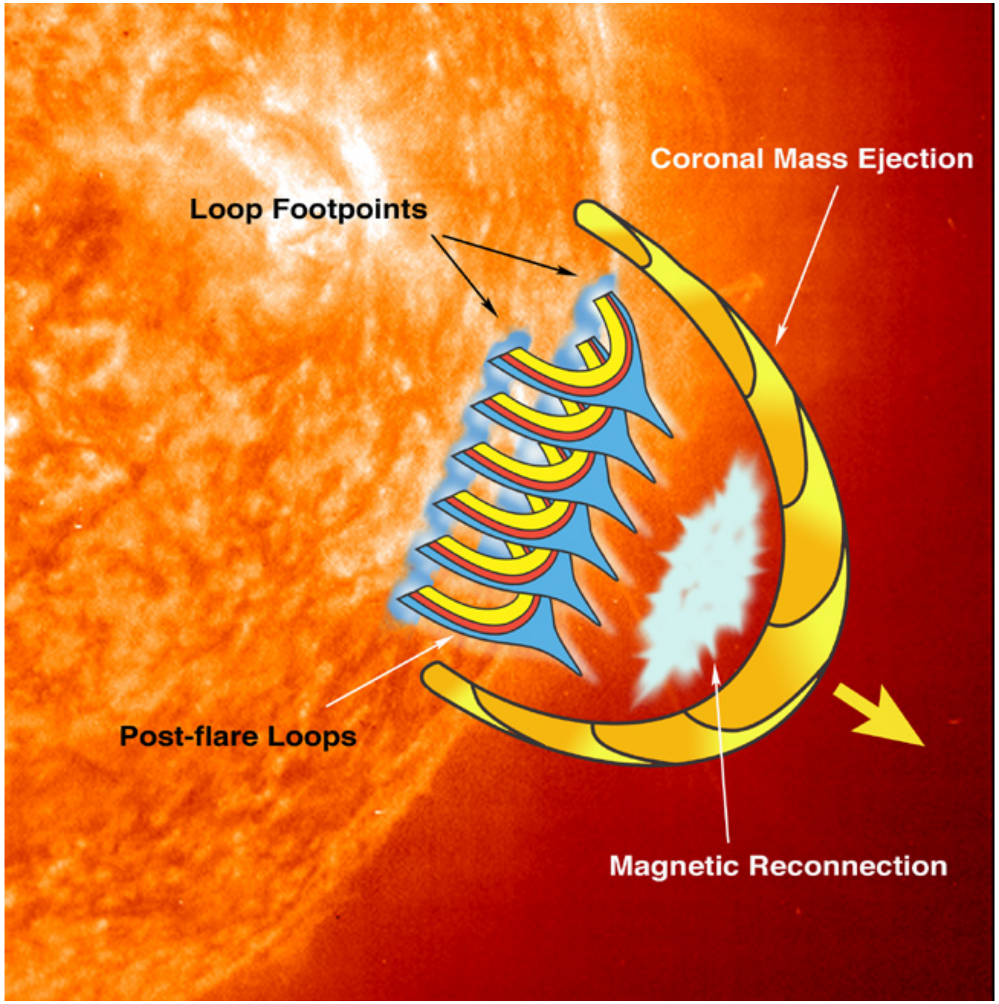
Astrofisica Nucleare e Subnucleare

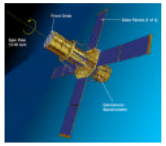
The Sun in Gamma-rays

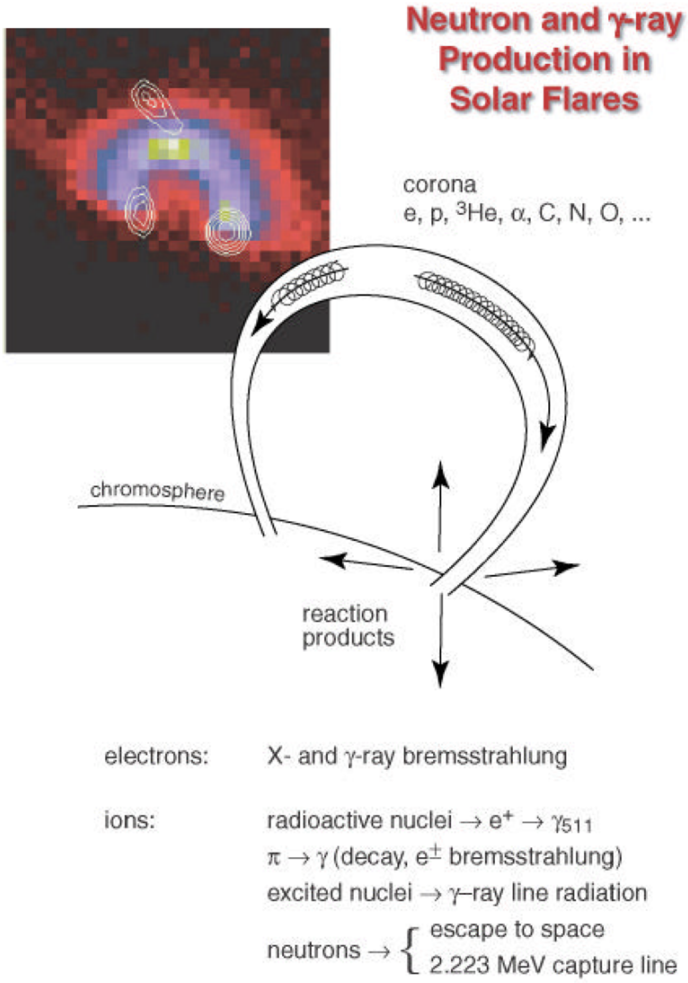
Solar Flares



Solar Flares in Gamma-rays

Solar γ-Ray Physics Comes of Age





Neutron and γ-ray Production in Solar Flares

electrons:	X- and γ-ray bremsstrahlung
ions:	radioactive nuclei → e^+ → γ_{511}
	π → γ (decay, $e^\pm$ bremsstrahlung)
	excited nuclei → γ–ray line radiation
	neutrons → { escape to space 2.223 MeV capture line

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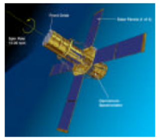
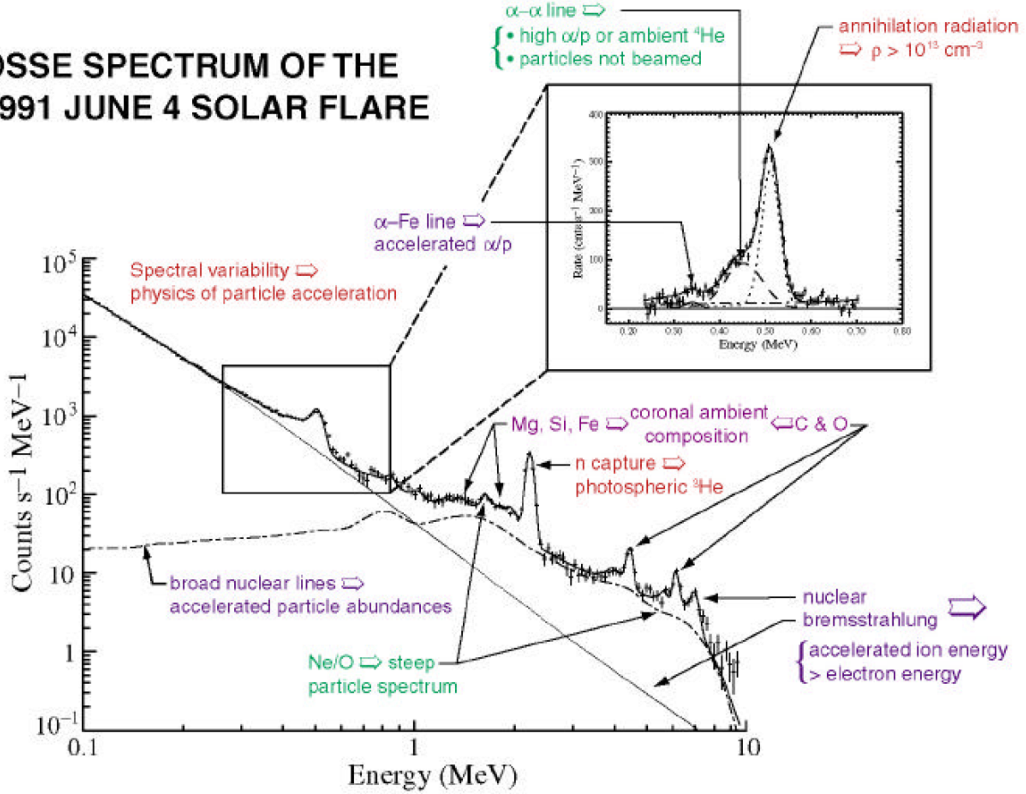
Solar Flares in Gamma-rays

Solar γ-Ray Physics Comes of Age



The Physics of Flares Revealed by
γ-Ray Spectroscopy



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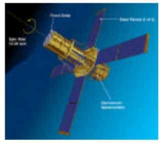
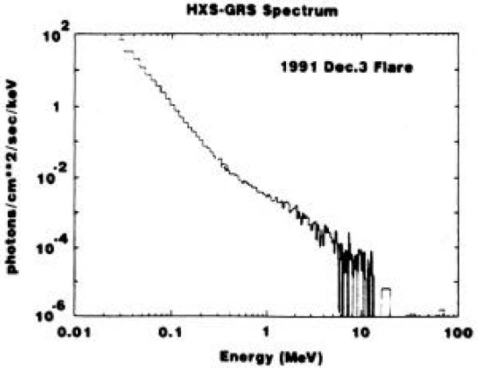
Solar γ-Ray Physics Comes of Age

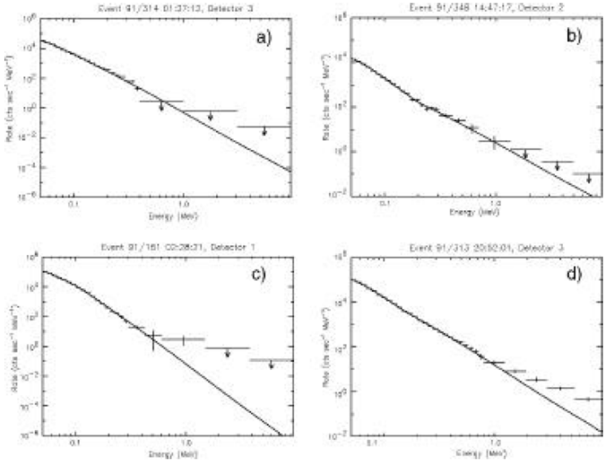


Shape of Bremsstrahlung Continuum >100 keV

Yohkoh



OSSE



Hardening found in spectra >100 keV by combined analysis of *SMM* GRS/HXRBS spectra.

Similar hardening observed in combined spectrum from *Yohkoh* HRS/GRS.

Important for measurements to be made with the same instrument.

Best instruments BATSE, OSSE, and HESSI.

OSSE continuum spectra exhibit: single power laws, broken power laws with hardening and softening between ~100 and 200 keV, and additional hardening above ~1 MeV.

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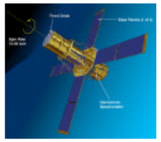
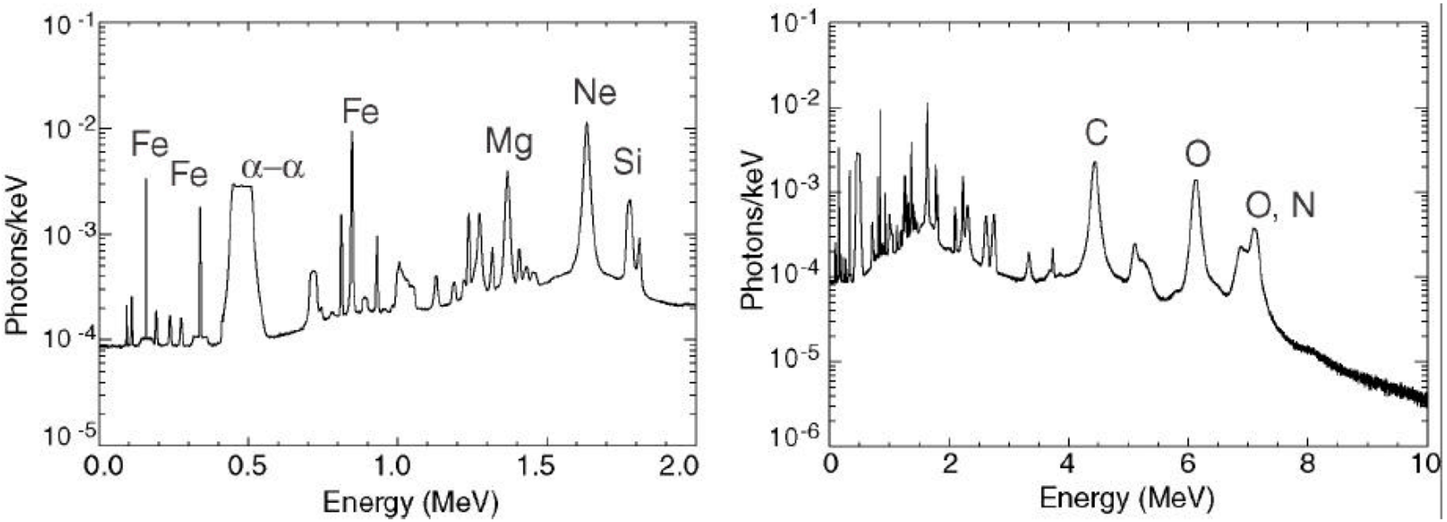
Solar Flares in Gamma-rays

Solar γ-Ray Physics Comes of Age



Theoretical Nuclear Line Spectrum



Ramaty, Kozlovsky, Lingenfelter, and Murphy

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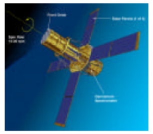
Solar γ-Ray Physics Comes of Age



Narrow γ-Ray Lines Observed in Flare Spectra

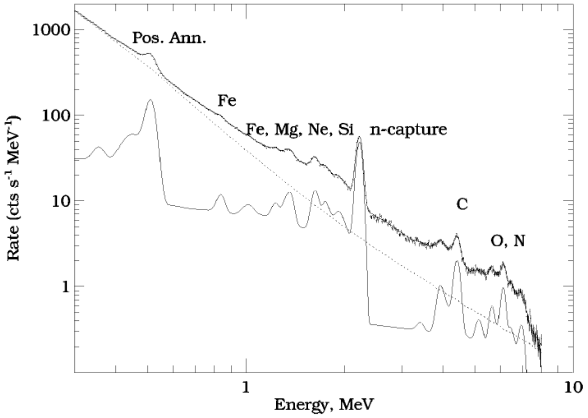
Produced by p and α interactions with ambient material.

At least 30% of flares with emission >0.3 MeV exhibit γ-ray line features. *HESSI* will make more definitive measurement.

At least 19 de-excitation lines have been identified in fits to flare spectra.

Widths of de-excitation lines measured to be ~2-4% in the summed spectrum. This exceeds theory in some cases suggesting presence of blended lines (e.g. ^{14}N near ^{20}Ne) or different Doppler shifts in the flares (see later discussion).

HESSI can resolve these lines and determine intrinsic widths.



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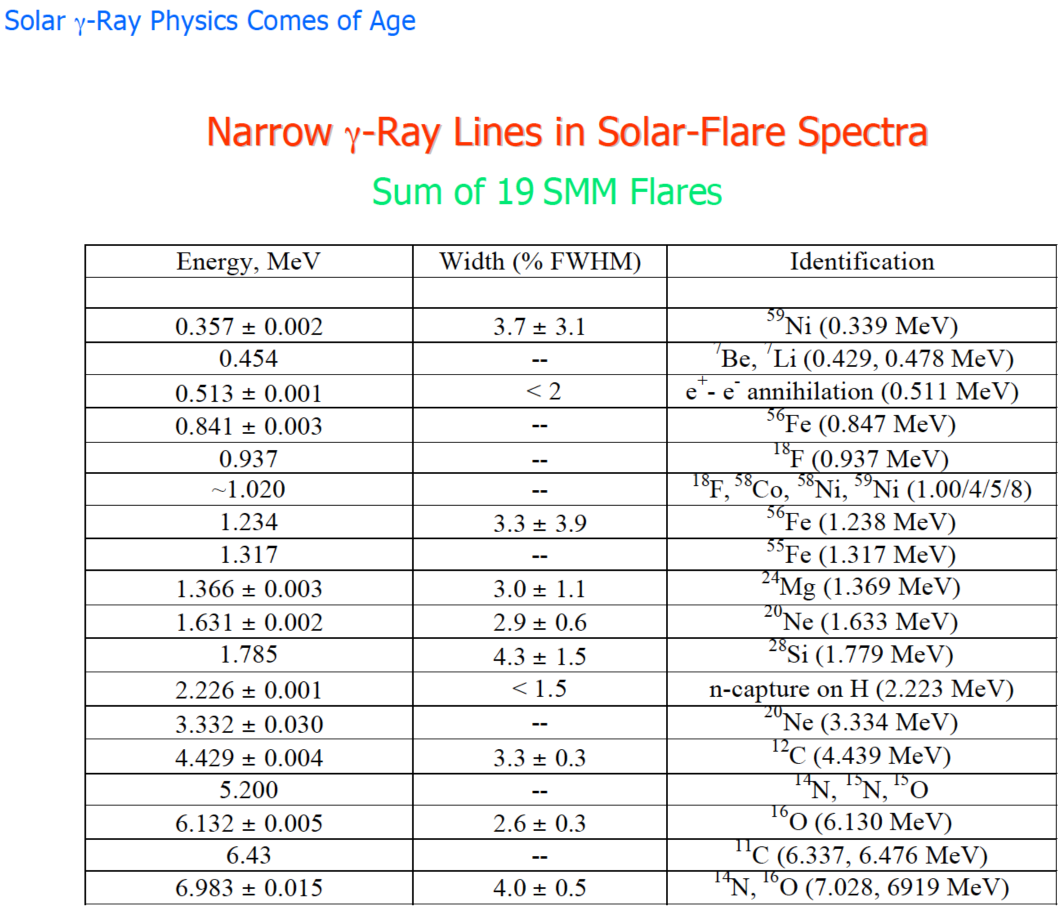
Solar Flares in Gamma-rays

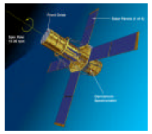


Solar γ-Ray Physics Comes of Age

Narrow γ-Ray Lines in Solar-Flare Spectra

Sum of 19 SMM Flares

Energy, MeV	Width (% FWHM)	Identification
0.357 ± 0.002	3.7 ± 3.1	^{59}Ni (0.339 MeV)
0.454	--	^{7}Be, ^{7}Li (0.429, 0.478 MeV)
0.513 ± 0.001	< 2	e^{+}- e^{-} annihilation (0.511 MeV)
0.841 ± 0.003	--	^{56}Fe (0.847 MeV)
0.937	--	^{18}F (0.937 MeV)
~1.020	--	^{18}F, ^{58}Co, ^{58}Ni, ^{59}Ni (1.00/4/5/8)
1.234	3.3 ± 3.9	^{56}Fe (1.238 MeV)
1.317	--	^{55}Fe (1.317 MeV)
1.366 ± 0.003	3.0 ± 1.1	^{24}Mg (1.369 MeV)
1.631 ± 0.002	2.9 ± 0.6	^{20}Ne (1.633 MeV)
1.785	4.3 ± 1.5	^{28}Si (1.779 MeV)
2.226 ± 0.001	< 1.5	n-capture on H (2.223 MeV)
3.332 ± 0.030	--	^{20}Ne (3.334 MeV)
4.429 ± 0.004	3.3 ± 0.3	^{12}C (4.439 MeV)
5.200	--	^{14}N, ^{15}N, ^{15}O
6.132 ± 0.005	2.6 ± 0.3	^{16}O (6.130 MeV)
6.43	--	^{11}C (6.337, 6.476 MeV)
6.983 ± 0.015	4.0 ± 0.5	^{14}N, ^{16}O (7.028, 6919 MeV)



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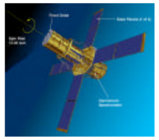
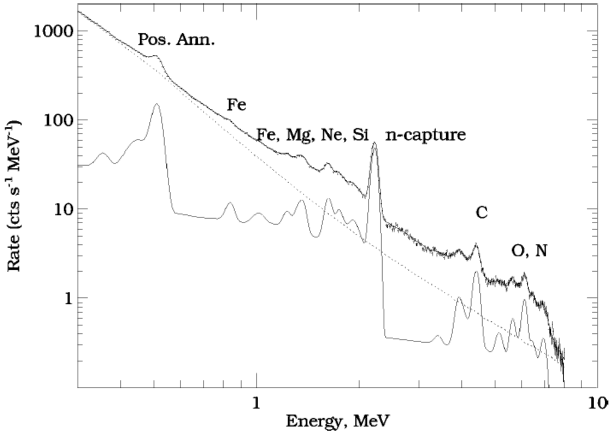
Solar Flares in Gamma-rays

Solar γ-Ray Physics Comes of Age



Revealing the Spectrum from Accelerated Heavy Ions



Accelerated heavy ions are excited by interaction with ambient H.

De-excitation lines from these ions are expected to be Doppler broadened by ~25%.

Broad line spectrum is revealed by subtracting best fitting narrow-line and bremsstrahlung components shown for sum of 19 flares observed by the *SMM*/GRS.

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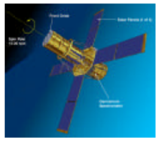
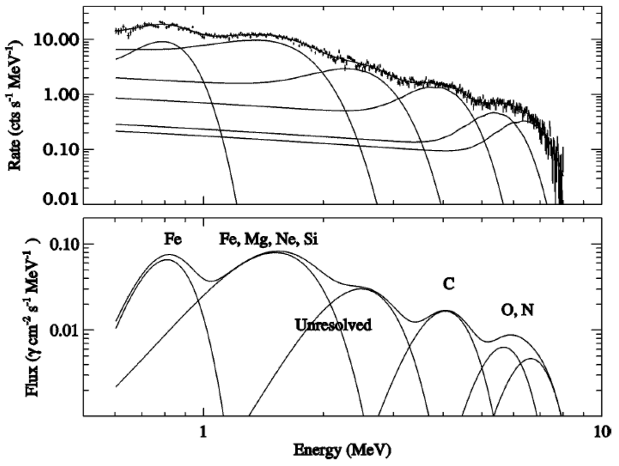
Solar Flares in Gamma-rays

Solar γ-Ray Physics Comes of Age



Gamma-Ray Spectrum from Accelerated Heavy Ions



Residual spectrum after subtracting contributions from bremsstrahlung and narrow lines reveals broadened lines from accelerated ions.

Best fit to spectrum contains six Gaussian features that can be identified with different ions.

Fe and C are resolved. The Fe, Mg, Ne, and Si lines between 1 - 2 MeV cannot be resolved.

Major uncertainty is the shape of the 'unresolved line' component that is expected to peak in the 1 - 3 MeV region.

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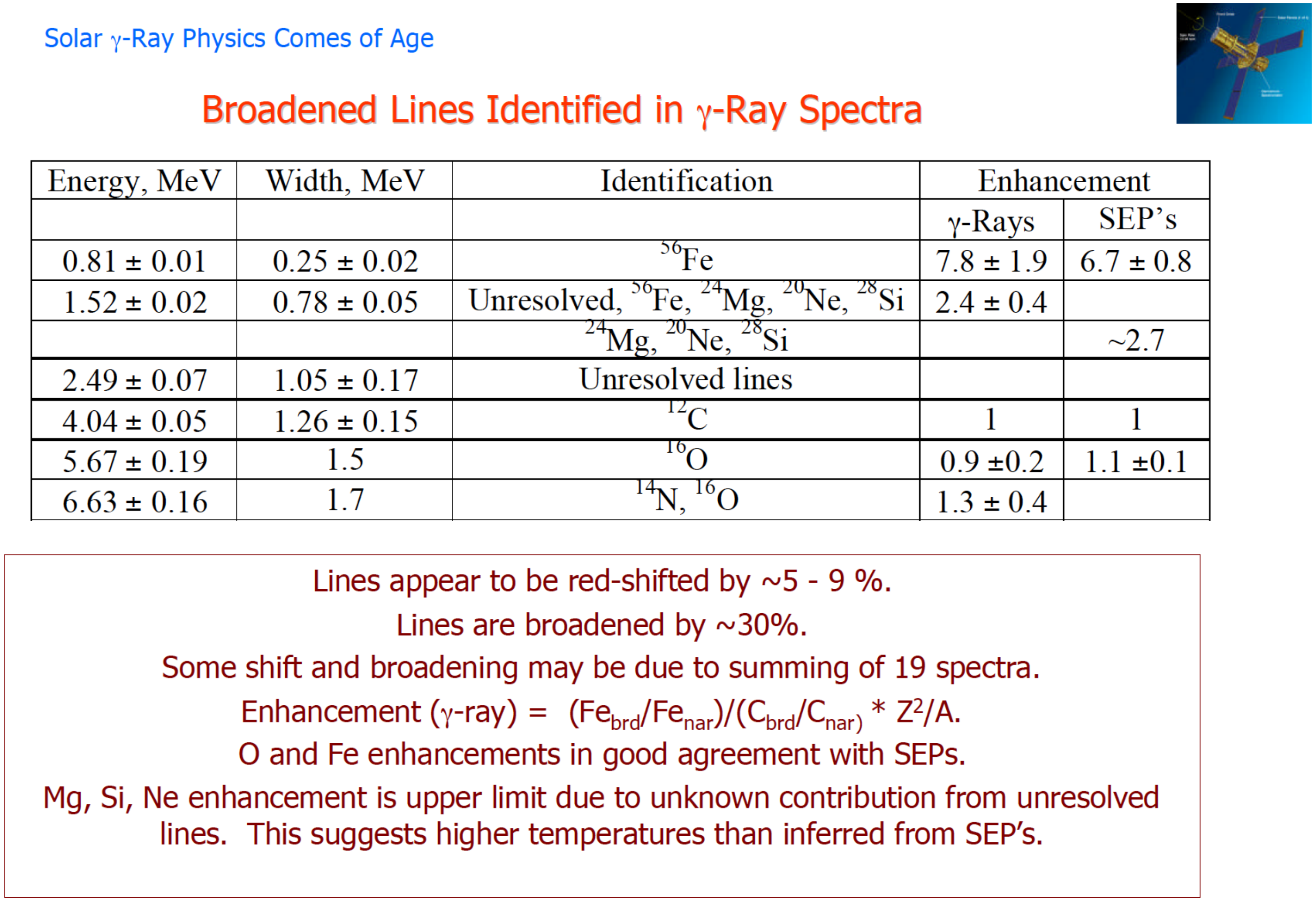
Solar Flares in Gamma-rays



Solar γ-Ray Physics Comes of Age

Broadened Lines Identified in γ-Ray Spectra

Energy, MeV	Width, MeV	Identification	Enhancement	
			γ-Rays	SEP's
0.81 ± 0.01	0.25 ± 0.02	^{56}Fe	7.8 ± 1.9	6.7 ± 0.8
1.52 ± 0.02	0.78 ± 0.05	Unresolved, ^{56}Fe, ^{24}Mg, ^{20}Ne, ^{28}Si	2.4 ± 0.4	
		^{24}Mg, ^{20}Ne, ^{28}Si		~2.7
2.49 ± 0.07	1.05 ± 0.17	Unresolved lines		
4.04 ± 0.05	1.26 ± 0.15	^{12}C	1	1
5.67 ± 0.19	1.5	^{16}O	0.9 ±0.2	1.1 ±0.1
6.63 ± 0.16	1.7	^{14}N, ^{16}O	1.3 ± 0.4	

Lines appear to be red-shifted by ~5 - 9 %.

Lines are broadened by ~30%.

Some shift and broadening may be due to summing of 19 spectra.

Enhancement (γ-ray) = $(Fe_{brd}/Fe_{nar})/(C_{brd}/C_{nar}) * Z^2/A$.

O and Fe enhancements in good agreement with SEPs.

Mg, Si, Ne enhancement is upper limit due to unknown contribution from unresolved lines. This suggests higher temperatures than inferred from SEP's.

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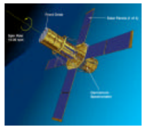
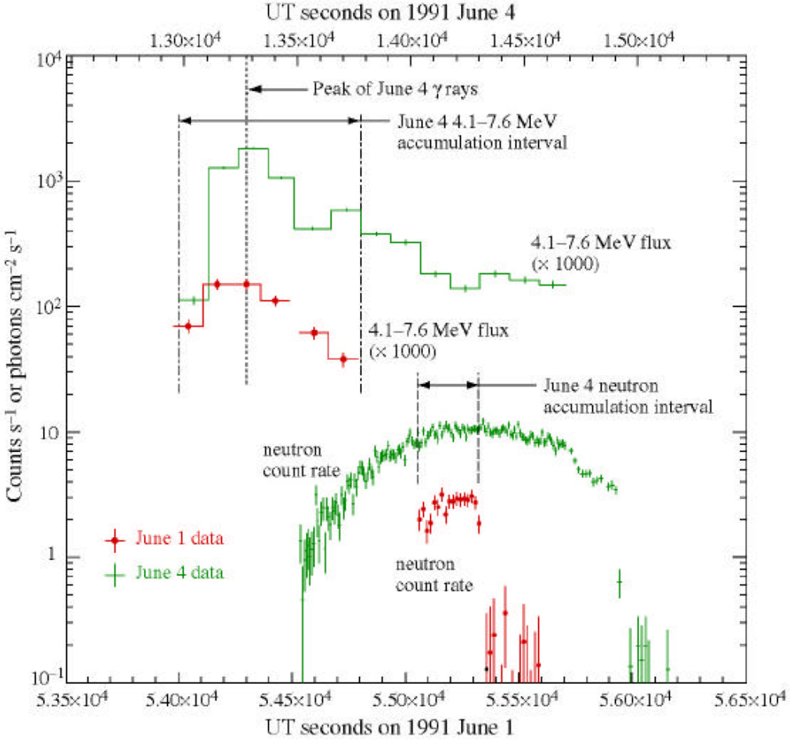
Solar Flares in Gamma-rays

Solar γ-Ray Physics Comes of Age



γ Rays and Neutrons Observed from the 1 & 4 June 1991 Flares



OSSE and GRANAT

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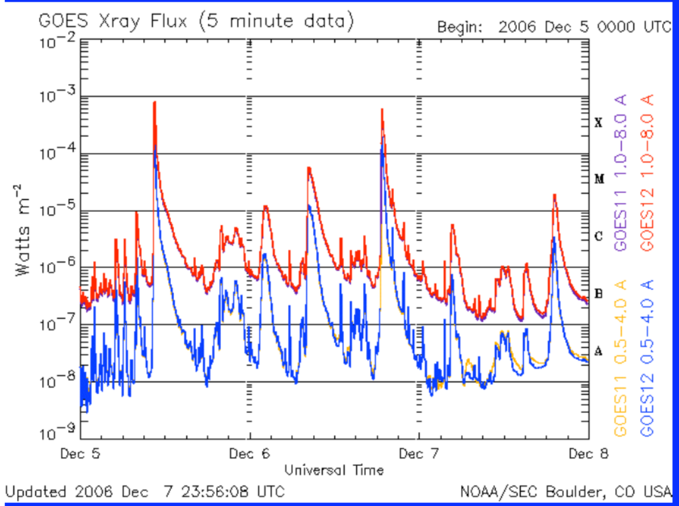
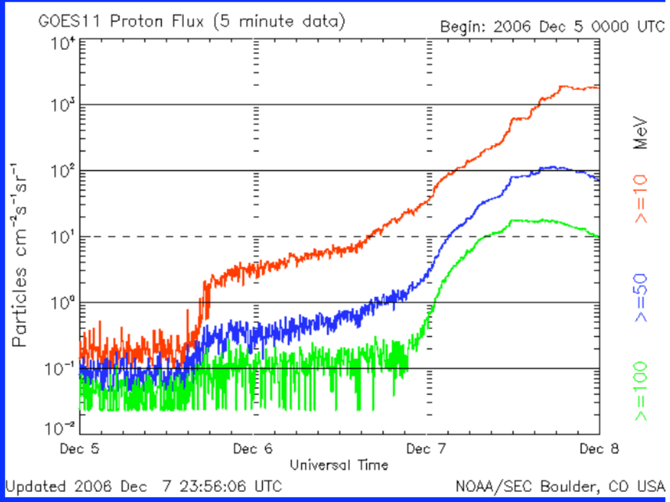
Solar Flares in Gamma-rays

Surprises though!
Active regions in January 2005 and December 2006 produced
intense X-Class Flares





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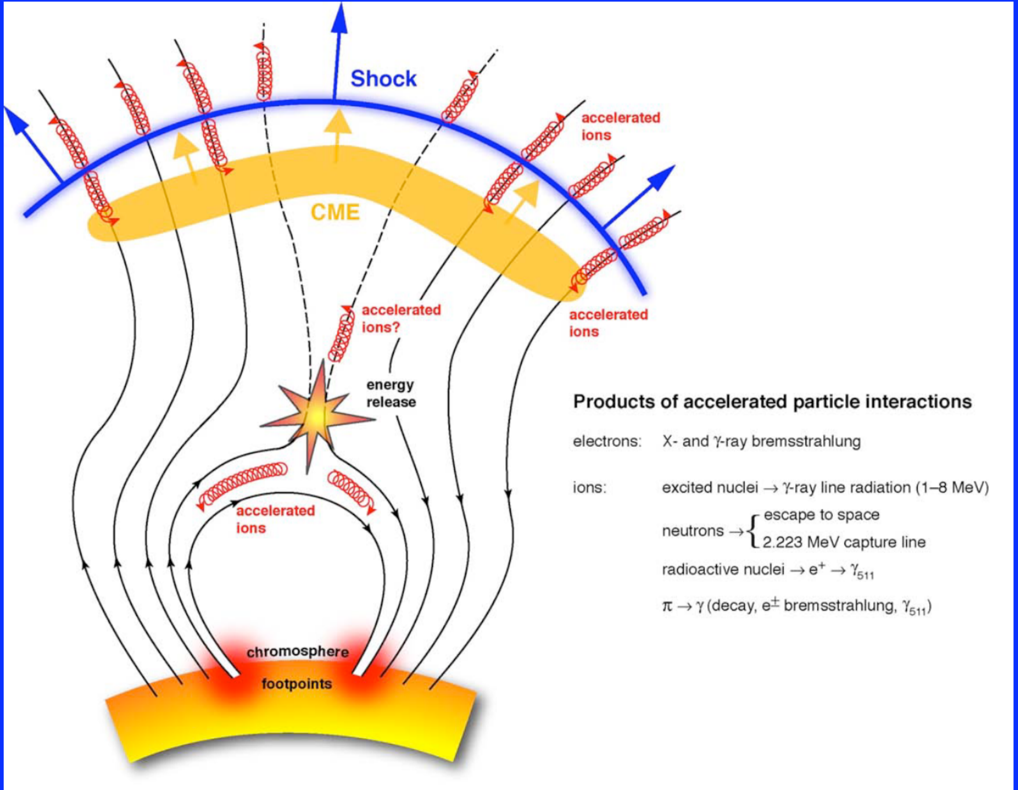
Solar Flares in Gamma-rays



Products of accelerated particle interactions

electrons:	X- and γ-ray bremsstrahlung
ions:	excited nuclei → γ-ray line radiation (1–8 MeV)
	neutrons $\rightarrow \begin{cases} \text{escape to space} \\ \text{2.223 MeV capture line} \end{cases}$
	radioactive nuclei $\rightarrow e^+ \rightarrow \gamma_{511}$
	$\pi \rightarrow \gamma$ (decay, $e^{\pm}$ bremsstrahlung, γ_{511})

Study how particles are accelerated at the Sun and their relationship to Solar Energetic Particles (SEP) and Ground Level Events (GLE).

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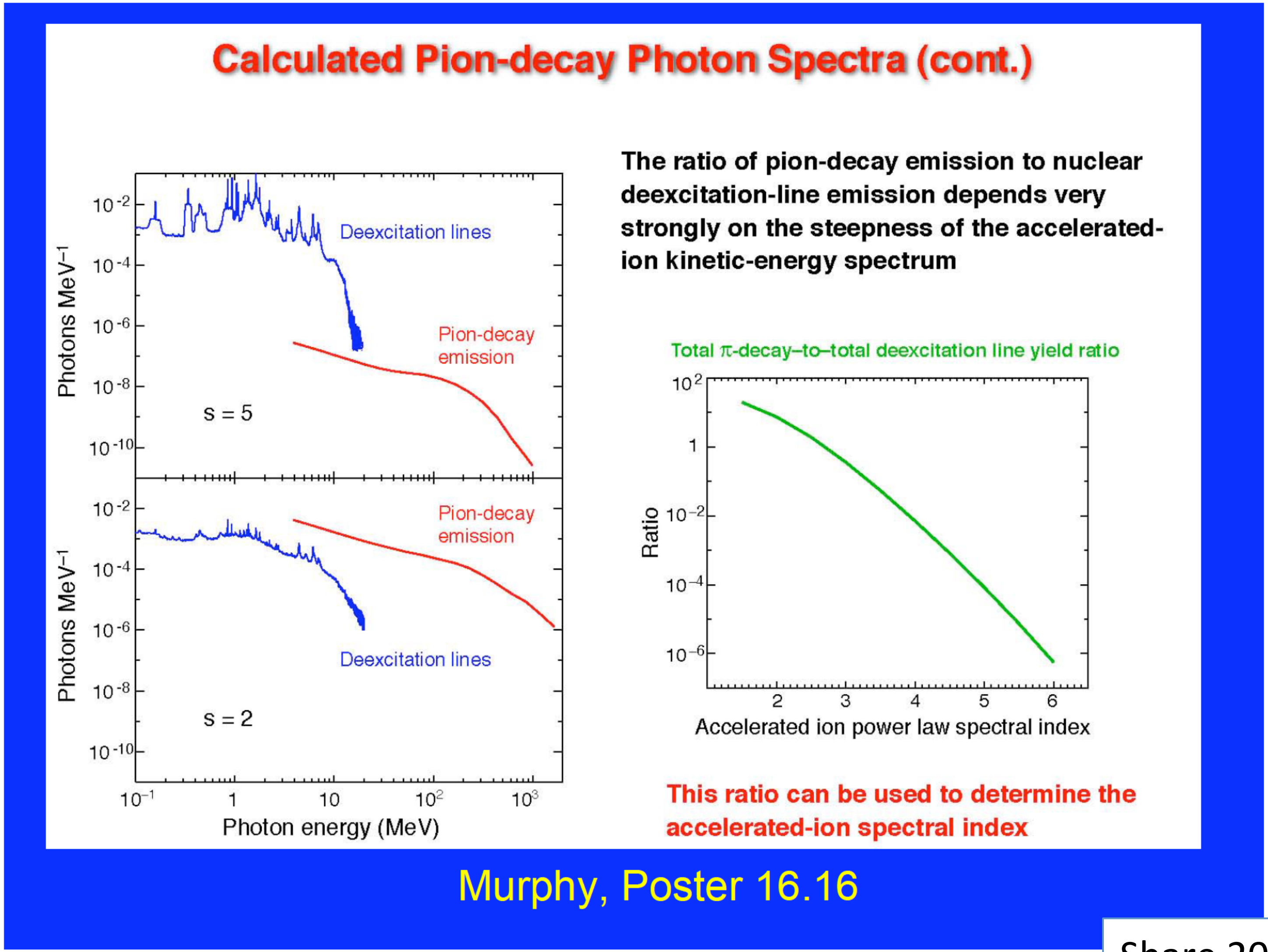
Solar Flares in Gamma-rays



Calculated Pion-decay Photon Spectra (cont.)

The ratio of pion-decay emission to nuclear deexcitation-line emission depends very strongly on the steepness of the accelerated-ion kinetic-energy spectrum

This ratio can be used to determine the accelerated-ion spectral index

Murphy, Poster 16.16

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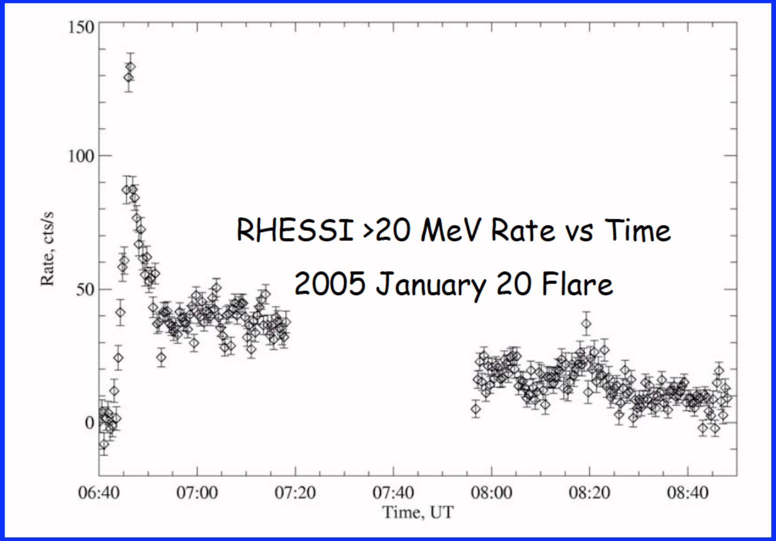
Solar Flares in Gamma-rays

Study particle acceleration and magnetic trapping of high-energy ions from minutes to hours after flares (e.g. EGRET observation on June 11, 1991; Kanbach et al.)


RHESSI >20 MeV Rate vs Time

2005 January 20 Flare

LAT is 10^4 times more sensitive to pion radiation than RHESSI

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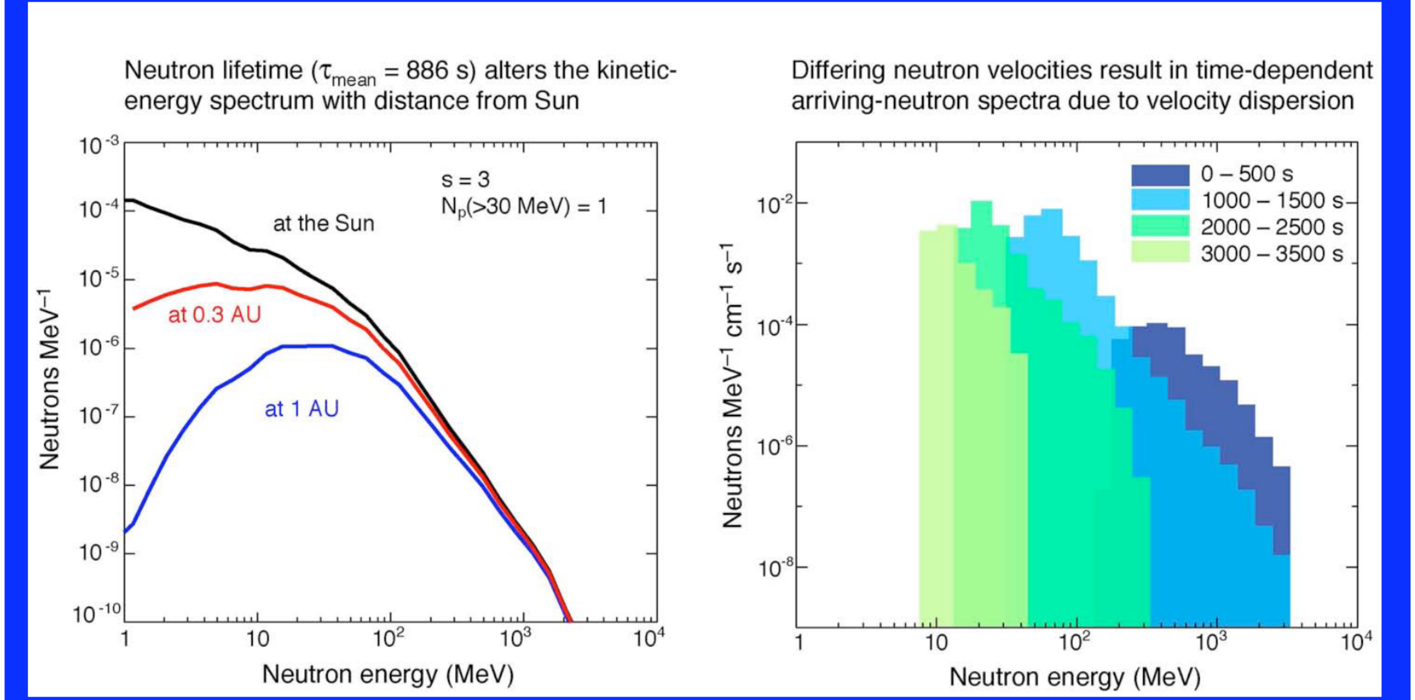
Solar Flares in Gamma-rays



Neutron lifetime (τ_{mean} = 886 s) alters the kinetic-energy spectrum with distance from Sun

Differing neutron velocities result in time-dependent arriving-neutron spectra due to velocity dispersion

Murphy, Poster 16.16

GBM will also detect an increase minutes after the impulsive phase of the flare.

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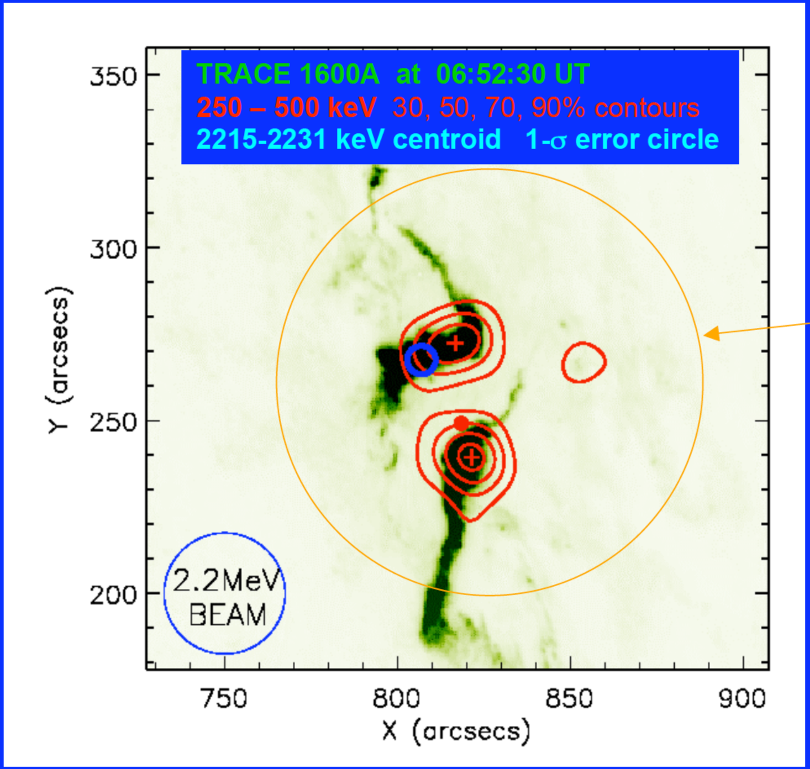
Solar Flares in Gamma-rays

20 January 2005 06:44-06:56

RHESSI, Hurford et al. 2007



GLAST Location

Localize the source of >1 GeV photons to ~30 arc sec

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Astrofisica Nucleare e Subnucleare

Gravitational Waves

Exercise on GW

- Find recent information on the status of LIGO, Virgo, KAGRA
- Find the status of eLISA GW observatory
- Find the status of PTA GW methods

Introduzione

Gravitational wave astronomy

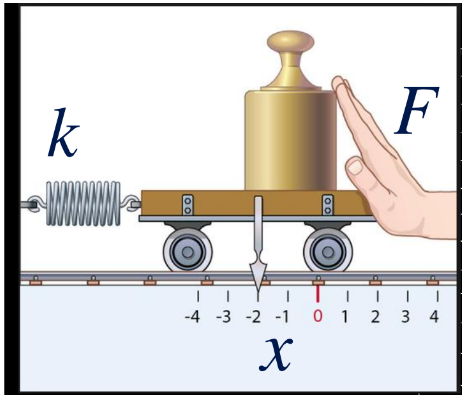
past, present and future

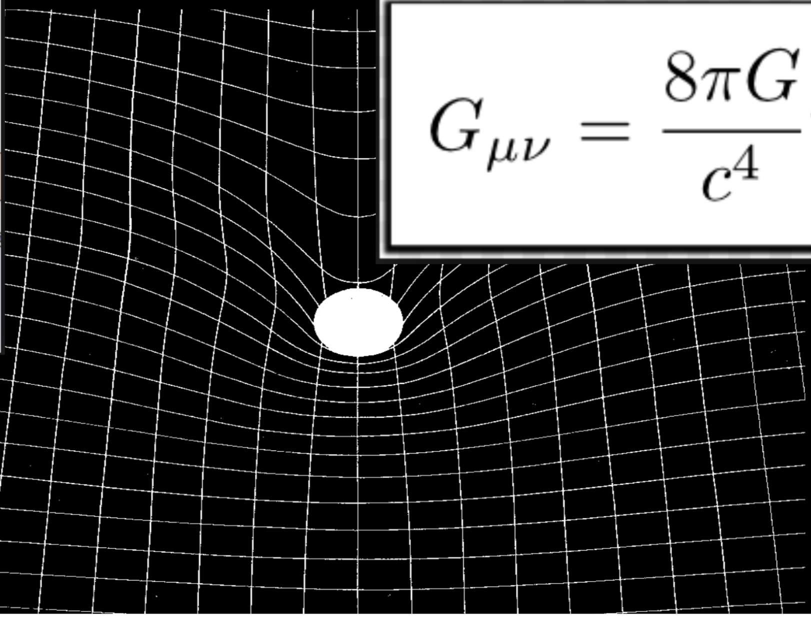
Eugenio Coccia

Gran Sasso Science Institute and INFN

SciNeGHE 2016
High-energy gamma-ray experiments at the dawn of gravitational wave astronomy
Pisa,18 October 2016

Introduzione





$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$F = -kx$$

$$F \Leftrightarrow T_{\mu\nu}$$

$$x \Leftrightarrow G_{\mu\nu}$$

$$k \Leftrightarrow \frac{c^4}{8\pi G}$$

$$c = 299\ 792\ 458\ \ m/s = 3\times10^8\ \ m/s$$

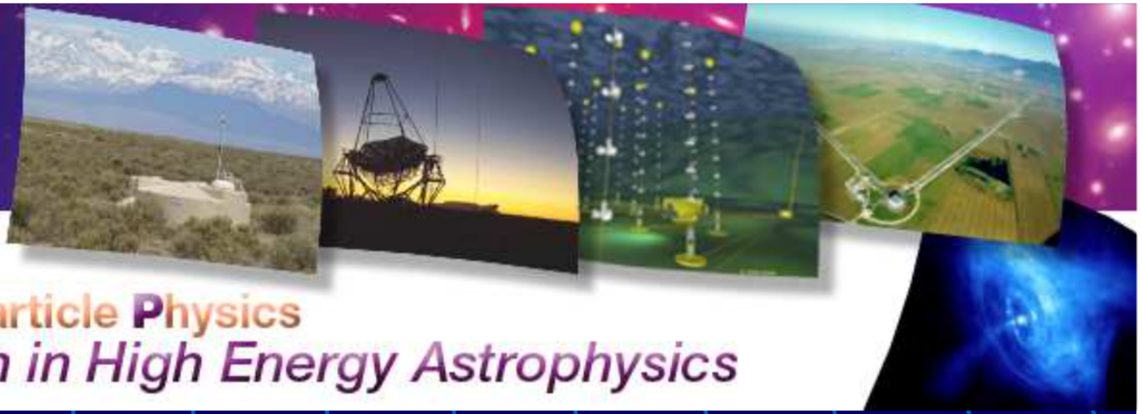
$$G = 0{,}000\ 000\ 000\ 066\ 7\ \frac{m^3}{kg\ s^2} = 6{,}67\times10^{-11}\ \frac{m^3}{kg\ s^2}$$

$$k \approx 10^{45}\ \frac{kg}{s^2} \qquad STIFF!$$

ISAPP
International School on AstroParticle Physics

2 - 13 July 2012
Paris (France)

European Doctorate School



International School of AstroParticle Physics
Multi-Messenger Approach in High Energy Astrophysics

Gravitational Waves Detection And Fourier Methods

ISAPP2012
Paris, France, July 2012

Patrice Hello

Laboratoire de l'Accélérateur Linéaire
Orsay-France

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Comprendre le monde, construire l'avenir®

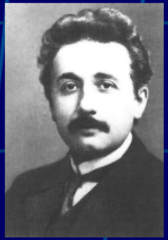
What are Gravitational Waves ?

Gravitational Waves (GW) are ripples of space-time

Theory of GW :

1. Einstein equations: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$
2. Far from sources: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$
3. Linearization: $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
4. Gauge TT: $$\nabla^2 h_{\mu\nu}^{TT} = 0$$

Propagation of some tensor field – *h* - on flat space-time



Prediction in 1916 !

Gravitational Wave general properties

- GW propagate at speed of light
- GW have two polarizations “+” and “x”
- GW emission is quadrupolar at lowest order

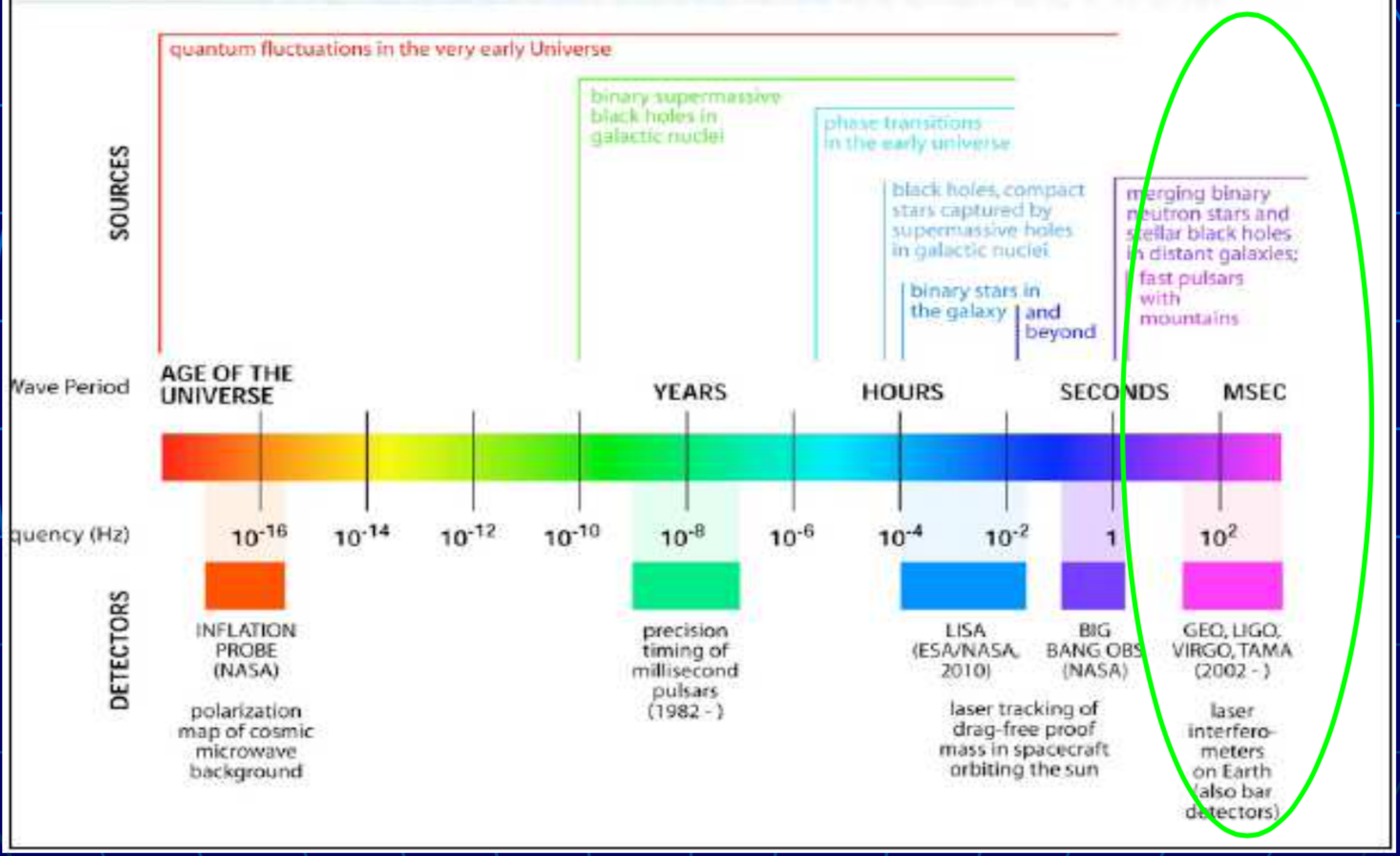
Example: plane wave propagating along z axis with 2 polarization amplitudes h_+ and h_x:

$$h_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Corresponding *Graviton* properties:

- Graviton has null mass
- Graviton has spin 2

THE GRAVITATIONAL WAVE SPECTRUM



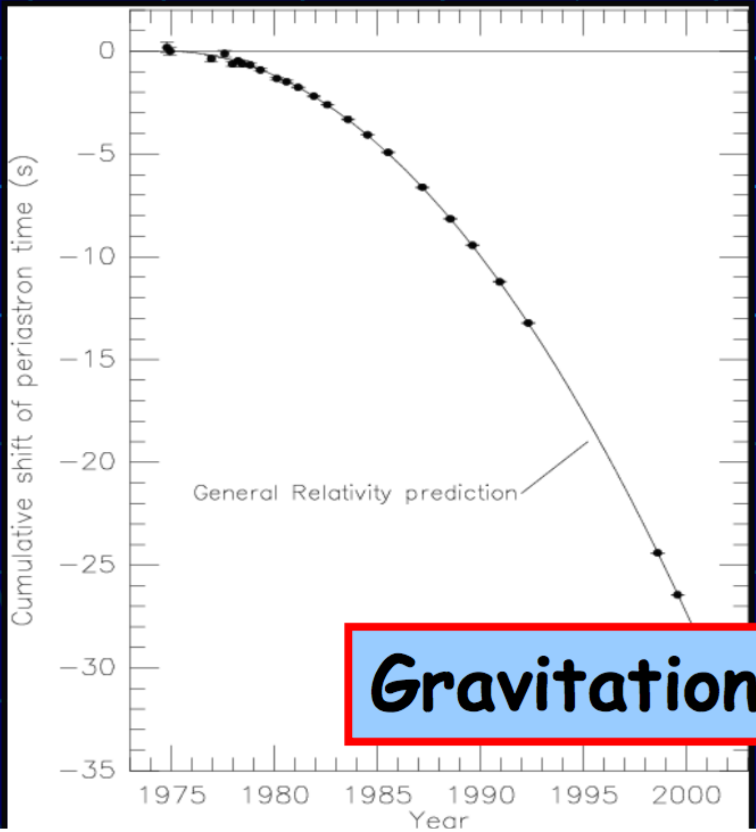
GW (indirect) discovery PSR 1913+16

(Hulse & Taylor, Nobel'93)

PSR 1913+16 : binary pulsar (system of 2 neutron stars, one being a radio pulsar seen by radiotelescopes) at ~ 7 kpc from Earth.

⇒ tests of Gravitation theory in strong field and dynamical regime

Loss of energy by GW emission : orbital period decreases



(merge in 300 billions years)

P (s)	**27906.9807807(9)**
dP/dt	**$-2.425(10)\cdot10^{-12}$**
dω/dt (º/yr)	**4.226628(18)**
M_p	**$1.442 \pm 0.003\ M_{\odot}$**
M_c	**$1.386 \pm 0.003\ M_{\odot}$**

Gravitational Waves do exist !

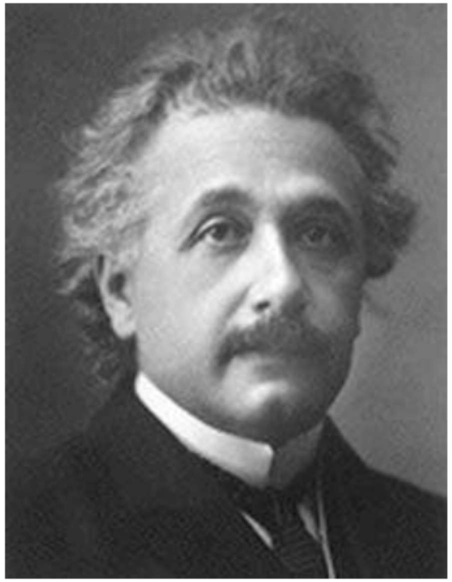
SUPAGWD

An Introduction to General Relativity, Gravitational Waves and Detection Principles

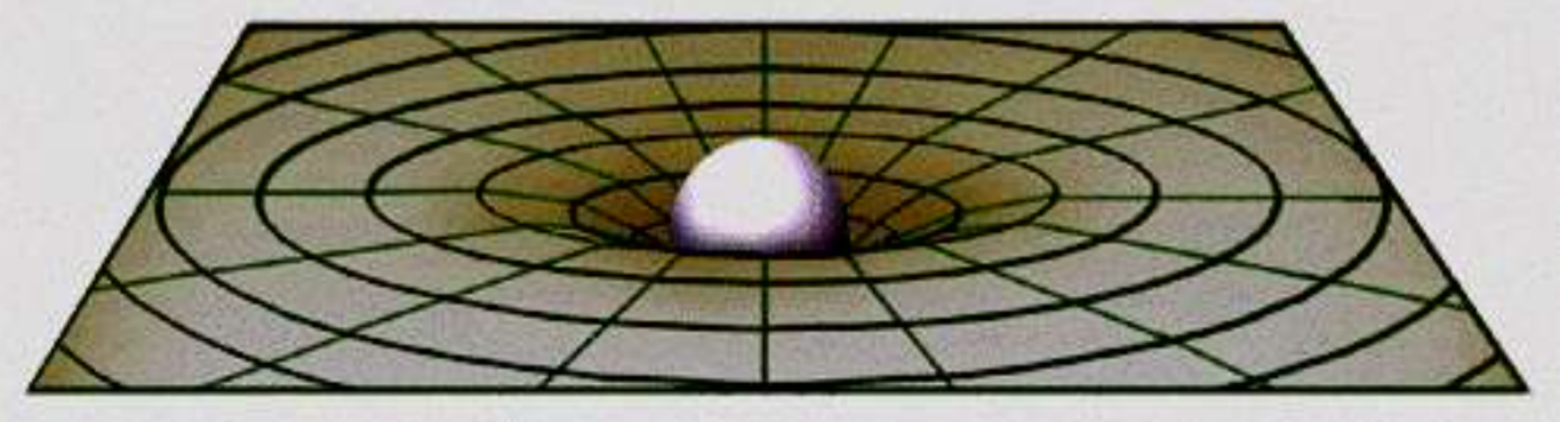
Prof Martin Hendry
University of Glasgow
Dept of Physics and Astronomy

October 2012

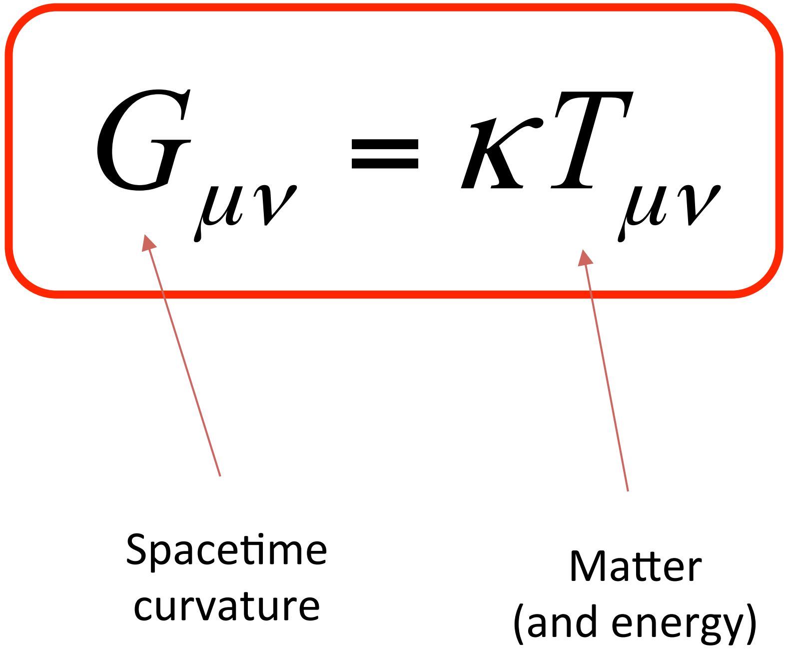
Gravity in Einstein's Universe



Spacetime tells matter how to move, and matter tells spacetime how to curve



"…joy and amazement at the beauty and grandeur of this world of which man can just form a faint notion."



$$\boxed{G_{\mu\nu} = \kappa T_{\mu\nu}}$$

Spacetime curvature

Matter (and energy)

6. Wave Equation for Gravitational Radiation (pgs.46 – 57)

Weak gravitational fields

In the absence of a gravitational field, spacetime is flat. We define a weak gravitational field as one is which spacetime is 'nearly flat'

i.e. we can find a coord system such that

$$\boxed{g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}}$$

where

$$\eta_{\alpha\beta} = \text{diag}\ (-1, 1, 1, 1)$$

$$|h_{\alpha\beta}| << 1 \ \text{for all} \ \alpha \ \text{and} \ \beta$$

This is known as a Nearly Lorentz coordinate system.

Einstein's equations for a weak gravitational field

The Einstein tensor is the (rather messy) expression

$$G_{\mu\nu} = \frac{1}{2}\left[h_{\mu\alpha,\nu}{}^{,\alpha} + h_{\nu\alpha,\mu}{}^{,\alpha} - h_{\mu\nu,\alpha}{}^{,\alpha} - h_{,\mu\nu} - \eta_{\mu\nu}\left(h_{\alpha\beta}{}^{,\alpha\beta} - h_{,\beta}{}^{,\beta}\right)\right]$$

but we can simplify this by introducing $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$

So that

$$G_{\mu\nu} = -\frac{1}{2}\left[\bar{h}_{\mu\nu,\alpha}{}^{,\alpha} + \eta_{\mu\nu}\bar{h}_{\alpha\beta}{}^{,\alpha\beta} - \bar{h}_{\mu\alpha,\nu}{}^{,\alpha} - \bar{h}_{\nu\alpha,\mu}{}^{,\alpha}\right]$$

And we can choose the **Lorentz gauge** to eliminate the last 3 terms

Einstein's equations for a weak gravitational field

To first order, the R-C tensor for a weak field reduces to

$$\boxed{R_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\alpha\gamma,\beta\delta} - h_{\beta\delta,\alpha\gamma}\right)}$$

and is invariant under gauge transformations.

Similarly, the Ricci tensor is

$$R_{\mu\nu} = \frac{1}{2}\left(h^{\alpha}_{\mu\ ,\nu\alpha} + h^{\alpha}_{\nu\ ,\mu\alpha} - h_{\mu\nu,\alpha}{}^{,\alpha} - h_{,\mu\nu}\right)$$

where

$$h \equiv h^{\alpha}_{\alpha} = \eta^{\alpha\beta} h_{\alpha\beta}$$

$$h_{\mu\nu,\alpha}{}^{,\alpha} \; = \; \eta^{\alpha\sigma}\left(h_{\mu\nu,\alpha}\right)_{,\sigma} \; = \; \eta^{\alpha\sigma}\, h_{\mu\nu,\alpha\sigma}$$

In the Lorentz gauge, then Einstein's equations are simply

$$-\bar{h}_{\mu\nu,\alpha}{}^{,\alpha} = 16\pi T_{\mu\nu}$$

And in free space this gives

$$\boxed{\bar{h}_{\mu\nu,\alpha}{}^{,\alpha} = 0}$$

Writing $\bar{h}_{\mu\nu,\alpha}{}^{,\alpha} \equiv \eta^{\alpha\alpha}\bar{h}_{\mu\nu,\alpha\alpha}$

or

$$\boxed{\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = 0}$$

Remembering that we are taking $c = 1$, if instead we write

$$\eta^{00} = -\frac{1}{c^2}$$

then

$$\boxed{\left(-\frac{\partial^2}{\partial t^2} + c^2 \nabla^2 \right) \overline{h}_{\mu\nu} \quad = \quad 0}$$

This is a key result. It has the mathematical form of a wave equation, propagating with speed c.

We have shown that the metric perturbations – the 'ripples' in spacetime produced by disturbing the metric – propagate at the speed of light as waves in free space.

7. The Transverse Traceless Gauge (pgs.57 – 62)

Simplest solutions of our wave equation are **plane waves**

$$\bar{h}_{\mu\nu} = \mathrm{Re}\left[A_{\mu\nu}\ \exp\left(ik_\alpha x^\alpha\right)\right]$$

Wave amplitude

Wave vector

Note the wave amplitude is symmetric → 10 independent components.

In the transverse traceless gauge,

$$\overline{h}_{\mu\nu}^{(\mathrm{TT})} = A_{\mu\nu}^{(\mathrm{TT})} \cos\left[\omega(t-z)\right]$$

where

$$A_{\mu\nu}^{(\mathrm{TT})} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx}^{(\mathrm{TT})} & A_{xy}^{(\mathrm{TT})} & 0 \\ 0 & A_{xy}^{(\mathrm{TT})} & -A_{xx}^{(\mathrm{TT})} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Also, since the perturbation is traceless $\overline{h}_{\alpha\beta}^{(\mathrm{TT})} = h_{\alpha\beta}^{(\mathrm{TT})}$

8. Effect of Gravitational Waves on Free Particles (pgs.63 – 75)

Choose Background Lorentz frame in which test particle initially at rest. Set up coordinate system according to the TT gauge.

Initial acceleration satisfies

$$\left(\frac{dU^\beta}{d\tau}\right)_0 = 0$$

i.e. coordinates do not change, but adjust themselves as wave passes so that particles remain ‘attached’ to initial positions.

Coordinates are frame-dependent labels.

What about **proper distance** between neighbouring particles?

Consider two test particles, both initially at rest, one at origin and the other at $x = \epsilon\,,\ y = z = 0$

$$\Delta\ell = \int \left| g_{\alpha\beta} dx^{\alpha} dx^{\beta} \right|^{1/2}$$

i.e. $$\Delta\ell = \int_0^{\epsilon} \left| g_{xx} \right|^{1/2} \quad \simeq \quad \sqrt{g_{xx}(x=0)}\ \epsilon$$

Now $$g_{xx}(x=0) = \eta_{xx} + h_{xx}^{(\mathrm{TT})}(x=0)$$

so $$\boxed{\Delta\ell \simeq \left[1 + \frac{1}{2} h_{xx}^{(\mathrm{TT})}(x=0) \right] \epsilon}$$

In general, this is time-varying

More formally, consider geodesic deviation ξ^α between two particles, initially at rest

i.e. initially with $U^\mu \quad = \quad (1,0,0,0)^T \qquad \xi^\beta \quad = \quad (0,\epsilon,0,0)^T$

Then

$$\frac{\partial^2 \xi^\alpha}{\partial t^2} = \epsilon R^\alpha_{ttx} = -\epsilon R^\alpha_{txt}$$

and

$$R^x_{txt} = \eta^{xx} R_{xtxt} = -\frac{1}{2} h^{(\mathrm{TT})}_{xx,tt}$$

$$R^y_{txt} = \eta^{yy} R_{ytxt} = -\frac{1}{2} h^{(\mathrm{TT})}_{xy,tt}$$

Hence

$$\boxed{\frac{\partial^2}{\partial t^2}\xi^x \quad = \quad \frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h^{(\mathrm{TT})}_{xx} \qquad \frac{\partial^2}{\partial t^2}\xi^y \quad = \quad \frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h^{(\mathrm{TT})}_{xy}}$$

Similarly, two test particles initially separated by ϵ in the y- direction satisfy

$$\frac{\partial^2}{\partial t^2}\xi^x = \frac{1}{2}\epsilon\frac{\partial^2}{\partial t^2}h_{xy}^{(\mathrm{TT})} \qquad \frac{\partial^2}{\partial t^2}\xi^y = -\frac{1}{2}\epsilon\frac{\partial^2}{\partial t^2}h_{xx}^{(\mathrm{TT})}$$

We can further generalise to a ring of test particles: one at origin, the other initially a :

$$x = \epsilon\cos\theta \qquad y = \epsilon\sin\theta \qquad z = 0$$

$$\frac{\partial^2}{\partial t^2}\xi^x = \frac{1}{2}\epsilon\cos\theta\frac{\partial^2}{\partial t^2}h_{xx}^{(\mathrm{TT})} + \frac{1}{2}\epsilon\sin\theta\frac{\partial^2}{\partial t^2}h_{xy}^{(\mathrm{TT})}$$

$$\frac{\partial^2}{\partial t^2}\xi^y = \frac{1}{2}\epsilon\cos\theta\frac{\partial^2}{\partial t^2}h_{xy}^{(\mathrm{TT})} - \frac{1}{2}\epsilon\sin\theta\frac{\partial^2}{\partial t^2}h_{xx}^{(\mathrm{TT})}$$

Solutions are:

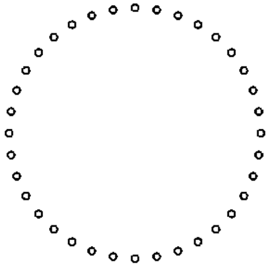
$$\xi^x = \epsilon\cos\theta + \frac{1}{2}\epsilon\cos\theta\, A_{xx}^{(\mathrm{TT})}\cos\omega t \;+\; \frac{1}{2}\epsilon\sin\theta\, A_{xy}^{(\mathrm{TT})}\cos\omega t$$

$$\xi^y = \epsilon\sin\theta + \frac{1}{2}\epsilon\cos\theta\, A_{xy}^{(\mathrm{TT})}\cos\omega t \;-\; \frac{1}{2}\epsilon\sin\theta\, A_{xx}^{(\mathrm{TT})}\cos\omega t$$

Suppose we now vary θ between 0 and 2π, so that we are considering an initially circular ring of test particles in the x-y plane, initially equidistant from the origin.

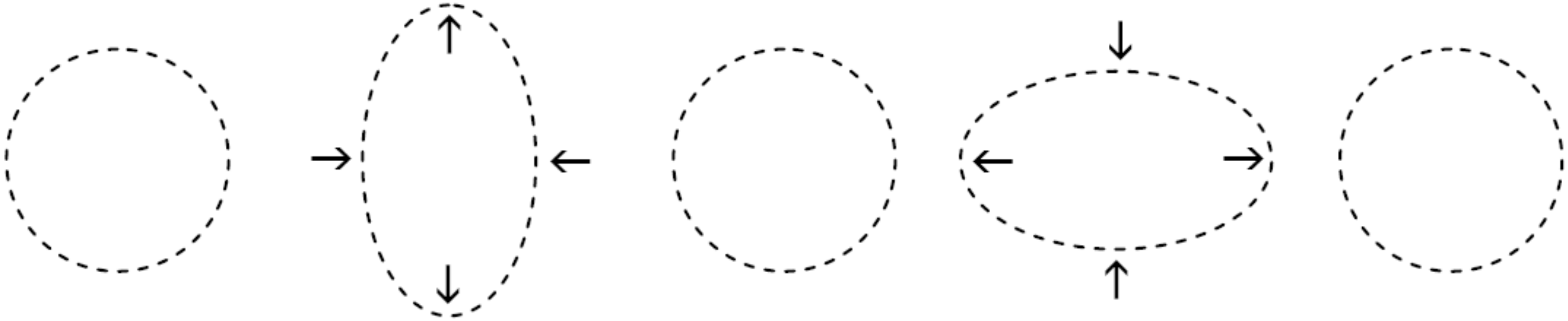
$$A_{xx}^{(\mathrm{TT})} \neq 0 \qquad A_{xy}^{(\mathrm{TT})} = 0$$

$$\xi^x = \epsilon\cos\theta\left(1 + \frac{1}{2}A_{xx}^{(\mathrm{TT})}\cos\omega t\right)$$



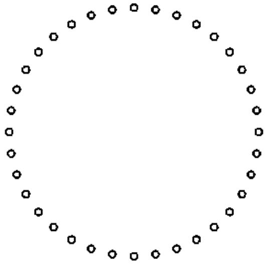
$$\xi^y = \epsilon\sin\theta\left(1 - \frac{1}{2}A_{xx}^{(\mathrm{TT})}\cos\omega t\right)$$

$A_{xx}^{(\mathrm{TT})} \neq 0$ **+ Polarisation**



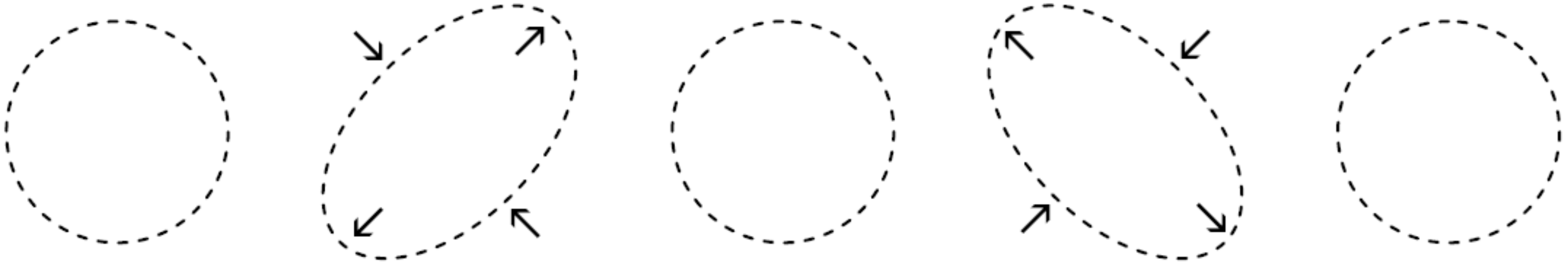
$$A_{xy}^{(\mathrm{TT})} \neq 0 \qquad A_{xx}^{(\mathrm{TT})} = 0$$

$$\xi^x = \epsilon \cos\theta + \frac{1}{2}\epsilon \sin\theta \, A_{xy}^{(\mathrm{TT})} \cos\omega t$$



$$\xi^y = \epsilon \sin\theta + \frac{1}{2}\epsilon \cos\theta \, A_{xy}^{(\mathrm{TT})} \cos\omega t$$

$A_{xy}^{(\mathrm{TT})} \neq 0$ × **Polarisation**



- The two solutions, for $A_{xx}^{(\mathrm{TT})} \neq 0$ and $A_{xy}^{(\mathrm{TT})} \neq 0$ represent two independent gravitational wave **polarisation states**, and these states are usually denoted by '+' and '×' respectively. In general any gravitational wave propagating along the z-axis can be expressed as a linear combination of the '+' and '×' polarisations, i.e. we can write the wave as

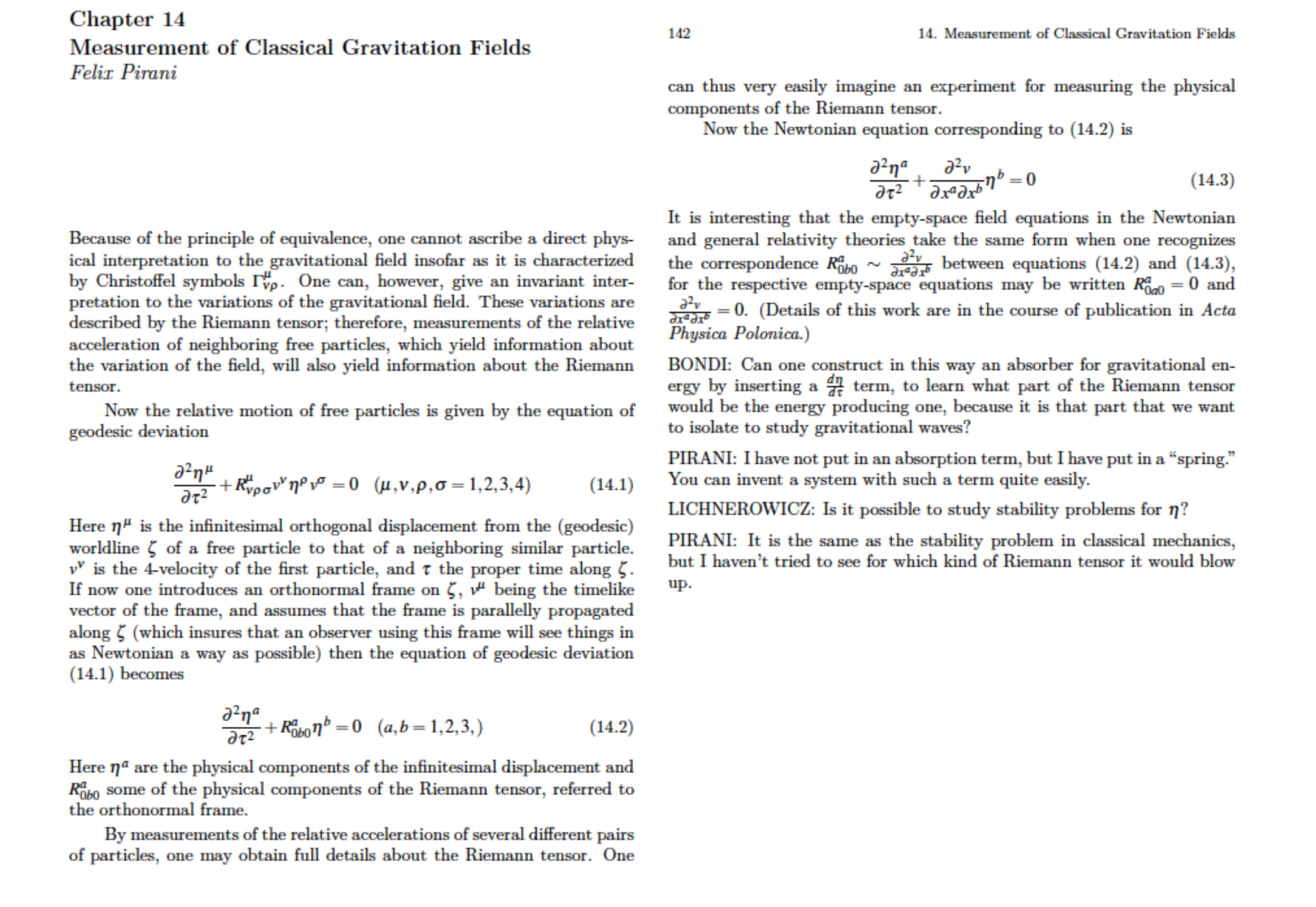
$$\mathbf{h} = a\,\mathbf{e}_+ + b\,\mathbf{e}_\times$$

where a and b are scalar constants and the *polarisation tensors* $\mathbf{e}_+$ and $\mathbf{e}_\times$ are

$$\mathbf{e}_+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{e}_\times = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Distortions are **quadrupolar** - consequence of fact that acceleration of geodesic deviation non-zero only for tidal gravitational field.

Metodi di rivelazione



Chapter 14

Measurement of Classical Gravitation Fields

Felix Pirani

Because of the principle of equivalence, one cannot ascribe a direct physical interpretation to the gravitational field insofar as it is characterized by Christoffel symbols $\Gamma^{\mu}_{\nu\rho}$. One can, however, give an invariant interpretation to the variations of the gravitational field. These variations are described by the Riemann tensor; therefore, measurements of the relative acceleration of neighboring free particles, which yield information about the variation of the field, will also yield information about the Riemann tensor.

Now the relative motion of free particles is given by the equation of geodesic deviation

$$\frac{\partial^2\eta^\mu}{\partial\tau^2}+R^{\mu}_{\nu\rho\sigma}v^\nu\eta^\rho v^\sigma=0 \quad (\mu,\nu,\rho,\sigma=1,2,3,4) \tag{14.1}$$

Here η^μ is the infinitesimal orthogonal displacement from the (geodesic) worldline ζ of a free particle to that of a neighboring similar particle. v^ν is the 4-velocity of the first particle, and τ the proper time along ζ. If now one introduces an orthonormal frame on ζ, v^μ being the timelike vector of the frame, and assumes that the frame is parallelly propagated along ζ (which insures that an observer using this frame will see things in as Newtonian a way as possible) then the equation of geodesic deviation (14.1) becomes

$$\frac{\partial^2\eta^a}{\partial\tau^2}+R^{a}_{0b0}\eta^b=0 \quad (a,b=1,2,3,) \tag{14.2}$$

Here η^a are the physical components of the infinitesimal displacement and R^a_{0b0} some of the physical components of the Riemann tensor, referred to the orthonormal frame.

By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One

142 14. Measurement of Classical Gravitation Fields

can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor.

Now the Newtonian equation corresponding to (14.2) is

$$\frac{\partial^2\eta^a}{\partial\tau^2}+\frac{\partial^2 v}{\partial x^a\partial x^b}\eta^b=0 \tag{14.3}$$

It is interesting that the empty-space field equations in the Newtonian and general relativity theories take the same form when one recognizes the correspondence $R^a_{0b0}\sim\frac{\partial^2 v}{\partial x^a\partial x^b}$ between equations (14.2) and (14.3), for the respective empty-space equations may be written $R^a_{0a0}=0$ and $\frac{\partial^2 v}{\partial x^a\partial x^b}=0$. (Details of this work are in the course of publication in *Acta Physica Polonica.*)

BONDI: Can one construct in this way an absorber for gravitational energy by inserting a $\frac{d\eta}{d\tau}$ term, to learn what part of the Riemann tensor would be the energy producing one, because it is that part that we want to isolate to study gravitational waves?

PIRANI: I have not put in an absorption term, but I have put in a "spring." You can invent a system with such a term quite easily.

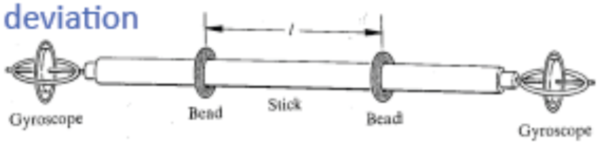
LICHNEROWICZ: Is it possible to study stability problems for η?

PIRANI: It is the same as the stability problem in classical mechanics, but I haven't tried to see for which kind of Riemann tensor it would blow up.

Metodi di rivelazione

F.A.E. Pirani, GW geodesic deviation

H. Bondi : beads on rod, energy in GW



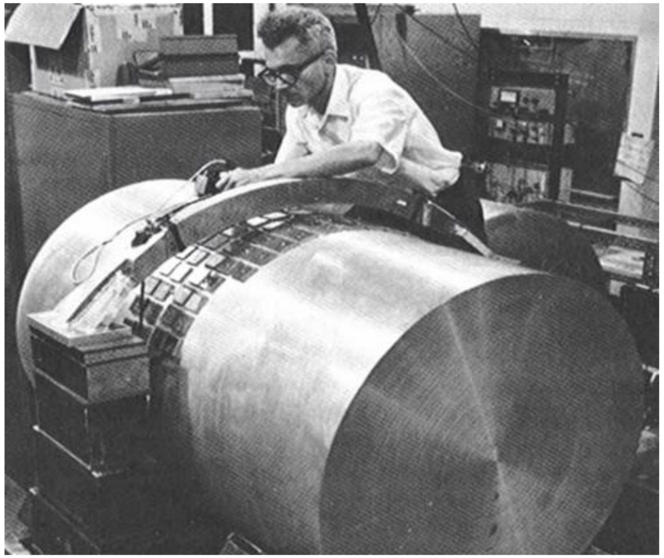
The main point of this presentation was that it is relative accelerations of neighboring free particles that are the physically meaningful (i.e.,measurable) ways to observe gravitational effects. Pirani points out the transparent connection between the equation of geodesic deviation and Newton's Second Law, as long as one identifies $\mathbf{R}_{a0b0}$ with the second derivative of the Newtonian potential (i.e., as the tidal field.)

To make sure everyone sees how important and simple this is, he remarks, "By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor".

from: P. Saulson, Gen Relativ Gravit (2011) 43:3289–3299

Metodi di rivelazione

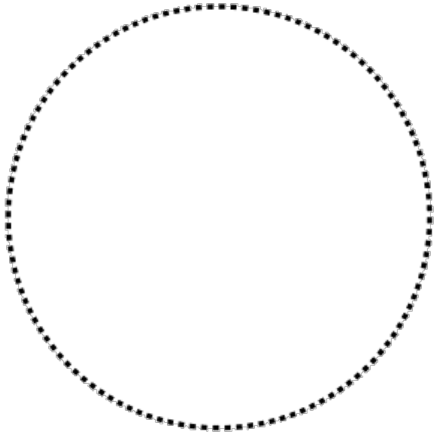
Weber's bar

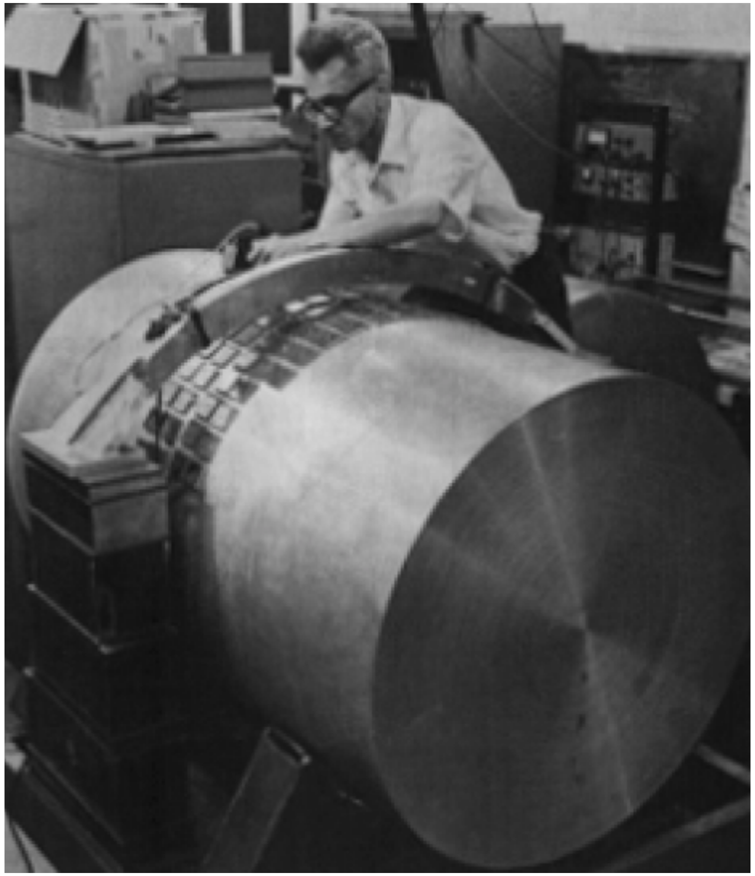


Weber's detector embodied Pirani's *gedankenexperiment*.

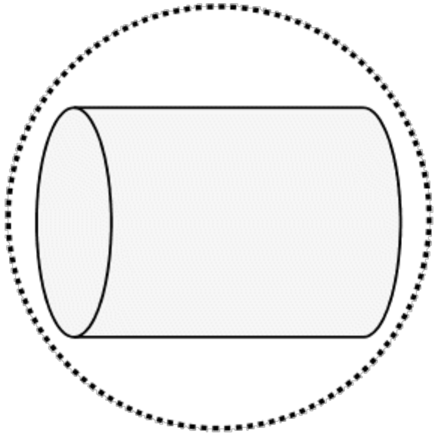
It was a cylinder of aluminum, each end of which is like a test mass, while the center is like a spring. PZT's around the midline absorb energy to send to an electrical amplifier.

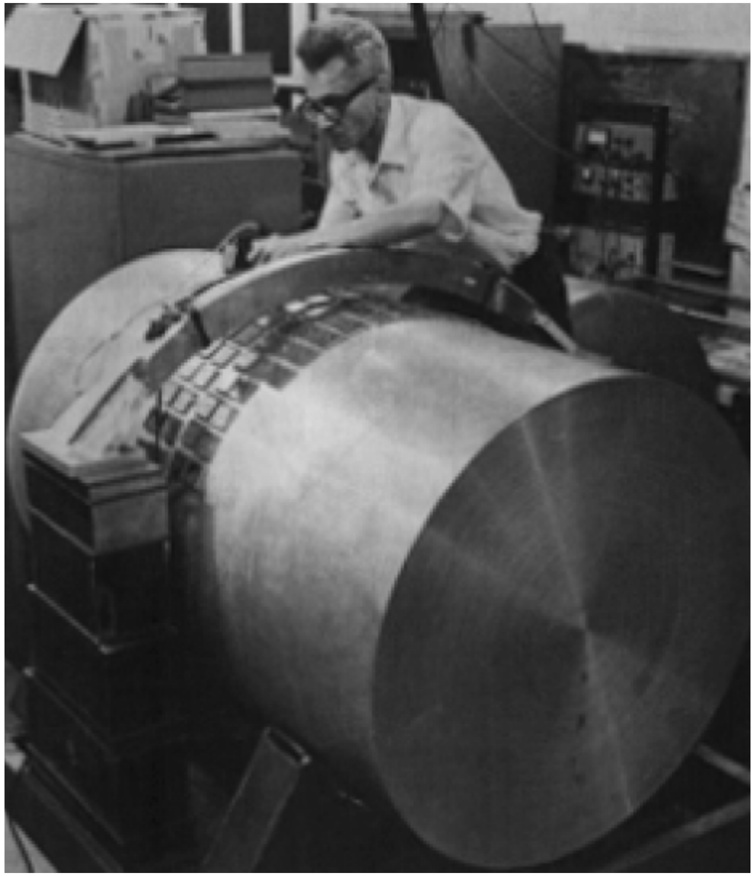
Design of gravitational wave detectors





Design of gravitational wave detectors





Metodi di rivelazione

Weber started seeing things

In 1969, Weber made his first of many announcements that he was seeing coincident excitations of two detectors.

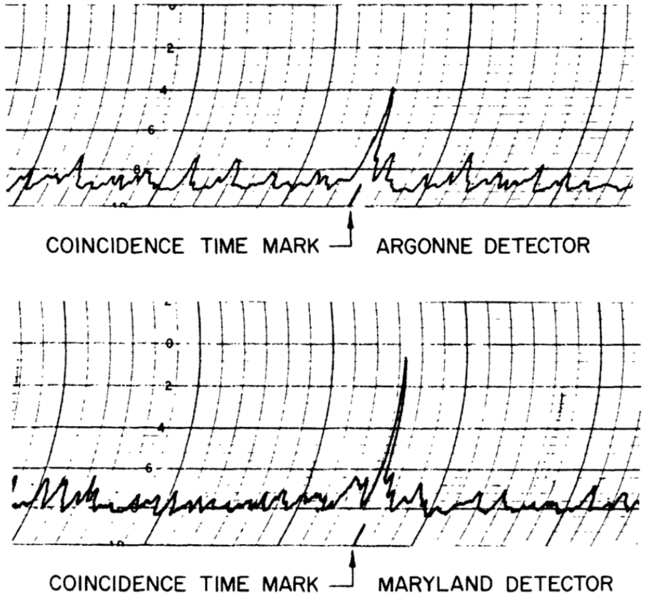
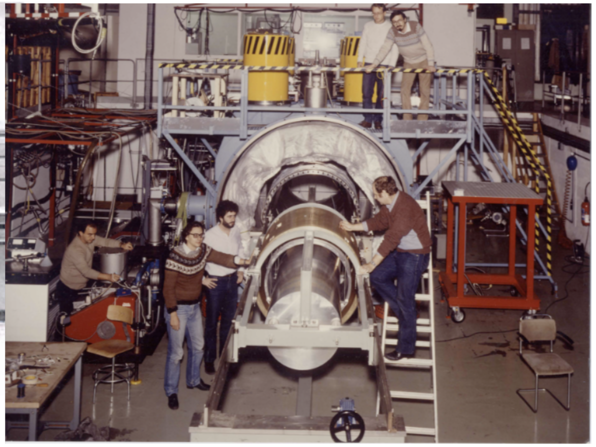


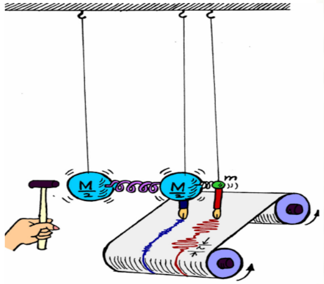
FIG. 2. Argonne National Laboratory and University of Maryland detector coincidence.

Metodi di rivelazione

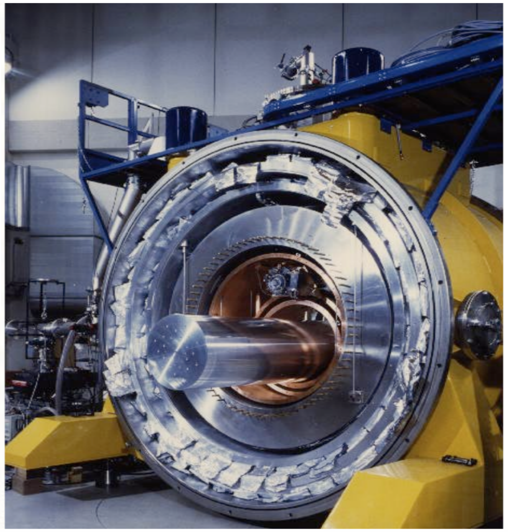


Explorer, CERN

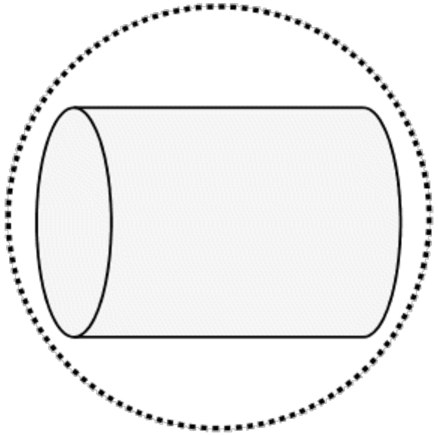
Nautilus, LNF

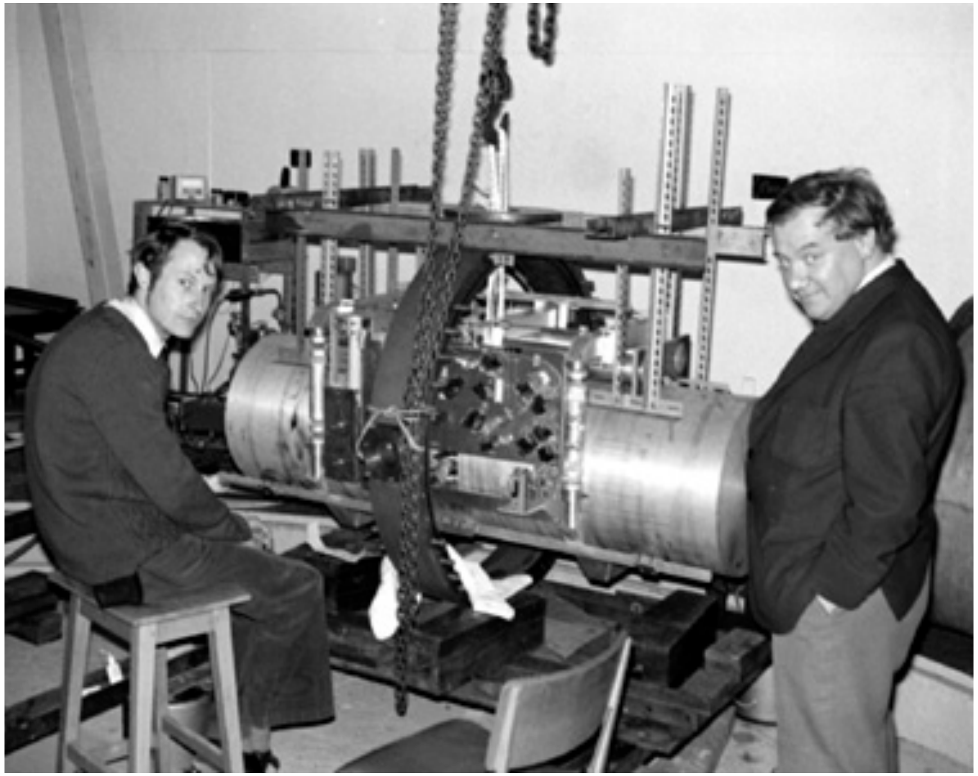


Auriga, LNL

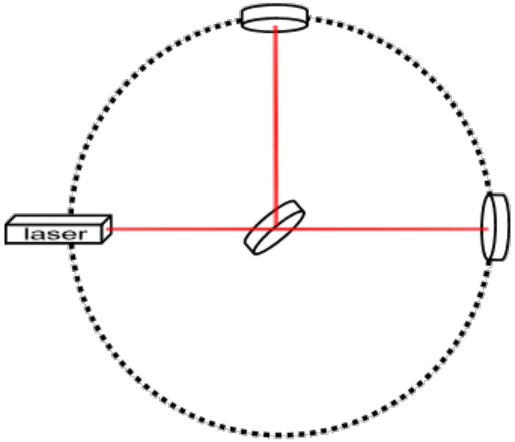


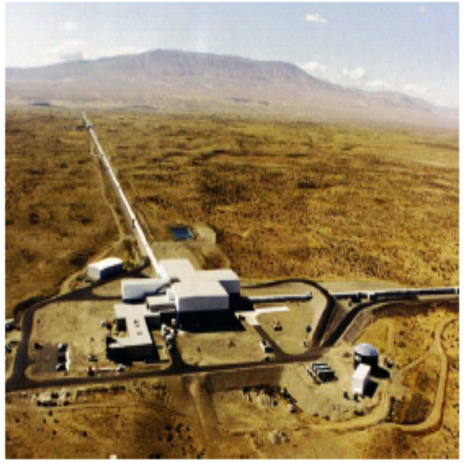
Design of gravitational wave detectors

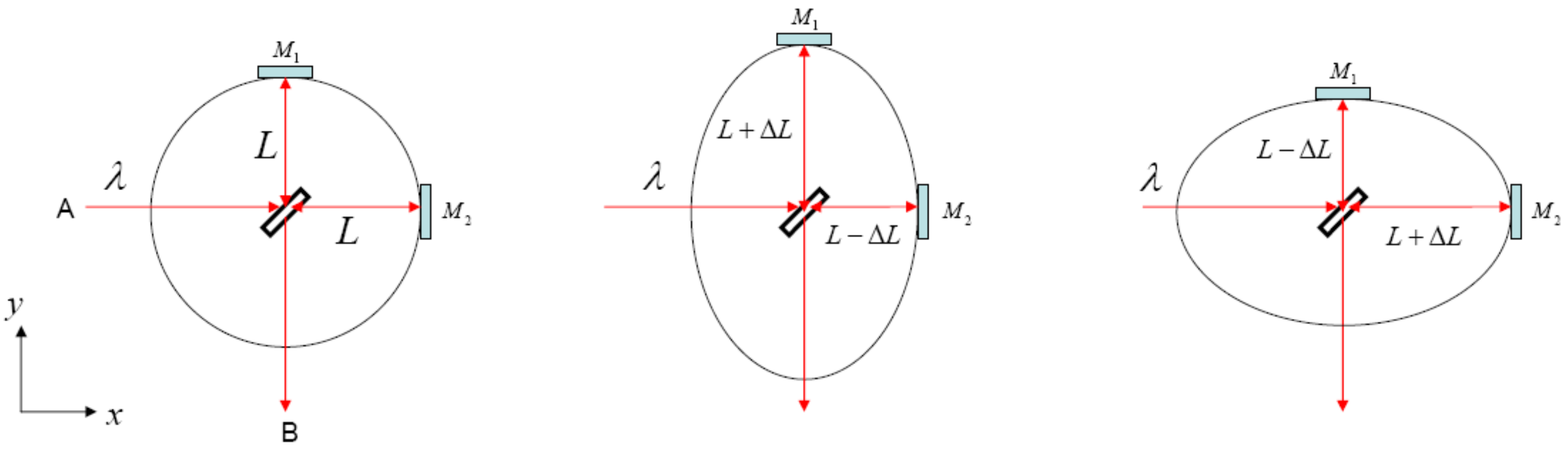




34 yrs on - Interferometric ground-based detectors







Gravitational wave $\mathbf{h} = h\mathbf{e}_+$ propagating along z axis.

Fractional change in proper separation

$$\boxed{\frac{\Delta L}{L} = \frac{h}{2}}$$

More generally, for $\mathbf{h} = h\,\mathbf{e}_+$

Detector 'sees'

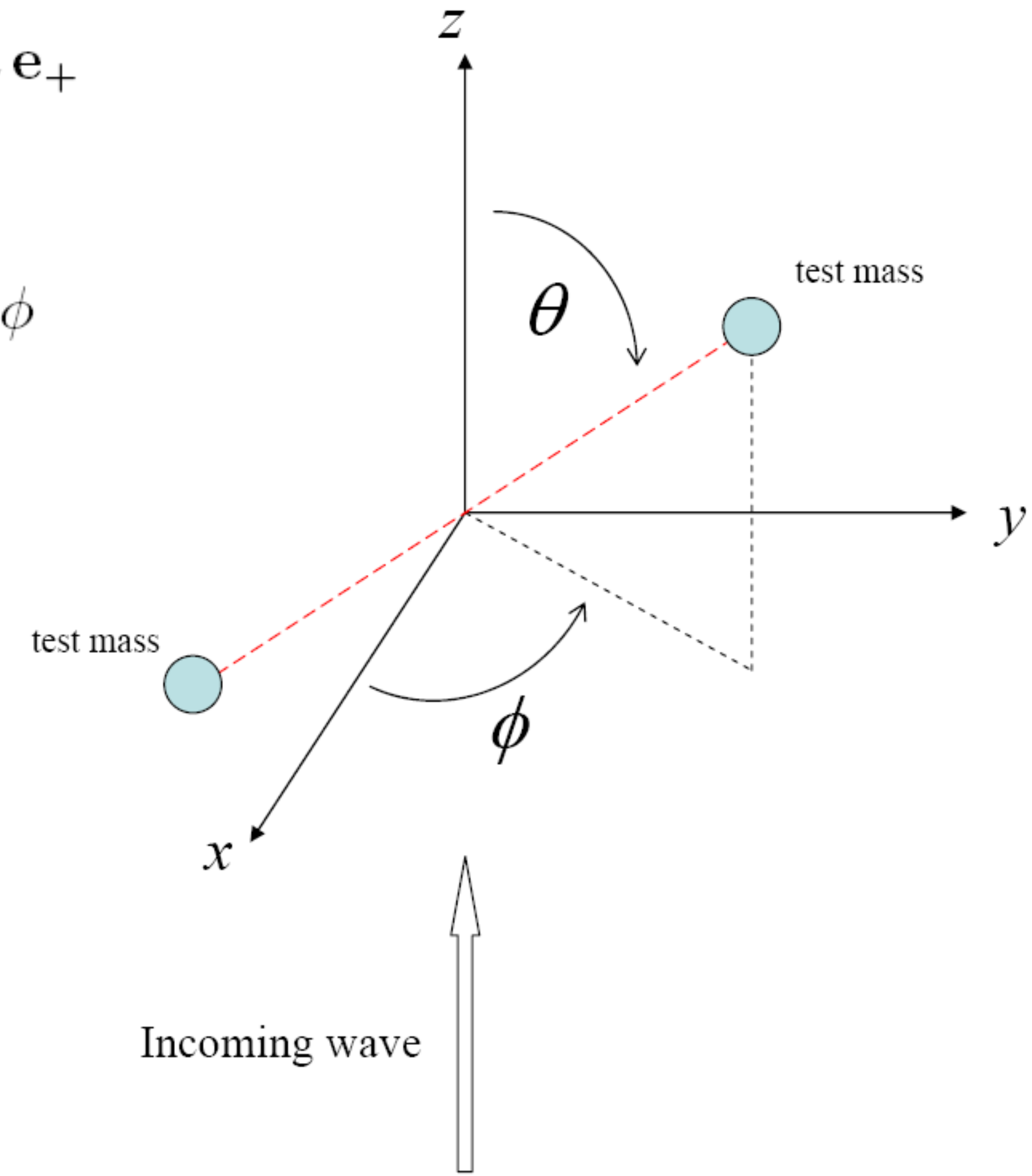
$$h_+ = h \sin^2\theta \cos 2\phi$$

Maximum response for

$$\theta = \pi/2 \qquad \phi = 0$$

Null response for

$$\theta = 0 \qquad \phi = \pi/4$$



More generally, for $\mathbf{h} = h\,\mathbf{e}_{\times}$

Detector 'sees'

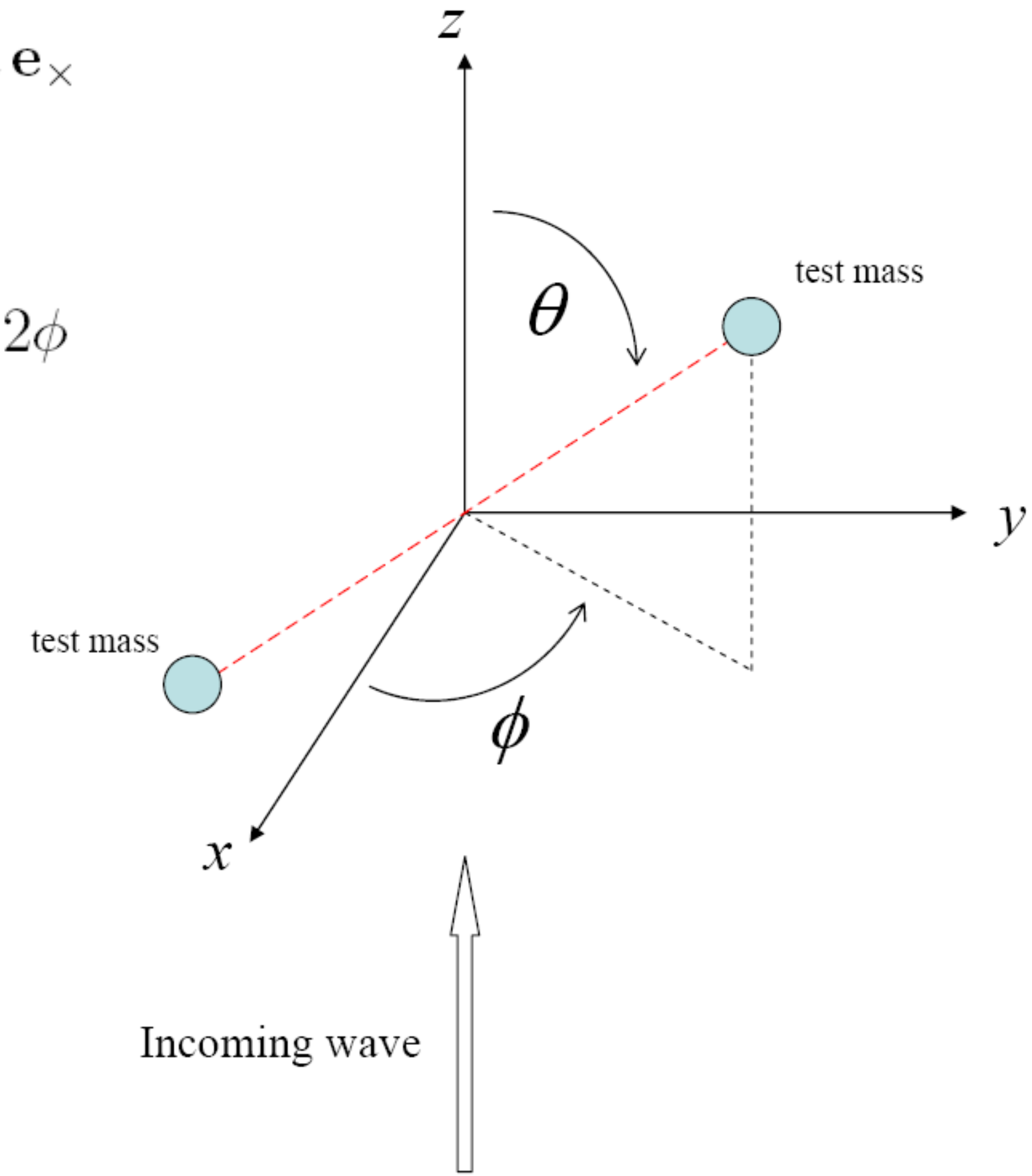
$$h_{\times} = h \sin^2\theta \sin 2\phi$$

Maximum response for

$$\theta = \pi/2 \qquad \phi = \pi/4$$

Null response for

$$\theta = 0 \qquad \phi = 0$$



Astrofisica Nucleare e Subnucleare

Sources of GW

Gravitational Wave emission

(quadrupole formalism)

Emission equation in the TT Gauge: $\nabla^2 h_{\mu\nu}^{TT} = -\frac{16\pi G}{c^4} T_{\mu\nu}$

Retarded solution: $h_{\mu\nu}^{TT}(\vec{x},t) = \frac{2G}{Rc^4}\ \ddot{Q}_{\mu\nu}^{TT}(t - R/c)$

Hence: $h_{+}^{TT}(\vec{x},t) = \frac{G}{Rc^4}\left[\ddot{Q}_{11}^{TT} - \ddot{Q}_{22}^{TT}\right](t-R/c)$ $h_{\times}^{TT}(\vec{x},t) = \frac{2G}{Rc^4}\left[\ddot{Q}_{12}^{TT}\right](t-R/c)$

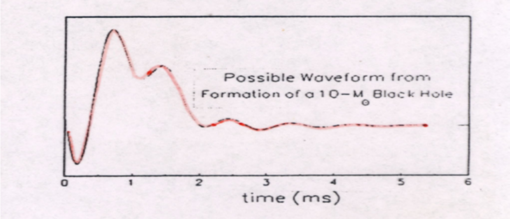
Where the ***reduced quadrupole*** moment:

$$Q_{\mu\nu}^{TT} = \iiint d^3x\ \rho\ (x_\mu\ x_\nu - \tfrac{1}{3}\delta_{\mu\nu} r^2)$$

Regular quadrupole (inertia) moment: $q_{\mu\nu} = \iiint d^3x\ \rho\ x_\mu x_\nu$

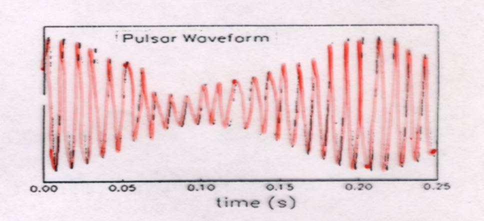
$\rho \sim T_{00}/c^2$: density of the source

Tipi di segnale



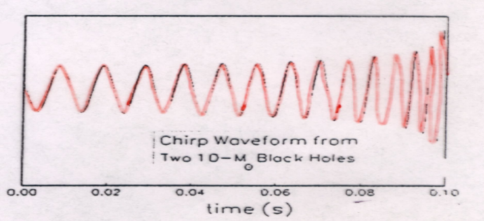
SUPERNOVAE.
If the collapse core is non-symmetrical, the event can give off considerable radiation in a millisecond timescale.

Information
Inner detailed dynamics of supernova
See NS and BH being formed
Nuclear physics at high density



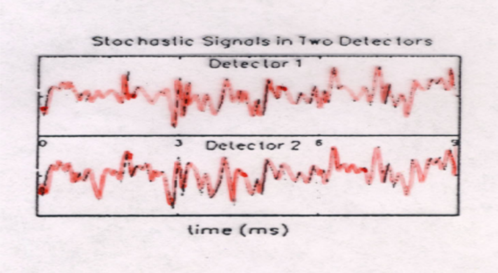
SPINNING NEUTRON STARS.
Pulsars are rapidly spinning neutron stars. If they have an irregular shape, they give off a signal at constant frequency (prec./Dpl.)

Information
Neutron star locations near the Earth
Neutron star Physics
Pulsar evolution



COALESCING BINARIES.
Two compact objects (NS or BH) spiraling together from a binary orbit give a chirp signal, whose shape identifies the masses and the distance

Information
Masses of the objects
BH identification
Distance to the system
Hubble constant
Test of strong-field general relativity



STOCHASTIC BACKGROUND.
Random background, relic of the early universe and depending on unknown particle physics. It will look like noise in any one detector, but two detectors will be correlated.

Information
Confirmation of Big Bang, and inflation
Unique probe to the Planck epoch
Existence of cosmic strings

9. The Production of Gravitational Waves (pgs 76 – 80)

We can understand something important about the nature of gravitational radiation by drawing analogies with the formulae that describe electromagnetic radiation. This approach is crude at best since the electromagnetic field is a vector field while the gravitational field is a tensor field, but it is good enough for our present purposes. Essentially, we will take familiar electromagnetic radiation formulae and simply replace the terms which involve the Coulomb force by their gravitational analogues from Newtonian theory.

Net electric dipole moment

$$L_{\text{electric dipole}} \propto e^2\, \ddot{\mathrm{d}}^2$$

$$L_{\text{magnetic dipole}} \propto \ddot{\mu}$$

$$\mu \;=\; \sum_{q_i} \; (\text{position of } q_i) \times (\text{current due to } q_i)$$

Gravitational analogues?...

Mass dipole moment: $$\mathbf{d} \;=\; \sum_{A_i} \; m_i \mathbf{x}_i$$

But $$\dot{\mathbf{d}} \;=\; \sum_{A_i} \; m_i \dot{\mathbf{x}}_i \quad \equiv \quad \mathbf{p}$$

Conservation of linear momentum implies no mass dipole radiation

$$L_{\text{magnetic dipole}} \propto \ddot{\mu}$$

$$\mu \;=\; \sum_{q_i} \; (\text{position of } q_i) \times (\text{current due to } q_i)$$

Gravitational analogues?...

$$\mu \;=\; \sum_{A_i} \; (\mathbf{x}_i) \times (m_i \mathbf{v}_i) \quad \equiv \quad \mathbf{J}$$

Conservation of angular momentum implies no mass dipole radiation

Also, the quadrupole of a **spherically symmetric mass distribution** is zero.

Metric perturbations which are spherically symmetric don't produce gravitational radiation.

Example: binary neutron star system.

$$h_{\mu\nu} = \frac{2G}{c^4 r} \ddot{I}_{\mu\nu}$$

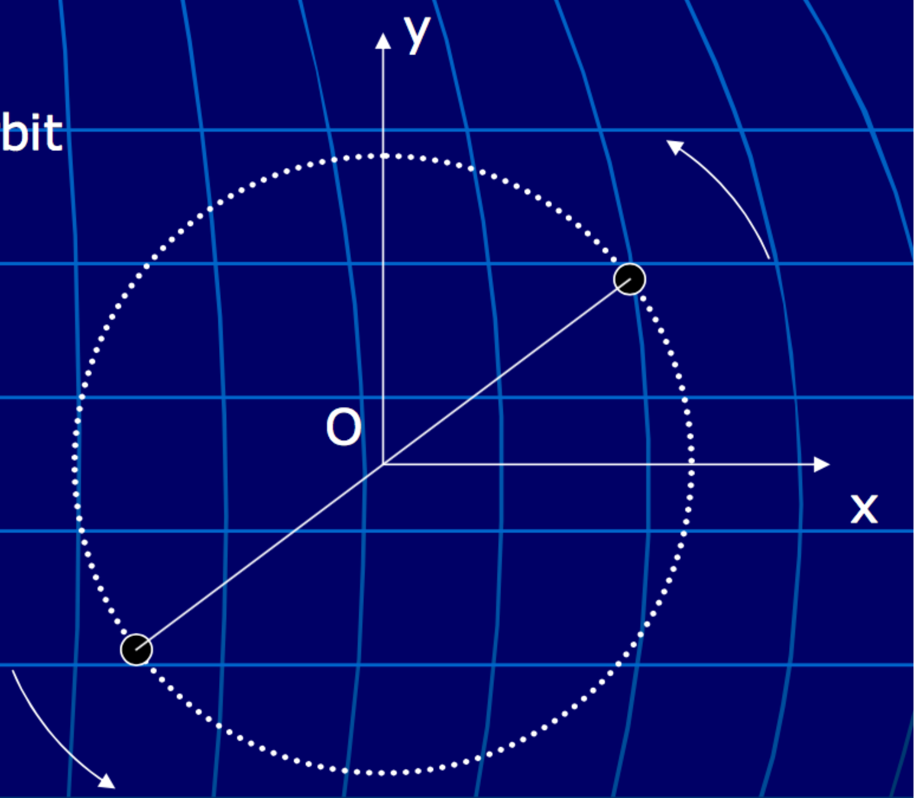
where $I_{\mu\nu}$ is the **reduced quadrupole moment** defined as

$$I_{\mu\nu} = \int \rho(\vec{r}) \left(x_\mu x_\nu - \frac{1}{3}\delta_{\mu\nu} r^2 \right) dV$$

Gravitational Wave emission: an example

2 identical point masses in circular orbit around their center of mass

- Orbital plane : xOy
- Mass : M
- Orbit radius : a
- Orbital frequency : $f_0=2\pi\omega_0$



Q: Compute the 2 amplitudes $h_+(t)$ and $h_x(t)$ at a distance r on the z axis (*without taking into account the radiation reaction !*)

Gravitational Wave emission: an example

Positions of the two masses:

$$x_1(\mathrm{t}) = a\cos(\omega_0 \mathrm{t}) \qquad x_2(\mathrm{t}) = -\,a\cos(\omega_0 \mathrm{t})$$
$$y_1(\mathrm{t}) = a\sin(\omega_0 \mathrm{t}) \qquad y_2(\mathrm{t}) = -\,a\sin(\omega_0 \mathrm{t})$$

So compute the reduced inertia tensor:

$$Q = \begin{pmatrix} ma^2\left(\frac{1}{3} + \cos(2\omega_0 t)\right) & ma^2 \sin(2\omega_0 t) & 0 \\ ma^2 \sin(2\omega_0 t) & ma^2\left(\frac{1}{3} - \cos(2\omega_0 t)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

After projection on the z direction:

$$h_+(t) = -\frac{2G}{rc^4} ma^2\omega^2 \cos\left(\omega(t - r/c)\right)$$
$$h_\times(t) = -\frac{2G}{rc^4} ma^2\omega^2 \sin\left(\omega(t - r/c)\right)$$

Where $\omega=2\omega_0$ is **TWICE** the orbital angular frequency

Note that if we look on the x direction:

$$h_+(t) = -\frac{G}{rc^4} ma^2\omega^2 \cos\left(\omega(t - r/c)\right)$$
$$h_\times(t) = 0$$

Face-on binary => **circular** polarization

Edge-on binary => **linear** polarization

Gravitational Wave emission:
Orders of magnitude

source	distance	h
Steel bar, 500 T, Ø = 2 m L = 20 m, 5 cycles/s	1 m	$2x10^{-34}$
H bomb, 1 megatonne Asymmetry 10%	10 km	$2x10^{-39}$
Supernova 10 $M_{\odot}$ asymmetry 3%	10 Mpc	10^{-21}
Coalescence 2 black holes 10 $M_{\odot}$	10 Mpc	10^{-20}

Hertz experiment is impossible for GWs ...

Gravitational Wave sources

Compact stars

"High frequency" sources ($f > 1$ Hz)

- supernovae (bursts)
- binary inspirals (chirps)
- black holes ringdowns (damped sine)
- isolated neutron stars, pulsars (periodic sources)
- stochastic background (stochastic)
- …

Amplitudes $h(t)$ on Earth ?
Rate of events ?

Gravitational Supernovae

type II SN = gravitational collapse of the core (Fe) of a massive star (> 10 $M_\odot$) after having burned all the H fuel → neutron star formation

GW Emission ? Depends on asymmetry (poorly known)

Sources of asymmetry
- fast rotation (instabilities)
- companion star

Modern models :

$h \sim 10^{-23}$ @ 10 Mpc
f peaks between 0.3 and 1 kHz
1 SN/ 40 yrs / galaxy

Black hole formation:
Progenitor too massive → collapse → black hole

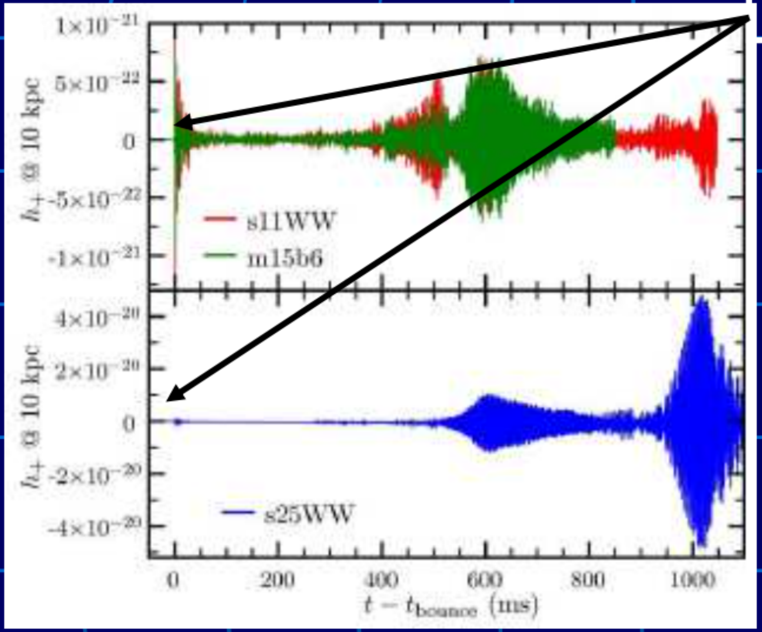
$h \sim 10^{-22}$ @ 10 Mpc
$f > 1$ kHz

+ oscillations…

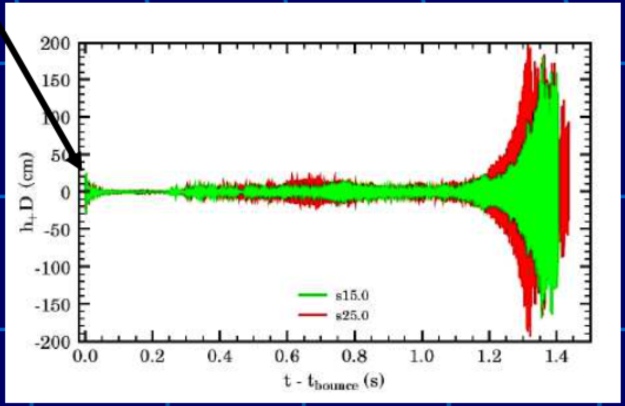
Gravitational Supernovae: GW amplitudes

+ coupling between the proto-neutron star and the envelope (rotation instabilities induced by turbulence and accretion)

collapse



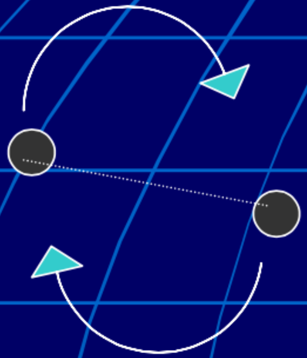
Ott and Burrows, 2006.



Marek et al., 2008.

Main conclusions:
+ Waveforms not well predicted
+ weak amplitudes -> only Galactic Supernova detectable ?

Binary inspirals:
GW amplitudes



System of 2 close compact stars

- Varying quadrupole -> GW emission
- GW emission -> loss of energy and angular momentum
- Loss of (gravitational) energy -> stars become closer
- Finally 2 stars merge (or disrupt)

Spiraling phase
(lowest order)

$$h_+^{TT}(t) = \frac{4(GM)^{5/3}}{Rc^4}\,\frac{1+\cos^2 i}{2}\,(\pi f(t))^{2/3}\cos\varphi(t)$$

$$h_\times^{TT}(t) = \frac{4(GM)^{5/3}}{Rc^4}\,\cos i\,(\pi f(t))^{2/3}\sin\varphi(t)$$

A “chirp”

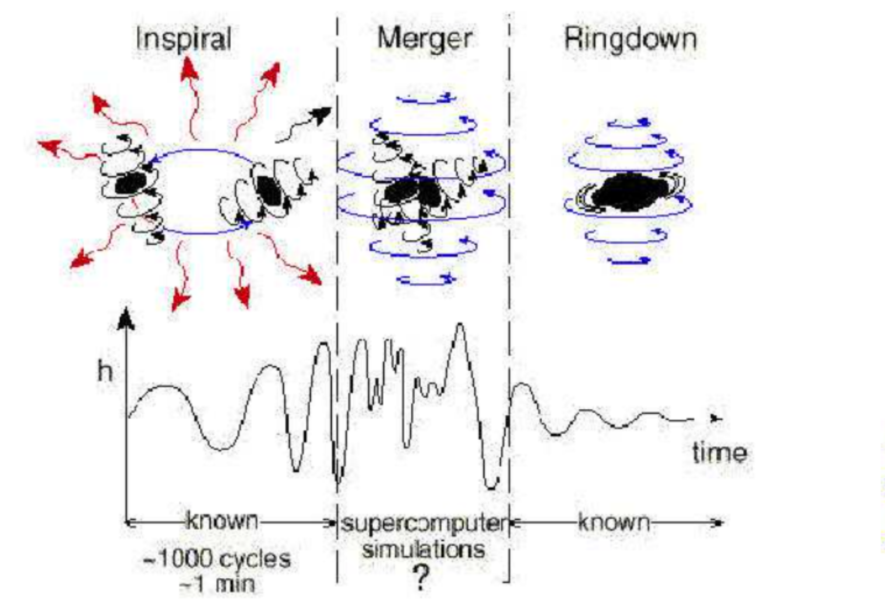
$$h(t) \propto (t_c - t)^{-1/4}$$

where

- Chirp mass: $M = \mu^{3/5} M_{tot}^{2/5}$
- frequency: $f(t) = \frac{1}{\pi}\left(\frac{256}{5}\frac{(GM)^{5/3}}{c^5}(t_c - t)\right)^{-3/8}$
- Phase: $\varphi(t) = -2\left(\frac{G^{5/3}}{c^5}\right)^{-3/8}\left(\frac{t_c - t}{5M}\right)^{5/8} + cste$

t_c : coalescence time

Binary inspirals: the chirp signal

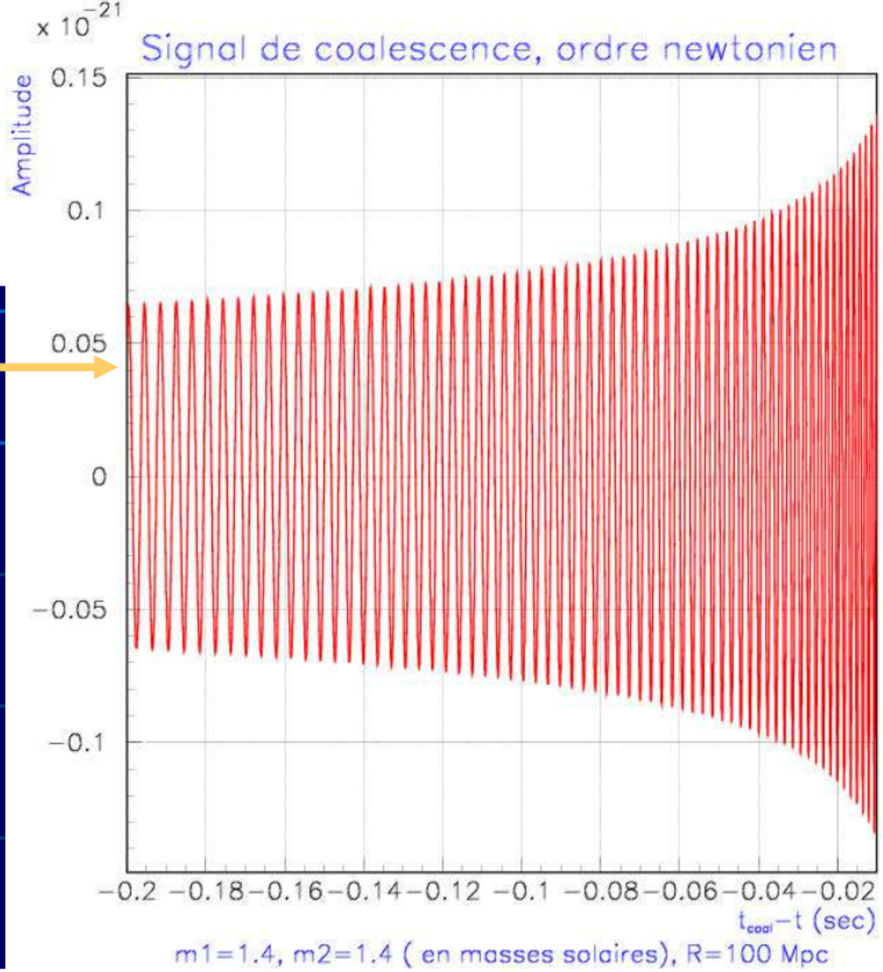


2 neutron stars @ 10Mpc

h_{max} ~10^{-21}

f_{max} (last stable orbit)~ 1 kHz

- « inspiral » : *h(t)* is a chirp
- « merger » : recent numerical progress
- « ringdown » : black hole quasi normal modes



Binary inspirals: rate of events

(first generation detectors)

- NS-NS: $1.4M_{\Theta} + 1.4M_{\Theta}$ (Kalogera et al astro-ph/0111452)
  - 0.001 – 1 / yr -> 20 Mpc
- NS-BH: $1.4M_{\Theta} + 10M_{\Theta}$
  - 0.001 – 1 / yr -> 43 Mpc
- BH-BH: $10M_{\Theta} + 10M_{\Theta}$
  - 0.001- 1 / yr -> 100 Mpc

- **Gain Factor 10** on detector sensitivity

  ➡ gain **factor 1000** on the event rate

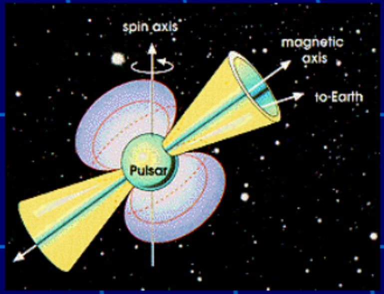
Other sources

Pulsars and rotating Neutron Stars

10^5 pulsars in the Galaxy, several thousands rapidly rotating.

Source of asymmetry ?

- rotation instabilities
- magnetic stress
- "mountains" on the solid crust …



Radio-astronomy observation of pulsar slowdown sets upper limits on GW emission and neutron star asymmetry (if rate of slowdown totally assigned to GW emission)

⇒Expected amplitudes are weak **($h<10^{-24}$)**

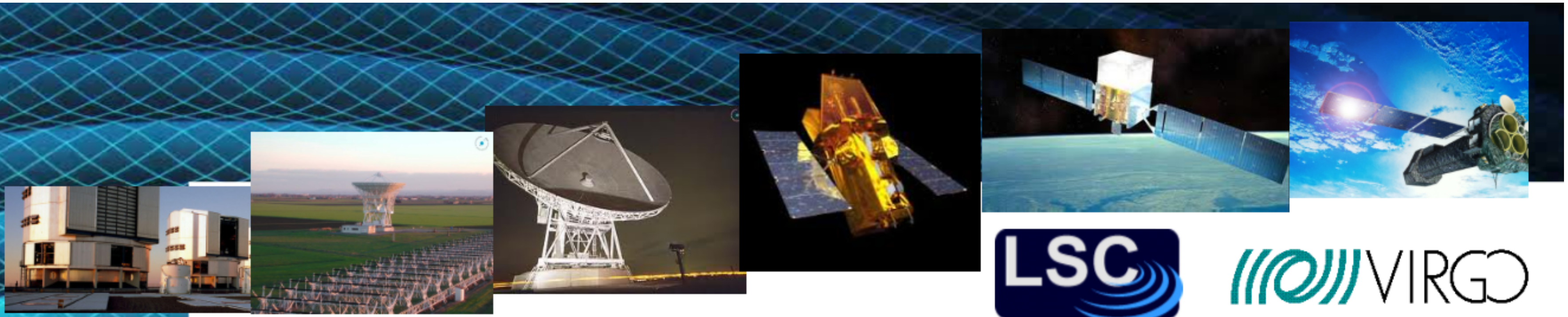
$$h \sim 10^{-26} \left(\frac{10\,\mathrm{kpc}}{\mathrm{distance}}\right)\left(\frac{f}{100\mathrm{Hz}}\right)^2\left(\frac{\varepsilon}{10^{-6}}\right)$$

But the signal is periodic ! ("simple" Fourier analysis)

Signal to noise ratio $\mathrm{S/N} \propto \sqrt{T}$ where *T* is the observation time

Astrofisica Nucleare e Subnucleare

Ground Detectors for GW



Gravitational waves – Experiments and sources

I PART

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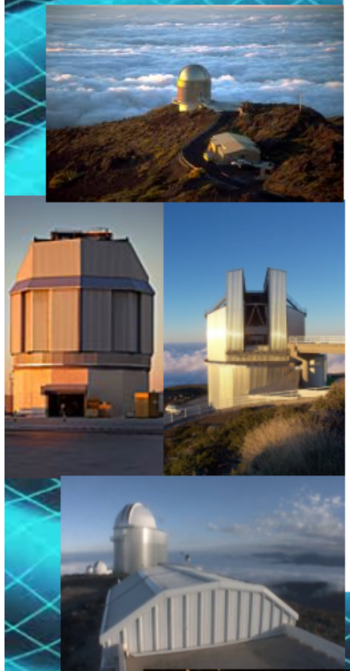
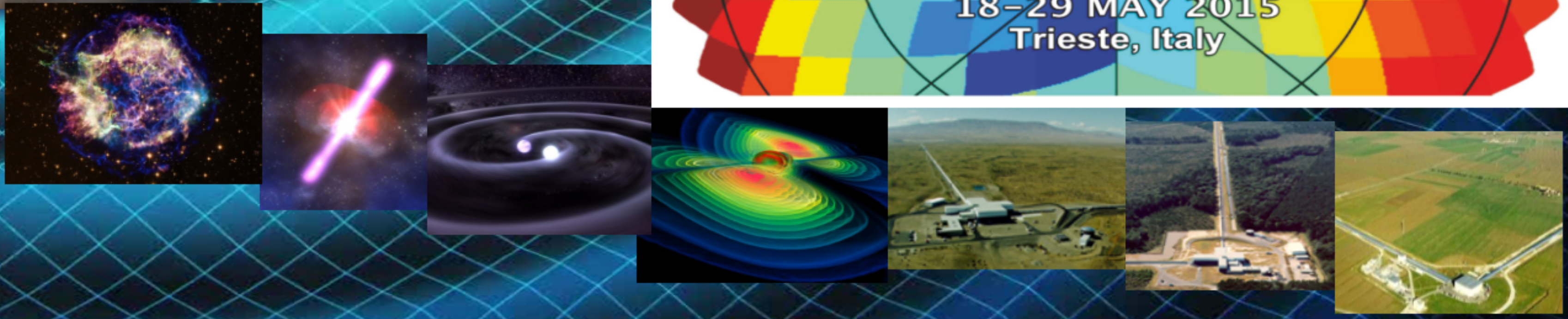
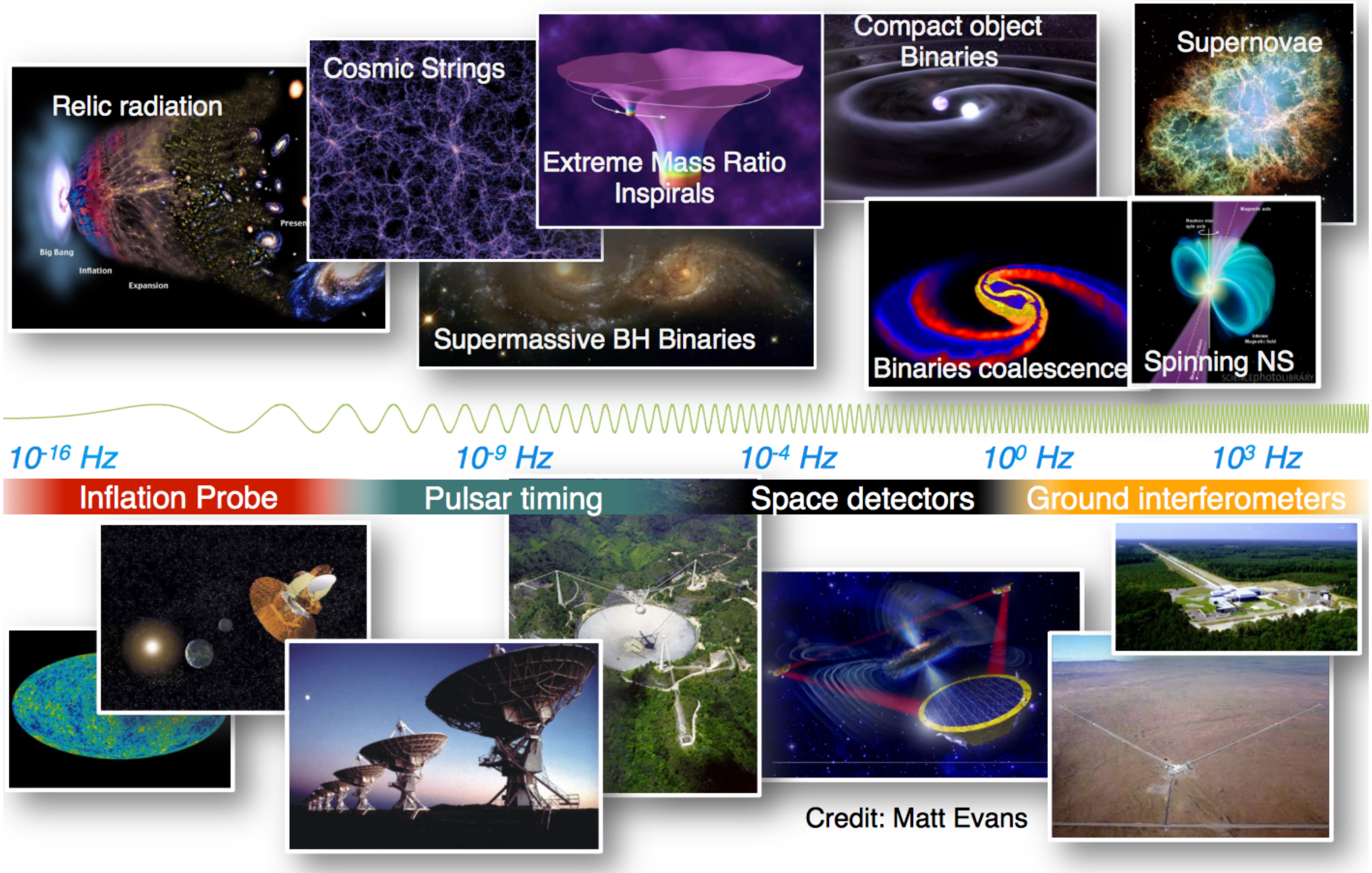




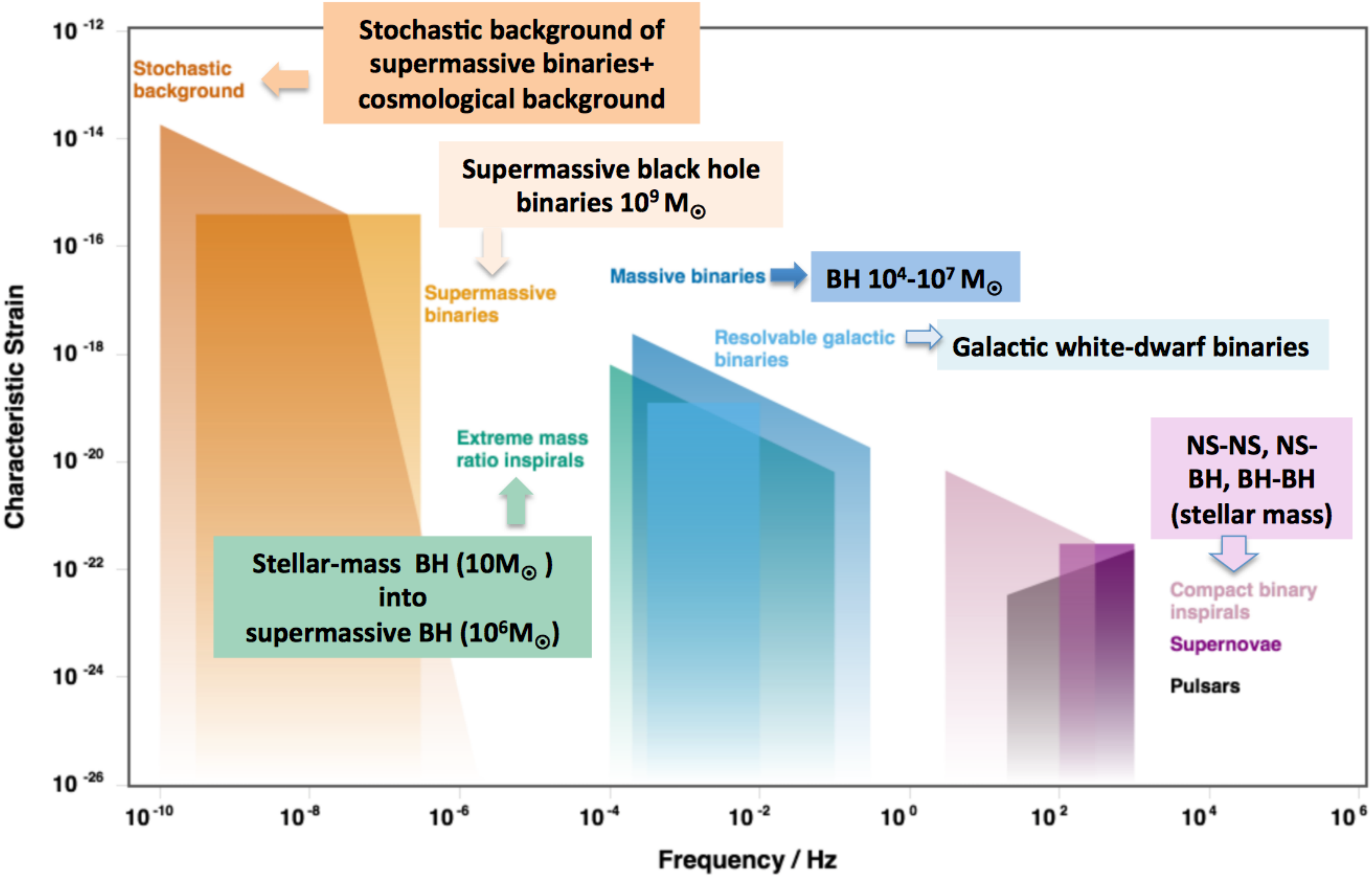
The GW Spectrum



Credit: Matt Evans

Astrophysical Sources

Now at: http://www.sr.bham.ac.uk/~cplb/GWplotter/



http://rhcole.com/apps/GWplotter/

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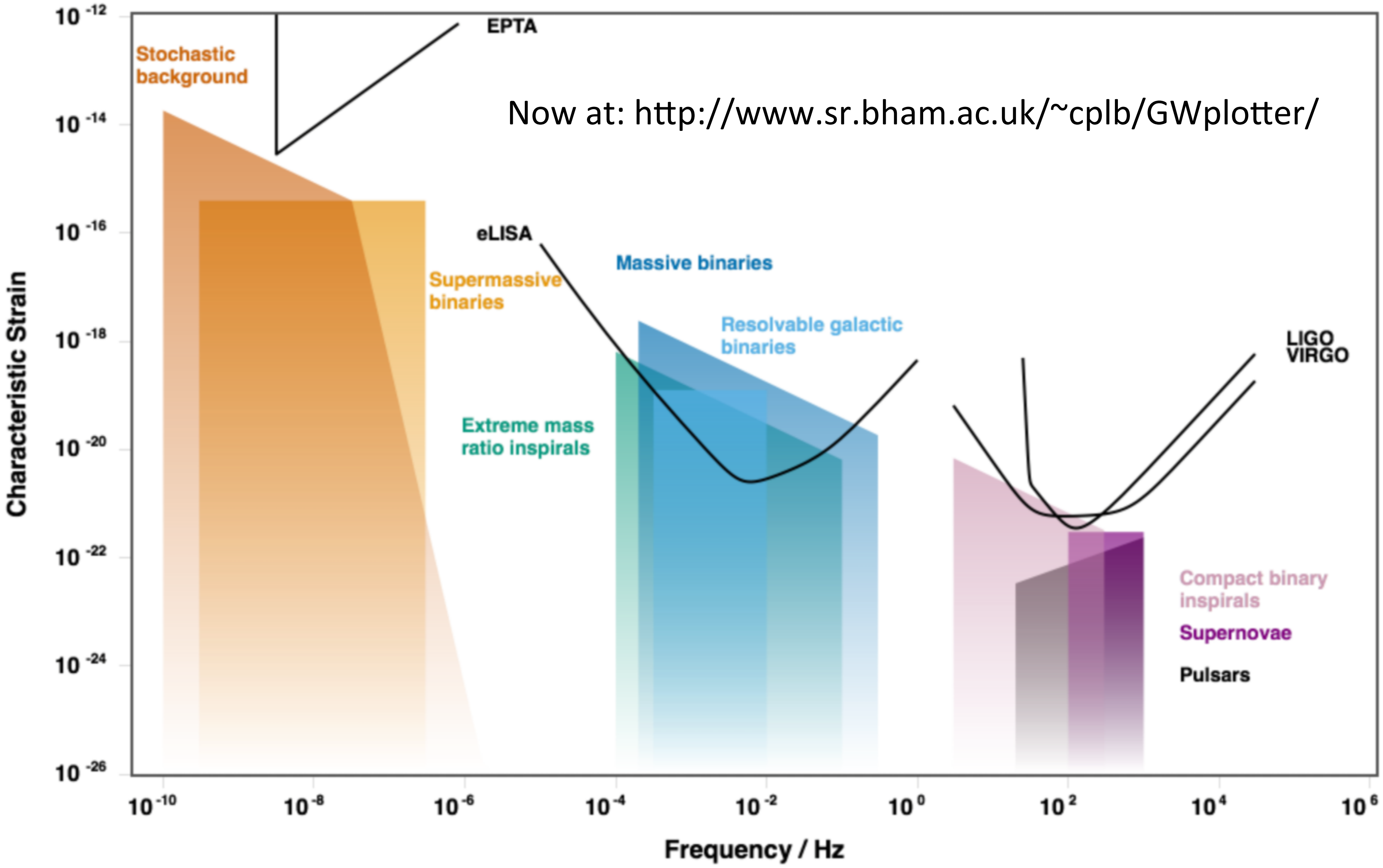

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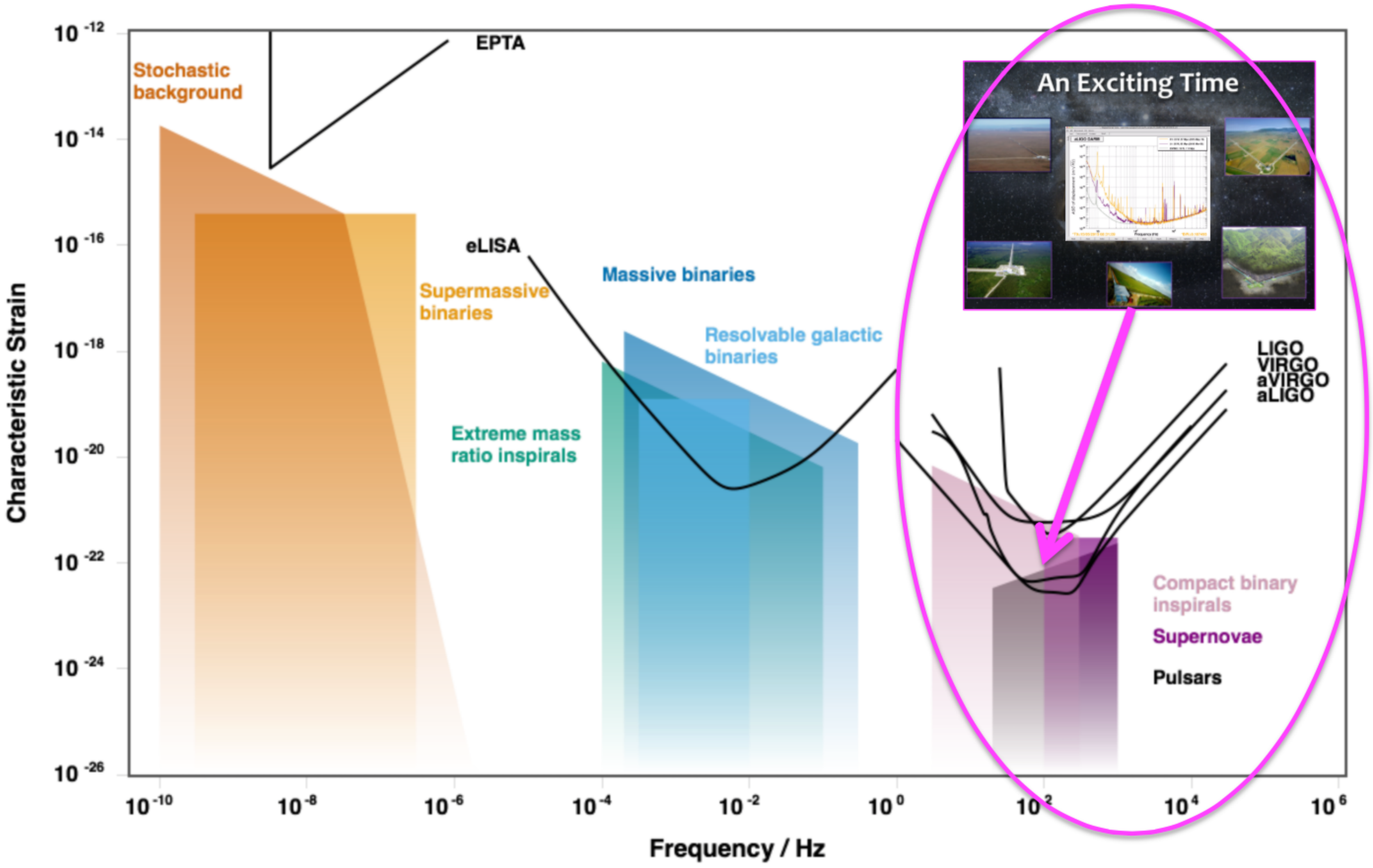

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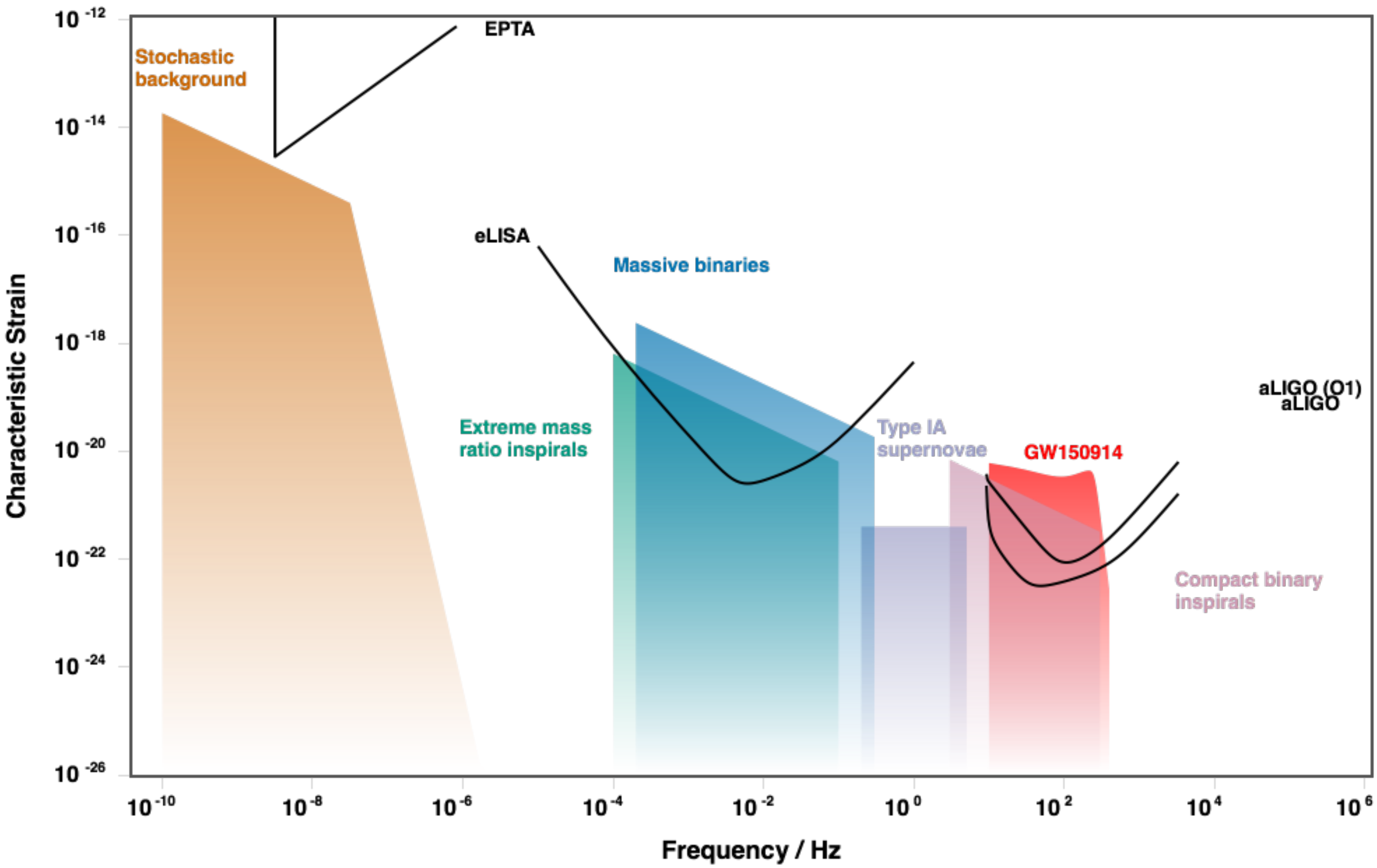

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