An introduction to Compact Binary Coalescences in GW data

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Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
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On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

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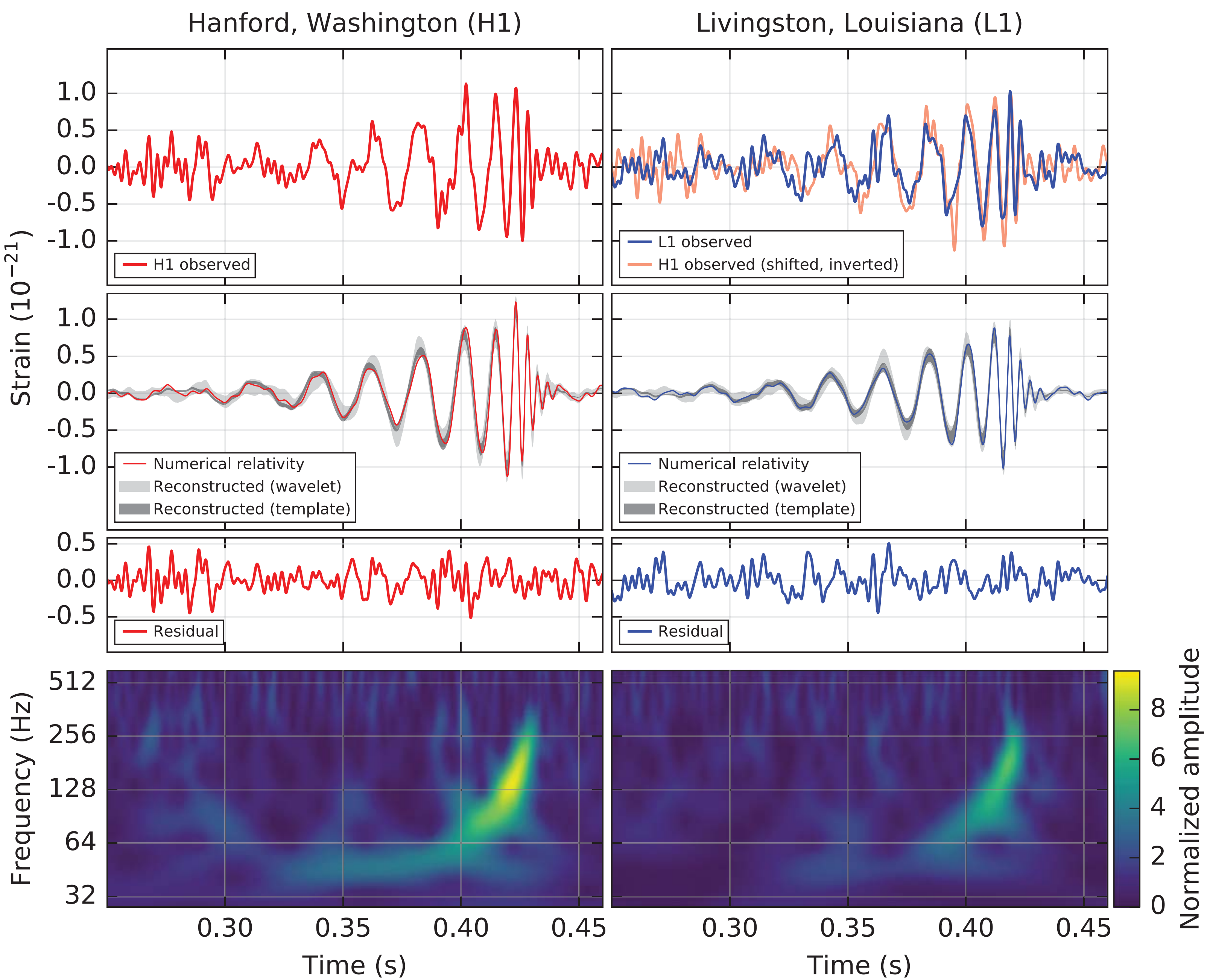
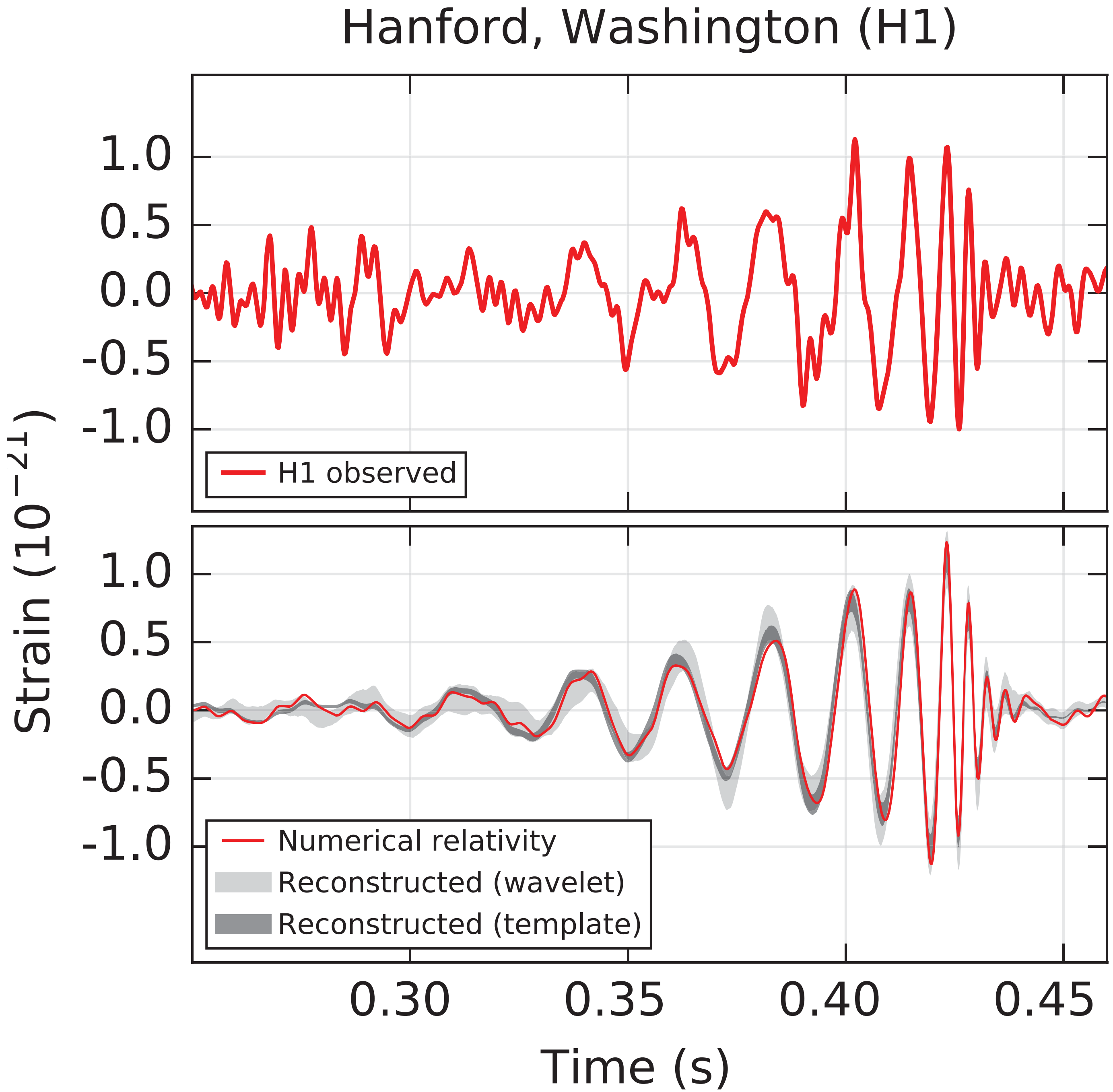


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
Time (s)
0.30
0.35
0.40
0.45
Normalized amplitude
1.0
0.5
0.0
-0.5
-1.0
0
2
4
6
8



Hanford, Washington (H1)
Strain (10^{-21})
1.0
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0.0
-0.5
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H1 observed
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
0.30
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0.40
0.45
Time (s)

Understanding chirps in very simple terms

Consider a body of mass *m* orbiting around a center of mass with associated mass *M* in a circular orbit of radius *r*.

The total energy is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

and equating the centripetal force to the gravitational force, we find

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad E = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

Now, assume that there is a simple frictional energy loss at each orbit, due to the frictional force

$$F_r = -\gamma v = -\gamma\sqrt{\frac{GM}{r}}$$

then the energy loss per orbit is

$$\Delta E \approx F_r \times 2\pi r = -2\pi\gamma\sqrt{GMr}$$

Since the period of each orbit is

$$\Delta T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\frac{r^{3/2}}{\sqrt{GM}}$$

then

$$\frac{dE}{dt} \approx \frac{\Delta E}{\Delta T} \approx -\gamma\frac{GM}{r} = \frac{2\gamma}{m}E \quad \Rightarrow \quad E(t) \approx E_0 e^{2\gamma t/m}$$

so that the initial negative total energy becomes more and more negative

We can also compute radius of the orbit, speed and orbital frequency

$$r = -\frac{1}{2}\frac{GMm}{E} = \frac{GMm}{2|E_0|}e^{-2\gamma t/m}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{2|E_0|}{m}}e^{\gamma t/m}$$

$$\omega = \frac{v}{r} \approx \frac{1}{GM}\left(\frac{2|E_0|}{m}\right)^{3/2} e^{3\gamma t/m}$$

Now, assume that the frictional force is proportional to acceleration, then

$$F_r = -\gamma \frac{v^2}{r} = -\gamma \frac{GM}{r}$$

then

$$\Delta E \approx F_r \times 2\pi r = -2\pi\gamma\sqrt{GMr}$$

and the differential equation for the energy loss is

$$\frac{dE}{dt} \approx \frac{\Delta E}{\Delta T} \approx -\gamma \left(\frac{GM}{r}\right)^{3/2} = -\gamma \left(\frac{2|E|}{m}\right)^{3/2}$$

or also

$$\frac{d|E|}{dt} \approx \gamma \left(\frac{2|E|}{m}\right)^{3/2} \Rightarrow E(t) = -\left[\frac{1}{\sqrt{|E_0|}} - \gamma \left(\frac{2}{m}\right)^{3/2} t\right]^{-2}$$

This form of the total energy means that there is a extreme time that corresponds to "infinite bounding energy" and which is given by

$$t_{\max} = \frac{2}{\gamma\sqrt{|E_0|}} \left(\frac{m}{2}\right)^{3/2}$$

In addition, we also find

$$r = \frac{1}{2} GMm \left[\frac{1}{\sqrt{|E_0|}} - \gamma \left(\frac{2}{m}\right)^{3/2} t \right]^2$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{2}{m}} \left[\frac{1}{\sqrt{|E_0|}} - \gamma \left(\frac{2}{m}\right)^{3/2} t \right]^{-1}$$

$$\omega = \frac{1}{GM} \left(\frac{2}{m}\right)^{3/2} \left[\frac{1}{\sqrt{|E_0|}} - \gamma \left(\frac{2}{m}\right)^{3/2} t \right]^3$$

To understand chirps in GR binary systems we need deeper physical concepts

The kind of two body system that we want to consider contains two compact objects (black holes or neutron stars)

The first well-studied system was discovered in 1974, the binary pulsar PSR 1913+16, in the constellation Aquila.

THE ASTROPHYSICAL JOURNAL, **195**:L51–L53, 1975 January 15

DISCOVERY OF A PULSAR IN A BINARY SYSTEM

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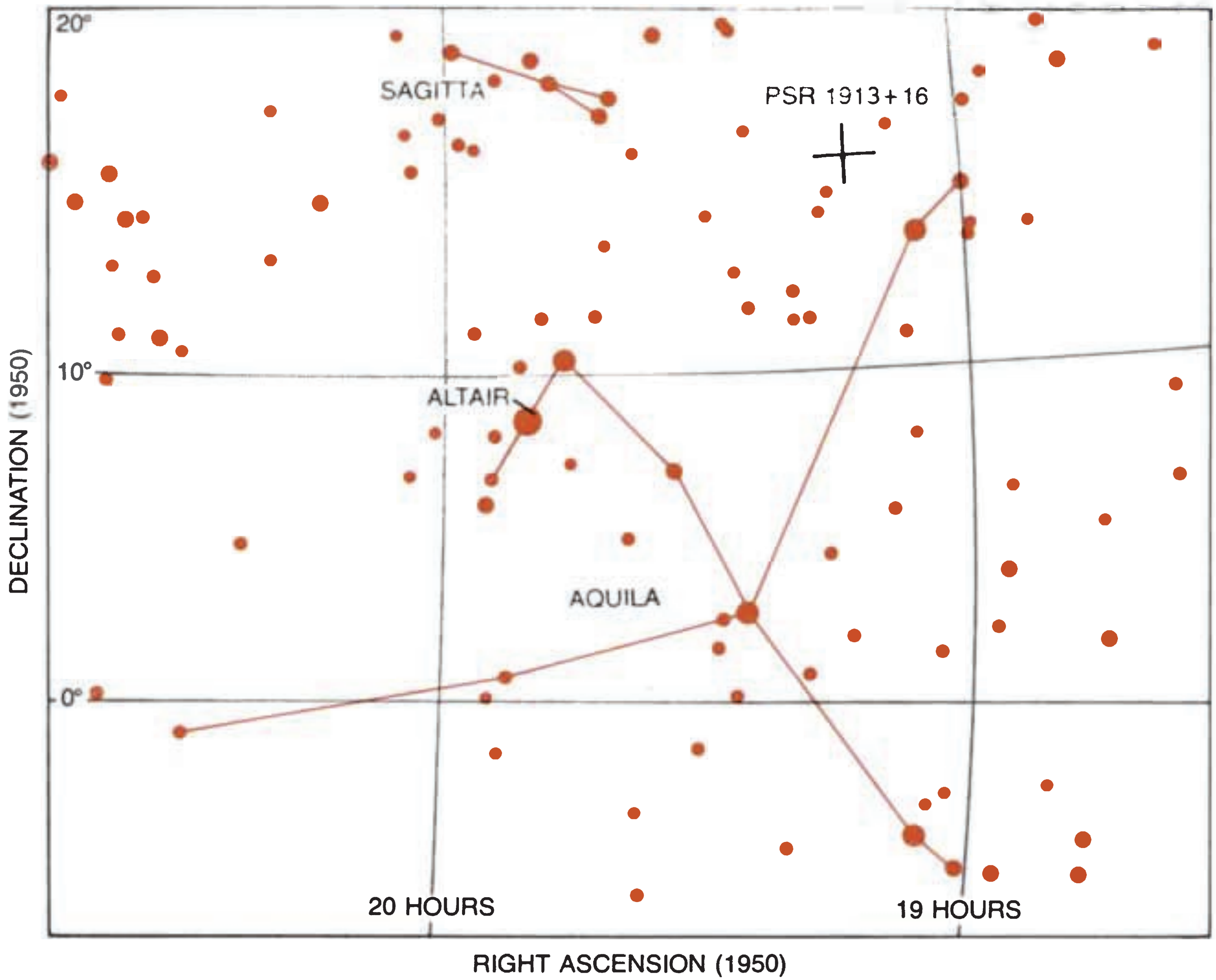
Received 1974 October 18

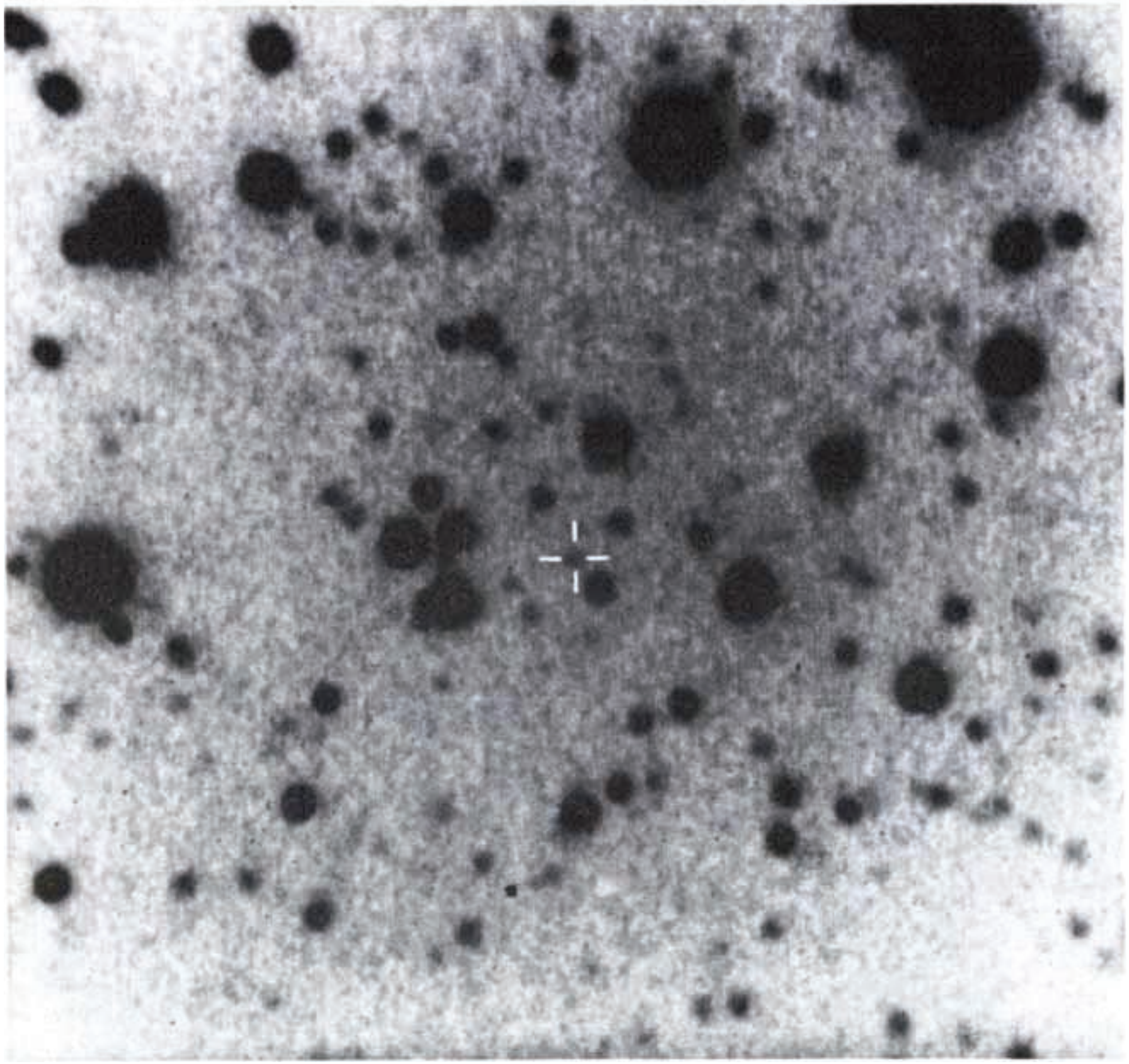
ABSTRACT

We have detected a pulsar with a pulsation period that varies systematically between $0^{s}.058967$ and $0^{s}.059045$ over a cycle of $0^{d}.3230$. Approximately 200 independent observations over 5-minute intervals have yielded a well-sampled velocity curve which implies a binary orbit with projected semimajor axis $a_1 \sin i = 1.0\,R_{\odot}$, eccentricity $e = 0.615$, and mass function $f(m) = 0.13\,M_{\odot}$. No eclipses are observed. We infer that the unseen companion is a compact object with mass comparable to that of the pulsar. In addition to the obvious potential for determining the masses of the pulsar and its companion, this discovery makes feasible a number of studies involving the physics of compact objects, the astrophysics of close binary systems, and special- and general-relativistic effects.

Subject headings: binaries — black holes — neutron stars — pulsars — relativity



BINARY PULSAR PSR 1913 + 16 is in the constellation Aquila at the coordinates that supply its designation: right ascension 19 hours 13 minutes and declination +16 degrees. Its position is marked by the reticle. It is estimated that the binary pulsar is some 15,000 light-years away, too far for it to be observed optically even with the most powerful existing telescopes.



POSSIBLE SILENT COMPANION OF PSR 1913 + 16 is marked by a reticle in this computer plot of visible-light photons recorded with a video camera on the four-meter telescope of the Kitt Peak National Observatory. The observation was made by J. A. Tyson of Bell Laboratories. It has been suggested that the object is a helium-core star: a star near the end of its life that has ejected its outer layers into space, leaving behind a dense core consisting mostly of helium. If the companion is actually another neutron star, as the authors suspect, its visible radiation could not be detected by existing optical telescopes. In that case the object here would probably be a faint star that happens to lie at nearly the same position as the binary pulsar.

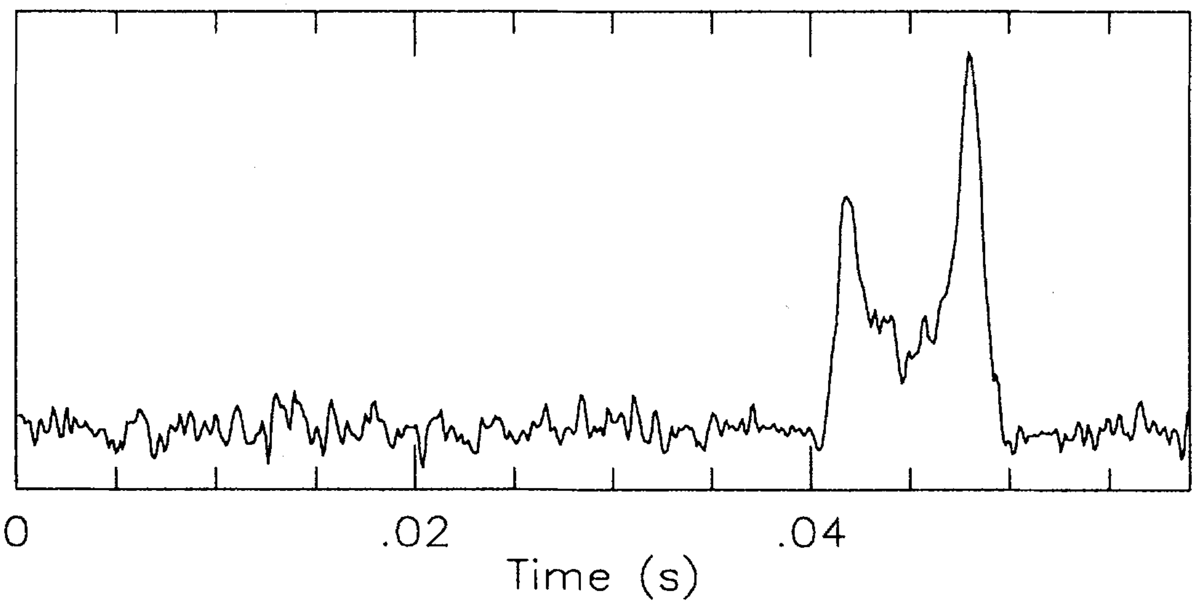
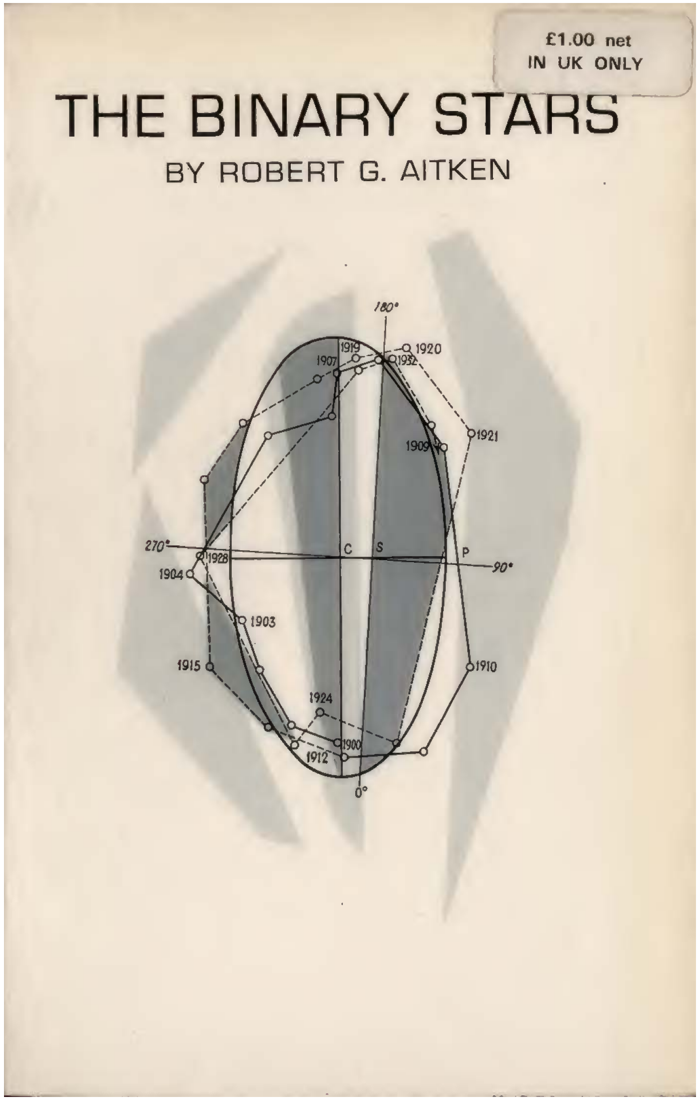
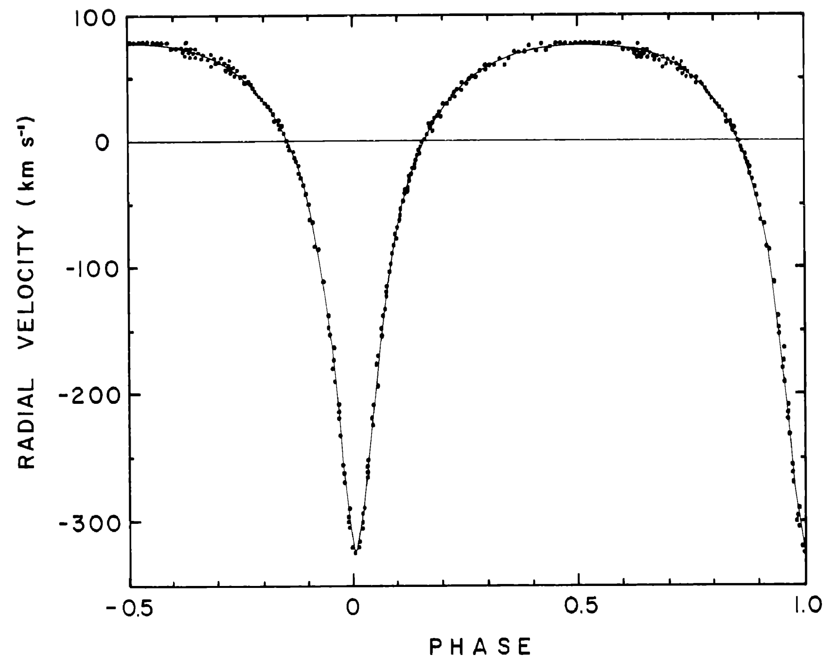


FIG. 4. Pulse profiles obtained on April 24, 1992 during a five-minute observation of PSR 1913+16. The characteristic double-peaked shape, clearly seen in the de-dispersed profile at the bottom, is also discernible in the 32 individual spectral channels.

First steps: accurate determination of orbital parameters according to classical Keplerian theory



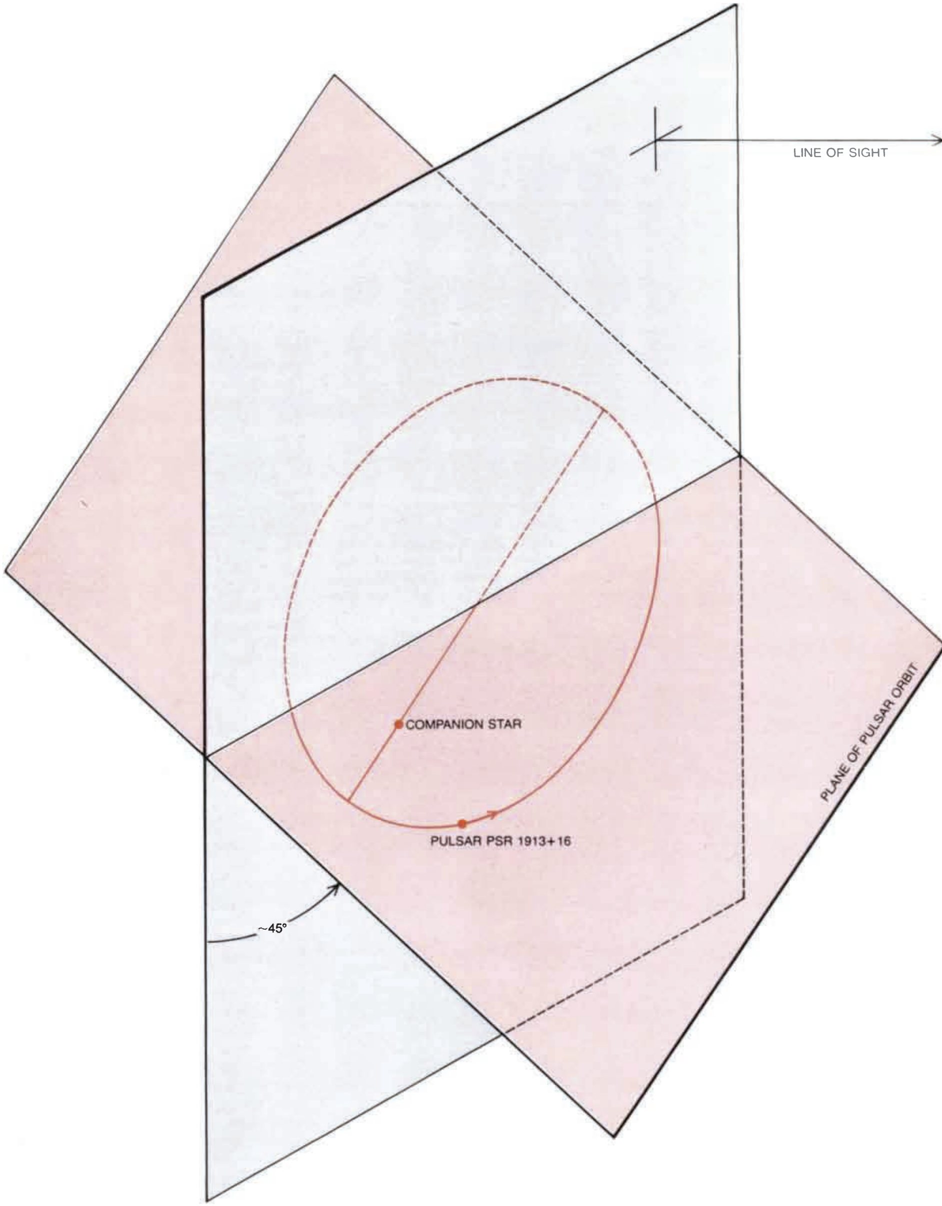
second ed. 1935
(https://archive.org/details/binarystars00aitk/page/n8)



Velocity curve for the binary pulsar. Points represent measurements of the pulsar period distributed over parts of 10 different orbital periods. The curve corresponds to equations from Aitken, with the following orbital parameters:

Parameters of the Binary Pulsar
$\alpha(1950.0) = 19^h 13^m 13^s \pm 4^s$
$\delta(1950.0) = +16°00'24'' \pm 60''$
$l = 49.^\circ 9$
$b = 2.^\circ 1$
$P_{cm} = 0.^s 059030 \pm 0.^s 000001$
$dP_{cm}/dt < 1 \times 10^{-12}$
$DM = 167 \pm 5\ cm^{-3}\ pc$
$S_{430} = 0.006 \pm 0.003$ Jy
$W_e < 10$ ms

Elements of the Orbit
$K_1 = 199 \pm 5\ km\ s^{-1}$
$P_b = 27908 \pm 7$ s
$e = 0.615 \pm 0.010$
$\omega = 179° \pm 1°$
$T = $ JD 2,442,321.433 $\pm$ 0.002
$a_1 \sin i = 1.00 \pm 0.02\ R_\odot$
$f(m) = 0.13 \pm 0.01\ M_\odot$



ORBIT OF PULSAR PSR 1913 + 16 lies in a plane tilted about 45 degrees from the line of sight. Like the more than 300 other pulsars discovered since 1967, PSR 1913 + 16 is thought to be a neutron star, 20 to 30 kilometers in diameter, that emits a radio beam that sweeps past the earth at precisely spaced intervals synchronized with the star's rate of spin. For PSR 1913 + 16 the spin rate is 16.94 revolutions per second. Unlike the large majority of other pulsars, PSR 1913 + 16 travels in an orbit around a companion star whose presence was inferred from a Doppler shift in the arrival time of the pulsar's "beeps." The beeps arrive slightly more frequently when the pulsar is traveling toward the earth and less frequently when the pulsar is receding. A complete picture of the pulsar's orbit around the center of mass of the binary system was derived through careful measurements of the Doppler shift in combination with an analysis of subtle gravitational effects predicted by the general theory of relativity. The theory made it possible to calculate that the pulsar and its companion are both 1.4 times as massive as the sun and that the separation of the two stars varies from 1.1 to 4.8 times the radius of the sun.

Further refinement: use GR to constrain all the remaining parameters

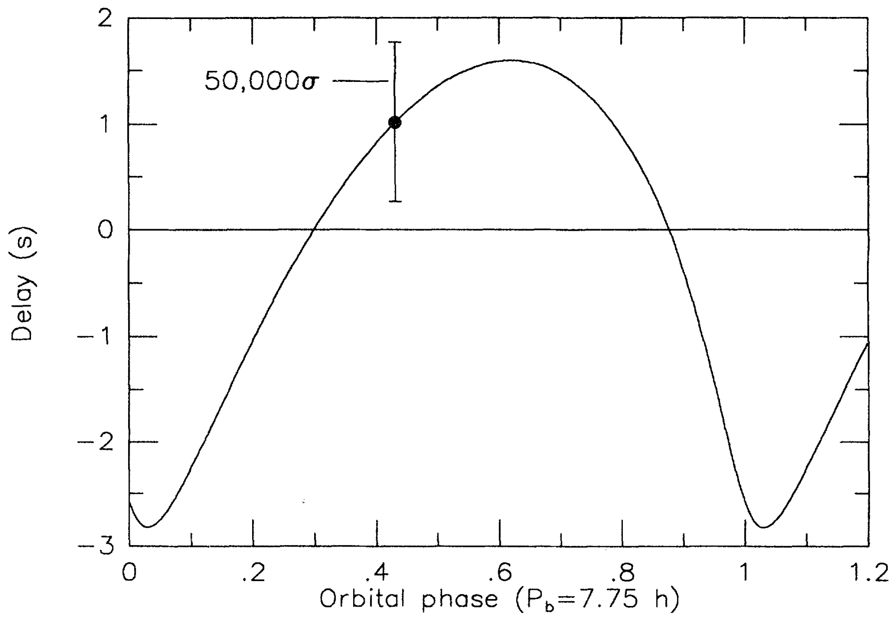


FIG. 5. Orbital delays observed for PSR 1913+16 during July, 1988. The uncertainty of an individual five-minute measurement is typically 50 000 times smaller than the error bar shown.

Many theoretical corrections, one of them is the Shapiro time delay in the binary system. This delay can be used to determine the mass of the companion NS.

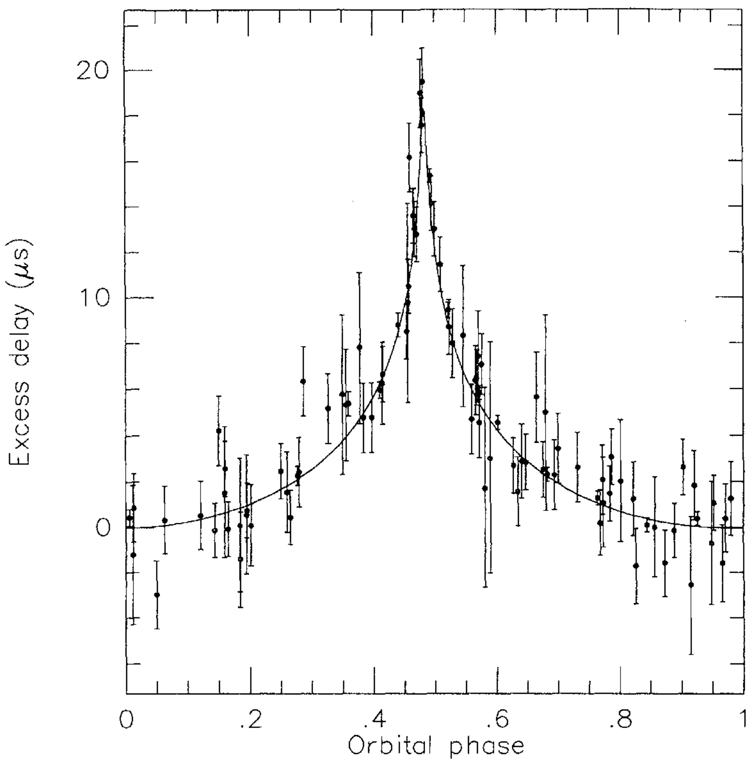
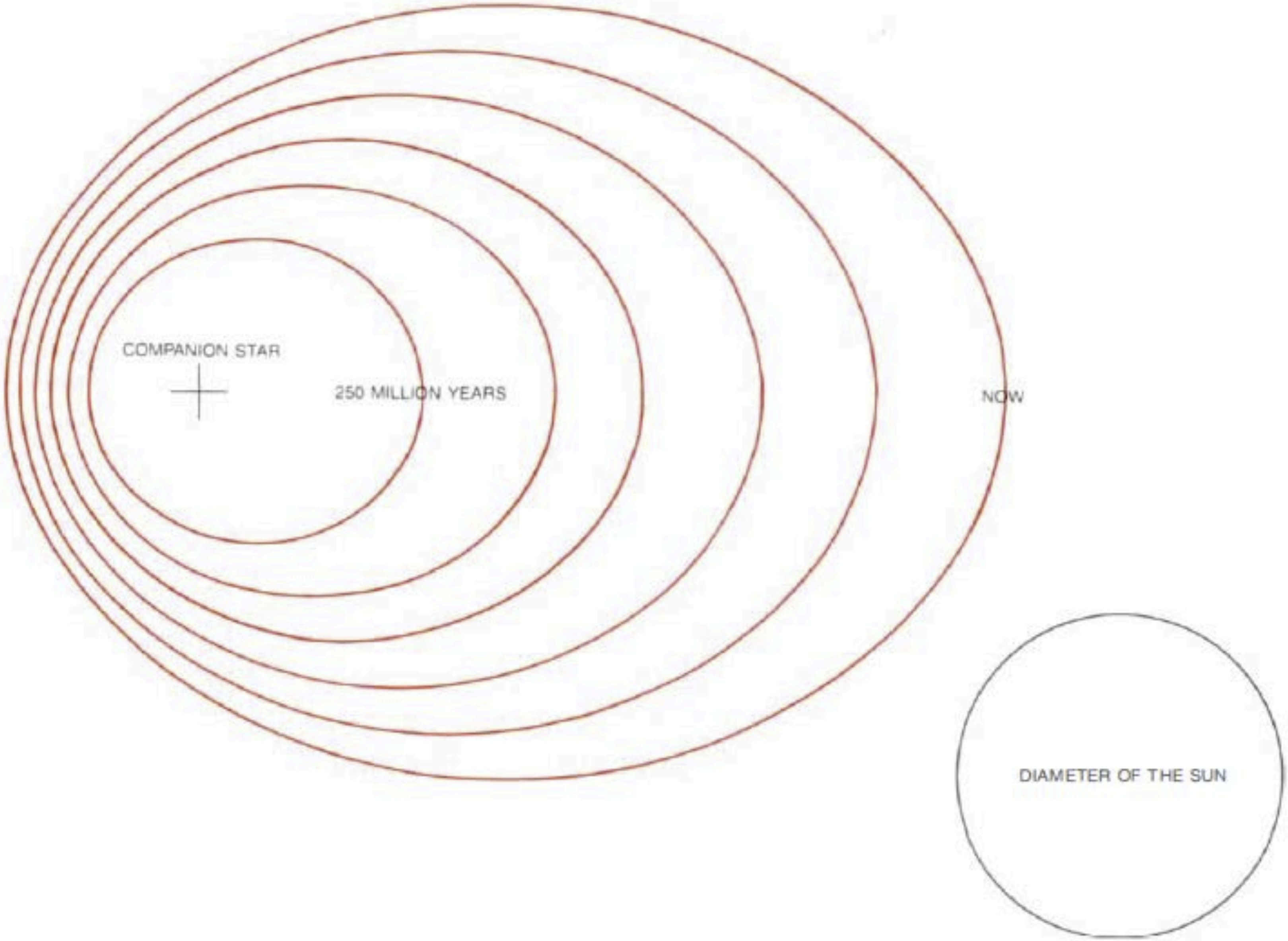
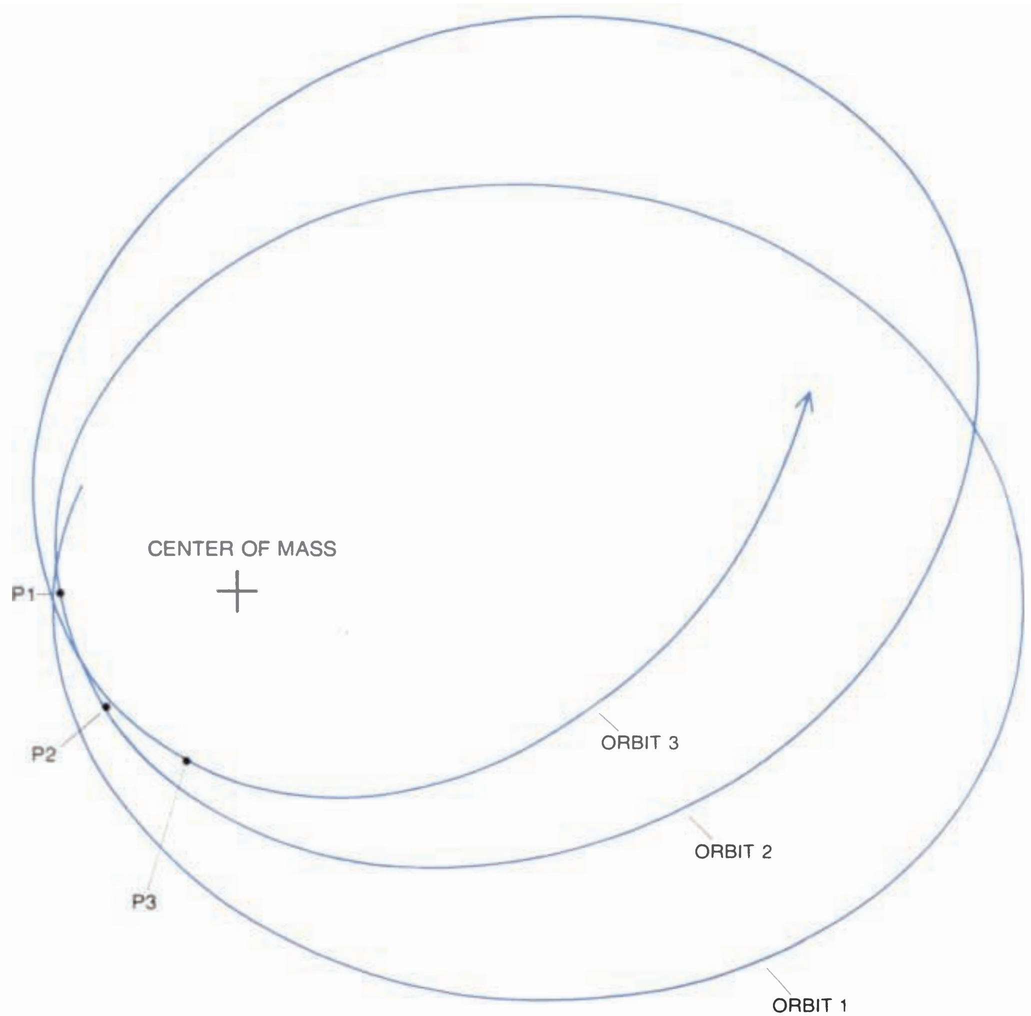


FIG. 8. Measurements of the Shapiro time delay in the PSR 1855+09 system. The theoretical curve corresponds to Eq. (10), and the fitted values of *r* and *s* can be used to determine the masses of the pulsar and companion star.



SHRINKING OF PULSAR'S ORBIT is projected on the basis of evidence that orbital energy is being converted into gravitational radiation, as is predicted by the general theory of relativity. According to the theory, the orbit of PSR 1913 + 16 should shrink by 3.1 millimeters per orbital revolution, or 3.5 meters per year. The orbital period should decrease accordingly by 6.7×10^{-8} second per orbit, or 7.6×10^{-5} second per year. This tiny change is measurable because it leads to a constantly accumulating deviation in the time of periastron passage. Here the orbit is drawn to scale as it will appear every 50 million years in the future until the two stars coalesce 300 million years from now. For comparison the sun is shown at the same scale. PSR 1913 + 16 is thought to be a 50,000th the diameter of the sun.



ADVANCE OF PERIASTRON in the orbit of PSR 1913 + 16 has provided one of the first clear observations of a general-relativistic effect involving bodies outside the solar system. The periastron advances, or rotates, as the elliptical orbit of PSR 1913 + 16 itself rotates in a plane because of the curvature of space-time in the vicinity of the pulsar's massive companion. In this diagram the effect is greatly exaggerated. The general theory of relativity predicts a periastron advance of about four degrees per year in the orbit of PSR 1913 + 16, the exact value depending on the total mass of the pulsar and its companion. The authors' measurements show the periastron is advancing 4.2 degrees per year, in good agreement with the prediction.

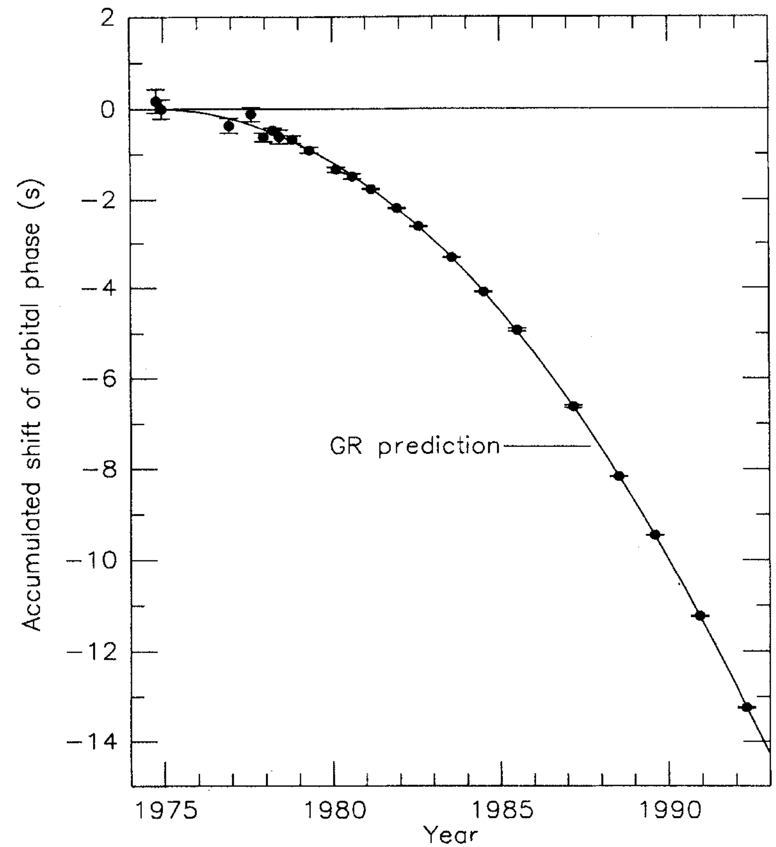


FIG. 10. Accumulated shift of the times of periastron in the PSR 1913+16 system, relative to an assumed orbit with constant period. The parabolic curve represents the general relativistic prediction for energy losses from gravitational radiation.

Now we consider in greater detail the physics of binary systems

The emission of GW's produces a circularization of the orbit, and here we consider only circular orbits.

In the CM system, conservation of momentum requires

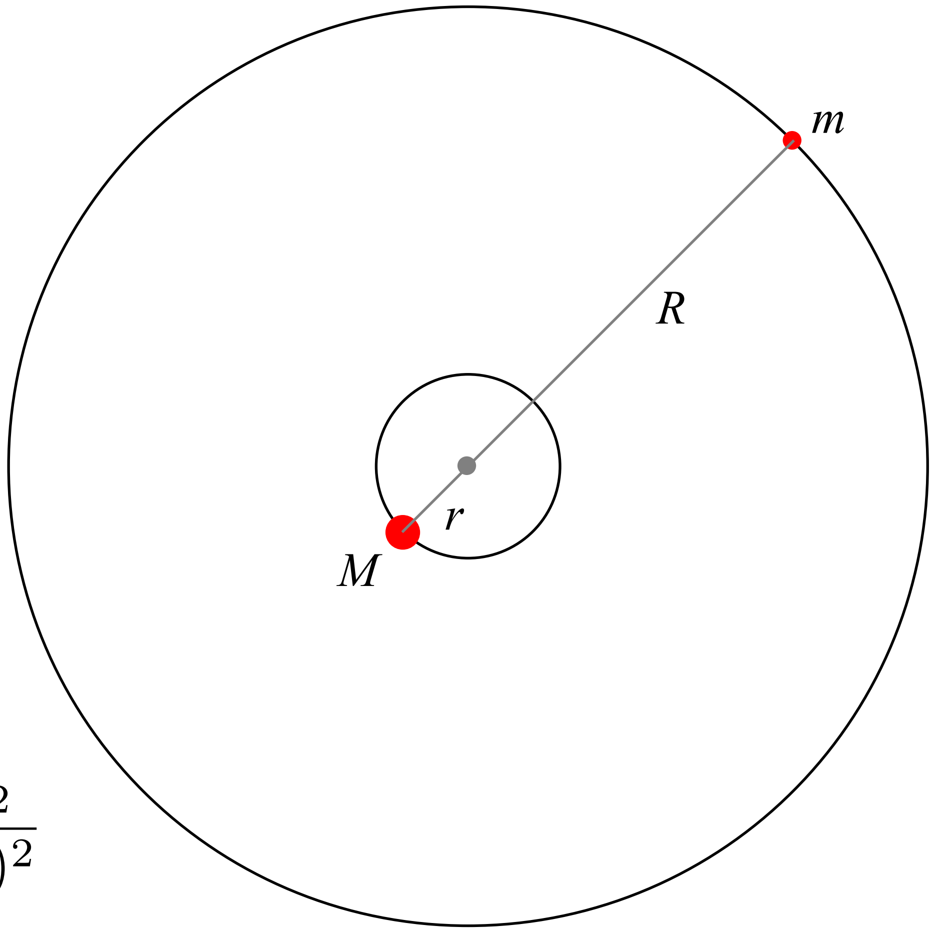
$$m_1 r_1 \omega = m_2 r_2 \omega$$

$$\Downarrow$$

$$m_1 r_1 = m_2 r_2$$

Moreover, the masses experience a centrifugal acceleration that must be balanced by the gravitational force, so that

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{G m_1 m_2}{(r_1 + r_2)^2}$$



The last formula can be rearranged to obtain

$$r_1\omega^2 = \frac{Gm_2}{(r_1+r_2)^2}; \quad r_2\omega^2 = \frac{Gm_1}{(r_1+r_2)^2}$$

and therefore we find the following variant of Kepler's third law

$$\omega^2 = \frac{G(m_1+m_2)}{(r_1+r_2)^3}$$

Notice also that the moment of inertia of the system about the CM is

$$\begin{aligned} I &= m_1 r_1^2 + m_2 r_2^2 = m_2 r_1 r_2 + m_1 r_1 r_2 = (m_1+m_2) r_1 r_2 \\ &= \frac{(m_1+m_2)^2}{m_1+m_2} r_1 r_2 = \frac{m_1^2 r_1 r_2 + m_2^2 r_1 r_2 + 2 m_1 m_2 r_1 r_2}{m_1+m_2} \\ &= \frac{m_1 m_2}{m_1+m_2}(r_1+r_2)^2 \end{aligned}$$

Then we find

$$E_{\rm tot} = \frac{1}{2}I\omega^2 - \frac{Gm_1m_2}{(r_1+r_2)} = \frac{1}{2}\frac{Gm_1m_2}{(r_1+r_2)} - \frac{Gm_1m_2}{(r_1+r_2)} = -\frac{1}{2}\frac{Gm_1m_2}{(r_1+r_2)}$$

or also

$$E_{\rm tot} = -\frac{1}{2}\;\frac{G^{2/3}\;m_1m_2}{(m_1+m_2)^{1/3}}\;\omega^{2/3}$$

What about gravitational waves?

There is no dipole radiation with GW.

Recall the definition of the dipole moment for a distribution of masses

$$\mathbf{p} = \sum_i m_i \mathbf{r}_i$$

and its time derivative in a system of masses which are under the action of internal forces only is

$$\dot{\mathbf{p}} = \sum_i m_i \mathbf{v}_i = \text{constant}$$

from the conservation of total momentum. Finally, this means

$$\ddot{\mathbf{p}} = 0$$

i.e., there cannot be any radiation that is proportional to the second derivative of the dipole moment, as one would expect from an analogy with EM radiation.

Now recall that the multipole expansion is derived from the following identity

$$\frac{1}{|\mathbf{R}-\mathbf{r}|} = \frac{1}{\sqrt{R^2+r^2-2Rr\cos\theta}} = \sum_{\ell=0}^{\infty} \frac{r^\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

Legendre polynomials

The quadrupole moment is associated with the Legendre polynomial of degree 2

$$P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$$

in particular it is derived from

$$\begin{aligned} r^2 P_2(\cos\theta_i) &= \frac{1}{2} r^2 (3\cos^2\theta - 1) \\ &= \frac{1}{2}\left[3(\mathbf{r}\cdot\hat{\mathbf{R}})^2 - (\mathbf{r}\cdot\mathbf{r})\right] = \frac{1}{2}\left[3(r_j\hat{R}_j)(r_k\hat{R}_k) - (r^2\delta_{j,k}\hat{R}_j\hat{R}_k)\right] \\ &= \frac{1}{2}\left(3r_j r_k - r^2\delta_{j,k}\right)\hat{R}_j\hat{R}_k \end{aligned}$$

When we extend the last calculation to many masses, we find

$$\sum_i \frac{m_{(i)}}{2} \left(3\, r_{(i),j} r_{(i),k} - r_{(i)}^2 \delta_{j,k}\right) \hat{R}_j \hat{R}_k = Q_{i,j} \hat{R}_j \hat{R}_k$$

Quadrupole tensor

The quadrupole tensor is related to the tensor of inertia

$$Q_{i,j} = \sum_i m_{(i)} \left(\frac{3}{2}\, r_{(i),j} r_{(i),k} - \frac{1}{2} r_{(i)}^2 \delta_{j,k}\right) = \frac{3}{2} I_{i,j} - \frac{1}{2} \mathrm{tr}(I) \delta_{j,k}$$

By analogy with the EM radiation and on dimensional grounds, we expect then that the radiated power is proportional to the "square" of the quadrupole moment, i.e. to the "square" of the tensor of inertia:

$$P_{\rm rad} = \alpha I^2 \omega^\gamma G^\eta c^\zeta$$

with α adimensional constant (this representation holds in the frequency domain).
From dimensional analysis we find

$$P_{\rm rad} = \alpha \frac{G I^2 \omega^6}{c^5}$$

The constant can be found only with a full GR treatment and one finds $\alpha = 32/5$.

In terms of the quadrupole tensor, the full formula in the time domain is

$$P_{\rm rad} = \frac{1}{5}\,\frac{G}{c^5}\sum_{j,k}\left(\frac{d^3 Q_{j,k}}{dt^3}\right)^2$$

Since the quadrupole tensor is symmetric, the emitted radiation has frequency 2ω

After the general considerations let's get back to the binary system

We can replace the expression of the moment of inertia and that of its rotation frequency to eliminate both the moment of inertia and the spatial separation between the masses in the formula for the radiated power:

$$P_{\text{binary}} = \alpha \frac{G^{7/3}}{c^5} \frac{(m_1 m_2)^2}{(m_1 + m_2)^{2/3}} \omega^{10/3}$$

This radiated power corresponds to the rate of change of the total energy of the binary system

$$E_{\text{tot}} = -\frac{1}{2} \frac{G^{2/3} m_1 m_2}{(m_1 + m_2)^{1/3}} \omega^{2/3} \quad \Rightarrow \quad \left| \frac{dE_{\text{tot}}}{dt} \right| = \frac{G^{2/3}}{3} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}} \omega^{-1/3} \frac{d\omega}{dt}$$

and **equating the expressions we find**

$$\frac{m_1 m_2}{(m_1 + m_2)^{1/3}} = \frac{c^5}{G^{5/3}} \frac{1}{3\alpha} \omega^{-11/3} \frac{d\omega}{dt}$$

It is customary to define the ``chirp mass'' as follows:

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left(\frac{1}{3\alpha}\,\omega^{-11/3}\,\frac{d\omega}{dt}\right)^{3/5}$$

Then, using the chirp mass and the emitted frequency we can write

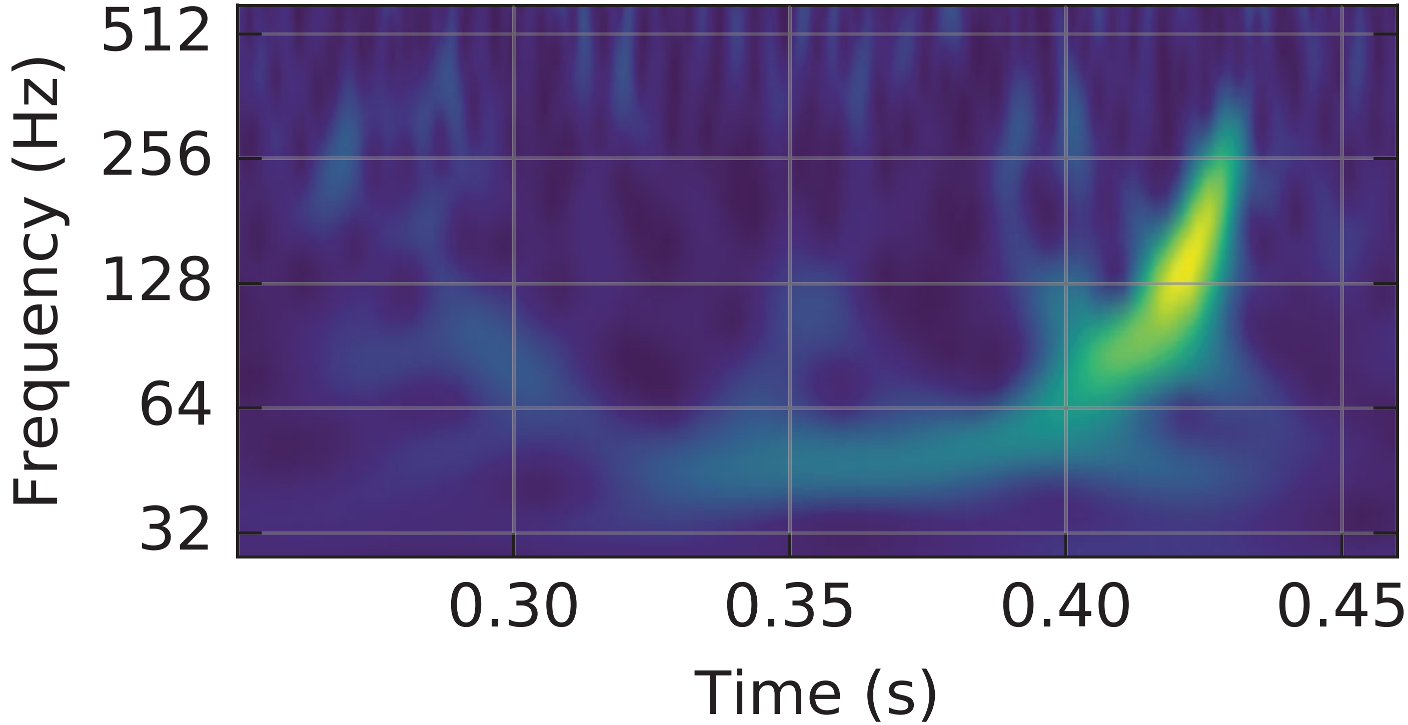
$$\mathcal{M} = \frac{c^3}{G}\left(\frac{5}{96}\pi^{-8/3}\,f^{-11/3}\,\frac{df}{dt}\right)^{3/5}$$

which is the only equation in the "discovery paper". This shows that the chirp mass can be derived from the observation of the frequency alone.

After integration, one can also approximate as follows

$$8\alpha\pi^{8/3}\mathcal{M}^{5/3}\frac{G^{5/3}}{c^5}(t - t_0) = f^{-8/3} - f_0^{-8/3}$$

$$\Rightarrow \quad \mathcal{M} \approx \frac{c^3}{G}\,\frac{1}{(8\alpha t)^{3/5}\,(\pi f_0)^{8/5}}$$



$f_0 \approx 42$ Hz

@ $t = 0.35$ s

$f \gg f_0$

@ $t = 0.43$ s

Compute the value of the chirp mass in units of solar masses, where $M_\odot \approx 2 \times 10^{30}$ kg

Recall the formula for the Schwarzschild radius

$$R_S = \frac{2GM}{c^2}$$

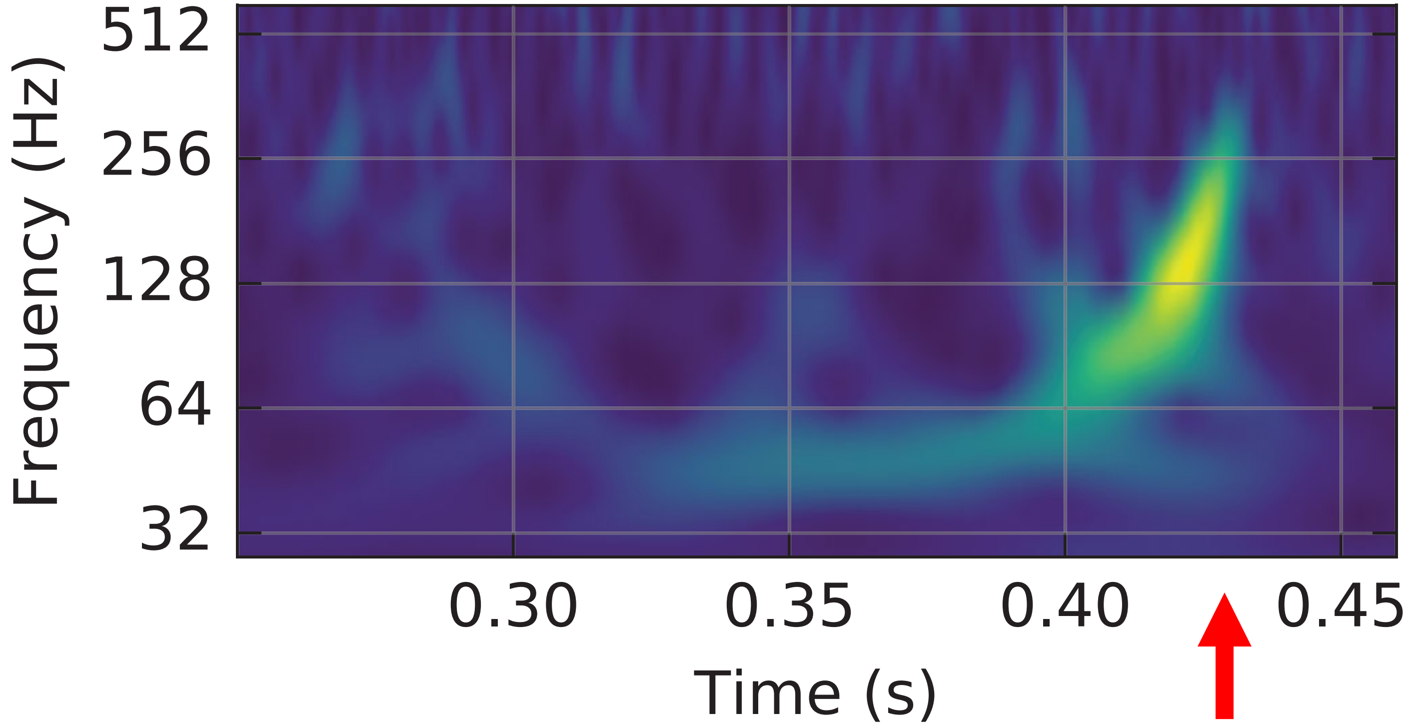
We can assume that coalescence stops when the separation between the two masses is equal to the sum of their Schwarzschild radii

$$r_1 + r_2 = \frac{2G}{c^2}(m_1 + m_2)$$

at this separation

$$\omega_S^2 = \frac{G(m_1 + m_2)}{(r_1 + r_2)^3} = \frac{c^6}{8G^2} \frac{1}{(m_1 + m_2)^2}$$

$$\Rightarrow \quad m_1 + m_2 = \frac{c^3}{\sqrt{8}G} \frac{1}{\pi f_S}$$



$f \approx 300$ Hz

@ $t = 0.43$ s

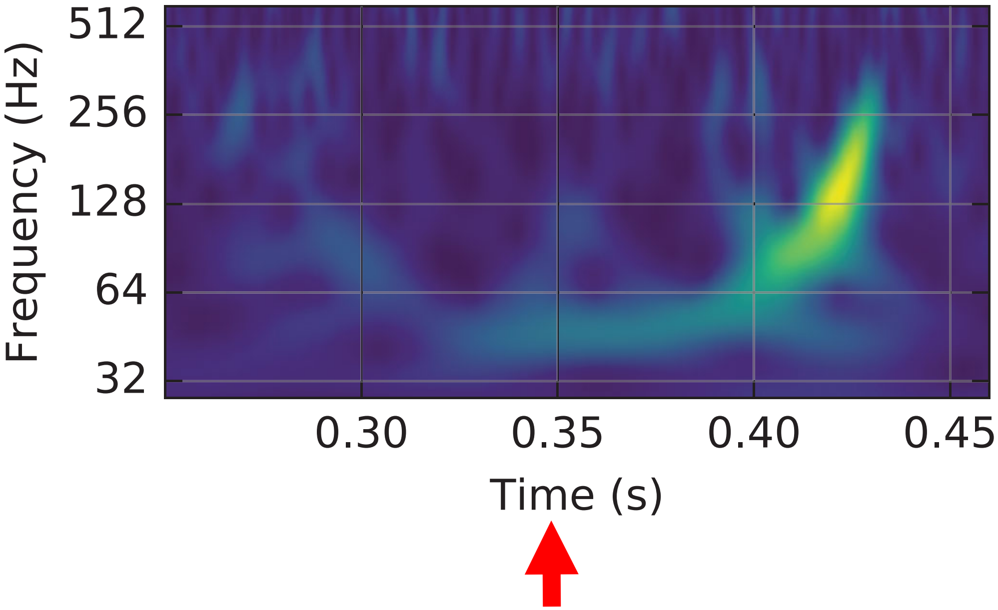
Use the equations below and the value of the chirp mass to estimate the two masses

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \qquad m_1 + m_2 = \frac{c^3}{\sqrt{8}G} \frac{1}{\pi f_S}$$

What is the initial separation of the two black holes? How does this compare with the Schwarzschild radii?

We can use the equation of the frequency as a function of the separation between masses

$$\omega^2 = \frac{G(m_1+m_2)}{(r_1+r_2)^3} \quad \Rightarrow \quad r_1 + r_2 = \left[\frac{G(m_1+m_2)}{\pi^2 f^2}\right]^{1/3}$$



$f_0 \approx 42\ \mathrm{Hz}$

$@\ t = 0.35\ \mathrm{s}$

Recall also

$$R_S = \frac{2GM}{c^2}$$

We can use the formula for the total energy to approximate the total radiated energy

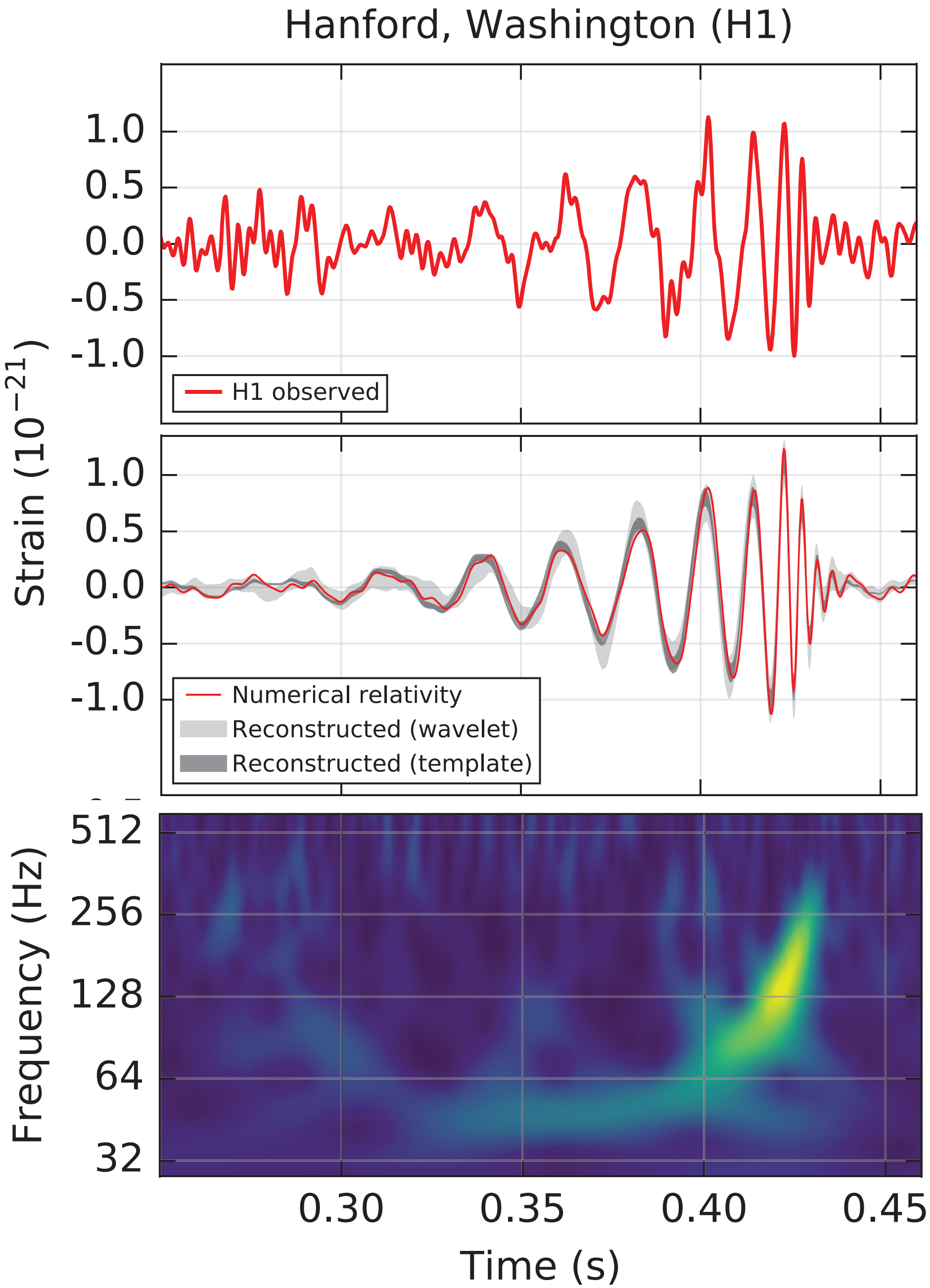
$$E_{\rm tot} = -\frac{1}{2}\frac{Gm_1m_2}{(r_1+r_2)}$$

At coalescence, this becomes

$$r_1 + r_2 = \frac{2G}{c^2}(m_1+m_2) \quad \Rightarrow \quad E_{\rm tot} = -\frac{c^2}{4}\frac{m_1m_2}{(m_1+m_2)}$$

and this is roughly equal to the total radiated energy.

Compute this estimate of the total radiated energy for GW150914 (in solar masses)



Largest strain

$$h \approx 10^{-21} \quad @ \ f \approx 250 \text{ Hz}$$

Amplitude and intensity of the GW

From elementary dimensional analysis we find

$$\mathcal{I}_{\rm rad} = \beta h^2 \frac{f^2 c^3}{G}$$

where the adimensional constant can determined by the full GR calculation:

$$\beta = \pi/2$$

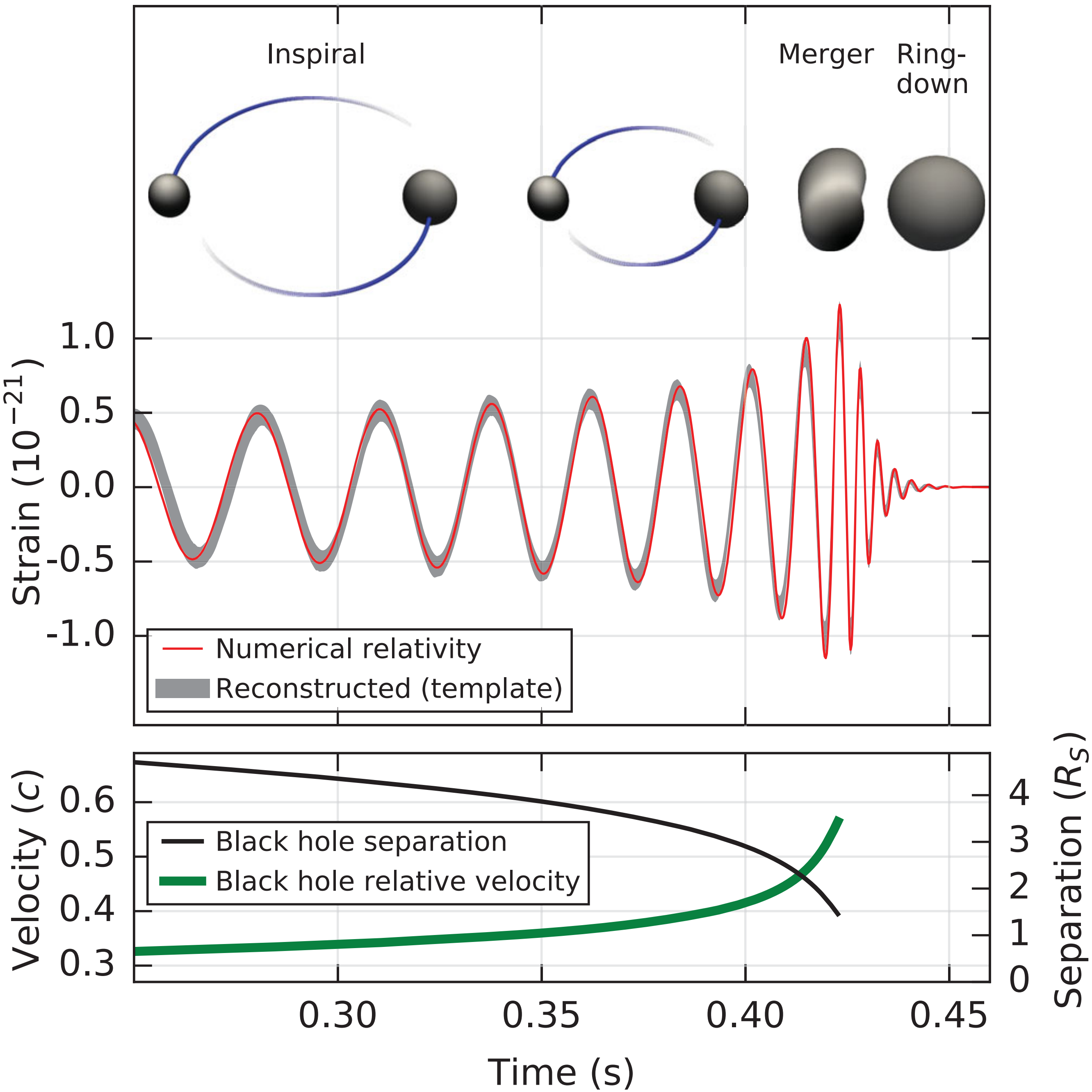
Using the inverse square law

$$\mathcal{I}_{\rm rad} = P_{\rm rad}/4\pi R^2$$

and the equation for the radiated power

$$P_{\rm rad} = \frac{32}{5} \frac{G^{7/3}}{c^5} \; \frac{(m_1 m_2)^2}{(m_1 + m_2)^{2/3}} \; (\pi f)^{10/3}$$

find the distance, using $h \approx 10^{-21} \quad @ \; f \approx 250 \ {\rm Hz}$



Inspiral
Merger
Ring-
down
1.0
0.5
0.0
-0.5
-1.0
Strain (10^{-21})
Numerical relativity
Reconstructed (template)
0.6
0.5
0.4
0.3
Velocity (c)
4
3
2
1
0
Separation (R_S)
Black hole separation
Black hole relative velocity
0.30
0.35
0.40
0.45
Time (s)

The Last Three Minutes: Issues in Gravitational-Wave Measurements of Coalescing Compact Binaries

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(Received 24 August 1992)

Gravitational-wave interferometers are expected to monitor the last three minutes of inspiral and final coalescence of neutron star and black hole binaries at distances approaching cosmological, where the event rate may be many per year. Because the binary's accumulated orbital phase can be measured to a fractional accuracy $\ll 10^{-3}$ and relativistic effects are large, the wave forms will be far more complex and carry more information than has been expected. Improved wave form modeling is needed as a foundation for extracting the waves' information, but is not necessary for wave detection.

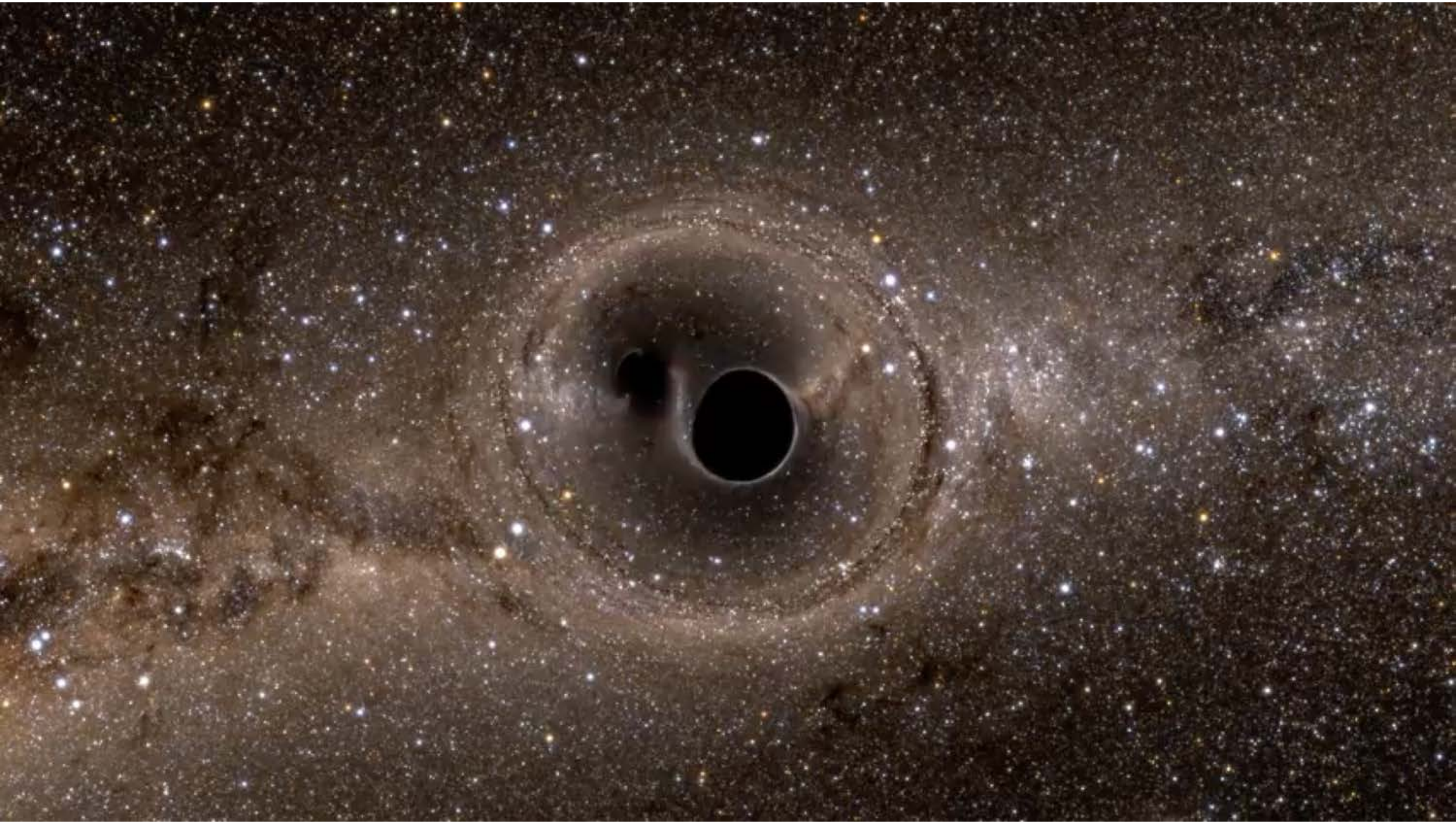
We shall assume (as almost always is the case) that the binary's orbit has been circularized by radiation reaction [10]. Then the only parameters λ_i that can significantly influence the inspiral template's phasing are the bodies' masses, vectorial spin angular momenta, and spin-induced quadrupole moments (which we shall ignore because, even for huge spins, they produce orbital phase shifts no larger than ~ 1 [8]). More specifically, the number of cycles spent in a logarithmic interval of frequency, $d\mathcal{N}_{\rm cyc}/d\ln f = (1/2\pi)(d\Phi/d\ln f)$, is

$$\frac{d\mathcal{N}_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1 + \left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)x \right. \\ \left. -[4\pi + \text{S.O.}]x^{1.5} + [\text{S.S.}]x^2 + O(x^{2.5})\right\}. \quad (1)$$

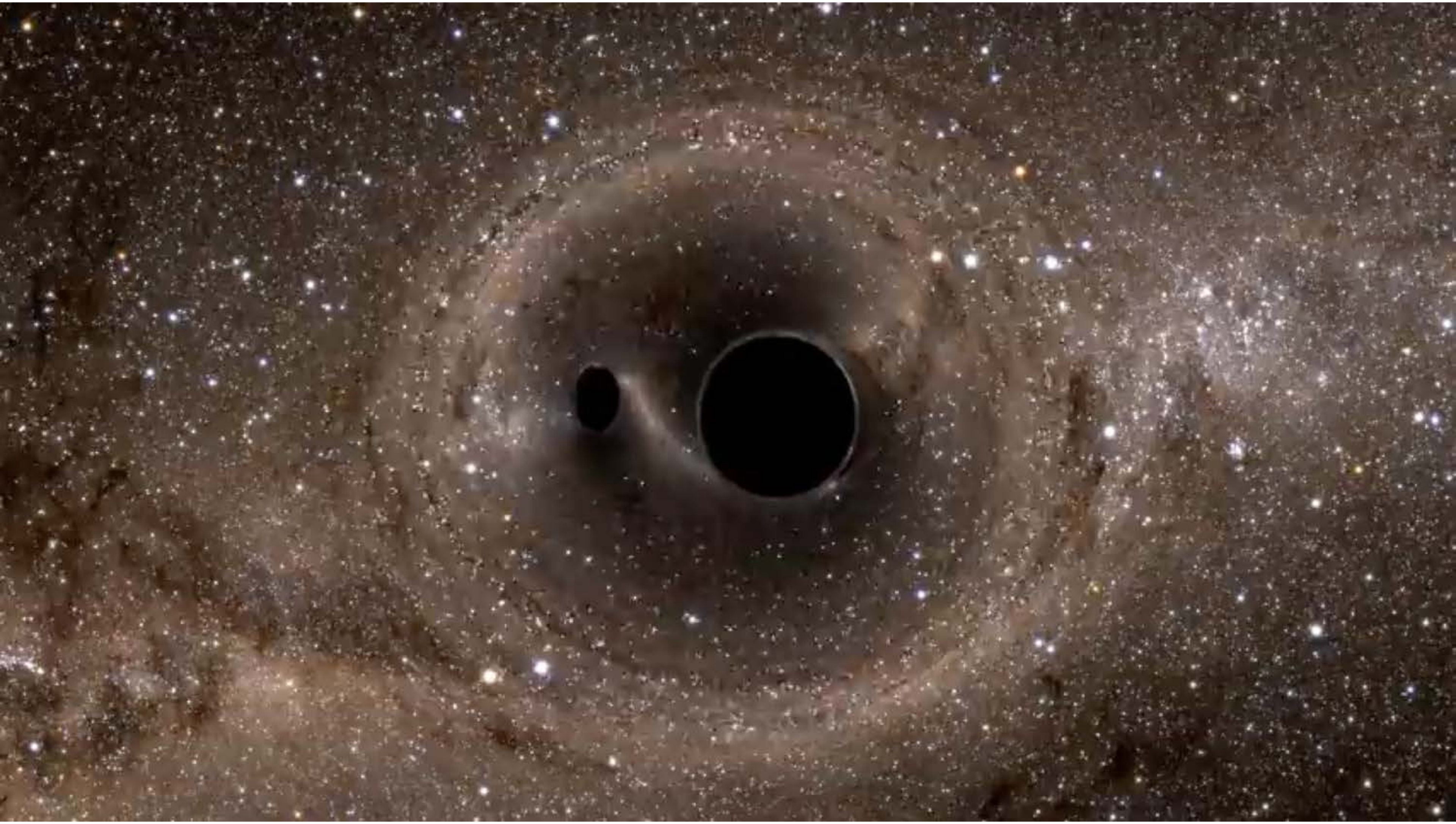
Here M is the binary's total mass, μ its reduced mass, and $x \equiv (\pi M f)^{2/3} \simeq M/D$ the PN expansion parameter (with D the bodies' separation and $c = G = 1$). The PN correction [$O(x)$ term] is from [13]. In the $\text{P}^{1.5}\text{N}$ correction [$O(x^{1.5})$ term], the 4π is created by the waves' interaction with the binary's monopolar gravitational field as they propagate from the near zone to the radiation zone [14], and the "S.O." denotes contributions due to spin-orbit coupling [15]. In the P^2N correction the "S.S." includes spin-spin coupling effects [15] plus an expression quadratic in μ/M. (For bodies with sizes comparable to their separations, the spin-orbit and spin-spin terms are of PN order; but the compactness of a BH or NS boosts them up to $\text{P}^{1.5}\text{N}$ and P^2N, respectively; cf. [15].)

Numerical relativity and post-Newtonian calculations

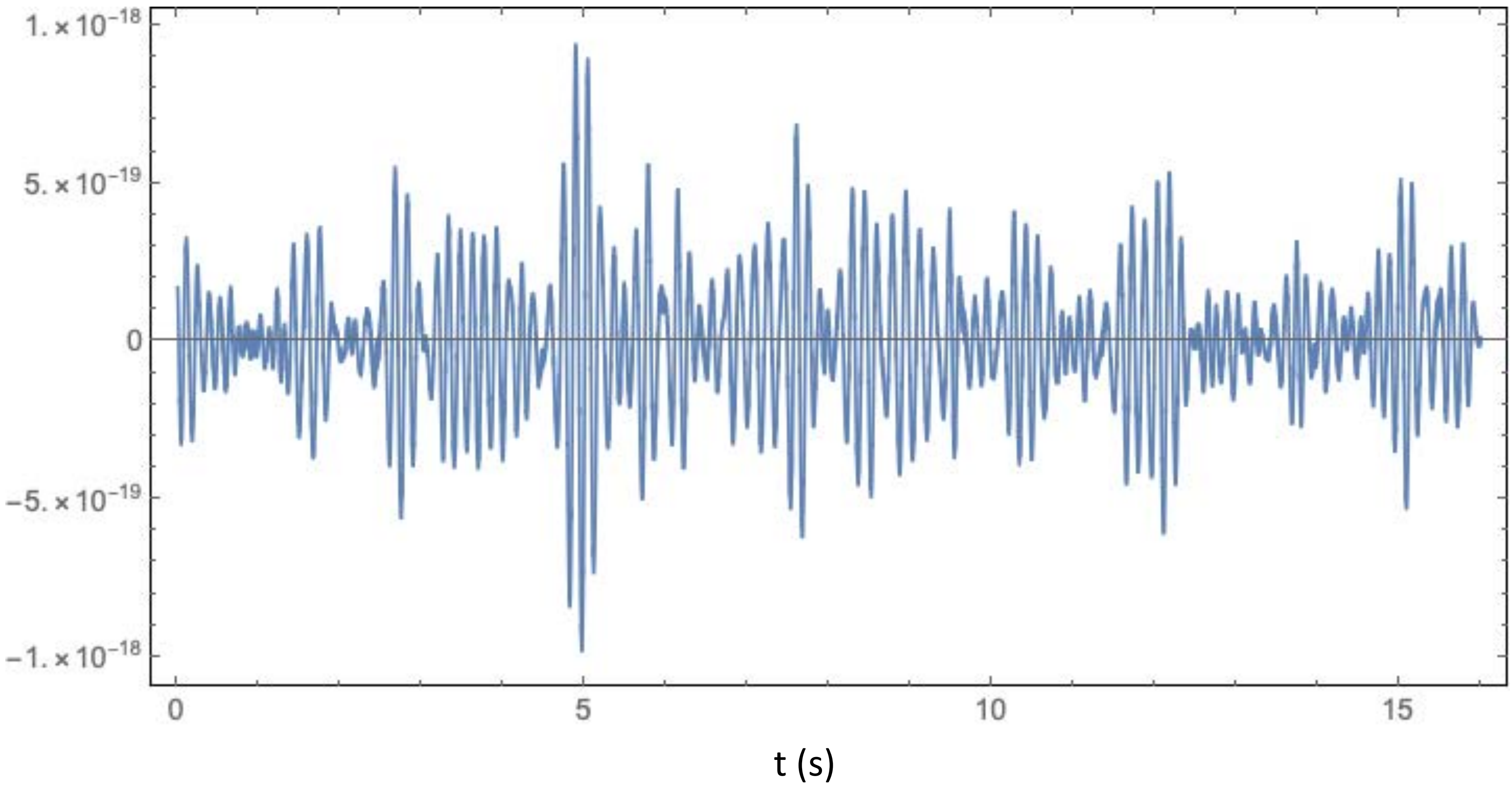
Merging black holes, side view

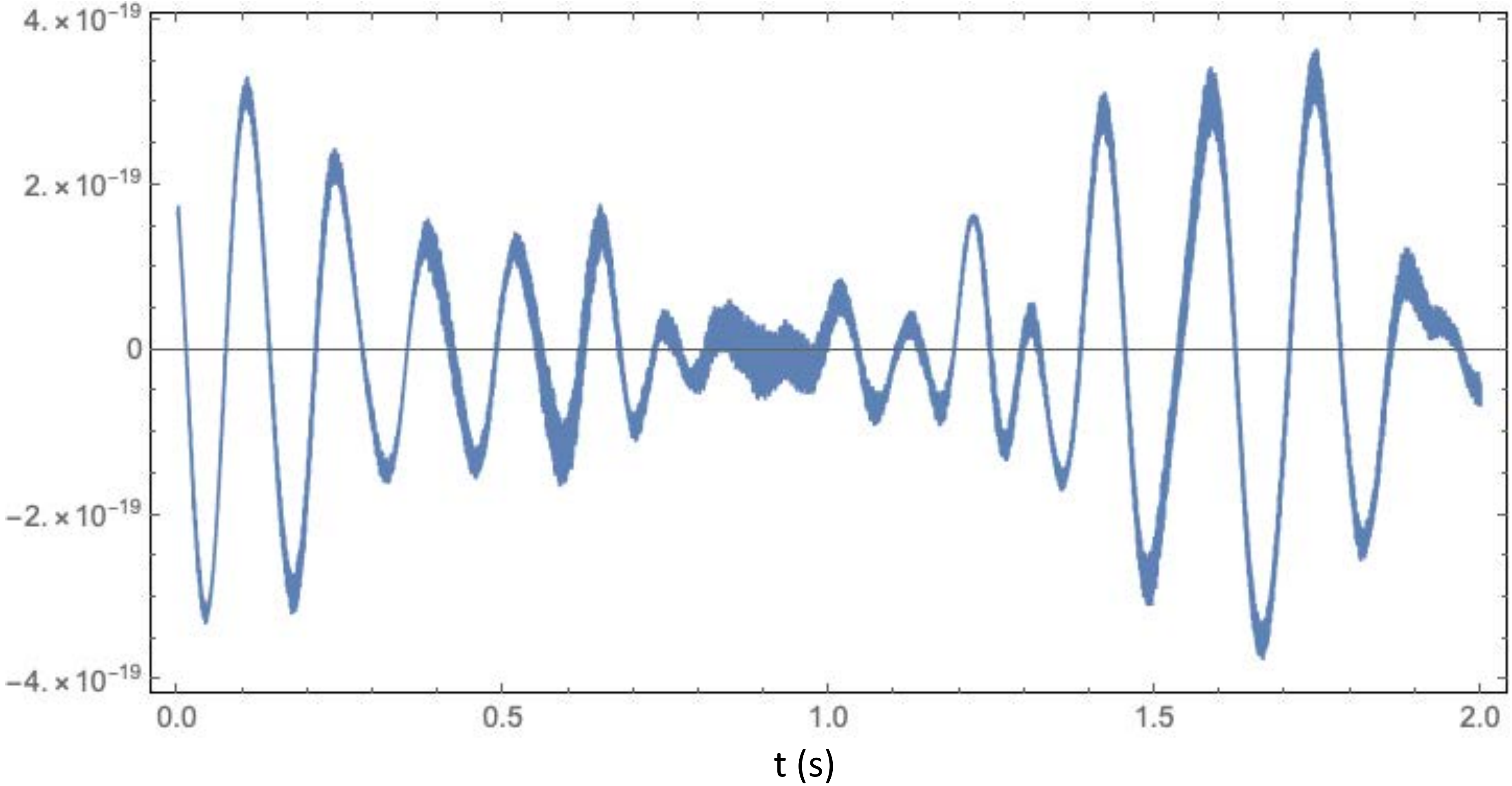


Merging black holes, top view

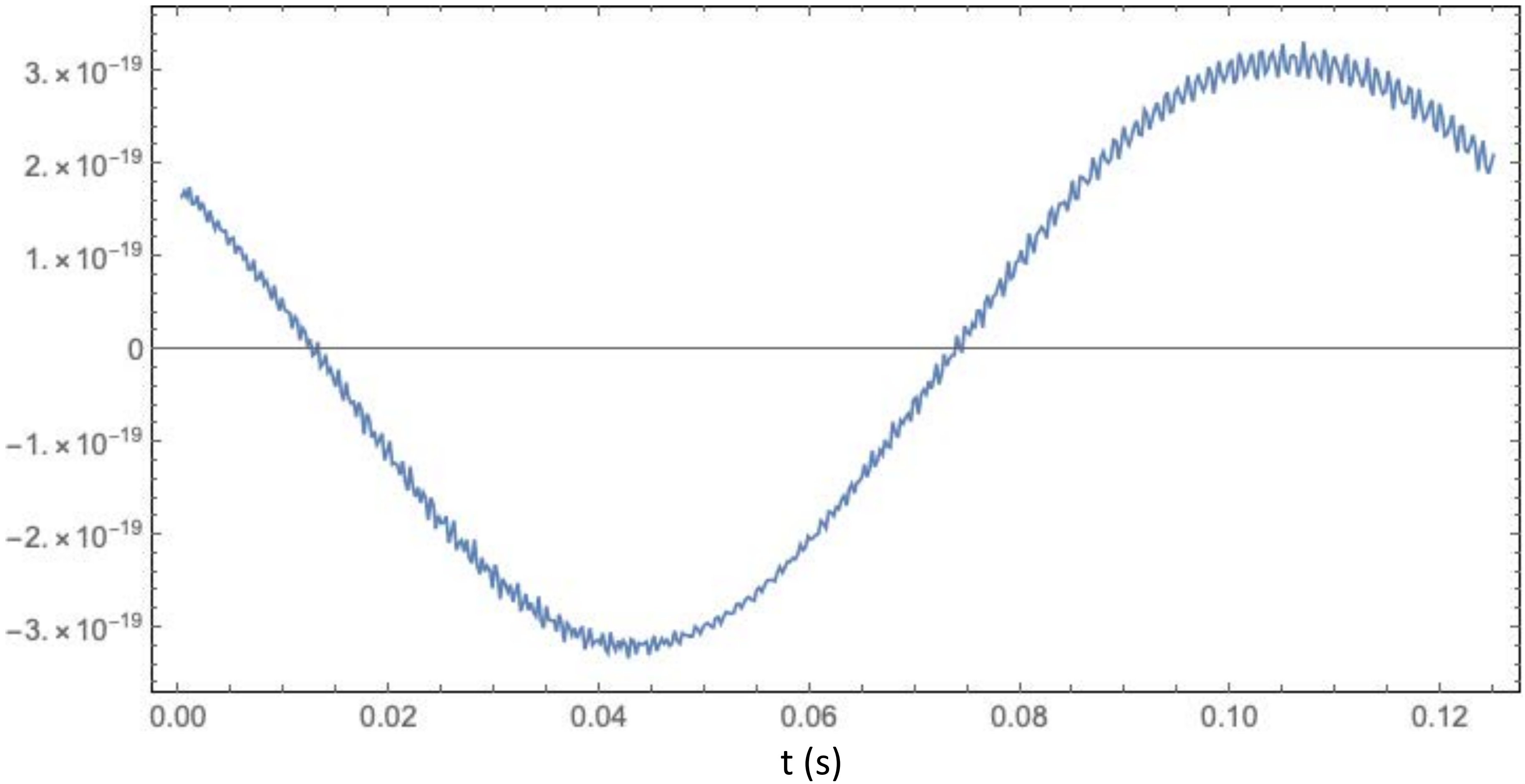


What is the role of signal filtering?



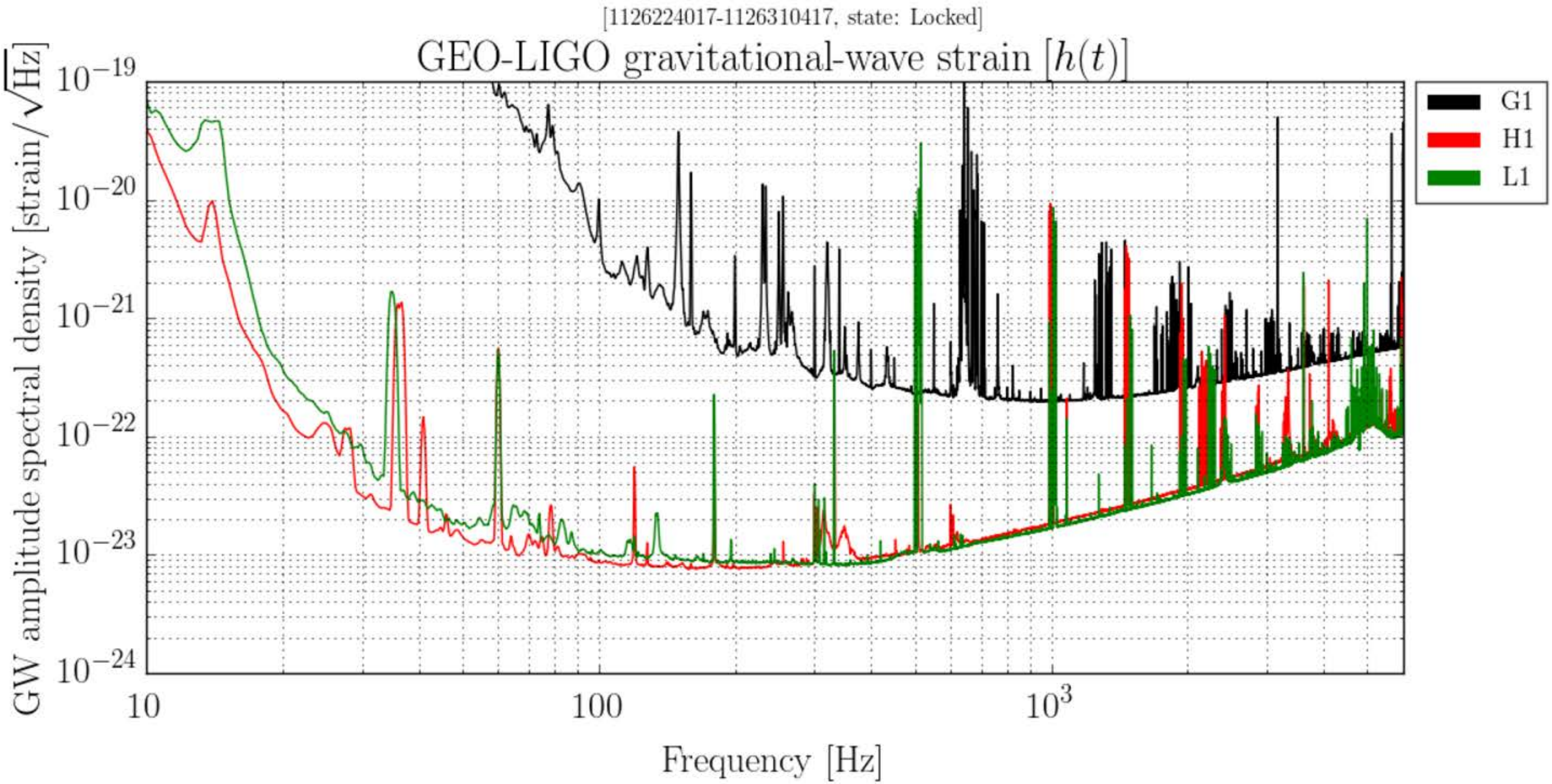


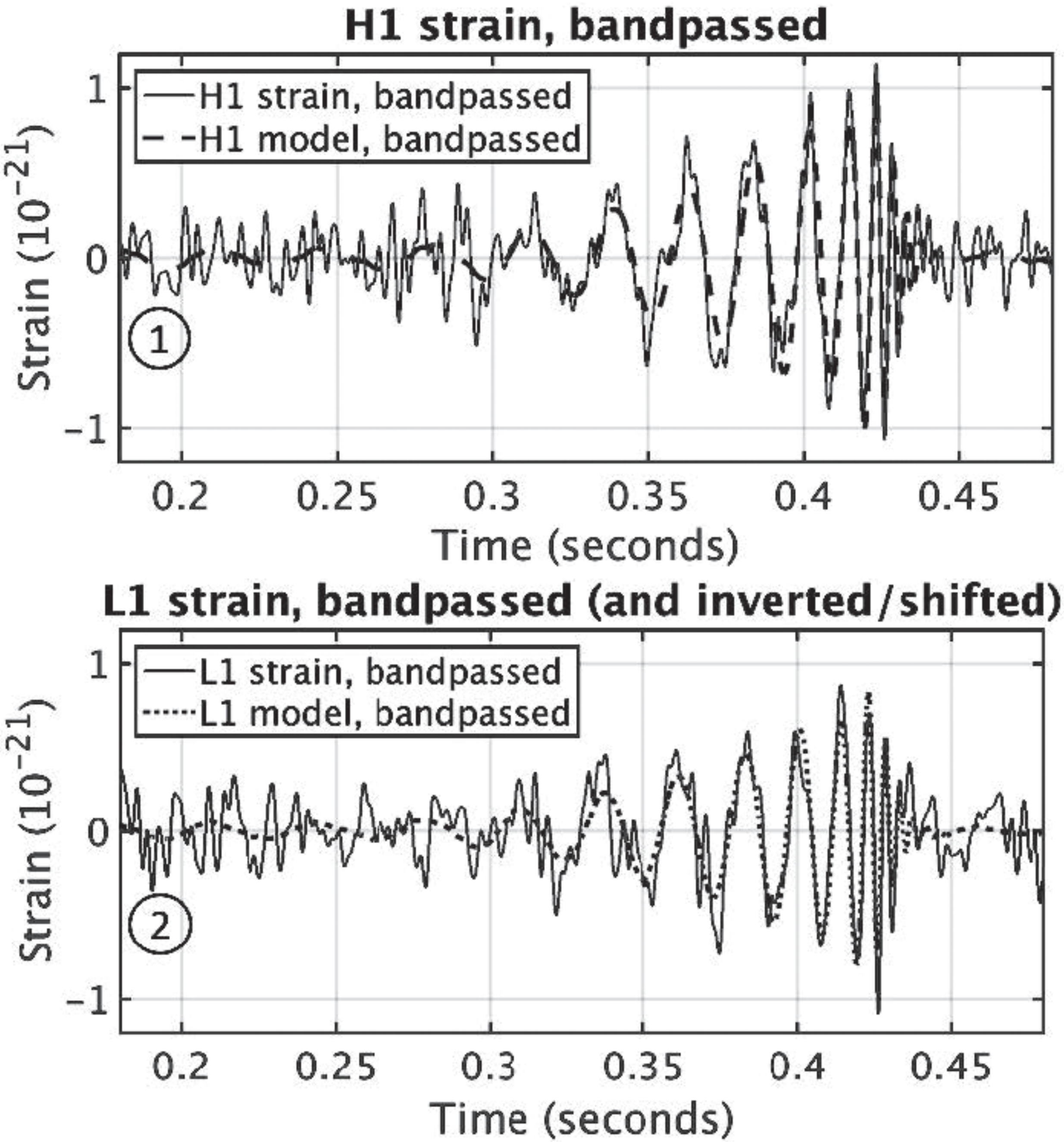
4. × 10^-19
2. × 10^-19
0
-2. × 10^-19
-4. × 10^-19
0.0
0.5
1.0
1.5
2.0
t (s)



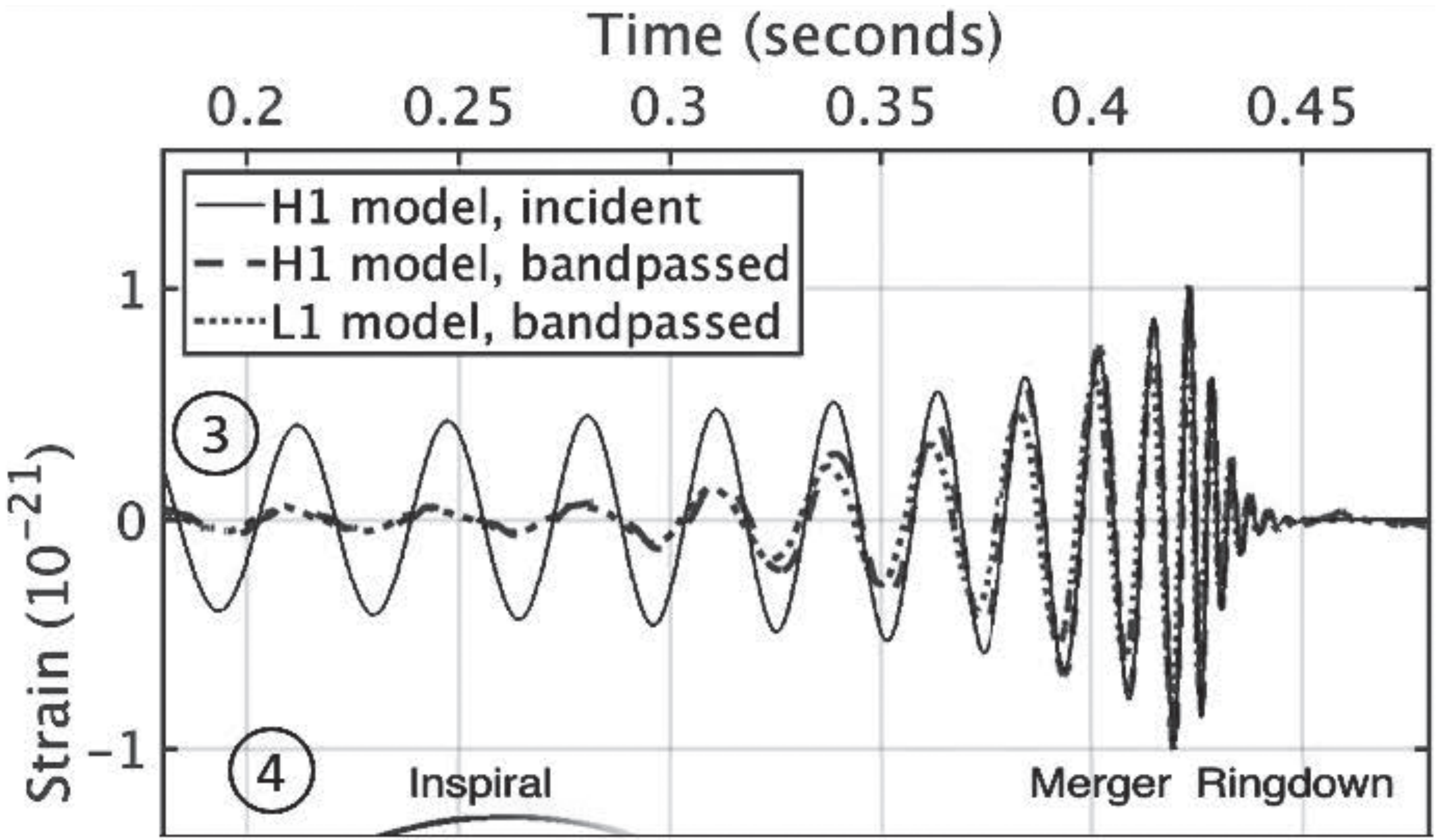
3. × 10^−19
2. × 10^−19
1. × 10^−19
0
−1. × 10^−19
−2. × 10^−19
−3. × 10^−19
0.00
0.02
0.04
0.06
0.08
0.10
0.12
t (s)

GW amplitude spectral density on Sept. 14 2015





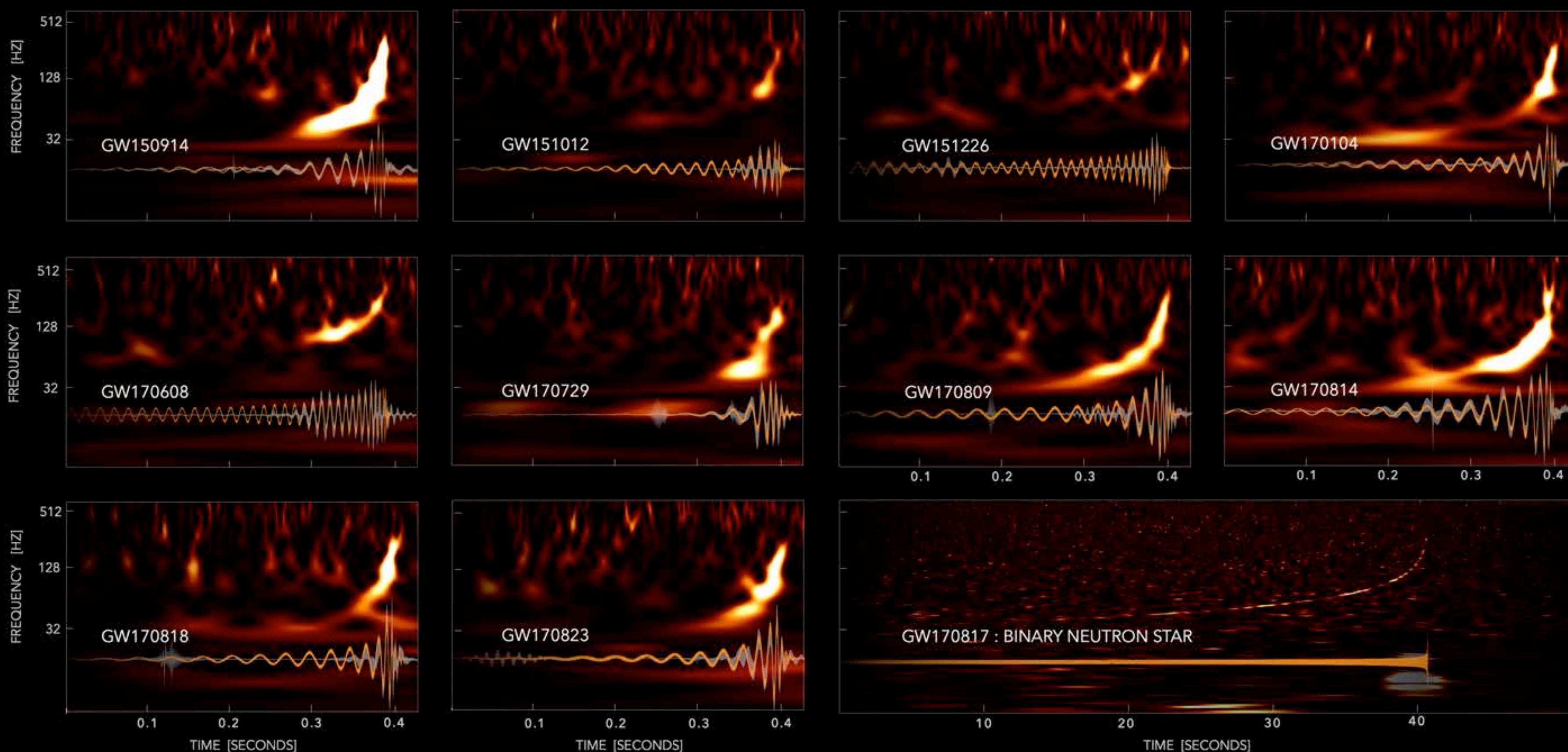
H1 strain, bandpassed
H1 strain, bandpassed
H1 model, bandpassed
Strain (10^{-21})
Time (seconds)
1
0
−1
0.2
0.25
0.3
0.35
0.4
0.45
①
L1 strain, bandpassed (and inverted/shifted)
L1 strain, bandpassed
L1 model, bandpassed
Strain (10^{-21})
Time (seconds)
②



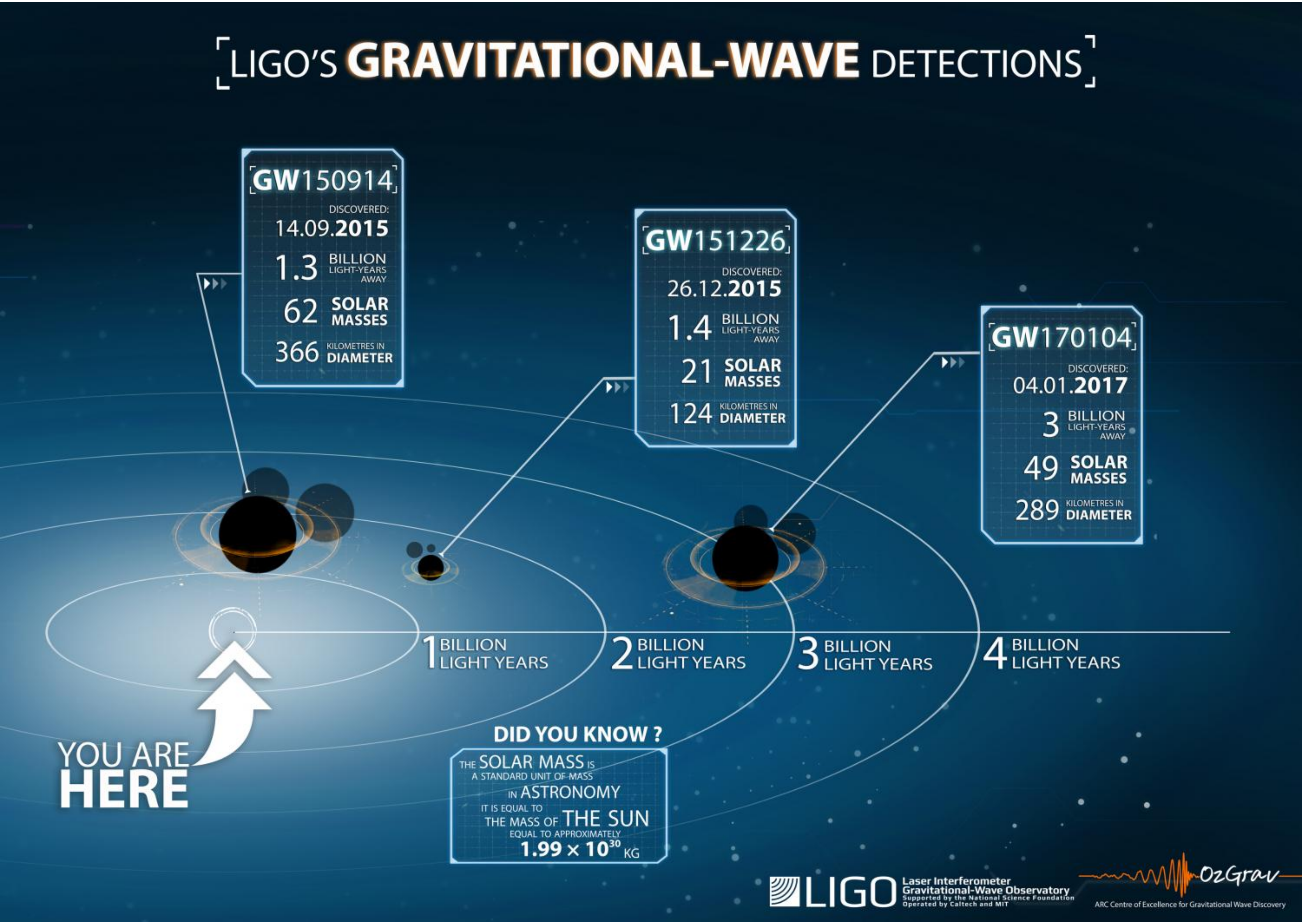
Time (seconds)
0.2
0.25
0.3
0.35
0.4
0.45
H1 model, incident
H1 model, bandpassed
L1 model, bandpassed
Strain (10^{-21})
1
0
−1
3
4
Inspiral
Merger
Ringdown

GRAVITATIONAL-WAVE TRANSIENT CATALOG-1

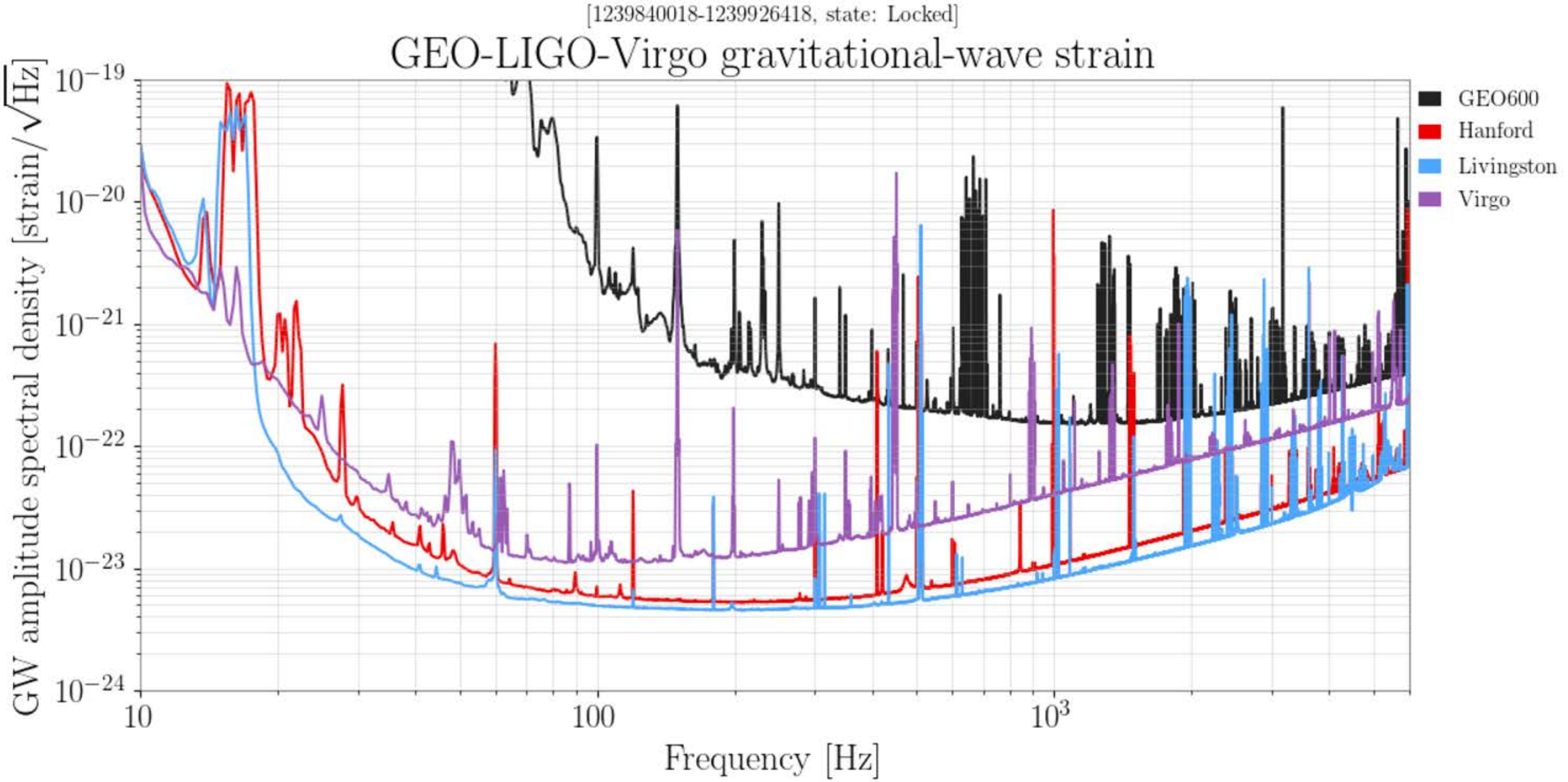
LIGO VIRGO NSF Georgia Tech



LIGO-VIRGO DATA: HTTPS://DOI.ORG/10.7935/82H3-HH23 **WAVELET (UNMODELED)** **EINSTEIN'S THEORY** **S. GHONGE, K. JANI | GEORGIA TECH**


LIGO'S GRAVITATIONAL-WAVE DETECTIONS
GW150914
DISCOVERED: 14.09.2015
1.3 BILLION LIGHT-YEARS AWAY
62 SOLAR MASSES
366 KILOMETRES IN DIAMETER
GW151226
DISCOVERED: 26.12.2015
1.4 BILLION LIGHT-YEARS AWAY
21 SOLAR MASSES
124 KILOMETRES IN DIAMETER
GW170104
DISCOVERED: 04.01.2017
3 BILLION LIGHT-YEARS AWAY
49 SOLAR MASSES
289 KILOMETRES IN DIAMETER
1 BILLION LIGHT YEARS
2 BILLION LIGHT YEARS
3 BILLION LIGHT YEARS
4 BILLION LIGHT YEARS
YOU ARE HERE
DID YOU KNOW ?
THE SOLAR MASS IS A STANDARD UNIT OF MASS IN ASTRONOMY
IT IS EQUAL TO THE MASS OF THE SUN
EQUAL TO APPROXIMATELY 1.99×10^{30} KG
LIGO Laser Interferometer Gravitational-Wave Observatory
Supported by the National Science Foundation
Operated by Caltech and MIT
OzGrav
ARC Centre of Excellence for Gravitational Wave Discovery

GW amplitude spectral density on April 21 2019



GraceDB — Gravitational Wave Candidate Event Database

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Latest — as of 3 May 2019 07:12:31 UTC

Test and MDC events and superevents are not included in the search results by default; see the query help for information on how to search for events and superevents in those categories.

Query:

Search for: Superevent

Search

UID	Labels	t_start	t_0	t_end	FAR (Hz)	UTC Created
S190426c	DQOK EMBRIGHT_READY PASTRO_READY SKYMAP_READY ADVOK GCN_PRELIM_SENT PE_READY	1240327332.331668	1240327333.348145	1240327334.353516	1.947e-08	2019-04-26 15:22:15 UTC
S190425z	DQOK SKYMAP_READY EMBRIGHT_READY PASTRO_READY ADVOK	1240215502.011549	1240215503.011549	1240215504.018242	4.538e-13	2019-04-25 08:18:26 UTC
S190421ar	DQOK EMBRIGHT_READY PASTRO_READY SKYMAP_READY GCN_PRELIM_SENT ADVOK	1239917953.250977	1239917954.409180	1239917955.409180	1.489e-08	2019-04-21 21:39:16 UTC
S190412m	DQOK SKYMAP_READY PASTRO_READY EMBRIGHT_READY ADVOK GCN_PRELIM_SENT PE_READY	1239082261.146717	1239082262.222168	1239082263.229492	1.683e-27	2019-04-12 05:31:03 UTC
S190408an	DQOK ADVOK SKYMAP_READY PASTRO_READY EMBRIGHT_READY GCN_PRELIM_SENT PE_READY	1238782699.268296	1238782700.287958	1238782701.359863	2.811e-18	2019-04-08 18:18:27 UTC
S190405ar	DQOK SKYMAP_READY EMBRIGHT_READY PASTRO_READY ADVNO	1238515307.863646	1238515308.863646	1238515309.863646	2.141e-04	2019-04-05 16:01:56 UTC

LIGO VIRGO LSC

MSc and PhD theses are available in the Virgo-TS group

The work done in the group deals with data analysis, in particular unmodeled analysis with the cWB (coherent WaveBurst) pipeline. The current focus of research in this field is on the full use of the many clues that are present in the signals to test GR and gather as much knowledge as possible about the sources.

If you are interested in the interferometer hardware, other theses topics are available in collaboration with several Italian groups.

For further information:

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milotti@ts.infn.it
Physics Dept., room 228, Via Valerio 2, Trieste