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Corso di Termofluidodinamica Computazionale

Homework No. 2
AA 2021/2022

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Proposed problem

1. Following *Homework 1 - AA. 2021-22*, first case, consider a cylindrical (pin) fin, as shown in figure 1, which is made with a uniform, isotropic material with a thermal conductivity value of $k = 40$ W/(m K). The fin has a length $L = 40$ mm and a diameter $d = 4$ mm. The fin is cooled only by convection with a convective heat transfer coefficient h =400 W/(m^2 K), and the temperature of the surrounding fluid is $T_\infty = 25$ °C. The temperature of the base of the fin is maintained at a temperature $T_b = 200$ °C, while also the tip of the fin contributes, with the same heat transfer coefficient, to the overall heat flux.

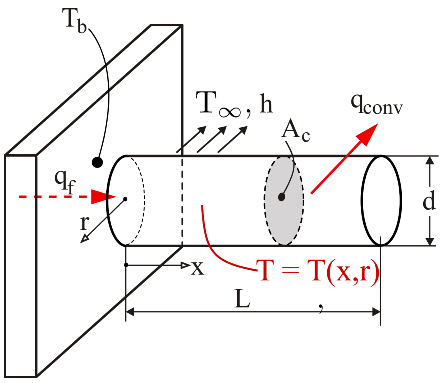

Figure 1: Axisymmetrical cylindrical (pin) fin.

In this case, disregard the usual assumption of 1D temperature distribution (see [1, 2]), i.e.

$$T \approx T(x)$$

and consider a full *2D, axisymmetric* temperature distribution [1, 2]:[1]

$$T = T(x, r)$$

Using the MATLAB *PDE Toolbox*, develop a 2D axisymmetric steady numerical model for the fin and, using an *adequate* number of finite elements, compute the heat flux q_{num2D} [W]. Compare the result with that obtained with the 1D model of *Homework 1*. What is the $\%$ error using the 1D assumption?
Plot a contour map of the temperature field.

[1] The general heat (conduction) equation for an isotropic material in cylindrical coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q}_g = \rho\, c_p \frac{\partial T}{\partial \tau}$$

which, under the assumption of steady, 2D axisymmetric temperature field with no heat generation, reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = 0$$

2. Repeat the same analysis for the second case of *Homework 1*, e.g. fin length $L = 40$ mm and diameter $d = 20$ mm, and again compare the result with that from the 1D model. What is the $\%$ error using the 1D assumption in this second case? Is it lower or higher? Why?
   Plot a contour map of the temperature field.

References

[1] G. Comini, G. Cortella, *Fondamenti di trasmissione del calore*, 4a Ed., S.G.E. Editore, (2013).

[2] F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Ed., Wiley, (2007).