Link 07/05/2020

- Link alle due lezioni registrate:
  - https://drive.google.com/open?id=1juVxhpCZDPCA_J5pDnL08f6_BILqAF1e

Es. 5.8 (5.19)

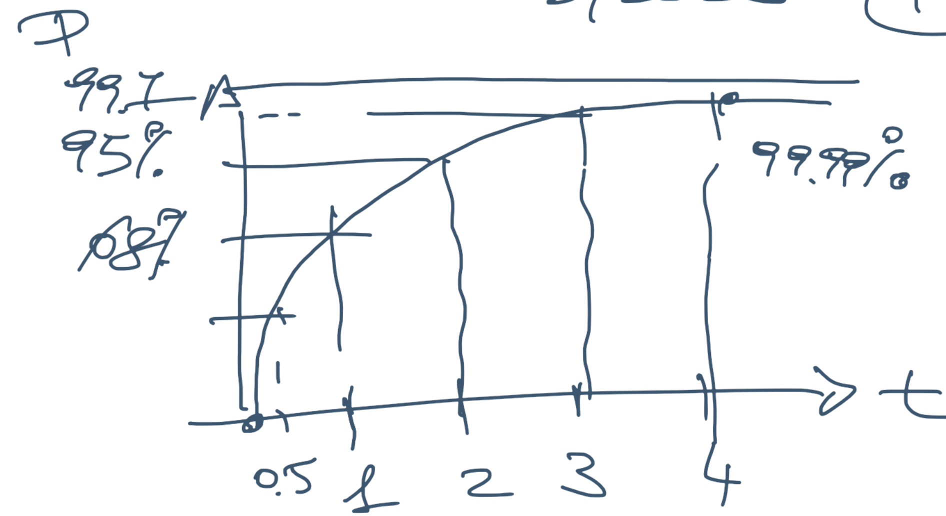
5.19. ★★ A student measures a quantity y many times and calculates his mean as $\bar{y} = 23$ and his standard deviation as $\sigma_y = 1$. What fraction of his readings would you expect to find between

- **(a)** 22 and 24?
- **(b)** 22.5 and 23.5?
- **(c)** 21 and 25?
- **(d)** 21 and 23?
- **(e)** 24 and 25?

Finally, **(f)** within what limits (equidistant on either side of the mean) would you expect to find 50% of his readings? (The necessary information for all parts of this question is in Figure 5.13. More detailed information on these kinds of probabilities is in Appendixes A and B.)

f) $t = 0.7$ $22.3 \div 23.7$ 07/05/2020 (1)

$\bar{y} = 23$ $\sigma_y = 1$



a) $22 \div 24$ $\bar{y} = 23$

-1σ $+1\sigma$ $\bar{y} \pm 1\sigma$ 68%

b) $\underbrace{22.5 \div 23.5}$ 23 ± 0.5

$t = 1/2$ 38%

c) $21 \div 25$ 23 ± 2

$t = 2$ $\underline{95\%}$

95.45

d) $21 \div 23$ $\bar{y} = 23$

$\rightarrow 2\sigma$ $5x$ Δ $1x, \sigma$

$\overline{48\%}$

e) $24 \div 25$ 23 1σ Δx

2σ Δx

Probab.$(23 \div 25) = 48\%$

Probab.$(23 \div 24) = \frac{68}{2}\% = 34\%$

Probab $= 48\% - 34\% = 14\%$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-X)^2}{2\sigma^2}}$$
X, σ

$(x_1, \dots, x_N)$

$$\bar{x} = \frac{\sum x_i}{N}$$ miglior stima di X

$$\sigma^2 = \frac{\sum_i (x_i - \bar{x})^2}{N-1}$$

$x, y, z \dots$

(X, σ_x) (Y, σ_y) (Z, σ_z)

$q = f(x, y, z)$ $\quad Q, \sigma_q$

$q = x + A$ $\quad X, \sigma_x$

$$x = q - A \quad \rightsquigarrow \quad e^{-\frac{(q-A-X)^2}{2\sigma_x^2}} = e^{-\frac{(q-(X+A))^2}{2\sigma_x^2}}$$

$Q = X + A$ $\quad \sigma_q = \sigma_x$

3

$q = B \cdot x \qquad x = \frac{q}{B}$

$$P \propto e^{-\frac{\left(\frac{q}{B} - x\right)^2}{2\sigma_x^2}} = e^{-\frac{1}{B^2}\frac{(q - Bx)^2}{2\sigma_x^2}} = e^{-\frac{(q-Bx)^2}{2(B\sigma_x)^2}}$$

$Q = BX \qquad \sigma_q = B\sigma_x$

$q = x + y$

$$P(q) = P(x) \cdot P(y) =$$

$\hookrightarrow e^{-\frac{(x-X)^2}{2\sigma_x^2}} \qquad \hookrightarrow e^{-\frac{(y-Y)^2}{2\sigma_y^2}}$

$$= e^{-\left[\frac{(x-X)^2}{2\sigma_x^2} + \frac{(y-Y)^2}{2\sigma_y^2}\right]}$$

$$e^{-\frac{((x+y)-(X-Y))^2}{2\sigma^2}}$$

$X, Y = 0$

$$e^{-\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\frac{1}{2}}$$

$A=\sigma_x^2 \quad B=\sigma_y^2$

4

$$\frac{x^2}{A}+\frac{y^2}{B}=\frac{(x+y)^2}{A+B}+\frac{(Bx-Ay)^2}{AB(A+B)}=$$

$$\frac{AB(x^2+2xy+y^2)+(B^2x^2-2ABxy+A^2y^2)}{AB(A+B)}=$$

$$=\frac{ABx^2+\cancel{2ABxy}+\boxed{ABy^2}+B^2x^2\cancel{-2ABxy}+\boxed{A^2y^2}}{AB(A+B)}=$$

$$=\frac{\cancel{AB}\left(x^2+\frac{B}{A}x^2+y^2+\frac{A}{B}y^2\right)}{\cancel{AB}(A+B)}=\frac{x^2\frac{\cancel{(A+B)}}{A}+y^2\frac{\cancel{(B+A)}}{B}}{\cancel{A+B}}$$

$$=\frac{x^2}{A}+\frac{y^2}{B}$$

(5)

$$\boxed{\frac{x^2}{A}+\frac{y^2}{B}=\boxed{\frac{(x+y)^2}{A+B}}+\frac{(Bx-Ay)^2}{AB(A+B)}}$$

z^2

$$e^{-\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\frac{1}{2}} \qquad A=\sigma_x^2 \quad B=\sigma_y^2$$

$$q=x+y$$

$$P(q)=P(x)\cdot P(y)$$

$$e^{-\frac{x^2}{2\sigma_x^2}} \qquad e^{-y^2/2\sigma_y^2}$$

$$e^{-\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\frac{1}{2}}$$

$$P(q)=e^{-\left[\frac{(x+y)^2}{A+B}+z^2\right]\frac{1}{2}}$$

$$=e^{-\frac{(x+y)^2}{2(A+B)}}\,e^{-z^2/2} \quad A \quad \sqrt{2\pi}$$

$$\propto e^{-\frac{(x+y)^2}{2(A+B)}}=e^{-\frac{q^2}{2\sigma_q^2}}$$

$$\boxed{\sigma_q^2=A+B=\sigma_x^2+\sigma_y^2}$$

$$X,Y=\phi$$

$$q=x+y$$

$$\boxed{g(x,y) \sim g(X,Y) + \left.\frac{\partial g}{\partial x}\right|_{X,Y} (x-X) + \left.\frac{\partial g}{\partial y}\right|_{X,Y} (y-Y)}$$

x,y $\quad (X,Y)$ $\quad (\sigma_x, \sigma_y)$

numeri

Taylor

piccole

numero

$$A + \underbrace{B\,(x-X)} + \underbrace{C\,(y-Y)}$$

σ_x, σ_y $\quad x,y$ $\quad X,Y$

$$\boxed{\begin{aligned}\sigma_g^2 &= B^2\sigma_x^2 + C^2\sigma_y^2\\ &= \left(\frac{\partial g}{\partial x}\right)_{X,Y}^2 \sigma_x^2 + \left(\frac{\partial g}{\partial y}\right)_{X,Y}^2 \sigma_y^2\end{aligned}}$$

$(x_1, \ldots, x_N)$ $\quad x, \sigma_x$ $\quad e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$ $\quad$ (7

$\bar{x} \pm \sigma_{\bar{x}}$

$\bar{x} = \frac{\sum_i x_i}{N} \to X$

$\sigma_x^2 = \frac{\sum_i (x-\bar{x})^2}{N-1}$

$\boxed{\bar{x} = \frac{\sum_i x_i}{N}}$ $\quad x_1, x_2, \ldots, x_N$

$= f(x_1, \ldots, x_N)$

$\sigma_{\bar{x}}^2 = \sum_i \left(\frac{\partial \bar{x}}{\partial x_i}\right)^2 \cdot \sigma_{x_i}^2 = \left(\frac{\partial \bar{x}}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \ldots + \left(\frac{\partial \bar{x}}{\partial x_N}\right)^2 \sigma_{x_N}^2 =$

σ_x^2

$\frac{\partial \bar{x}}{\partial x_1} = \frac{1}{N} = \frac{\partial \bar{x}}{\partial x_2} = \ldots = \frac{\partial \bar{x}}{\partial x_N}$

$= \frac{1}{N^2}\sigma_x^2 + \frac{1}{N^2}\sigma_x^2 + \ldots + \frac{1}{N^2}\sigma_x^2 =$

$= \frac{\cancel{N}\sigma_x^2}{N^{\cancel{2}}} = \boxed{\sigma_x^2 / N}$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

$\boxed{\text{CAP. 7}}$

$\boxed{\text{MAXIMUM LIKELIHOOD}}$

(8

$x_A \pm \sigma_A \qquad x_B \pm \sigma_B$

$\frac{x_A + x_B}{2} \qquad \sigma_A, \sigma_B$

$$P(x_A, x_B) = \underset{X,\ \sigma_A}{P(x_A)} \cdot \underset{X\ \sigma_B}{P(x_B)} \propto \frac{1}{\sigma_A \sigma_B}\, e^{-\frac{(x_A - X)^2}{2\sigma_A^2}} \cdot e^{-\frac{(x_B - X)^2}{2\sigma_B^2}}$$

$$X \qquad = \frac{1}{\sigma_A \sigma_B}\, e^{-\left[\frac{(x_A - X)^2}{\sigma_A^2} + \frac{(x_B - X)^2}{\sigma_B^2}\right]}$$

X

$$\boxed{\chi^2 = \frac{(x_A - X)^2}{\sigma_A^2} + \frac{(x_B - X)^2}{\sigma_B^2}}$$

CHI QUADRO

$$\frac{d\chi^2}{dX} = \frac{\cancel{2}(x_A - X)}{\sigma_A^2}\cancel{(-1)} + \frac{\cancel{2}(x_B - X)}{\sigma_B^2}\cancel{(-1)} = 0$$

$$\underbrace{\frac{x_A}{\sigma_A^2} - \frac{X}{\sigma_A^2}} + \underbrace{\frac{x_B}{\sigma_B^2} - \frac{X}{\sigma_B^2}} = 0$$

$$w_A = \frac{1}{\sigma_A^2} \qquad w_B = \frac{1}{\sigma_B^2}$$

$$x_A w_A + x_B w_B = X(w_A + w_B)$$

$$X = \frac{x_A w_A + x_B w_B}{w_A + w_B}$$

weight pesi

MEDIA PESATA

$R_1 = 11 \pm 1 \longrightarrow w_1 = 1$ $\quad \sum w_i = \frac{19}{9} \sim 2$ $\quad$ 10

$R_2 = 12 \pm 1 \longrightarrow w_2 = 1$

$\cancel{R_3 = 10 \pm 3 \longrightarrow w_3 = \frac{1}{9}}$

$\bar{R} = \frac{\sum w_i \cdot R_i}{\sum w_i} = 11.4 \pm 0.7$

$R_{12} = 11.5 \pm 0.7$