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Proposed problem

A plane fin of uniform cross-section, as shown in figure 1, is made with a uniform, isotropic material with a thermal conductivity $k = 30$ W/(m K); it has a thickness $t = 1$ mm, a length $L = 20$ mm and its width w is large in comparison with its thickness. The fin is cooled only by convection but the heat transfer coefficient h is spatially variable with the following law:

$$h(x) = (\gamma + 1)\ h_0 \left(\frac{x}{L}\right)^{\gamma} \tag{1}$$

where h_0 is the average heat transfer coefficient and $\gamma \geq 0$ is a constant. When $\gamma = 0$, the coefficient $h(x)$ is constant and equal to h_0 over all the faces of the fin. When $\gamma = 1$, the coefficient increases linearly from $x = 0$ to $x = L$ while values of $\gamma \geq 2$ produce parabolic distributions. In all cases when $\gamma \geq 1$, the magnitude of the convective heat transfer coefficient is zero at the fin base $x = 0$.

The base of the fin is maintained at a temperature $T_b = 400$ °C and the surrounding fluid temperature is T_∞ = 25 °C. The tip of the fin can be assumed perfectly insulated.

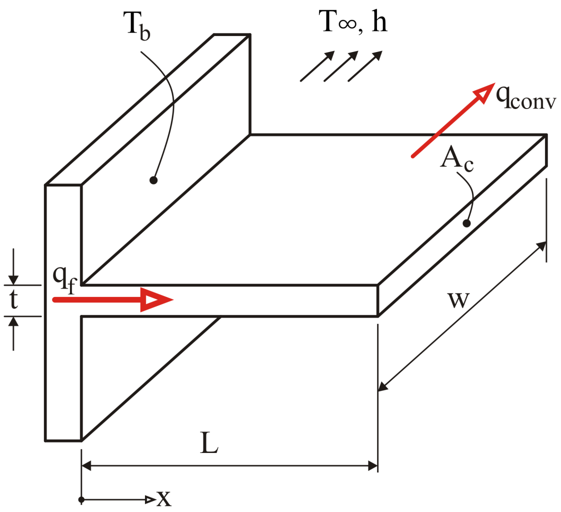

Figure 1: Straight fin with uniform cross-section.

Assuming a *1D* temperature distribution, i.e. $T \approx T(x)$, compute, with the Finite Volume method (FV), the heat flux per unit width of the fin q'_{num} [W/m], using a number N of FVs equal to $N = 10$, 20, 40, and 80.

Consider the two following cases:

1. In the first case $\gamma = 0$ and $h_0 = 15\ W/(m^2K)$, corresponding *e.g.* to the standard case of a straight fin with constant heat transfer coefficient.

2. For the second case assume $\gamma = 4$ and $h_0 = 15\ W/(m^2K)$.

Plot in a *log-log* graph the behavior of the error *vs* N, verifying its quadratic trend. The error is defined as the difference between the numerical value of the heat flux per unit width q'_{num} and its analytical (exact) [1] solution q'_f given by:

$$q'_f = h_0\, 2L\, (T_b - T_\infty) \left[\frac{(\gamma+2)^\gamma (\gamma+1)}{(mL)^{2(\gamma+1)}} \right]^{1/(\gamma+2)} \times \frac{I_{(\gamma+1)/(\gamma+2}(u_b)}{I_{-(\gamma+1)/(\gamma+2)}(u_b)} \frac{\Gamma((\gamma+1)/(\gamma+2))}{\Gamma(1/(\gamma+2))} \tag{2}$$

where:

$$m = \sqrt{2\, h_0 / k\, t}$$

and

$$u_b = \frac{2\sqrt{\gamma+1}}{\gamma+2}\, mL$$

In (2) I represents the modified *Bessel function* of the first kind (in MATLAB: function `besseli`), while Γ is the *Gamma function* (in MATLAB: function `gamma`).

TIP

As a further check, for the first case with constant heat transfer coefficient h_0, the analytical solution [2, 3] is also provided by the following simpler equation:

$$q'_f = \sqrt{h_0 2kt}\, \tanh mL\, (T_b - T_\infty) \tag{3}$$

References

[1] A. D. Kraus, A. Aziz, J. Welty, EXTENDED SURFACE HEAT TRANSFER, J. Wiley & Sons, (2001).

[2] G. Comini, G. Cortella, *Fondamenti di trasmissione del calore*, 4a Ed., S.G.E. Editore, (2013).

[3] F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Ed., Wiley, (2007).