11 gennaio

$$\int_1^{+\infty} \frac{1}{x^3+x^2+x+1}\, dx$$

$$\int (x^2+x)\arctan(x)\, dx = \qquad (\arctan x)' = \frac{1}{1+x^2}$$

$$= \int \left(\frac{x^3}{3}+\frac{x^2}{2}\right)' \arctan(x)\, dx =$$

$$= \left(\frac{x^3}{3}+\frac{x^2}{2}\right)\arctan(x) - \int \left(\frac{x^3}{3}+\frac{x^2}{2}\right)\frac{1}{1+x^2}\, dx$$

$$\int x^2 \frac{1}{1+x^2}\, dx, \qquad \int x^3 \frac{1}{1+x^2}\, dx$$

$$\int \frac{x^2}{1+x^2}\, dx = \int \frac{\cancel{x^2+1}}{\cancel{1+x^2}}\, dx - \int \frac{1}{1+x^2}\, dx$$

$$= x - \arctan(x) + c$$

$$x(x^2+1)$$

$$\int \frac{x^3}{1+x^2}\, dx = \int \frac{x^3+x}{1+x^2} - \frac{1}{2}\int \frac{2x}{1+x^2}$$

$$= \int x\, dx - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$$

$$= \frac{x^2}{2} - \frac{1}{2}\lg(1+x^2) + c$$

$$\frac{1}{x \lg^p(x)}$$ per quali $p > 0$ e' integrabile

in $[2, +\infty)$ $\lim_{R \to +\infty} \int_2^R \frac{1}{x \lg^p(x)} \, dx$

$p \neq 1$

$$\int_2^R \frac{1}{x \lg^p(x)} \, dx =$$

$y = \lg x$

$dy = \frac{1}{x} dx$

$$= \int_{\lg 2}^{\lg R} \frac{1}{y^p} \, dy$$

$p \neq 1$ $p > 0$

$$\lim_{R \to +\infty} \int_2^R \frac{1}{x \lg^p(x)} \, dx = \lim_{R \to +\infty} \int_{\lg 2}^{\lg R} \frac{1}{y^p} \, dy$$

il limite e' finito per $p > 1$ ed e' infinito

per $0 < p \leq 1$

$$\int_2^R \frac{1}{x \lg(x)} \, dx$$

$y = \lg x$

$dy = \frac{1}{x} dx$

$$= \int_{\lg 2}^{\lg R} \frac{1}{y} \, dy \xrightarrow{R \to +\infty} +\infty$$

$\sin(\frac{1}{x}) \in L(0,1]$

$\lim_{\varepsilon\to 0^+} \int_\varepsilon^1 \sin(\frac{1}{x})\,dx$ esiste finito

$\int_\varepsilon^1 \sin(\frac{1}{x})\,dx =$

$t = \frac{1}{x}$

$dt = -\frac{1}{x^2}\,dx$

$dx = -x^2\,dt = -\frac{1}{t^2}\,dt$

$= -\int_{\frac{1}{\varepsilon}}^{1} \frac{\sin(t)}{t^2}\,dt$

$= \int_1^{\frac{1}{\varepsilon}} \frac{\sin(t)}{t^2}\,dt \xrightarrow{\varepsilon\to 0^+}$

$\sin(\frac{1}{x}) \in L_{loc}(0,1]$

$0 \le |\sin(\frac{1}{x})| \le 1$ $|\sin(\frac{1}{x})|$ è integrabile

per il criterio del confronto $\Rightarrow$ $\sin(\frac{1}{x})$ è assolutamente integrabile in $(0,1]$ $\Rightarrow$ $\sin(\frac{1}{x}) \in L(0,1]$

$\lim_{x\to 0^+} \frac{\int_x^{2x} \sin(\frac{1}{t})\,dt}{x}$

$\frac{\int_x^{2x} \sin(\frac{1}{t})\,dt}{x} = \frac{\int_0^{2x} \sin(\frac{1}{t})\,dt - \int_0^{x} \sin(\frac{1}{t})\,dt}{x}$

$= 2\,\frac{\int_0^{2x} \sin(\frac{1}{t})\,dt}{2x} - \frac{\int_0^{x} \sin(\frac{1}{t})\,dt}{x}$

$= 2\,\frac{F(2x)}{2x} - \frac{F(x)}{x}$

$\lim_{x\to 0^+} \frac{\int_0^{x} \sin(\frac{1}{t})\,dt}{x}$ $\cancel{\lim_{x\to 0^+} \sin(\frac{1}{x})}$

$F(x) = \int_0^x \sin(\frac{1}{t})\,dt$ $F \in C^0(\mathbb{R})$

$F(0) = 0$ $F'(x) = \sin(\frac{1}{x})$ per $x \neq 0$

In classe si è dato un esempio di funzione G t.c. $G'(x) = \begin{cases} 0 & \text{se } x = 0 \\ \sin(\frac{1}{x}) & \text{se } x \neq 0 \end{cases}$

$G(0) = 0$

Risulta che $F(x) = G(x)$ $\forall x \in \mathbb{R}$ grazie a Lagrange $H(x) = F(x) - G(x)$

$H(0) = 0$ $H'(x) = 0$ $\forall x \neq 0$

se $x \neq 0$ $H(x) = \frac{H(x)}{x}\,x$

$0 \quad c_x \quad x$

$= \frac{H(x) - H(0)}{x}\,x$

$H(x) = \underbrace{H'(c_x)}_{0}\,x =$

dove c_x è un punto intermedio tra 0 e x

$\lim_{x\to 0^+} \frac{\int_x^{2x} \sin(\frac{1}{t})\,dt}{x} = \lim_{x\to 0^+} \left[2\,\underbrace{\frac{F(2x) - F(0)}{2x}}_{\to 0} - \underbrace{\frac{F(x) - F(0)}{x}}_{\to 0} \right]$

$\lim_{x\to 0^+} \frac{F(x) - F(0)}{x} = F'(0) = 0$

$\lim_{x\to 0^+} \frac{F(2x) - F(0)}{2x} \underset{y=2x}{=} \lim_{y\to 0^+} \frac{F(y) - F(0)}{y} = F'(0) = 0$

$f(x) = \tan(x)$ Mc Laurin ordine 6

$$P_6(x) = \sum_{j=0}^{6} \frac{\tan^{(j)}(0)}{j!} x^j$$

$$f(x) = \frac{\sin(x)}{\cos x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)}{1 \boxed{- \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + o(x^6)}}$$

y

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + o(y^3)$$

$y =$

$$f(x) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right) \left[1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} + \right.$$

$$\left. + \left(-\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right)^2 - \left(-\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right)^3 + o(x^6)\right]$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right) \left[1 + \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} + \right.$$

$$\left. + \frac{x^4}{4} - \frac{x^6}{4!} + \frac{x^6}{8} + o(x^6)\right]$$

$$f(x) = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^6)\right) \left[1 + \frac{x^2}{2} + x^4\left(\frac{1}{4} - \frac{1}{4!}\right) + x^6\left(\frac{1}{6!} - \frac{1}{4!} + \frac{1}{8}\right) + o(x^6)\right]$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^3}{2} - \frac{x^5}{2 \cdot 3!} + x^5\left(\frac{1}{4} - \frac{1}{4!}\right) + o(x^6)$$

$$= x + \left(\frac{1}{2} - \frac{1}{3!}\right)x^3 + \left(\frac{1}{5!} - \frac{1}{2 \cdot 3!} + \frac{1}{4} - \frac{1}{4!}\right)x^5 + o(x^6)$$

$$\underbrace{y'' + 4y}_{L[y]} = \sin(2x)$$

$y'' + 4y = 0$ $\quad y_h$ soluzione generale dell'omogenea

e poi cerchiamo una soluzione particolare

$$L[y_p] = \sin(2x)$$

$$y_g = y_h + y_p \qquad y^{(j)} \qquad r^j$$

$$y'' + 4y = 0 \qquad r^2 + 4 = 0 \Rightarrow r = \pm 2 \cdot i$$

$$y_g = A\cos(2x) + B\sin(2x)$$

$$y'' + by' + cy = 0 \qquad r^2 + br + c = 0$$

$$r_\pm = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2} \qquad \text{ie} \qquad b^2 - 4c < 0$$

$$= -\frac{b}{2} \pm i \frac{\sqrt{|b^2 - 4c|}}{2}$$

$$e^{-\frac{b}{2}x} \cdot \cos\left(\frac{\sqrt{|b^2 - 4c|}}{2} x\right)$$

$$e^{-\frac{b}{2}x} \sin(\qquad)$$

$$L[y_p] = \sin(2x)$$

$$y_p = \alpha\, x\cos(2x) + \beta\, x\sin(2x)$$

$$L[y_p] = \alpha\, L[x\cos(2x)] + \beta\, L[x\sin(2x)]$$

$$= -4\alpha\sin(2x) + 4\beta\cos(2x) = \sin(2x)$$

$$L[x\cos(2x)] = (x\cos(2x))'' + 4(x\cos(2x))$$

$$= x\underbrace{L[\cos(2x)]}_{0} + 2(\cos(2x))' = -4\sin(2x)$$

$$L[x\sin(2x)] = x\,L[\sin(2x)] + 2(\sin(2x))' = 4\cos(2x)$$

$$-4\alpha = 1 \qquad 4\beta = 0 \qquad \alpha = -\frac{1}{4}$$

$$y_p = -\frac{1}{4}x\cos(2x)$$