12 gennaio

$$\int_1^{\infty} \underbrace{\frac{x^2+1}{x^3+x^2+x+2}}_{R(x)}\,dx$$

$$\lim_{x\to+\infty} \frac{R(x)}{\frac{1}{x}} = \lim_{x\to+\infty} x\,R(x) =$$

$$= \lim_{x\to+\infty} x\,\frac{x^2}{x^3} = 1$$

$\frac{1}{x}$ non è integrabile in $[1,+\infty)$

$\Rightarrow$ R non è integrabile $[\cancel{0}1,+\infty)$

$$\int_{\cancel{0}1}^{+\infty} \frac{x^2+1}{x^3+x^2+x+2}\,dx = \lim_{R\to+\infty}\int_1^R \frac{x^2+1}{x^3+x^2+x+2}\,dx$$

$$= +\infty$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\int \sin^4(x)\,dx$$

$$= \int\left(\frac{1-\cos(2x)}{2}\right)^2 dx = \frac{1}{4}\int dx - \frac{1}{2}\int \cos(2x)\,dx + \frac{1}{4}\int \cos^2(2x)\,dx$$

$$= \frac{x}{4} - \frac{1}{2}\,\frac{\sin(2x)}{2} + \frac{1}{4}\int \frac{1+\cos(4x)}{2}\,dx$$

$$= \frac{x}{4} - \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{8}\int \cos(4x)\,dx$$

$$= \frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{8}\,\frac{\sin(4x)}{4} + C$$

$\frac{\sin(x)}{\lg x}$ integrabile in $[2,+\infty)$?

$\sin x$

$$\int_2^R \frac{\sin(x)}{\lg x}\,dx = -\int_2^R \frac{\cos'(x)}{\lg x}\,dx =$$

$$= -\frac{\cos(R)}{\lg(R)} + \frac{\cos(2)}{\lg(2)} + \int_2^R \cos x \left(\frac{1}{\lg x}\right)' dx$$

$$= -\frac{\cos(R)}{\lg(R)} + \frac{\cos(2)}{\lg(2)} - \boxed{\int_2^R \cos(x) \frac{1}{\lg^2 x} \frac{1}{x}\,dx}$$

$\downarrow$

0

$$\frac{1}{x \lg^p(x)} \in L[2,+\infty) \iff p > 1$$

$$\frac{1}{x \lg^2(x)} \in L[2,+\infty)$$

$$0 \le |\cos x| \frac{1}{x \lg^2(x)} \le \frac{1}{x \lg^2(x)}$$

$$\in L[2,+\infty) \implies \frac{\cos x}{x \lg^2(x)} \in L[2,+\infty)$$

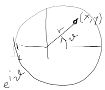
$$\int_2^R \cos'(x) \frac{1}{\lg x}\,dx = \left.\frac{\cos(x)}{\lg x}\right]_2^R$$

$$- \int_2^R \cos(x) \left(\frac{1}{\lg x}\right)' dx$$

Si $\int_1^{+\infty} \frac{1}{1+x^4}\,dx$

$z^4+1=0$

$z = x+iy = r(\cos\vartheta + i\sin\vartheta)$
$= r\,e^{i\vartheta}$
$x = r\cos\vartheta$
$y = r\sin\vartheta$



$z = r\,e^{i\vartheta}$
$z^4 = r^4\,e^{i4\vartheta}$

$z^4=-1$ $\qquad r^4\,e^{i4\vartheta} = -1 = e^{i\pi}$

$r^4\,e^{4i\vartheta} = e^{i\pi} = -1$ $\qquad r^4 = 1 \Rightarrow r=1$

$e^{4i\vartheta} = e^{i\pi} = -1$

$\cos(4\vartheta) + i\sin(4\vartheta) = -1 + i\,0$

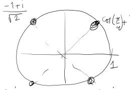
$$\begin{cases}\cos(4\vartheta) = -1\\ \sin(4\vartheta) = 0\end{cases}$$

$4\vartheta = \pi + 2\pi k$

$\vartheta = \frac{\pi}{4} + \frac{2\pi}{4}k$

$k \in \mathbb{Z}$

$\frac{-1+i}{\sqrt2}$ $\qquad \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right) = \frac{1+i}{\sqrt2}$



$\frac{-1-i}{\sqrt2}$ $\qquad \frac{1-i}{\sqrt2}$

$$\cancel{x}^4+1 = \left(\cancel{x} - \frac{1+i}{\sqrt2}\right)\left(x - \frac{1-i}{\sqrt2}\right)\left(x+\frac{1+i}{\sqrt2}\right)\left(x+\frac{1-i}{\sqrt2}\right)$$

$$= \left(\left(x-\frac{1}{\sqrt2}\right)-\frac{i}{\sqrt2}\right)\left(\left(x-\frac{1}{\sqrt2}\right)+\frac{i}{\sqrt2}\right)\left(\left(x+\frac{1}{\sqrt2}\right)+\frac{i}{\sqrt2}\right)\left(\left(x+\frac{1}{\sqrt2}\right)-\frac{i}{\sqrt2}\right)$$

$$= \left(\left(x-\frac{1}{\sqrt2}\right)^2+\frac12\right)\left(\left(x+\frac{1}{\sqrt2}\right)^2+\frac12\right)$$

$$x^4+1 = \left(x^2-\sqrt2 x+1\right)\left(x^2+\sqrt2 x+1\right)$$

$$\frac{1}{1+x^4} = \frac{Ax+B}{x^2-\sqrt2x+1} + \frac{Cx+D}{x^2+\sqrt2x+1} =$$

$$= \frac{Ax(x^2+\sqrt2x+1)+B(x^2+\sqrt2x+1)+Cx(x^2-\sqrt2x+1)+D(x^2-\sqrt2x+1)}{1+x^4}$$

$$\frac{1}{1+x^4} = \frac{(A+C)x^3 + x^2(\sqrt2A+B-C\sqrt2+D)+x(A+\sqrt2B+C-\sqrt2D) + B+D}{1+x^4}$$

$$\begin{cases}A+C=0\\ \sqrt2A+B-\sqrt2C+D=0\\ \cancel{A}+\sqrt2B+\cancel{C}-\sqrt2D=0\\ B+D=1\end{cases}$$

$$\begin{cases}A+C=0\\ B-D=0\\ \sqrt2A-\sqrt2C=-1\\ B+D=1\end{cases}$$

$$\begin{cases}A+C=0\\ A-C=-\frac{1}{\sqrt2}\end{cases} \qquad \begin{cases}B+D=1\\ B-D=0\end{cases} \qquad B=D=\frac12$$

$A = -\frac{1}{2\sqrt2}$ $\qquad C = \frac{1}{2\sqrt2}$

$$\lim_{x\to+\infty} \int_x^{3x} \lg\left(1+\frac{1}{\sqrt t}+\frac{1}{t}\right) dt$$

$$\lg\left(1+\frac{1}{\sqrt t}+\frac{1}{t}\right)$$

$$\lg(1+y) = y + o(y) = y - \frac{y^2}{2} + o(y^2)$$

$$\lg\left(1+\frac{1}{\sqrt t}+\frac{1}{t}\right) = \frac{1}{\sqrt t}+\frac{1}{t} - \frac12\left(\frac{1}{\sqrt t}+\frac1t\right)^2 + o\left(\left(\frac{1}{\sqrt t}+\frac1t\right)^2\right)$$

$$= \frac{1}{\sqrt t} + \frac1t - \frac12 \frac1t - \frac12 \frac{1}{t^2} - \frac{1}{t^{3/2}} + o\left(\frac1t\right)$$

$$= \frac{1}{\sqrt t} + \frac12 \frac1t + o\left(\frac1t\right)$$

$$\int_x^{3x} \lg\left(1+\frac{1}{\sqrt t}+\frac1t\right) dt = \int_x^{3x} t^{-\frac12}\, dt + \frac12\int_x^{3x} \frac1t + \int_x^{3x} o\left(\frac1t\right)$$

$$= 2\, t^{\frac12}\Big]_x^{3x} + \frac{\lg 3}{2} + \int_x^{3x} o\left(\frac1t\right) dt$$

$$= 2\left(\sqrt{3x}-\sqrt x\right) + \frac{\lg 3}{2} + \int_x^{3x} o\left(\frac1t\right) dt$$

$$= 2\left(\sqrt3 - 1\right)\sqrt x + \frac{\lg 3}{2} + \int_x^{3x} o\left(\frac1t\right) dt$$

$\downarrow$ $+$ $\qquad\qquad$ $\downarrow_{x\to+\infty}$?

$$\int_x^{3x} o\left(\frac1t\right) dt = 2\frac{x}{c_x}\,\frac{o\left(\frac{1}{c_x}\right)}{\frac{1}{c_x}} \xrightarrow[x\to+\infty]{} 0 \qquad x \le c_x \le 3x$$

$$\frac13 \le \frac{x}{c_x} \le 1 \qquad \downarrow 0$$

$$\lim_{x\to+\infty} \frac{o\left(\frac{1}{c_x}\right)}{\frac{1}{c_x}} = \lim_{y\to+\infty} \frac{o\left(\frac1y\right)}{\frac1y} = 0$$