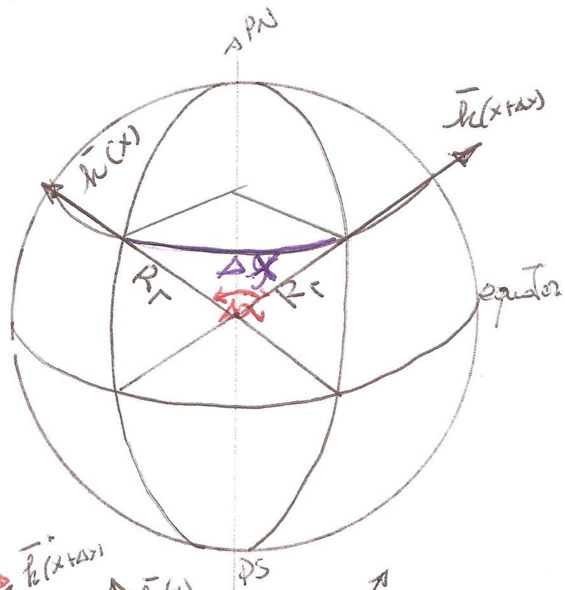
Analisi della variazione del versore $\vec{k}$

$$\frac{d\bar{k}}{dt} = \frac{\partial \bar{k}}{\partial t} + u\frac{\partial \bar{k}}{\partial x} + v\frac{\partial \bar{k}}{\partial y} + w\frac{\partial \bar{k}}{\partial z}$$

nulla in quanto $\bar{k}(x,y,z)$ ma non dipende esplicitamente da t.

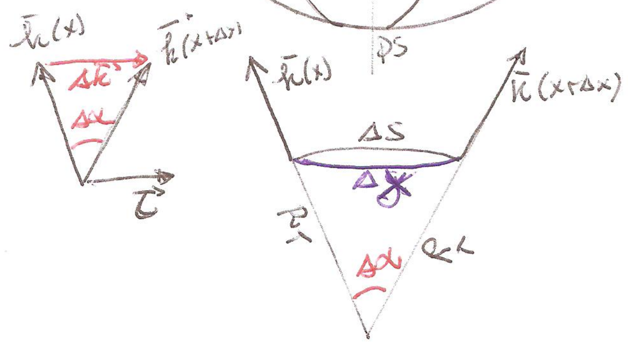


$$\frac{\partial \bar{k}}{\partial x} = \lim_{\Delta x \to 0} \frac{\bar{k}(x+\Delta x) - \bar{k}(x)}{\Delta x}$$

Ricordiamo che $|\bar{k}| = 1$ (versore)

$$|\Delta \bar{k}| = |\bar{k}(x+\Delta x) - \vec{k}(x)| = 1 \cdot \Delta\alpha$$

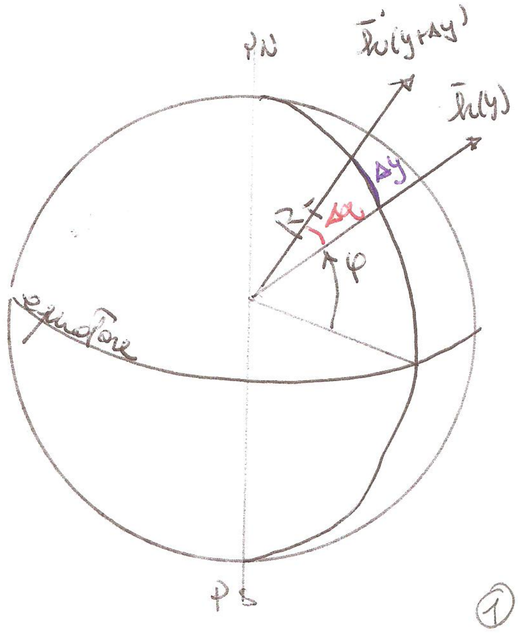
$$\Delta\alpha = \frac{\Delta S}{(R_T + z)}$$



Ricordiamo che $R_T + z \simeq R_T$ e che $\lim_{\Delta x \to 0} \Delta S = \Delta x$

da cui

$$\boxed{\frac{\partial \bar{k}}{\partial x}} = \lim_{\Delta x \to 0} \frac{\Delta \bar{k}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta S}{R_T} \frac{1}{\Delta x} \vec{i} = \boxed{\frac{1}{R_T} \vec{i}}$$



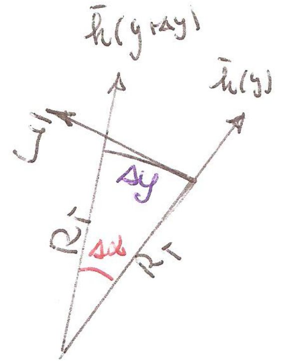
$$\frac{\partial \bar{k}}{\partial y} = \lim_{\Delta y \to 0} \frac{\bar{k}(y+\Delta y) - \bar{k}(y)}{\Delta y}$$

Ricordiamo che $|\bar{k}| = 1$ (versore)

$$|\Delta \bar{k}| = |\bar{k}(y+\Delta y) - \bar{k}(y)| = 1 \cdot \Delta\alpha$$

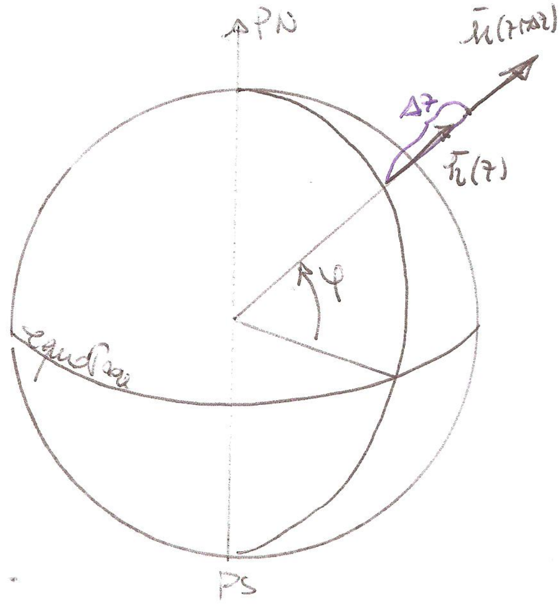
$$\Delta\alpha = \frac{\Delta y}{R_T + z} \simeq \frac{\Delta y}{R_T}$$

Al limite per $\Delta y \to 0$ $\Delta \bar{k}$ ha direzione e verso concorde con il versore $\vec{j}$



$$\boxed{\frac{\partial \vec{k}}{\partial y}} = \lim_{\Delta y \to 0} \frac{\Delta \vec{k}}{\Delta y} = \lim_{\Delta y \to 0} \frac{\Delta y}{\Delta y} \frac{1}{R_T} \vec{j}$$

$$= \boxed{\frac{1}{R_T} \vec{j}}$$



$$\frac{\partial \bar{k}}{\partial z} = \lim_{\Delta z \to 0} \frac{\bar{k}(z+\Delta z) - \bar{k}(z)}{\Delta z}$$

Osservazione i due versori $\bar{k}(z+\Delta z)$ e $\bar{k}(z)$ sono paralleli (identici

da cui $\Delta \vec{k} = \bar{k}(z+\Delta z) - \bar{k}(z) = 0$

Quindi $\boxed{\frac{\partial \vec{k}}{\partial z} = 0}$

Sostituendo tutti gli addendi alla $\frac{d\bar{k}}{dt}$ si ottiene:

$$\frac{d\bar{k}}{dt} = 0 + u \frac{1}{R_T} \bar{i} + v \frac{1}{R_T} \vec{j} + w \cdot 0$$

Quindi

$$\boxed{\frac{d\bar{k}}{dt} = \frac{u}{R_T} \bar{i} + \frac{v}{R_T} \vec{j}}$$

Osservazione

La variazione di $\bar{k}$ dà due contributi ai assi distinti da $\vec{k}$