CHIRAL ANOMALY IN 4d

$P = \bar{\psi}\gamma_5\psi$

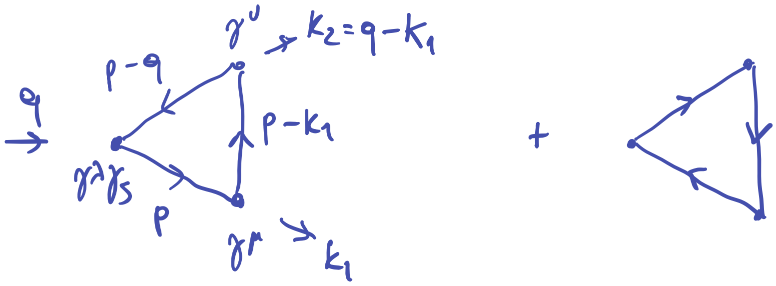
Ci calcoliamo i correlatori $\langle \underbrace{j^\mu j^\nu j_A^\lambda}_{T^{\mu\nu\lambda}} \rangle$ e $\langle \underbrace{P j^\mu j^\nu}_{T^{\mu\nu}} \rangle$

Perché anomalia solo in correlatori con 3 correnti? VEDI (*)

AWI : $q_\lambda T^{\mu\nu\lambda} = 2m\, T^{\mu\nu}$

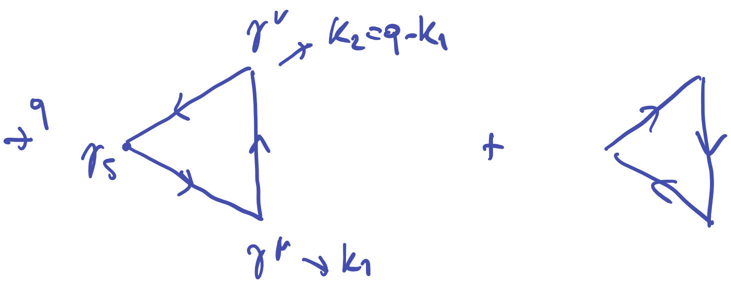
VWI : $k_{1\mu} T^{\mu\nu\lambda} = 0 = k_{2\nu} T^{\mu\nu\lambda}$

$\langle j^\mu j^\nu j_A^\lambda \rangle$



$$T^{\mu\nu\lambda} = i\int \frac{d^4p}{(2\pi)^4}(-)\, tr\left[\frac{i}{\not p - m}\gamma^\lambda\gamma_5 \frac{i}{\not p - \not q - m}\gamma^\nu \frac{i}{\not p - \not k_1 - m}\gamma^\mu\right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$\langle j^\mu j^\nu P \rangle$



$$T^{\mu\nu} = i\int \frac{d^4p}{(2\pi)^4}(-)\, tr\left[\frac{i}{\not p - m}\gamma_5 \frac{i}{\not p - \not q - m}\gamma^\nu \frac{i}{\not p - \not k_1 - m}\gamma^\mu\right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

Avremo bisogno della seguente identità:

$$\not q\gamma_5 = \gamma_5(\not p - \not q - m) + (\not p - m)\gamma_5 + 2m\gamma_5$$

$$q_\lambda T^{\mu\nu\lambda} = 2m\, T^{\mu\nu} + R_1^{\mu\nu} + R_2^{\mu\nu}$$

$$R_1^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\overbrace{\frac{1}{\not p - \not k_2 - m}\,\gamma_5\gamma^\nu\, \frac{1}{\not p - \underbrace{\not q}_{=k_1+k_2} - m}\,\gamma^\mu}^{1°} - \overbrace{\frac{1}{\not p - m}\,\gamma_5\gamma^\nu\, \frac{1}{\not p - \not k_1 - m}\,\gamma^\mu}^{2°} \right)$$

$$R_2^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\frac{1}{\not p - \not k_1 - m}\,\gamma_5\gamma^\mu\, \frac{1}{\not p - \not q - m}\,\gamma^\nu - \frac{1}{\not p - m}\,\gamma_5\gamma^\mu\, \frac{1}{\not p - \not k_2 - m}\,\gamma^\nu \right)$$

Se $R_1 = R_2 = 0 \Rightarrow$ AWI è soddisfatta

→ Formalmente $R_1 \to 0$ se shiftiamo $p \to p + k_2$ nel 1° integrale e $p \to p - k_2$ nel 2° integrale

$$R_1^{\mu\nu} = \int \frac{d^4p}{(2\pi)^4}\, \mathrm{tr}\left(\frac{1}{\not p - m}\,\gamma_5\gamma^\nu\, \frac{1}{\not p - \not k_1 - m}\,\gamma^\mu - \frac{1}{\not p - \not k_2 - m}\,\gamma_5\gamma^\nu\, \frac{1}{\not p - \not q - m}\,\gamma^\mu \right)$$

$$= -R_1^{\mu\nu} \Rightarrow R_1^{\mu\nu} = 0$$

Stesso conto per $R_2^{\mu\nu}$ $\quad (p \to p \pm k_1)$

siccome l'integrale diverge (div. lineare), le manipolazioni interne all'integrale non sono giuste $(\infty - \infty)$

→ dobbiamo regolarizzare la teoria.

Per questo conto useremo la regolarizzaz. di PAULI-VILLARS:

Introduciamo una specie ausiliaria di fermioni Ψ con STATISTICA OPPOSTA e massa M; alla fine del conto bisogna prendere il lim. $M \to \infty$ (statistica errata)

$\hookrightarrow$ • $T^{\mu\nu\lambda}_{reg} = T^{\mu\nu\lambda}(m) - T^{\mu\nu\lambda}(M)$

$$T^{\mu\nu\lambda}_{phys} \equiv \lim_{M\to\infty} T^{\mu\nu\lambda}_{reg}$$

• $T^{\mu\nu}_{phy} = \lim_{M\to\infty} T^{\mu\nu}_{reg} = \lim_{M\to\infty} \left(T^{\mu\nu}(m) - T^{\mu\nu}(M)\right) = T^{\mu\nu}(m)$

$T^{\mu\nu}$ è convergente e $T^{\mu\nu}(M) \sim \frac{1}{M} \xrightarrow[M\to\infty]{} 0$

Questa regolarizzazione fa una scelta specifica su chi preservare tra AWI e VWI:

$$\left.\begin{aligned} K_{1\mu} T^{\mu\nu\lambda}_{phy} &= 0 \\ K_{2\nu} T^{\mu\nu\lambda}_{phys} &= 0 \end{aligned}\right\} \to \text{VWI è PRESERVATA}$$

(Euristicam. $\mathcal{L}_\Phi$ non rompe $U(1)_V$)

AWI ?

Ora $T^{\mu\nu\lambda}_{reg}$ è finito; in particolare $R_1{}^{\mu\nu}_{reg}$ e $R_2{}^{\mu\nu}_{reg}$ sono finiti $\longrightarrow$ quindi ora posso fare le manipolazioni all'interno dell'integrale $\Longrightarrow R_1{}^{\mu\nu}_{reg} = 0 = R_2{}^{\mu\nu}_{reg}$

$$\Longrightarrow \quad q_\lambda T^{\mu\nu\lambda}_{reg} = 2m\, T^{\mu\nu}(m) - 2M\, T^{\mu\nu}(M)$$

$$\Rightarrow \quad q_\lambda T^{\mu\nu\lambda}_{phys} = 2m\, T^{\mu\nu}(m) - \underbrace{\lim_{M\to\infty} 2M\, T^{\mu\nu}(M)}_{\substack{\text{ora calcoleremo qto} \\ \text{(violazione di AWI} \\ \text{se } \neq 0)}}$$

$$T^{\mu\nu}(M) = \int \frac{d^4p}{(2\pi)^4} (-) \, \mathrm{tr} \frac{1}{\not p - M} \gamma_5 \frac{1}{\not p - \not q - M} \gamma^\nu \frac{1}{\not p - \not k_1 - M} \gamma^\mu$$

$$+ \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$\frac{1}{a_1 a_2 a_3} = 2\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\left[a_1 x_2 + a_2(1-x_1-x_2) + a_3 x_1\right]^3}$$

$$= -\int \frac{d^4p}{(2\pi)^4} 2\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{\mathrm{tr}\left[(\not p + M)\gamma_5(\not p - \not q + M)\gamma^\nu(\not p - \not k_1 + M)\gamma^\mu\right]}{\left[(p^2-M^2)x_2 + \left((p-q)^2-M^2\right)(1-x_1-x_2) + \left((p-k_1)^2 - M^2\right)x_1\right]^3}$$

$$+ \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$\mathrm{Tr}\, \gamma_5 \gamma^{\alpha_1}\cdots\gamma^{\alpha_n} = 0 \quad \text{für} \quad n \neq 4$$

$$= 4i\epsilon^{\alpha_1\cdots\alpha_4} \quad \text{für} \quad n = 4$$

$$\mathrm{tr}\left(\not p \gamma_5 (\cancel{\not p} - \not q)\gamma^\nu\gamma^\mu\right) + \mathrm{tr}\left(\not p \gamma_5 \gamma^\nu (\cancel{\not p} - \not k_1)\gamma^\mu\right) + \mathrm{tr}\left(\gamma_5 (\not p - \not q)\gamma^\nu(\not p - \not k_1)\gamma^\mu\right)$$

$$= \mathrm{tr}\left(\gamma_5 \cancel{\not p \not q} \gamma^\nu\gamma^\mu\right) + \mathrm{tr}\left(\gamma_5 \cancel{\not p \gamma^\nu \not k_1} \gamma^\mu\right) - \mathrm{tr}\left(\cancel{\gamma_5 \not p \gamma^\nu \not k_1 \gamma^\mu}\right)$$

$$- \mathrm{tr}\left(\cancel{\gamma_5 \not q \gamma^\nu \not p \gamma^\mu}\right) + \mathrm{tr}\left(\gamma_5 \underset{\cancel{\not k_1} + \not k_2}{\underset{\parallel}{\not q}} \gamma^\nu \not k_1 \gamma^\mu\right) =$$

$$= 4i\epsilon^{\beta\nu\alpha\mu} k_{1\alpha} k_{2\beta} \qquad \epsilon^{\alpha\nu\beta\mu} = \epsilon^{\beta\alpha\nu\mu} = \epsilon^{\beta\mu\alpha\nu}$$

$$= 4i\epsilon^{\beta\mu\alpha\nu} k_{2\alpha} k_{1\beta}$$

$$= -\int \frac{d^4p}{(2\pi)^4} 2\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{M\left(-4i\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}\right)}{\left[x_2 p^2 - x_2 M^2 + (1-x_1-x_2)(p^2 - 2pq + q^2 - M^2) + x_1(p^2 - 2pk_1 + k_1^2 - M^2)\right]^3} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$$p^2 - M^2 - 2p\left(q(1-x_1-x_2) + k_1 x_1\right) + q^2(1-x_1-x_2) + k_1^2 x_1$$

$$= p^2 - 2pK - \bar M^2 \qquad K \equiv q(1-x_1-x_2) + k_1 x \qquad \bar M \equiv M^2 - q^2(1-x_1-x_2) - k_1^2 x_1$$

$$= 8iM\,\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \underbrace{\int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2-2pk-\bar{M}^2)^3}} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

$p^0 = il^0$

$\Downarrow$

$p^2 = -l^2$

$$-i \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2+2pk+\bar{M}^2)^3} = \frac{-i\ \Gamma(3-2)}{(\bar{M}-k^2)^{3-2}(4\pi)^2\,\Gamma(3)}$$

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2+2kp+b^2)^A} = \frac{\Gamma(A-d/2)}{(b^2-p^2)^{A-d/2}(4\pi)^{d/2}\Gamma(A)}$$

$$= \frac{-i}{(4\pi)^2} \frac{1}{2(\bar{M}^2-k^2)}$$

$$= \frac{1}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{M}{M^2 + \underbrace{f(x_1,x_2)}_{\text{non dip da } M}} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix}$$

stessa espressione del primo termine ma con $f(x_1,x_2)$ sostituita da una diversa $g(x_1,x_2)$

$$\lim_{M\to\infty} 2M\,T^{\mu\nu}(M) = \frac{1}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \underbrace{\lim_{M\to\infty} \left(\frac{2M^2}{M^2+f(x_1,x_2)} + \frac{2M^2}{M^2+g(x_1,x_2)} \right)}_{=4}$$

$$= \frac{1}{\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \underbrace{\int_0^1 dx_1 \int_0^{1-x_1} dx_2}_{= \int_0^1 dx_1 (1-x_1) = 1 - \frac{1}{2} = \frac{1}{2}}$$

$$q_\lambda T^{\mu\nu\lambda}_{phys} = 2m\,T^{\mu\nu}(m) \underbrace{- \frac{1}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}}_{\text{ANOMALIA}}$$

$$\partial^z_\lambda \langle j^\mu(x)\, j^\nu(y)\, j^\lambda_A(z)\rangle = -i \int \frac{dq}{(2\pi)^4}\int\frac{dk_1}{(2\pi)^4}\int\frac{dk_2}{(2\pi)^4}\, e^{-ik_1x-ik_2y+iqz}\,\cdot$$
$$\cdot\, i q_\lambda\, T^{\mu\nu\lambda}(q,k_1,k_2)$$

$$= \int\frac{dq}{(2\pi)^4}\int\frac{dk_2}{(2\pi)^4}\, e^{-ik_2(y-x)+iq(z-x)} \left(-\frac{1}{2\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\, i q_\alpha (-i) k_{2\beta}\right)$$

$$= -\frac{1}{2\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\partial^z_\alpha\,\delta(z-x)\,\partial^y_\beta\,\delta(y-x)$$

$$\partial^z_\lambda \langle j^\lambda_A(z)\rangle_A = \partial^z_\lambda\langle j^\lambda_A(z)\rangle_{free} + \dots - \frac{1}{2}\int dx_1\, dx_2\, A_{\rho_1}(x_1)\, A_{\rho_2}(x_2)\,\cdot$$
$$\cdot\, \partial_\lambda \langle j^{\rho_1}(x_1)\, j^{\rho_2}(x_2)\, j^\lambda_A(z)\rangle + \dots$$

campo di gauge esterno accoppiato alla corrente vettoriale j^μ

$$= -\int dx_1\, dx_2\, A_{\rho_1}(x_1)\, A_{\rho_2}(x_2)\left(-\frac{1}{4\pi^2}\right)\epsilon^{\rho_1\rho_2\alpha\beta}\,\partial^z_\alpha\,\delta(z-x_1)\,\partial^{x_2}_\beta\,\delta(x_2-\overset{=z}{x_1})$$

$$= \frac{1}{16\pi^2}\,\epsilon^{\rho_1\rho_2\alpha\beta}\,F_{\alpha\rho_1}\,F_{\beta\rho_2}$$

$$= -\frac{1}{16\pi^2}\,\epsilon^{\alpha\rho_1\beta\rho_2}\,F_{\alpha\rho_1}\,F_{\beta\rho_2}$$

$$= -\frac{1}{16\pi^2}\,\epsilon^{\mu_1\nu_1\mu_2\nu_2}\,F_{\mu_1\nu_1}\,F_{\mu_2\nu_2}$$

NON-ABELIAN FIELDS

$$\mathcal{L} = \bar{\Psi}\, i \not{D} \psi = \bar{\Psi}_i\, i \gamma^\mu (\partial_\mu \delta^{ij}) \psi_j$$

$U(1)_{AV}$
$SU(N)_{AV}$
$N = \dim R$

Campi ψ stanno in una rap R di G

$$\Rightarrow \quad j^{\mu a} = \bar{\Psi} \gamma^\mu t_R^a \psi$$

$a \in$ Adj rap.

$$j_A^{\mu a} = \bar{\Psi} \gamma^\mu \gamma_5 t_R^a \psi \qquad P = \bar{\Psi} \gamma_5 t_R^a \psi$$

In ph situazione ci sono correlatori a più pti che contribuiscono all'anomalia (anche $\langle \underbrace{j \cdots j}_{n} \rangle$ con $n \geq 3$)

$$T^{abc\,\mu\nu\lambda}(k_1 k_2) = i \int dx\, dy\, e^{ik_1 x + ik_2 y} \langle j^{a\mu}(x) j^{b\nu}(y) j_A^{c\lambda}(0) \rangle$$

$$T^{abc\,\mu\nu} \quad = \quad '' \qquad P^c(0)$$

$\downarrow$

$$\text{AWI:} \quad q_\lambda T^{abc\,\mu\nu\lambda} = 2m\, T^{abc\,\mu\nu}_{(m)} \underbrace{- \frac{C^{abc}}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta}}$$

$$C^{abc} = \frac{1}{2} \mathrm{tr}_R \{ t_R^a, t_R^b \} t_R^c$$

ABJ anomaly per il caso non-abeliano

Se manteniamo la CORRENTE ASSIALE ABELIANA $j_A^\lambda = \bar{\Psi} \gamma^\mu \gamma_5 \psi$

$\updownarrow$

legata a simm. $\psi \mapsto e^{i\gamma_5 \beta} \psi$

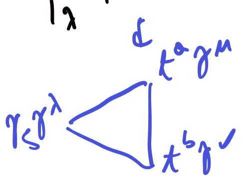
(j_A^λ è un "color singlet", cioè in rap. triviale di G),

e ph vett. non-ab. $j_V^{\mu a} = \bar{\Psi} \gamma^\mu t_R^a \psi$

allora :

(mentre $j_A^{\lambda a} = \bar{\Psi} \gamma^\mu \gamma_5 t_R^a \psi$ è legata a $\psi \mapsto e^{i\gamma_5 \beta^a t_R^a} \psi$)

$$q_\lambda T^{ab\,\mu\nu\lambda} = 2m\, T^{ab\,\mu\nu} - \frac{c(R)\, \delta^{ab}}{2\pi^2} \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \qquad (\ast)$$



$\mathrm{Tr}(t^a t^b \mathbb{1})$

Accoppiamo la corrente vettoriale $j^{\mu a}$ a un campo di gauge non-abeliano A_μ^a. L'anomalia (*) produrrà ancora termini del tipo $(\partial_\xi A_\beta^a)^2$. Come contribuiscono ~~le altre~~ gli altri termini dell' WI anomale?

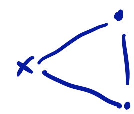
Devono contribuire in maniera consistente col fatto che $\partial_\lambda j_A^\lambda$ è un SINGOLETTO sotto G

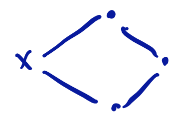
$$\langle \partial_\lambda j_A^\lambda \rangle_A = -\frac{c(R)\,\delta^{ab}}{16\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,F_{\mu\nu}^a F_{\alpha\beta}^b \qquad (m=0)$$

SINGLET ANOMALY

$$= -\frac{1}{16\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,F_{\mu\nu}F_{\alpha\beta}$$

$\epsilon^{\mu\nu\alpha\beta}\delta^{ab}F_{\mu\nu}^a F_{\alpha\beta}^b$

$\epsilon^{\mu\nu\alpha\beta}\,\partial_\mu A_\nu^a\,\partial_\alpha A_\beta^a \longrightarrow$  triangolo

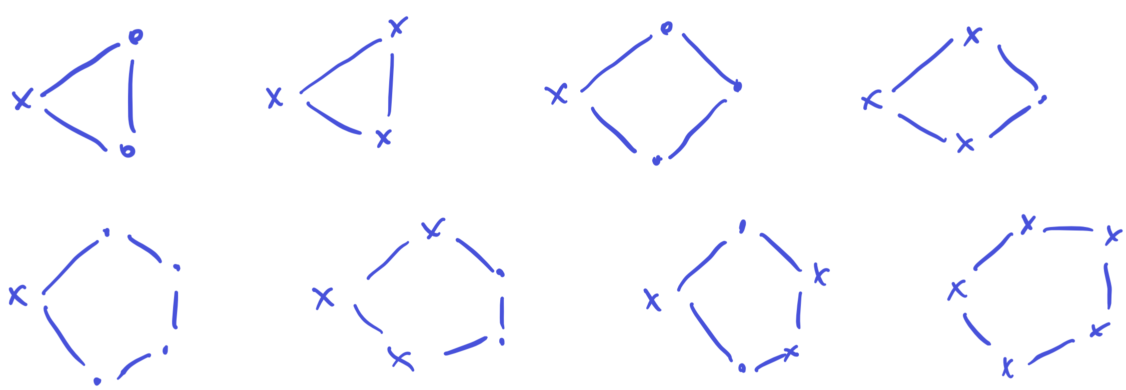
$\epsilon^{\mu\nu\alpha\beta}\,\partial_\mu A_\nu^a A_\alpha^c A_\beta^d f^{bcd} \longrightarrow$  quadrato

$\epsilon^{\mu\nu\alpha\beta}\,A_\mu^c A_\nu^d A_\alpha^e A_\beta^f\,\underbrace{f^{ecd}f^{aef}}$

$= \#\left(f^{acd}f^{aef} + f^{aed}f^{afc} + f^{afd}f^{ace}\right) = 0$ Jacobi id.

Se anche la corrente assiale viene presa non-ab. $j_A^{\lambda a} = \bar\psi\gamma^\mu\gamma_5 t_R^a\psi$

$\Rightarrow$ molti diagrammi contribuiscono $\rightarrow$ molte WI sono anomale nella teoria libera



Se accoppiamo $j^{\mu a}$ a A_μ^a :

$$\langle (D_\mu j_A^\mu)^a \rangle = -\frac{1}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{tr}(t^a F_{\mu\nu} F_{\rho\sigma}) \qquad (*)$$

COVARIANT ANOMALY

Finora abbiamo sempre accoppiato A_μ^a alla CORRENTE VETT., cioè abbiamo GAUGIATO la simmetria preservata. Infatti se la simm. di gauge è ANOMALA la teoria quantistica risulta INCONSISTENTE: 1) perdiamo la relazione di equivalenza necessaria per tener conto della ridondanza della descrizione 2) alcune proprietà come la rinormalizzabilità e l'unitarietà vengono violate.

Campi chirali

Pensiamo di accoppiare correnti R-handed e L-handed a diversi campi vett. A_μ^R e A_μ^L.

Otteniamo:

$$\langle (D_\mu^H j^{H\mu})^a \rangle = \eta_H \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{tr}\left(t^a \partial_\mu \left(A_\nu^H \partial_\rho A_\sigma^H + \frac{i}{2} A_\nu^H A_\rho^H A_\sigma^H\right)\right)$$

$H = L, R$

CONSISTENT ANOMALY

$$\eta_H = \begin{cases} -1 & H = L \\ +1 & H = R \end{cases}$$

Notiamo che otteniamo lo stesso risultato formale (a meno del segno) per L e R.

Esiste anche un altro tipo di anomalia COVARIANT ANOMALY

per correnti L, R (diversa regolarizzazione rispetto a CONS. AN.)

$$\langle (D_\mu j^{H\mu})^a \rangle = \eta_H \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{tr}\left(t^a F^H_{\mu\nu} F^H_{\rho\sigma} \right) \qquad H = L, R$$

$$j^{L,R\,\mu\,a} = \bar{\psi} \gamma^\mu t^a \left(\frac{1 \pm \gamma_5}{2} \right) \psi = \frac{1}{2} \left(j^{\mu a} \pm j_A^{\mu a} \right)$$

$$A_\mu^{L,R} = A_\mu^V \pm A_\mu^A$$

In $\mathcal{L}$

$\sim j^{L\mu a} A_\mu^{aL} + j^{R\mu a} A_\mu^{aR}$

$\Leftrightarrow j^\mu A_\mu^{aV} + j_A^\mu A_\mu^{aA}$

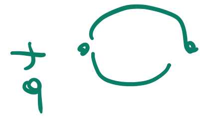
↳ se $A_\mu^A \equiv 0 \Rightarrow A_\mu^{L,R} = A_\mu$

e $\langle (D_\mu j_A^\mu)^a \rangle = \langle (D_\mu j^{L\mu})^a \rangle - \langle (D_\mu j^{R\mu})^a \rangle =$

$$= 2 \left[-\frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \operatorname{tr}\left(t^a F_{\mu\nu} F_{\rho\sigma} \right) \right] = -\frac{1}{16\pi^2} \epsilon \operatorname{tr}(tFF)$$

cioè (*).

(A) - Quando ho $\langle j^{\mu_1}(x_1)\cdots j^{\mu_n}(x_n)\rangle$ $n>3$ gli integrali sono convergenti e quindi manipolazioni formali sono lecite e portano a $\partial\langle j\cdots j\rangle = 0$.

- Per due correnti:  conto simile a funz. a 2 pti $\langle AA\rangle = \langle jj\rangle$ a 1-loop in QED

$$\langle j_A^\mu j_A^\lambda\rangle = (i^2)\, i\int \frac{d^4p}{(2\pi)^4}\,(-)\,\underbrace{\mathrm{tr}\left[(\not p+m)\gamma^\lambda\gamma_5(\not p-\not q+m)\gamma^\mu\gamma_5\right]}_{\mathrm{tr}\,(\not p+m)\gamma^\lambda(\not p-\not q-m)\gamma^\mu}\,\frac{1}{(p^2-m^2)((p-q)^2-m^2)}$$

$$\Rightarrow \langle j_A j_A\rangle = \langle jj\rangle\big|_{m^2\mapsto -m^2}$$

ma $\langle jj\rangle$ è proporzionale a $q^2 g^{\mu\nu} - q^\mu q^\nu$

$$\Rightarrow \partial\langle j_A j_A\rangle = 0$$

$$\langle j^\mu j_A^\lambda\rangle = (i^2)\, i\int \frac{d^4p}{(2\pi)^4}\,(-)\,\underbrace{\mathrm{tr}\left[(\not p+m)\gamma^\lambda\gamma_5(\not p-\not q+m)\gamma^\mu\right]}_{=-\mathrm{tr}\left[\not p\gamma^\lambda\gamma_5\not q\gamma^\mu\right]\propto q_\alpha p_\beta\epsilon^{\alpha\beta\mu\lambda}}\,\frac{1}{(p^2-m^2)((p-q)^2-m^2)}$$

$$\Rightarrow \partial_\lambda\langle j^\mu j_A^\lambda\rangle = 0$$

- $\langle j^\mu\rangle = \langle j_A^\mu\rangle = 0$ per invarianza di Lorentz.