# Link 11/05/2020

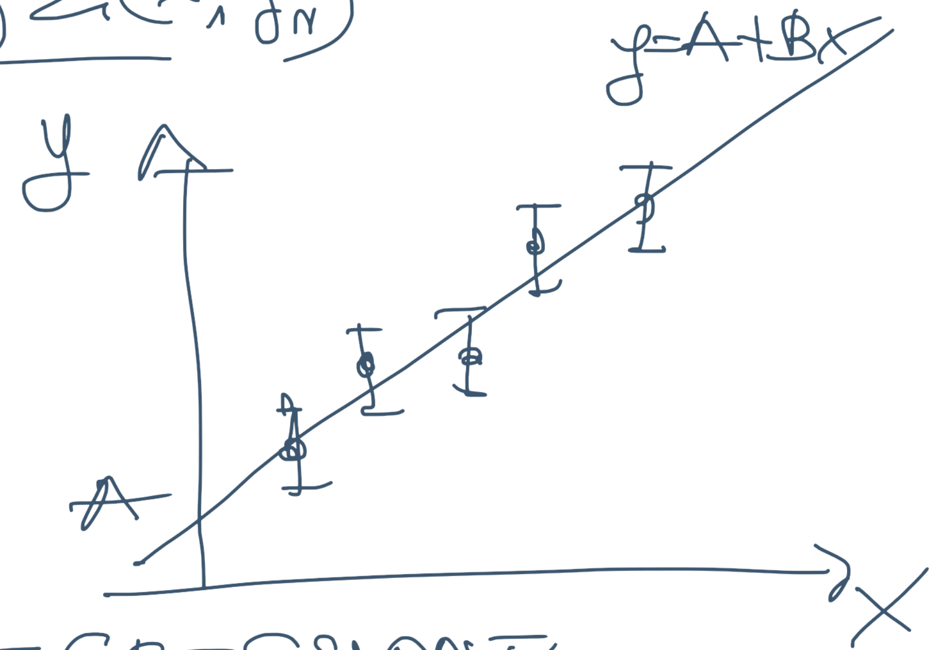
- Link alle due lezioni registrate:
  - https://drive.google.com/open?id=1tt7rw2lt-Tf2VYl0rJMG4MJFKQih0rPD

# Es. 7.3 (7.1)

**7.1.** ★ Find the best estimate and its uncertainty based on the following four measurements of a certain voltage:

$$1.4 \pm 0.5, \quad 1.2 \pm 0.2, \quad 1.0 \pm 0.25, \quad 1.3 \pm 0.2.$$

$$A = \frac{\left(\sum_i x_i^2\right)\left(\sum_i y_i\right) - \left(\sum x_i\right)\sum_i (x_i y_i)}{\Delta}$$




$$B =$$

$(x_i, y_i)$ $A, B$

$\sigma_A \quad \sigma_B$

FIT REGRESSIONE LINEARE

$$\sigma_A^2 = \sigma_y^2 \frac{\sum_i x_i^2}{\Delta}$$

$$\sigma_B^2 = N \frac{\sigma_y^2}{\Delta}$$

$$v = v_0 + g t$$

$v \quad\quad x$

$$h = \frac{1}{2} g t^2$$

$$z = t^2$$

$$\boxed{h = \frac{1}{2} g z}$$

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$$A = \frac{\sum y_i - B\sum x_i}{N}$$

$$\sum (y_i - A - Bx_i)x_i = 0$$

$$\sum y_i x_i - \sum A x_i - \sum B x_i^2 = 0$$

$$\sum x_i y_i - \left(\frac{\sum y_i - B\sum x_i}{N}\right)\sum x_i - B\sum x_i^2 = 0$$

$$N\sum x_i y_i - \frac{\sum y_i \sum x_i}{\cancel{N}} + \frac{B\sum x_i \sum x_i}{\cancel{N}} - B\sum x_i^2 = 0 \quad \times N$$

$$B\left(N\sum x_i^2 - \left(\sum x_i\right)^2\right) = N\sum x_i y_i - \sum x_i \sum y_i$$

$$\Delta = N\sum x_i^2 - \left(\sum x_i\right)^2$$

$$\boxed{B = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\Delta}}$$

$$A = \sum y_i - B$$

$$\chi^2 = \sum_i \frac{(y_i - A - Bx_i)^2}{\sigma_y^2}$$

$$\frac{\partial \chi^2}{\partial A} = \frac{1}{\cancel{\sigma_y^2}} \sum_i \cancel{2} (y_i - A - Bx_i)\cancel{(-1)} = \phi$$

$$\sum_i (y_i - A - Bx_i) = 0 \qquad \sum_i y_i - \underbrace{\sum_i A}_{NA} - \underbrace{\sum B x_i}_{B\sum x_i} = \phi$$

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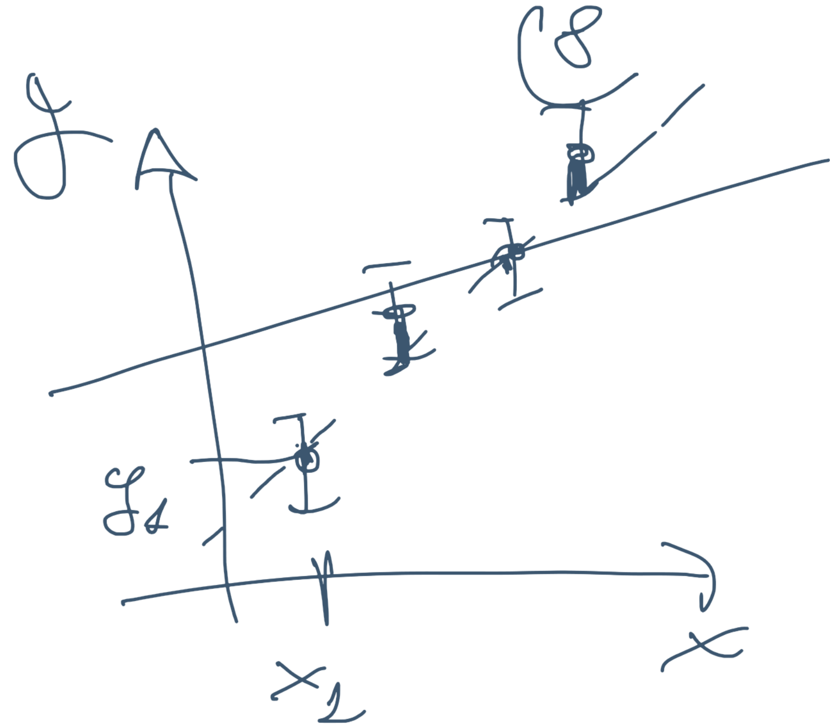
$$\boxed{A = \frac{\sum_i y_i - B\sum_i x_i}{N}}$$

$$\frac{\partial \chi^2}{\partial B} = \frac{1}{\cancel{\sigma_y^2}} \sum \cancel{2} (y_i - A - Bx_i)(-x_i) = 0$$

$$\sum (y_i - A - Bx_i)\, x_i = 0$$

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - A - Bx_i)^2}{\sigma_y^2}$$

$$P_{AB}(y_1, \dots y_N) \propto \frac{1}{\sigma_y^N} e^{-\chi^2/2}$$




$$\frac{\partial \chi^2}{\partial A} = 0 \qquad \frac{\partial \chi^2}{\partial B} = 0$$

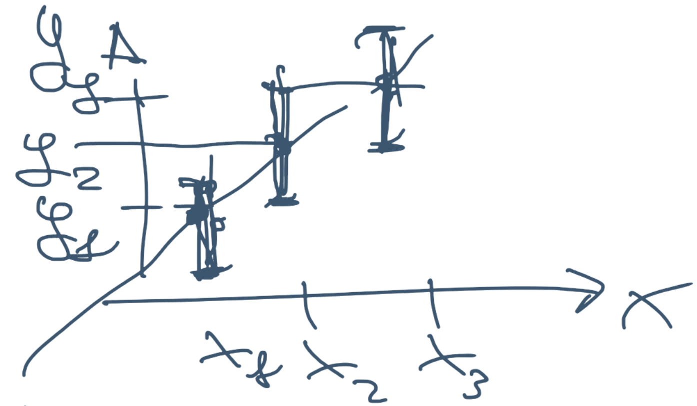
MAX. LIKELIHOOD

$v = v_0 + gt$

$x, y$    $x$: incertezza trascurabile    $x \sim 1\%$

$\cancel{\sigma_x}$

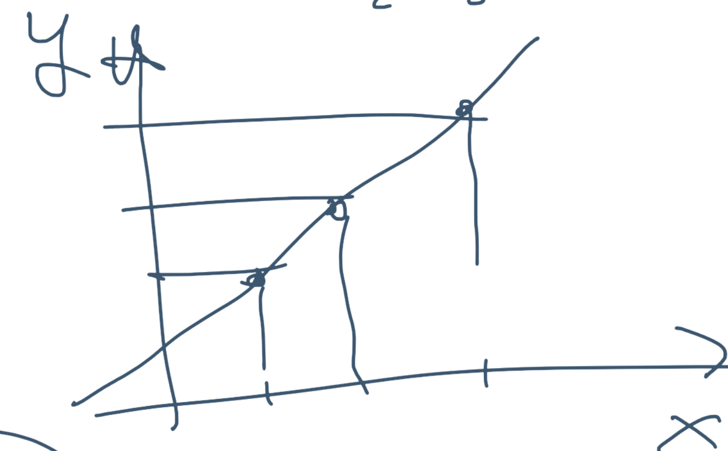
$y$: $\sigma_y$ costante



$y = A + Bx$

$y_i \rightsquigarrow \underbrace{A - Bx_i}$

$$P_{AB}(y_i) \propto \frac{1}{\sigma_y} \underbrace{e^{-\frac{(y_i - A - Bx_i)^2}{2\sigma_i^2}}}$$



$$P_{AB}(y_1, \dots y_N) = P_{AB}(y_1) \cdots P_{AB}(y_N)$$

$$= \frac{1}{\sigma_y^N} e^{-\left[\sum_i (y_i - A - Bx_i)\right]^2 \frac{1}{2\sigma_y^2}}$$

# FITTING AI MINIMI QUADRATI

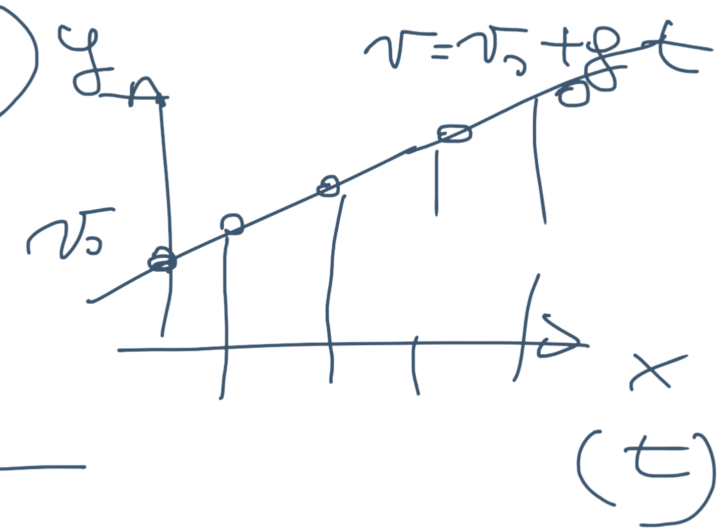
$x, y$ LEGGE FISICA CHE RELAZIONE VARIABILE

$$h = \frac{1}{2} g t^2$$

$$\boxed{v = v_0 + gt}$$

cost cost

LINEARE




$(x_1, \dots, x_N)$

$(y_1, \dots, y_N)$

$$y = A + Bx$$

$$A = v_0 \qquad B = g$$

| $x_i$ | $\sigma_i$ | $w_i$ | $x_i \cdot w_i$ |
|---|---|---|---|
| 1.4 | $\pm 0.5$ | 4 | 5.6 |
| 1.2 | $\pm 0.2$ | 25 | 30.0 |
| 1.00 | $\pm 0.25$ | 16 | 16.0 |
| 1.3 | $\pm 0.2$ | 25 | 32.5 |
| | | 70 | 84,1 |

$$\boxed{X_{best} = 1{,}20 \pm 0{,}12}$$

$$w_1 = \frac{1}{\sigma_1^2} = \frac{1}{(0.5)^2} = \left(\frac{1}{0.5}\right)^2 = 2^2 = 4$$

$$w_2 = \left(\frac{1}{0.2}\right)^2 = 5^2 = 25$$

$$\hookrightarrow \frac{10}{2} = 5$$

$$w_3 = \left(\frac{1}{0.25}\right)^2 = \left(\frac{100}{25}\right)^2 = 4^2 = 16$$

$$X = \frac{\sum x_i w_i}{\sum w_i} = \frac{84.1}{70} = 1{,}2$$

$$\sigma = \left(\sum w_i\right)^{-1/2} = \frac{1}{\sqrt{\sum w_i}} = \frac{1}{\sqrt{70}} = 0.12$$

4

$$X = \frac{\sum_i w_i x_i}{\sum_i w_i} = x_{best} \qquad w_i = \frac{1}{\sigma_i^2}$$

$$\sigma^2_{x_{best}} = \left(\frac{\partial x_{best}}{\partial x_1}\right)^2 \sigma_1^2 + \dots + \left(\frac{\partial x_{best}}{\partial x_N}\right)^2 \sigma_N^2$$

$$\frac{\partial X_{best}}{\partial x_1} = \frac{1}{\sum_i w_i} w_1 \qquad \frac{\partial X_{best}}{\partial x_N} = \frac{w_N}{\sum_i w_i}$$

$$\sigma^2_{best} = \left(\frac{1}{\sum_i w_i}\right)^2 \cdot \left[ w_1^{\cancel{2}} \cdot \frac{1}{\cancel{w_1}} + \dots + w_N^{\cancel{2}} \frac{1}{\cancel{w_N}} \right] = \frac{1}{(\sum w_i)^{\cancel{2}}} (\cancel{\sum w_i})$$

$$= \frac{1}{\sum w_i}$$

$$\sigma_{best} = \left(\sum_{i=1}^{N} w_i\right)^{-1/2}$$

③

$$X = \frac{w_A X_A + w_B X_B}{w_A + w_B}$$

$w_A = \frac{1}{\sigma_A^2}$ $w_B = \frac{1}{\sigma_B^2}$

$$X = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}$$

$R_1 = 11 \pm 1$

$R_2 = 12 \pm 1$

$R_3 = 10 \pm 3$

$R_{12,b} = 11.5 \pm 0.7$

$x_1, x_2, \dots x_N; \sigma_1, \sigma_2, \dots \sigma_N$

$w_1, w_2, \dots w_N$

$\to 1/\sigma_1^2$ $\to 1/\sigma_2^2$ $\to \frac{1}{\sigma_N^2}$

$w_1 = \frac{1}{1^2} = 1$ $w_2 = \frac{1}{1^2} = 1$ $w_3 = \frac{1}{9}$

$$R_V = \frac{11 \cdot 1 + 12 \cdot 1 + 10 \cdot \frac{1}{9}}{2.1} = 11.4$$

$\sum w_i = \underbrace{1 + 1 + \frac{1}{9}}_{=2.1} = \frac{19}{9}$

(2

$$\boxed{\chi^2 = \frac{(x_A - X)^2}{\sigma_A^2} + \frac{(x_B - X)^2}{\sigma_B^2} \qquad \frac{\partial \chi^2}{\partial X} = \emptyset}$$

$$\frac{\partial \chi^2}{\partial X} = 2(x_A - X) \cdot \frac{(-1)}{\sigma_A^2} + 2(x_B - X)\frac{(-1)}{\sigma_B^2} = \emptyset$$

$w_A = \dfrac{1}{\sigma_A^2}$ peso A $\qquad w_B = \dfrac{1}{\sigma_B^2}$ peso B

$$w_A(x_A - X) + w_B(x_B - X) = 0$$

$$w_A x_A - w_A \underline{X} + w_B x_B - w_B \underline{X} = 0 \qquad \emptyset$$

$$w_A x_A + w_B x_B = X(w_A + w_B) \longrightarrow X = \frac{w_A x_A + w_B x_B}{w_A + w_B}$$

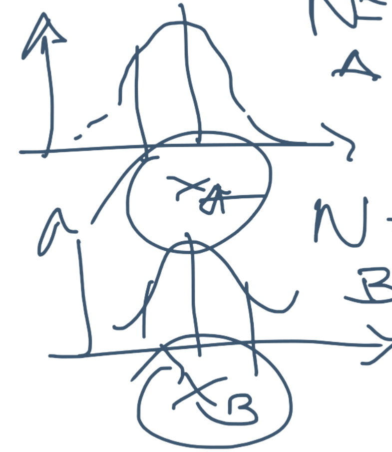
COMBINAZIONE DI MISURE

11/05/2020 (1)

$x_A \pm \sigma_A, \quad x_B \pm \sigma_B$

$X = X_A + X_B$

best



$N_A = 100$

$N_B = 80$

MAXIMUM LIKELIHOOD

$X$

$$P_X(x_A) \propto \frac{1}{\sigma_A} e^{-\frac{(x_A - X)^2}{2\sigma_A^2}}$$

$$P_X(x_B) \propto \frac{1}{\sigma_B} e^{-\frac{(x_B - X)^2}{2\sigma_B^2}}$$

$-(A) \quad -(B)$

$$P_X(x_A, x_B) \propto P_X(x_A) P_X(x_B) = \frac{1}{\sigma_A \sigma_B} e^{-(A)} \cdot e^{-(B)} =$$

$$= \frac{1}{\sigma_A \sigma_B} e^{\underbrace{-\left[\frac{(x_A - X)^2}{\sigma_A^2} + \frac{(x_B - X)^2}{\sigma_B^2}\right]\frac{1}{2}}_{\text{CHI-QUADRO}}}$$

$\boxed{x_C \quad N = 180}$

CHI-QUADRO

$$\chi^2 = \frac{(x_A - X)^2}{\sigma_A^2} + \frac{(x_B - X)^2}{\sigma_B^2}$$