Link 13/05/2020

- Link alle due lezioni registrate:
  - https://drive.google.com/open?id=1nZLVZktlgzsVpHZ8hd5ywMNgYKv2qub3

Es. 10.10 & 11.4

10.10. ★ A hospital admits four patients suffering from a disease for which the mortality rate is 80%. Find the probabilities of the following outcomes: **(a)** None of the patients survives. **(b)** Exactly one survives. **(c)** Two or more survive.

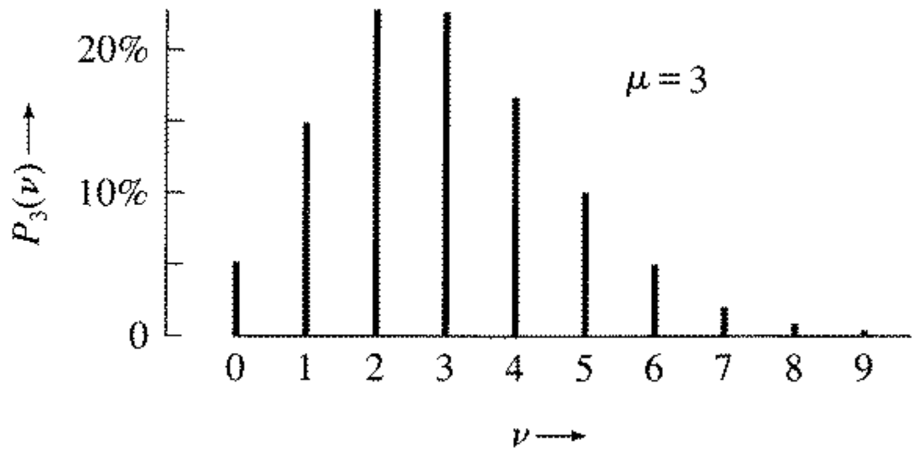
***11.4** (Section 11.1). A certain radioactive sample contains 1.5×10^{20} nuclei, each of which has a probability $p = 10^{-20}$ of decaying in any given minute.

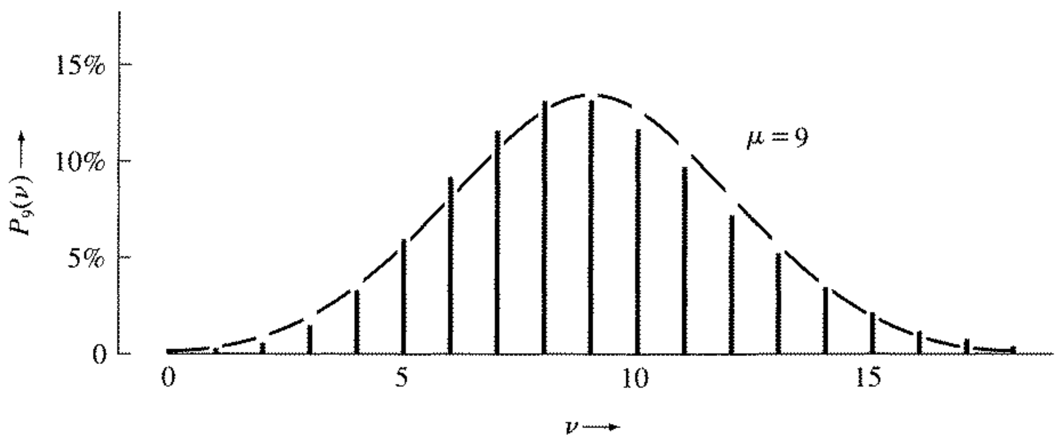
(a) What is the expected average number, μ, of decays from the sample in one minute?

(b) Compute the probability $p_\mu(\nu)$ of observing ν decays in a minute for $\nu = 0, 1, 2, 3$.

(c) What is the probability of observing four or more decays in one minute?

Es.





11.4

$n = 1.5 \times 10^{20}$ nuclei $\quad p = 10^{-20}$ in 1 minuto

$\mu = np = 1.5 \times 10^{20} \times 10^{-20} = 1.5$

a) $\mu = np = 1.5$ decadim. al minuto

b) $P_\mu(\nu)$ di osservare ν decad. in 1 min.

$\nu = 0, 1, 2, 3$

$$P_{1.5}(\nu) = e^{-1.5} \cdot \frac{(1.5)^\nu}{\nu!}$$

$e^{-1.5} \to 0.22$

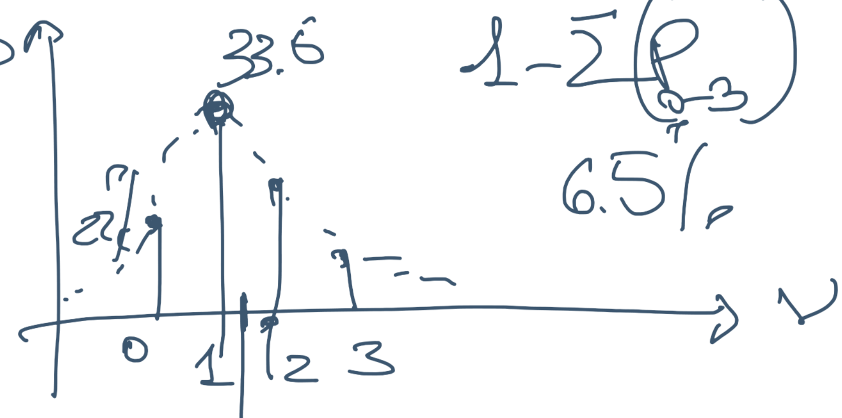
$\nu = 0 \quad P(0) = 0.22 \frac{(1.5)^0}{0!} = 22.3\%$

$P(1) = 0.22 \cdot \frac{(1.5)^1}{1} = 33.5\%$

$P(2) = 0.22 \frac{(1.5)^2}{2} = 25.1\%$

$P(3) = 0.22 \frac{(1.5)^3}{6} = 12.6\%$

93.5



$1 - \sum(P_{0-3})$

6.5%

$\mu = 64$

$\nu = 72$

$$P(72 \text{ cattogg}) = \frac{e^{-64}(64)^{72}}{72!} = 2.9\%$$

$$P(\nu \geq 72) = P(72) + P(73) + P(74) + \dots = 17.3\%$$

$\bar{x} = 64, \ \sigma = 8$

71, 72, 73

$$f_{64,8}(x \geq 71.5) = 17.4\%$$

$$= \frac{71.5 - 64}{8} = \frac{7.5}{8} \sim 1 \sim 0.94\sigma$$

$$f_{64,8}(72) = \frac{1}{\sqrt{2\pi}\cdot 8} e^{-\frac{(72-64)^2}{2\cdot 64}} = 3\%$$

$$P_\mu(\nu) \sim f_{x,\sigma}(\nu)$$

$$f_{\mu,\sqrt{\mu}}(\nu)$$

$$P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$$

$$\bar{\nu} = \mu$$

$$\sigma_\nu = \sqrt{\mu}$$

$$\frac{\sigma_\nu}{\mu} = \frac{\sqrt{\mu}}{\mu} = \frac{1}{\sqrt{\mu}}$$

$$\mu \gg$$

5

$$P_\mu(\nu) = e^{-\mu} \frac{\mu^\nu}{\nu!}$$

$$\bar{\nu} = \mu$$

$$\sigma^2_\nu \rightsquigarrow \frac{\sum (x-\bar{x})^2}{N-1} \rightsquigarrow \frac{\sum (\nu-\bar{\nu})^2}{N} \rightsquigarrow \overline{(\nu-\bar{\nu})^2}$$

$$\sigma^2_\nu = \bar{\nu} = \mu$$

$$\sigma_\nu = \sqrt{\mu}$$

$$\bar{\nu} = \mu$$

$$\sigma_\nu = \sqrt{\mu}$$

$$\mu$$

$$x \to \mu$$

$$\sigma \rightsquigarrow \sqrt{\mu}$$

$$P_\mu(\nu) = e^{-\mu} \frac{\mu^\nu}{\nu!} \quad (4$$

$$\bar{\nu} = \sum_{\nu=0}^{\infty} \nu P_\mu(\nu) = $$

$$= \sum_{\nu=0}^{\infty} \nu e^{-\mu} \frac{\mu^\nu}{\nu!} = \sum_{\nu=0}^{\infty} e^{-\mu} \cdot \mu \frac{\mu^{\nu-1}}{(\nu-1)!} = \circledast$$

$$\nu! = \nu \cdot (\nu-1) \cdot (\nu-2) \cdots 1$$

$$\nu! = \nu [(\nu-1)!]$$

$$\frac{\nu}{\nu!} = \frac{\nu}{\nu \cdot (\nu-1)!}$$

$$\circledast = e^{-\mu} \cdot \mu \cdot \sum_{\nu=0}^{\infty} \frac{\mu^{\nu-1}}{(\nu-1)!} \rightarrow e^{\mu}$$

$$= e^{-\mu} \cdot \mu \cdot e^{\mu} = \mu$$

POISSON DISTRIBUTION

1 MINUTO ν

n nuclei p prob. singolo di decadim. nucleo n

$b_{np}(\nu)$

- n numero grande $n \sim 10^{20}$
- p 10^{-20}

$$P_\mu(\nu) = e^{-\mu} \frac{\mu^\nu}{\nu!}$$

μ ←

(2

$x = 0.8 \pm 0.8$

$0 \div 1.6$ $68\% + 16\%$

$+32\%$ → 16%

$x \leq 1.6$ 84%

$x > 1.6$ 16%

Mortalità $= 80\%$

$p = 20\%$

$n = 4$

$$b_{n,p}(r) = \binom{n}{r} p^r q^{n-r}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$b_{4,0.2}(0) = \frac{4!}{0!\,4!}(0.2)^0 \cdot (0.8)^4 = 0.41 = 41\%$$

$$b_{4,0.2}(1) = \frac{4!}{1!\,3!}(0.2)^1 \cdot (0.8)^3 = 0.8 \cdot (0.8)^3 = (0.8)^4 = 41\%$$

$$b_{4,0.2}(r \geq 2) = 1 - b_{4,0.2}(0) - b_{4,0.2}(1) = 1 - 0.82 = 18\%$$

$$\bar{x} = np = 4 \cdot 0.2 = 0.8$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{4 \cdot 0.2 (0.8)} = \sqrt{(0.8)^2} = 0.8$$