2}\int_1^R \frac{1}{x^2+1}\,dx$$

$$= \frac{1}{2}\lg(R+1) - \frac{1}{2}\lg 2 - \frac{1}{4}\lg(x^2+1)\Big]_1^R +$$

$$+ \frac{1}{2}\operatorname{arctan} R - \frac{\pi}{8}$$

$$= \frac{1}{2}\lg(R+1) - \frac{1}{2}\lg(R^2+1)^{\frac{1}{2}} - \frac{1}{2}\lg 2 + \frac{1}{4}\lg 2$$

$$+ \frac{1}{2}\operatorname{arctan} R - \frac{\pi}{8}$$

$$= \frac{1}{2}\lg\frac{R+1}{\sqrt{R^2+1}} - \frac{1}{4}\lg 2 + \frac{1}{2}\operatorname{arctan} R - \frac{\pi}{8}$$

$$\xrightarrow{R\to+\infty} -\frac{1}{4}\lg 2 - \frac{\pi}{8} + \frac{\pi}{4} = \frac{\pi}{8} - \frac{1}{4}\lg 2 =$$

$$= \frac{1}{4}\left(\frac{\pi}{2} - \lg 2\right) > 0$$