

## FORMULARIO METODI

### Analisi complessa:

*Determinazione principale logaritmo:*  $\text{Log}(\omega) = \log(|\omega|) + i \text{Arg}(\omega)$

*Identità:*  $\sin(x + iy) = \sin(x)\cosh(y) + i \cos(x)\sinh(y)$

*Identità:*  $\cos(x + iy) = \cos(x)\cosh(y) - i \sin(x)\sinh(y)$

*Residuo polo ordine n:*  $\text{Res}(f, z_0) = \frac{1}{(n-1)!} \lim_{z \rightarrow z_0} \frac{d^{n-1}}{dz^{n-1}} ((z - z_0)^n f(z))$

### Laplace:

*Trasformate notevoli e proprietà:*

$$f(t) = \sin(at) \Rightarrow \mathcal{L}\{f(t)\}(s) = \frac{a}{s^2 + a^2}$$

$$f(t) = \sinh(at) \Rightarrow \mathcal{L}\{f(t)\}(s) = \frac{a}{s^2 - a^2}$$

$$f(t) = \cos(at) \Rightarrow \mathcal{L}\{f(t)\}(s) = \frac{s}{s^2 + a^2}$$

$$f(t) = \cosh(at) \Rightarrow \mathcal{L}\{f(t)\}(s) = \frac{s}{s^2 - a^2}$$

$$\mathcal{L}\{t^k u(t)\} = \frac{k!}{s^{k+1}} = \frac{\Gamma(k+1)}{s^{k+1}}$$

$$g(t) = f(t-a) \cdot u(t-a) \Rightarrow G(s) = F(s)e^{-sa}$$

$$g(t) = f(t)e^{\gamma t} \Rightarrow G(s) = F(s - \gamma)$$

$$g(t) = f(t\omega) \Rightarrow G(s) = \frac{1}{\omega} F\left(\frac{s}{\omega}\right)$$

*Derivata trasformata:*  $F^k(s) = (-1)^k \mathcal{L}\{t^k f(t)\}(s)$

*Trasformata segnali periodici:*  $F^*(s) = F(s)(1 - e^{-sT})$

*Convoluzione:*  $(f * g)(x) = \int_{\mathcal{R}^N} f(y)g(x-y)dy$

### Fourier:

*Coefficienti serie Fourier:*  $a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos(n\omega t)dt$   $b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sin(n\omega t)dt$   $c_n = \frac{1}{T} \int_0^T x(t)e^{-in\omega t}dt$

*Trasformate notevoli e proprietà:*

$$f(t) = e^{-a|t|} \Rightarrow \mathcal{F}\{f(t)\}(\omega) = \frac{2a}{a^2 + \omega^2}$$

$$f(t) = \frac{1}{a^2 + t^2} \Rightarrow \mathcal{F}\{f(t)\}(\omega) = \frac{\pi}{a} e^{-a|\omega|}$$

$$f(t) = e^{-at^2} \Rightarrow \mathcal{F}\{f(t)\}(\omega) = e^{-\frac{\omega^2}{4a}} \sqrt{\frac{\pi}{a}}$$

$$f(x) = \chi_{[a,b]}(x) \Rightarrow \mathcal{F}\{f(x)\}(\omega) = \frac{e^{-i\omega a} - e^{-i\omega b}}{i\omega}$$

$$\hat{p}_{2d}(\omega) = 2d \text{sinc}\left(\frac{d\omega}{\pi}\right)$$

$$\hat{\Lambda}_a = a^2 \text{sinc}^2\left(\frac{\omega a}{2\pi}\right)$$

$$g(x) = f(x - x_0) \Rightarrow \hat{g}(\omega) = e^{-i\omega x_0} \hat{f}(\omega)$$

$$g(x) = e^{i\omega_0 x} f(x) \Rightarrow \hat{g}(\omega) = \hat{f}(\omega - \omega_0)$$

$$\mathcal{F}\{f^k\}(\omega) = (i\omega)^k \mathcal{F}\{f\}(\omega)$$

$$g(x) = \overline{f(x)} \Rightarrow \hat{g} = \overline{\hat{f}(-\omega)}$$

$$g(x) = f(\alpha x) \Rightarrow \hat{g}(\omega) = \frac{1}{|\alpha|} \hat{f}\left(\frac{\omega}{\alpha}\right)$$

$$g(x) = x \cdot f(x) \Rightarrow \frac{d\hat{f}}{d\omega}(\omega) = -i\hat{g}(\omega)$$

*Plancherel:*  $\|\hat{f}\|_{L^2}^2 = 2\pi \|f\|_{L^2}^2$