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Proposed problem

An annular fin of rectangular profile, as shown in figure 1, is made with a uniform, isotropic material with a thermal conductivity $k = 40$ [W/(m K)]; it has a thickness $t = 10$ [mm], an inner radius $r_1 = 50$ [mm] and an outer radius $r_2 = 125$ [mm].

The fin is cooled only by convection, with a convective heat transfer coefficient $h = 40$ [W/(m^2 K)], and the surrounding fluid temperature $T_\infty = 25$ [°C]. The base of the fin is maintained at a temperature $T_b = 110$ [°C], while also the tip of the fin contribute, with the same heat transfer coefficient, to the overall heat flux.

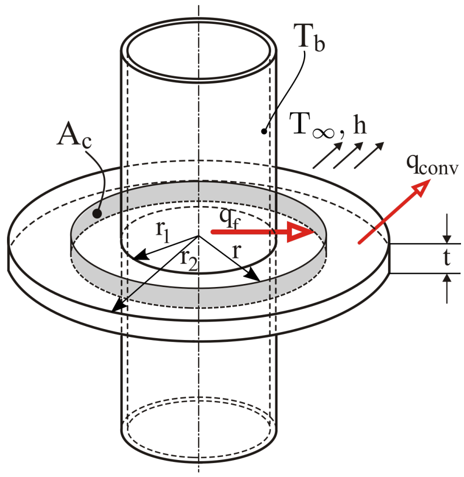

Figure 1: Annular (radial) fin of rectangular profile.

Assuming a *1D* temperature distribution, i.e. $T \approx T(r)$, compute, with the Finite Volume method (FV), the heat flux from the fin q_{num} [W], using a number N of FVs equal to $N = 10$, 20, 40, and 80.
Compare the numerical results with the analytical (exact) solution given in [1]:

$$q_f = 2\pi k r_1 t m \left(T_b - T_\infty\right) \frac{\gamma K_1(mr_1) - I_1(mr_1)}{I_0(mr_1) + \gamma K_0(mr_1)} \tag{1}$$

with

$$\gamma = \frac{(h/mk) I_0(mr_2) + I_1(mr_2)}{K_1(mr_2) - (h/mk) K_0(mr_2)} \tag{2}$$

and

$$m = \sqrt{2h/kt} \tag{3}$$

In (1) and (2) I_0 e I_1 represent, respectively, the modified Bessel function of the first kind of *order 0* and of *order 1* (in MATLAB: function `besseli`), while K_0 e K_1 are the modified

Bessel function of the second kind of *order 0* and of *order 1*, respectively (in MATLAB: function `besselk`).

TIP

Discretize the fin, assuming $T \approx T(r)$, with circular (ring-shaped) Finite Volumes of thickness t and width Δr, noting that, differently from linear fins, the cross-section and the external surface of each FV varies with r, i.e. $A_c = A_c(r)$ and $\Delta A_s = \Delta A_s(r)$.

References

[1] A. D. Kraus, A. Aziz, J. Welty, EXTENDED SURFACE HEAT TRANSFER, J. Wiley & Sons, (2001).