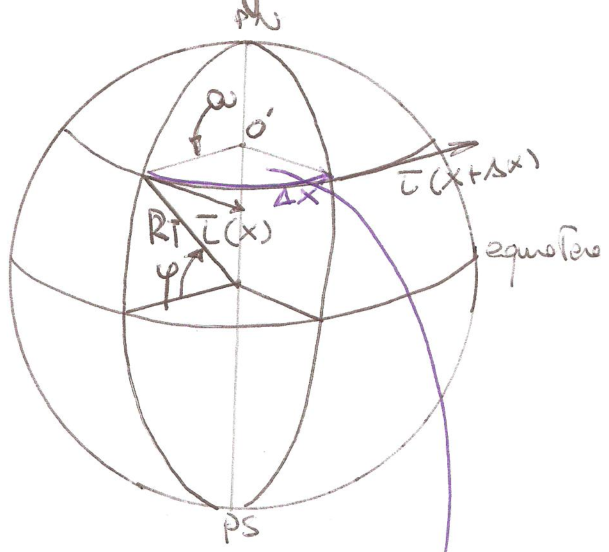
Analisi della variazione del versore $\boxed{\vec{i}}$

$$\frac{d\bar{i}}{dt} = \frac{\partial \bar{i}}{\partial t} + u\frac{\partial \bar{i}}{\partial x} + v\frac{\partial \bar{i}}{\partial y} + w\frac{\partial \bar{i}}{\partial z}$$

$\uparrow$ nulla in quanto $\bar{i}(x,y,z)$ ma non dipende (esplicitamente) da t

$$\boxed{\frac{\partial \bar{i}}{\partial x} = \lim_{\Delta x \to 0} \frac{\bar{i}(x+\Delta x) - \bar{i}(x)}{\Delta x}}$$



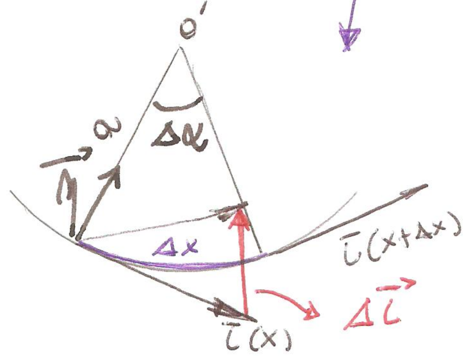
Ricordando che $|\vec{i}| = 1$ (versore)

$$|\Delta \bar{i}| = |\bar{i}(x+\Delta x) - \bar{i}(x)| = 1 \cdot \Delta\alpha$$

$$\Delta\alpha = \frac{\Delta x}{a}$$

con $a = (R_T + z)\cos\varphi \simeq R_T\cos\varphi$

Da cui $$|\Delta \bar{i}| = \frac{\Delta x}{R_T\cos\varphi}$$

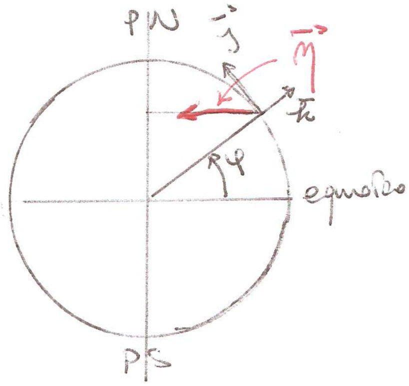


Quindi:

$$\boxed{\frac{\partial \bar{i}}{\partial x}} = \lim_{\Delta x \to 0} \frac{\Delta x}{R_T\cos\varphi}\frac{1}{\Delta x}\vec{\eta}$$ con $\bar{\eta}$ versore

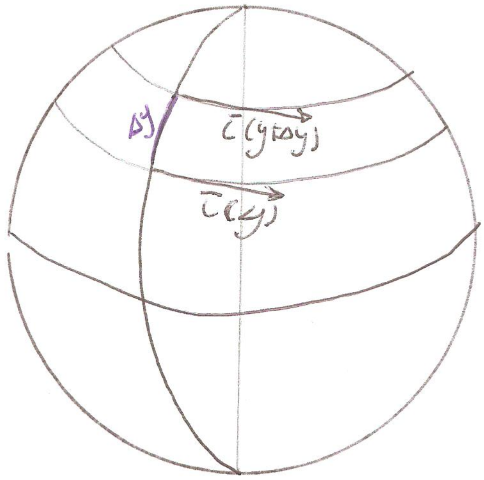
$$= \frac{1}{R_T\cos\varphi}\left(-\cos\varphi\,\vec{k} + \sin\varphi\,\bar{j}\right)$$

$$= \boxed{\frac{1}{R_T}\operatorname{tg}\varphi\,\vec{j} - \frac{1}{R_T}\vec{k}}$$



Osservando che $\bar{\eta} = 0\,\bar{i} + \sin\varphi\,\bar{j} - \cos\varphi\,\bar{k}$

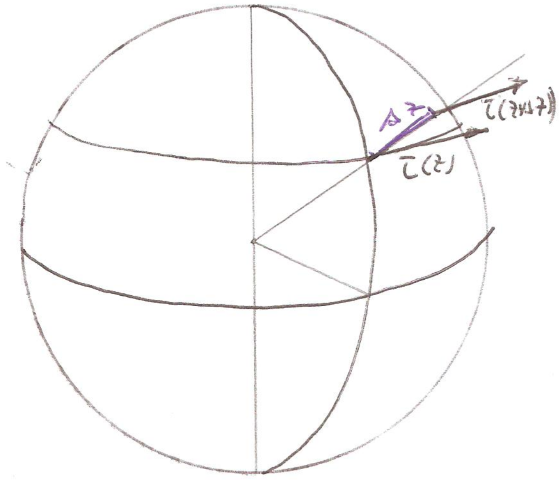
$$\boxed{\frac{\partial \bar{i}}{\partial y} = \lim_{\Delta y \to 0} \frac{\bar{i}(y+\Delta y) - \bar{i}(y)}{\Delta y}}$$



Osserviamo che $\bar{i}(y+\Delta y)$ e $\bar{i}(y)$ sono versori paralleli (identici)

da cui $\Delta \bar{i} = \bar{i}(y+\Delta y) - \bar{i}(y) = 0$

Quindi $$\boxed{\frac{\partial \bar{i}}{\partial y} = 0}$$



$$\frac{\partial \bar{i}}{\partial z} = \lim_{\Delta z \to 0} \frac{\bar{i}(z+\Delta z) - \bar{i}(z)}{\Delta z}$$

Osservazione: $\bar{i}(z+\Delta z)$ e $\bar{i}(z)$ sono due versori paralleli (identici)

da cui $\Delta \bar{i}_z = \bar{i}(z+\Delta z) - \bar{i}(z) = 0$

Quindi $$\boxed{\frac{\partial \bar{i}}{\partial z} = 0}$$

Sommando tutti gli addendi della $\frac{d\bar{i}}{dt}$ si ottiene

$$\frac{d\bar{i}}{dt} = 0 + u\frac{1}{R_T}\,tg\varphi\,\vec{j} - u\frac{1}{R_T}\bar{k} + v \cdot 0 + w \cdot 0$$

Quindi $$\boxed{\frac{d\bar{i}}{dt} = \frac{u}{R_T}\,tg\varphi\,\vec{j} - \frac{u}{R_T}\vec{k}}$$

<u>Osservazione</u> la variazione di $\bar{i}$ dai due contributi su assi diversi da $\vec{i}$