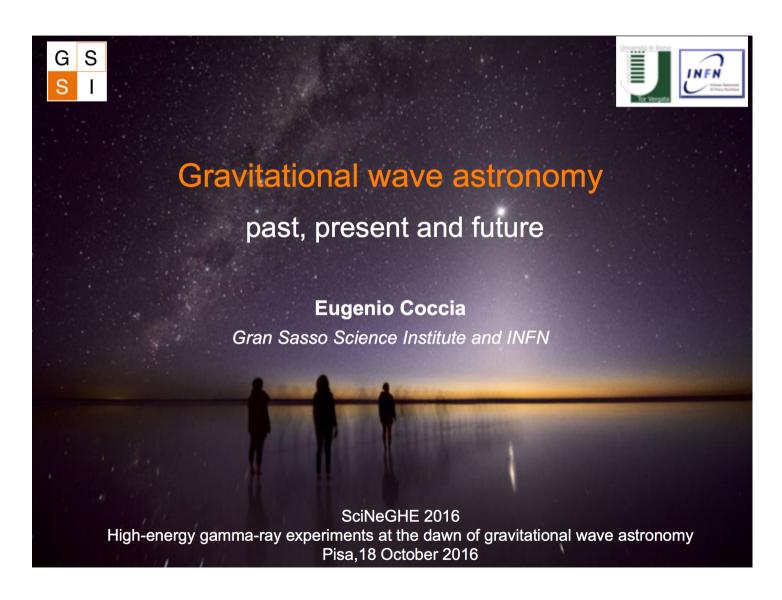
Astrofisica Nucleare e Subnucleare Gravitational Waves

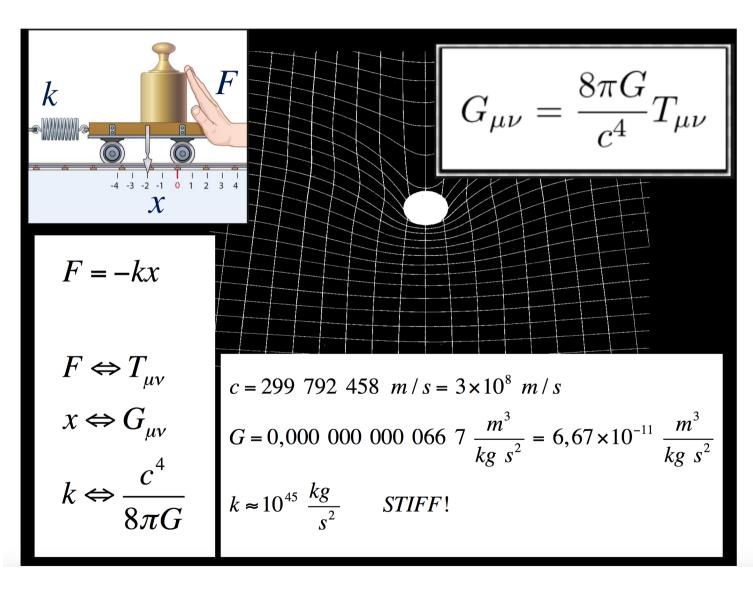
Exercise #3

- Find recent information on the status of LIGO,
 Virgo, KAGRA
- Find the status of eLISA GW observatory
- Find the status of PTA GW methods

Introduzione



Introduzione





Gravitational Waves Detection And Fourier Methods

ISAPP2012 Paris, France, July 2012

Patrice Hello

Laboratoire de l'Accélérateur Linéaire Orsay-France





What are Gravitational Waves?

Gravitational Waves (GW) are ripples of space-time

Theory of GW:

1. Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

2. Far from sources:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

3. Linearization:

$$g_{\mu\nu}$$
= $\eta_{\mu\nu}$ + $h_{\mu\nu}$

4. Gauge TT:

$$\nabla^2 \boldsymbol{h}_{\mu\nu}^{TT} = 0$$

Propagation of some tensor field -h - on flat space-time



Prediction in 1916!

Gravitational Wave general properties

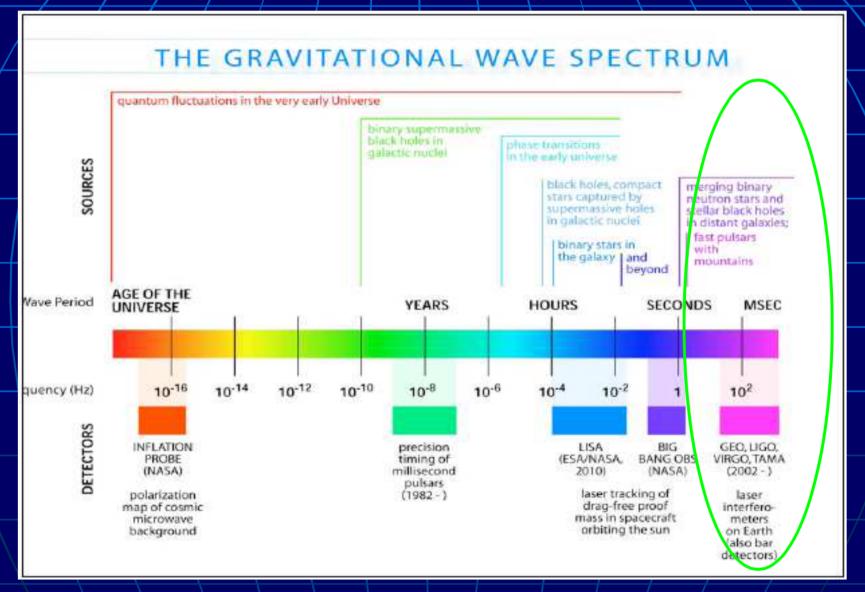
- GW propagate at speed of light
- GW have two polarizations "+" and "x"
- GW emission is quadrupolar at lowest order

Example: plane wave propagating along z axis with 2 polarization amplitudes h_+ and h_x :

$$h_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{+} & h_{\times} & 0 \\ 0 & h_{\times} & -h_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Corresponding *Graviton* properties:

- Graviton has null mass
- Graviton has spin 2





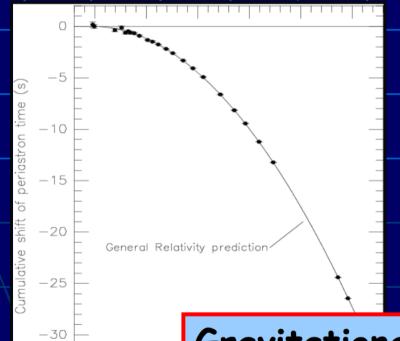
GW (indirect) discovery PSR 1913+16

(Hulse & Taylor, Nobel 93)

PSR 1913+16: binary pulsar (system of 2 neutron stars, one being a radio pulsar seen by radiotelescopes) at ~ 7 kpc from Earth.

— tests of Gravitation theory in strong field and dynamical regime

Loss of energy by GW emission : orbital period decreases



(merge in 300 billions years)

P (s)	27906.9807807(9)
dP/dt	-2.425(10)-10-12
dω/dt (º/yr)	4.226628(18)
M_{p}	1.442 ± 0.003 M
M_c	1.386 ± 0.003 M

Gravitational Waves do exist!

SUPAGWD

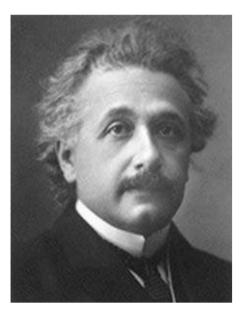
An Introduction to General Relativity, Gravitational Waves and Detection Principles

Prof Martin Hendry
University of Glasgow
Dept of Physics and Astronomy

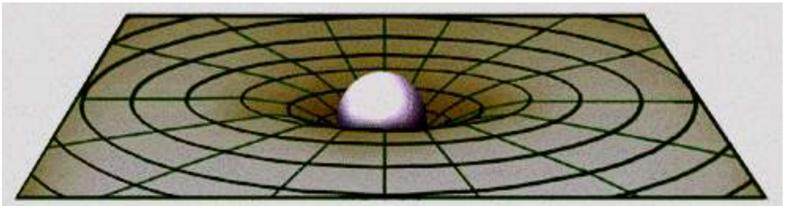
October 2012



Gravity in Einstein's Universe



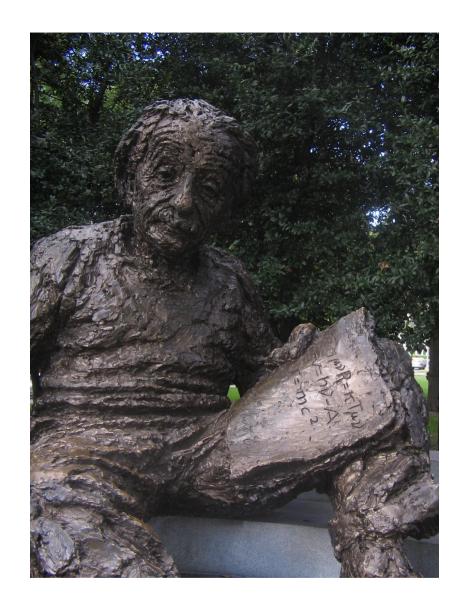
Spacetime tells matter how to move, and matter tells spacetime how to curve











"...joy and amazement at the beauty and grandeur of this world of which man can just form a faint notion."

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

Spacetime curvature Matter (and energy)







6. Wave Equation for Gravitational Radiation (pgs.46 - 57)

Weak gravitational fields

In the absence of a gravitational field, spacetime is flat. We define a weak gravitational field as one is which spacetime is 'nearly flat'

i.e. we can find a coord system such that

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

where

$$\eta_{\alpha\beta} = \text{diag} (-1, 1, 1, 1)$$

$$|h_{\alpha\beta}| \ll 1$$
 for all α and β

This is known as a Nearly Lorentz coordinate system.







1) Background Lorentz transformations

$$(t', x', y', z')^T = \begin{pmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (t, x, y, z)^T$$

i.e. Lorentz boost of speed v







1) Background Lorentz transformations

Hence, our original nearly Lorentz coordinate system remains nearly Lorentz in the new coordinate system. In other words, a spacetime which looks nearly flat to one observer still looks nearly flat to any other observer in uniform relative motion with respect to the first observer.







1) Background Lorentz transformations

Under this transformation

$$g'_{\alpha\beta} = \eta'_{\alpha\beta} + \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial x^{\nu}}{\partial x'^{\beta}} h_{\mu\nu} = \eta_{\alpha\beta} + h'_{\alpha\beta}$$

provided $v \ll 1$, then if $|h_{\alpha\beta}| \ll 1$ for all α and β , then $|h'_{\alpha\beta}| \ll 1$ also.







2) Gauge transformations

Suppose now we make a very small change in our coordinate system by applying a coordinate transformation of the form

$$x'^{\alpha} = x^{\alpha} + \xi^{\alpha}(x^{\beta})$$

we now demand that the ξ^{α} are small, in the sense that

$$|\xi^{\alpha}_{,\beta}| << 1$$
 for all α, β







2) Gauge transformations

Suppose now that the unprimed coordinate system is nearly Lorentz

Then
$${g'}_{\alpha\beta}=\eta_{\alpha\beta}+h_{\alpha\beta}-\xi_{\alpha,\beta}-\xi_{\beta,\alpha}$$

and we can write
$$h'_{\alpha\beta} = h_{\alpha\beta} - \xi_{\alpha,\beta} - \xi_{\beta,\alpha}$$







2) Gauge transformations

The above results tell us that – once we have identified a coordinate system which is nearly Lorentz – we can add an arbitrary small vector ξ^{α} to the coordinates x^{α} without altering the validity of our assumption that spacetime is nearly flat. We can, therefore, choose the components ξ^{α} to make Einstein's equations as simple as possible. We call this step choosing a gauge for the problem – a name which has resonance with a similar procedure in electromagnetism – and coordinate transformations of this type given by equation are known as gauge transformation. We will consider below specific choices of gauge which are particularly useful.







Einstein's equations for a weak gravitational field

The Einstein tensor is the (rather messy) expression

$$G_{\mu\nu} = \frac{1}{2} \left[h_{\mu\alpha,\nu}^{\ ,\alpha} + h_{\nu\alpha,\mu}^{\ ,\alpha} - h_{\mu\nu,\alpha}^{\ ,\alpha} - h_{,\mu\nu} - \eta_{\mu\nu} \left(h_{\alpha\beta}^{\ ,\alpha\beta} - h_{,\beta}^{\ ,\beta} \right) \right]$$

but we can simplify this by introducing

$$\overline{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

So that

$$G_{\mu\nu} = -\frac{1}{2} \left[\overline{h}_{\mu\nu,\alpha}^{,\alpha} + \eta_{\mu\nu} \overline{h}_{\alpha\beta}^{,\alpha\beta} - \overline{h}_{\mu\alpha,\nu}^{,\alpha} - \overline{h}_{\nu\alpha,\mu}^{,\alpha} \right]$$

And we can choose the **Lorentz gauge** to eliminate the last 3 terms







Einstein's equations for a weak gravitational field

To first order, the R-C tensor for a weak field reduces to

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2} \left(h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\alpha\gamma,\beta\delta} - h_{\beta\delta,\alpha\gamma} \right)$$

and is invariant under gauge transformations.

Similarly, the Ricci tensor is

$$R_{\mu\nu} = \frac{1}{2} \left(h^{\alpha}_{\mu,\nu\alpha} + h^{\alpha}_{\nu,\mu\alpha} - h_{\mu\nu,\alpha}^{\alpha} - h_{,\mu\nu} \right)$$

where

$$h \equiv h_{\alpha}^{\alpha} = \eta^{\alpha\beta} h_{\alpha\beta}$$

$$h_{\mu\nu,\alpha}^{\alpha,\alpha} = \eta^{\alpha\sigma} (h_{\mu\nu,\alpha})_{,\sigma} = \eta^{\alpha\sigma} h_{\mu\nu,\alpha\sigma}$$







In the Lorentz gauge, then Einstein's equations are simply

$$-\overline{h}_{\mu\nu,\alpha}^{,\alpha} = 16\pi T_{\mu\nu}$$

And in free space this gives

$$\overline{h}_{\mu\nu,\alpha}^{,\alpha} = 0$$

Writing
$$\overline{h}_{\mu\nu,\alpha}^{,\alpha} \equiv \eta^{\alpha\alpha}\overline{h}_{\mu\nu,\alpha\alpha}$$

or

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \overline{h}_{\mu\nu} = 0$$







Remembering that we are taking c = 1, if instead we write

$$\eta^{00} = -\frac{1}{c^2}$$

then

$$\left(-\frac{\partial^2}{\partial t^2} + c^2 \nabla^2\right) \overline{h}_{\mu\nu} = 0$$

This is a key result. It has the mathematical form of a wave equation, propagating with speed c.

We have shown that the metric perturbations – the 'ripples' in spacetime produced by disturbing the metric – propagate at the speed of light as waves in free space.







7. The Transverse Traceless Gauge (pgs.57 - 62)

Simplest solutions of our wave equation are plane waves

$$\overline{h}_{\mu\nu} = \operatorname{Re}\left[A_{\mu\nu} \, \exp\left(ik_{\alpha}x^{\alpha}\right)\right]$$

Wave vector Wave amplitude

Note the wave amplitude is symmetric \rightarrow 10 independent components.

Also, easy to show that

$$k_{\alpha} k^{\alpha} = 0$$

i.e. the wave vector is a **null** vector







Thus

$$\omega = k^t = (k_x^2 + k_y^2 + k_z^2)^{1/2}$$

Also, from the Lorentz gauge condition

$$\overline{h}^{\mu\alpha}_{,\alpha} = 0$$

which implies that
$$A_{\mu\alpha}\,k^{lpha}=0$$

i.e. the wave amplitude components must be orthogonal to the wave vector \mathbf{k} .

But this is 4 equations, one for each value of the index μ

Hence, we can eliminate 4 more of the wave amplitude components.







Can we do better? Yes

Our choice of Lorentz gauge, chosen to simplify Einstein's equations, was not unique. We can make small adjustments to our original Lorentz gauge transformation and still satisfy the Lorentz condition.

We can choose adjustments that will make our wave amplitude components even simpler – we call this choice the **Transverse Traceless** gauge:

$$A^{\mu}_{\mu} = \eta^{\mu\nu} A_{\mu\nu} = 0 \qquad \text{(traceless)}$$

$$A_{\alpha t} = 0$$
 for all α







Suppose we orient our coordinate axes so that the plane wave is travelling in the positive z direction. Then

$$k^t = \omega$$
, $k^x = k^y = 0$, $k^z = \omega$

and

$$A_{\alpha z} = 0$$
 for all α

i.e. there is no component of the metric perturbation in the direction of propagation of the wave. This explains the origin of the 'Transverse' part







So in the transverse traceless gauge,

$$\overline{h}_{\mu\nu}^{(\mathrm{TT})} = A_{\mu\nu}^{(\mathrm{TT})} \cos \left[\omega(t-z)\right]$$

where

$$A_{\mu\nu}^{(\text{TT})} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx}^{(\text{TT})} & A_{xy}^{(\text{TT})} & 0 \\ 0 & A_{xy}^{(\text{TT})} & -A_{xx}^{(\text{TT})} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Also, since the perturbation is traceless

$$\overline{h}_{\alpha\beta}^{(\mathrm{TT})} = h_{\alpha\beta}^{(\mathrm{TT})}$$







8. Effect of Gravitational Waves on Free Particles (pgs.63 - 75)

Choose Background Lorentz frame in which test particle initially at rest. Set up coordinate system according to the TT gauge.

Initial acceleration satisfies

$$\left(\frac{dU^{\beta}}{d\tau}\right)_0 = 0$$

i.e. coordinates do not change, but adjust themselves as wave passes so that particles remain 'attached' to initial positions.

Coordinates are frame-dependent labels.

What about **proper distance** between neighbouring particles?







Consider two test particles, both initially at rest, one at origin and the other at $\ x=\epsilon\,,\ y=z=0$

$$\Delta \ell = \int \left| g_{\alpha\beta} dx^{\alpha} dx^{\beta} \right|^{1/2}$$

i.e.
$$\Delta \ell = \int_0^\epsilon |g_{xx}|^{1/2} \simeq \sqrt{g_{xx}(x=0)} \ \epsilon$$

Now
$$g_{xx}(x=0) = \eta_{xx} + h_{xx}^{(TT)}(x=0)$$

$$\Delta \ell \simeq \left[1 + \frac{1}{2} h_{xx}^{(\mathrm{TT})}(x=0)\right] \epsilon$$

In general, this is time-varying







More formally, consider geodesic deviation ξ^{α} between two particles, initially at rest

i.e. initially with
$$U^{\mu}=(1,0,0,0)^T$$
 $\xi^{\beta}=(0,\epsilon,0,0)^T$

Then
$$\frac{\partial^2 \xi^\alpha}{\partial t^2} = \epsilon R^\alpha_{ttx} = -\epsilon R^\alpha_{txt}$$

and
$$R^x_{txt} = \eta^{xx} R_{xtxt} = -\frac{1}{2} h^{(\mathrm{TT})}_{xx,tt}$$

$$R_{txt}^y = \eta^{yy} R_{ytxt} = -\frac{1}{2} h_{xy,tt}^{(TT)}$$

Hence
$$\frac{\partial^2}{\partial t^2} \xi^x = \frac{1}{2} \epsilon \frac{\partial^2}{\partial t^2} h_{xx}^{(\mathrm{TT})} \qquad \frac{\partial^2}{\partial t^2} \xi^y = \frac{1}{2} \epsilon \frac{\partial^2}{\partial t^2} h_{xy}^{(\mathrm{TT})}$$







Similarly, two test particles initially separated by ϵ in the y-direction satisfy

$$\frac{\partial^2}{\partial t^2} \xi^x = \frac{1}{2} \epsilon \frac{\partial^2}{\partial t^2} h_{xy}^{(\text{TT})} \qquad \frac{\partial^2}{\partial t^2} \xi^y = -\frac{1}{2} \epsilon \frac{\partial^2}{\partial t^2} h_{xx}^{(\text{TT})}$$

We can further generalise to a ring of test particles: one at origin, the other initially a :

$$x = \epsilon \cos \theta$$
 $y = \epsilon \sin \theta$ $z = 0$

$$\frac{\partial^2}{\partial t^2} \xi^x = \frac{1}{2} \epsilon \cos \theta \frac{\partial^2}{\partial t^2} h_{xx}^{(TT)} + \frac{1}{2} \epsilon \sin \theta \frac{\partial^2}{\partial t^2} h_{xy}^{(TT)}$$

$$\frac{\partial^2}{\partial t^2} \xi^y = \frac{1}{2} \epsilon \cos \theta \frac{\partial^2}{\partial t^2} h_{xy}^{(\text{TT})} - \frac{1}{2} \epsilon \sin \theta \frac{\partial^2}{\partial t^2} h_{xx}^{(\text{TT})}$$







Solutions are:

$$\xi^{x} = \epsilon \cos \theta + \frac{1}{2} \epsilon \cos \theta A_{xx}^{(TT)} \cos \omega t + \frac{1}{2} \epsilon \sin \theta A_{xy}^{(TT)} \cos \omega t$$

$$\xi^{y} = \epsilon \sin \theta + \frac{1}{2} \epsilon \cos \theta A_{xy}^{(TT)} \cos \omega t - \frac{1}{2} \epsilon \sin \theta A_{xx}^{(TT)} \cos \omega t$$

Suppose we now vary θ between 0 and 2π , so that we are considering an initially circular ring of test particles in the x-y plane, initially equidistant from the origin.

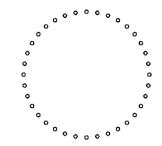






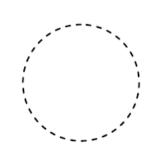
$$A_{xx}^{(\mathrm{TT})} \neq 0 \qquad A_{xy}^{(\mathrm{TT})} = 0$$

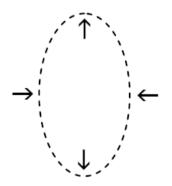
$$\xi^{x} = \epsilon \cos \theta \left(1 + \frac{1}{2} A_{xx}^{(TT)} \cos \omega t \right)$$

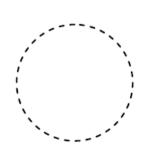


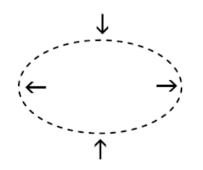
$$\xi^y = \epsilon \sin \theta \left(1 - \frac{1}{2} A_{xx}^{(TT)} \cos \omega t \right)$$

$$A_{xx}^{(TT)} \neq 0$$
 + Polarisation











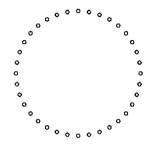






$$A_{xy}^{(\mathrm{TT})} \neq 0 \qquad A_{xx}^{(\mathrm{TT})} = 0$$

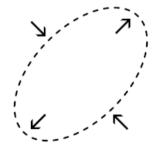
$$\xi^{x} = \epsilon \cos \theta + \frac{1}{2} \epsilon \sin \theta A_{xy}^{(TT)} \cos \omega t$$

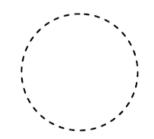


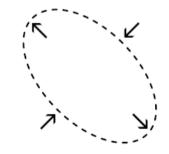
$$\xi^{y} = \epsilon \sin \theta + \frac{1}{2} \epsilon \cos \theta A_{xy}^{(TT)} \cos \omega t$$

$$A_{xy}^{(TT)} \neq 0$$
 \times Polarisation

















Rotating axes through an angle of $-\pi/4$ to define

$$x' = \frac{1}{\sqrt{2}} \left(x - y \right)$$

We find that

$$\xi'^{x} = \epsilon \cos \left(\theta + \frac{\pi}{4}\right) + \frac{1}{2} \epsilon \sin \left(\theta + \frac{\pi}{4}\right) A_{xy}^{(TT)} \cos \omega t$$

$$y' = \frac{1}{\sqrt{2}} \left(x + y \right)$$

$$\xi'^{y} = \epsilon \sin\left(\theta + \frac{\pi}{4}\right) + \frac{1}{2}\epsilon \cos\left(\theta + \frac{\pi}{4}\right) A_{xy}^{(TT)} \cos\omega t$$

These are identical to earlier solution, apart from rotation.







• The two solutions, for $A_{xx}^{(\text{TT})} \neq 0$ and $A_{xy}^{(\text{TT})} \neq 0$ represent two independent gravitational wave **polarisation states**, and these states are usually denoted by '+' and '×' respectively. In general any gravitational wave propagating along the z-axis can be expressed as a linear combination of the '+' and '×' polarisations, i.e. we can write the wave as

$$\mathbf{h} = a \mathbf{e}_{+} + b \mathbf{e}_{\times}$$

where a and b are scalar constants and the polarisation tensors $\mathbf{e_+}$ and $\mathbf{e_{\times}}$ are

$$\mathbf{e}_{+} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{e}_{\times} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$







- Distortions are quadrupolar consequence of fact that acceleration of geodesic deviation non-zero only for tidal gravitational field.
- At any instant, a gravitational wave is invariant under a rotation of 180 degrees about its direction of propagation.
 (c.f. spin states of gauge bosons; graviton must be S=2, tensor field)







Chapter 14
Measurement of Classical Gravitation Fields
Felix Pirani

Because of the principle of equivalence, one cannot ascribe a direct physical interpretation to the gravitational field insofar as it is characterized by Christoffel symbols $\Gamma^{\mu}_{\nu\rho}$. One can, however, give an invariant interpretation to the variations of the gravitational field. These variations are described by the Riemann tensor; therefore, measurements of the relative acceleration of neighboring free particles, which yield information about the variation of the field, will also yield information about the Riemann tensor.

Now the relative motion of free particles is given by the equation of geodesic deviation

$$\frac{\partial^2 \eta^{\mu}}{\partial \tau^2} + R^{\mu}_{\nu\rho\sigma} \nu^{\nu} \eta^{\rho} \nu^{\sigma} = 0 \quad (\mu, \nu, \rho, \sigma = 1, 2, 3, 4) \quad (14.1)$$

Here η^{μ} is the infinitesimal orthogonal displacement from the (geodesic) worldline ζ of a free particle to that of a neighboring similar particle. ν^{ν} is the 4-velocity of the first particle, and τ the proper time along ζ . If now one introduces an orthonormal frame on ζ , ν^{μ} being the timelike vector of the frame, and assumes that the frame is parallelly propagated along ζ (which insures that an observer using this frame will see things in as Newtonian a way as possible) then the equation of geodesic deviation (14.1) becomes

$$\frac{\partial^2 \eta^a}{\partial \tau^2} + R^a_{0b0} \eta^b = 0 \quad (a, b = 1, 2, 3,)$$
 (14.2)

Here η^a are the physical components of the infinitesimal displacement and R^a_{0b0} some of the physical components of the Riemann tensor, referred to the orthonormal frame.

By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One 14. Measurement of Classical Gravitation Fields

can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor.

Now the Newtonian equation corresponding to (14.2) is

$$\frac{\partial^2 \eta^a}{\partial \tau^2} + \frac{\partial^2 v}{\partial x^a \partial x^b} \eta^b = 0 \tag{14.3}$$

It is interesting that the empty-space field equations in the Newtonian and general relativity theories take the same form when one recognizes the correspondence $R^a_{0b0} \sim \frac{\partial^2 \nu}{\partial x^a \partial x^b}$ between equations (14.2) and (14.3), for the respective empty-space equations may be written $R^a_{0a0} = 0$ and $\frac{\partial^2 \nu}{\partial x^a \partial x^b} = 0$. (Details of this work are in the course of publication in Acta Physica Polonica.)

BONDI: Can one construct in this way an absorber for gravitational energy by inserting a $\frac{d\eta}{d\eta}$ term, to learn what part of the Riemann tensor would be the energy producing one, because it is that part that we want to isolate to study gravitational waves?

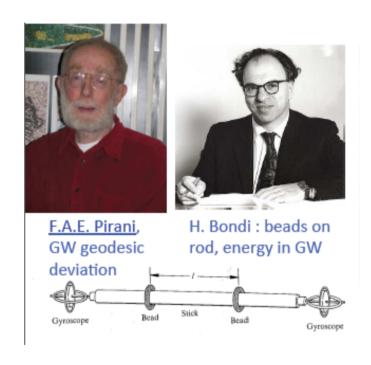
PIRANI: I have not put in an absorption term, but I have put in a "spring."

You can invent a system with such a term quite easily.

LICHNEROWICZ: Is it possible to study stability problems for η ?

PIRANI: It is the same as the stability problem in classical mechanics, but I haven't tried to see for which kind of Riemann tensor it would blow up.

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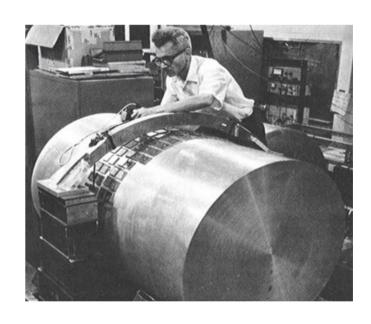


The main point of this presentation was that it is relative accelerations of neighboring free particles that are the physically meaningful (i.e.,measurable) ways to observe gravitational effects. Pirani points out the transparent connection between the equation of geodesic deviation and Newton's Second Law, as long as one identifies \mathbf{R}_{a0b0} with the second derivative of the Newtonian potential (i.e., as the tidal field.)

To make sure everyone sees how important and simple this is, he remarks, "By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor".

from: P. Saulson, Gen Relativ Gravit (2011) 43:3289-3299

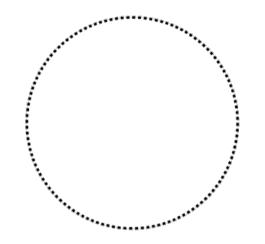
Weber's bar



Weber's detector embodied Pirani's gedankenexperiment.

It was a cylinder of aluminum, each end of which is like a test mass, while the center is like a spring. PZT's around the midline absorb energy to send to an electrical amplifier.

Design of gravitational wave detectors



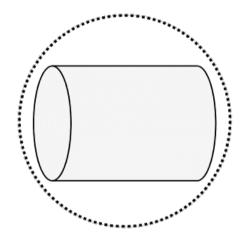








Design of gravitational wave detectors











Weber started seeing things

In 1969, Weber made his first of many announcements that he was seeing coincident excitations of two detectors.

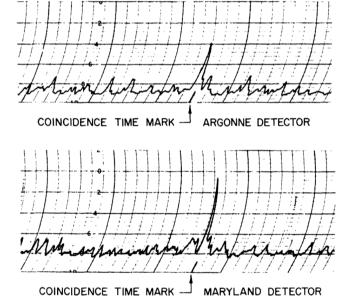
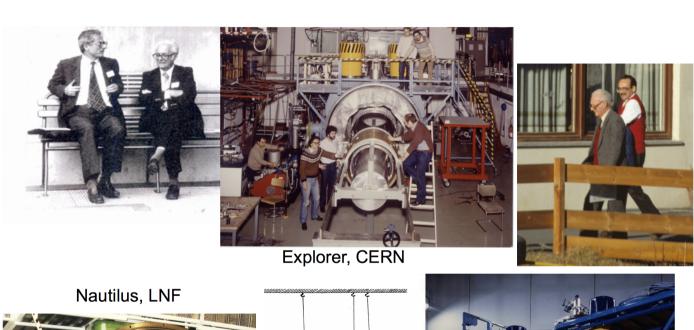
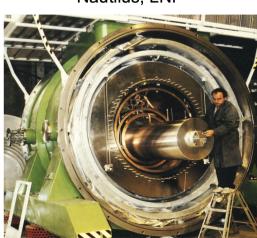
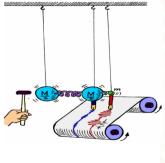


FIG. 2. Argonne National Laboratory and University of Maryland detector coincidence.



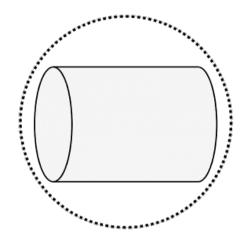


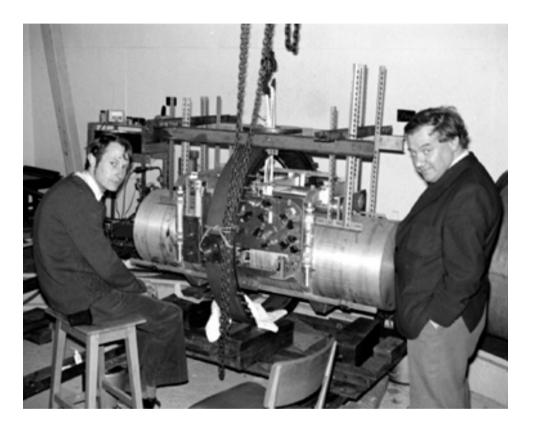




Auriga, LNL

Design of gravitational wave detectors



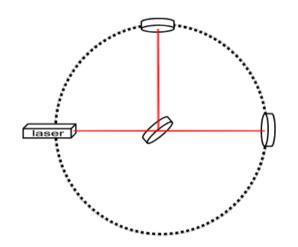








34 yrs on - Interferometric ground-based detectors







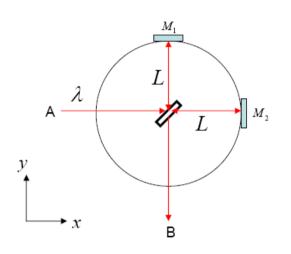


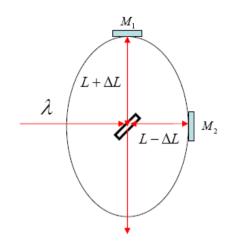


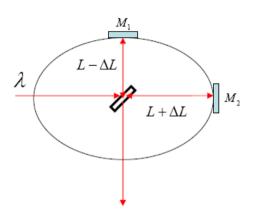












Gravitational wave

 $\mathbf{h} = h_{\mathbf{P}}$ popagating along z axis.

Fractional change in proper separation

$$\frac{\Delta L}{L} = \frac{h}{2}$$







More generally, for $h = h e_+$

$$\mathbf{h} = h \, \mathbf{e}_+$$

Detector 'sees'

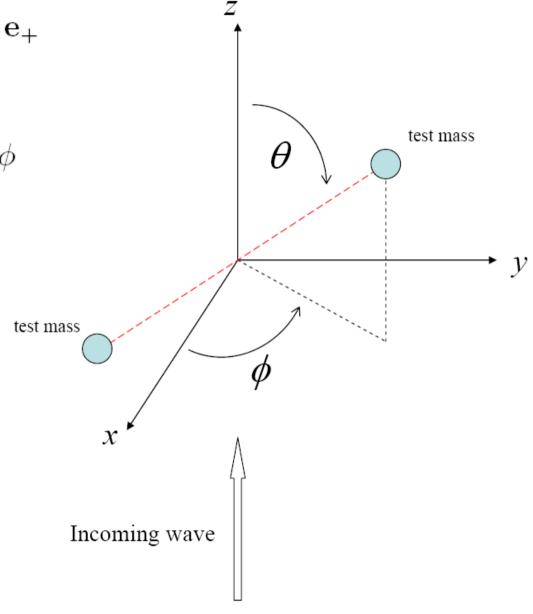
$$h_{+} = h \sin^2 \theta \cos 2\phi$$

Maximum response for

$$\theta = \pi/2$$
 $\phi = 0$

Null response for

$$\theta = 0$$
 $\phi = \pi/4$









More generally, for $\mathbf{h} = h \, \mathbf{e}_{\times}$

$$\mathbf{h} = h \, \mathbf{e}_{\times}$$

Detector 'sees'

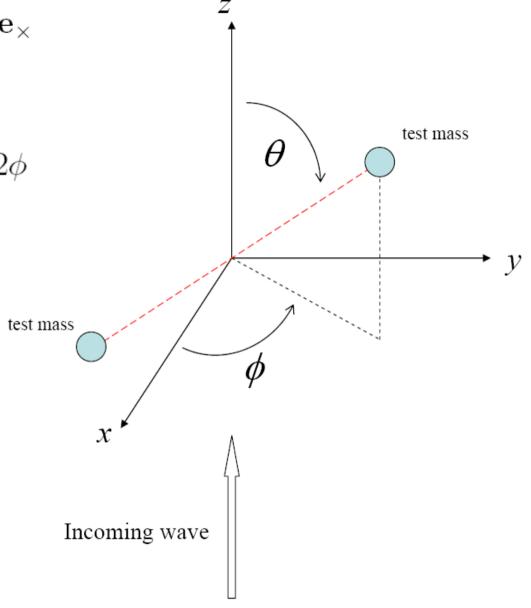
$$h_{\times} = h \sin^2 \theta \sin 2\phi$$

Maximum response for

$$\theta = \pi/2$$
 $\phi = \pi/4$

Null response for

$$\theta = 0$$
 $\phi = 0$









Astrofisica Nucleare e Subnucleare Sources of GW

Gravitational Wave emission

(quadrupole formalism)

Emission equation in the TT Gauge: $\nabla^2 h_{\mu\nu}^{TT} = -\frac{16\pi G}{4} T_{\mu\nu}$

$$\nabla^2 h_{\mu\nu}^{TT} = -\frac{16\pi G}{c} T_{\mu\nu}$$

Retarded solution:

$$h_{\mu\nu}^{TT}(\vec{x},t) = \frac{2G}{Rc^4} \ddot{Q}_{\mu\nu}^{TT}(t-R/c)$$

Hence:

$$h_{+}^{TT}(\vec{x},t) = \frac{G}{Rc^{4}} \left[\ddot{Q}_{11}^{TT} - \ddot{Q}_{22}^{TT} \right] (t - R/c) \qquad h_{\times}^{TT}(\vec{x},t) = \frac{2G}{Rc^{4}} \left[\ddot{Q}_{12}^{TT} \right] (t - R/c)$$

$$h_{\star}^{TT}(\vec{x},t) = \frac{2G}{Rc^4} \ddot{Q}_{12}^{TT} \int (t-R/c)$$

Where the **reduced quadrupole** moment:

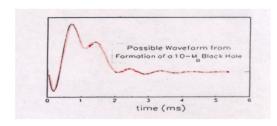
$$Q_{\mu\nu}^{TT} = \iiint d^{3}x \, \rho \, (x_{\mu} \, x_{\nu} - \frac{1}{3} \delta_{\mu\nu} r^{2})$$

Regular quadrupole (inertia) moment:

$$q_{\mu\nu} = \iiint d^3 x \rho x_{\mu} x_{\nu}$$

 $\rho \sim T_{00}/c^2$: density of the source

Tipi di segnale

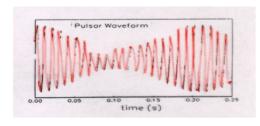


SUPERNOVAE.

If the collapse core is non-symmetrical, the event can give off considerable radiation in a millisecond timescale.

Information

Inner detailed dynamics of supernova See NS and BH being formed Nuclear physics at high density

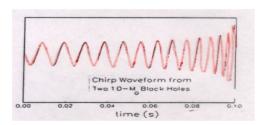


SPINNING NEUTRON STARS.

Pulsars are rapidly spinning neutron stars. If they have an irregular shape, they give off a signal at constant frequency (prec./Dpl.)

Information

Neutron star locations near the Earth Neutron star Physics Pulsar evolution

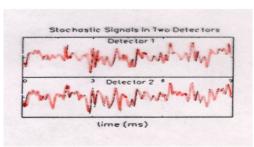


COALESCING BINARIES.

Two compact objects (NS or BH) spiraling together from a binary orbit give a chirp signal, whose shape identifies the masses and the distance

Information

Masses of the objects
BH identification
Distance to the system
Hubble constant
Test of strong-field general relativity



STOCHASTIC BACKGROUND.

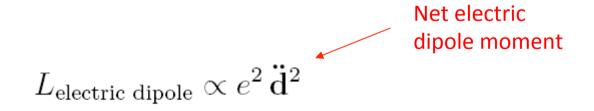
Random background, relic of the early universe and depending on unknown particle physics. It will look like noise in any one detector, but two detectors will be correlated.

Information

Confirmation of Big Bang, and inflation Unique probe to the Planck epoch Existence of cosmic strings

9. The Production of Gravitational Waves (pgs 76 - 80)

We can understand something important about the nature of gravitational radiation by drawing analogies with the formulae that describe electromagnetic radiation. This approach is crude at best since the electromagnetic field is a vector field while the gravitational field is a tensor field, but it is good enough for our present purposes. Essentially, we will take familiar electromagnetic radiation formulae and simply replace the terms which involve the Coulomb force by their gravitational analogues from Newtonian theory.









$$L_{\rm magnetic\ dipole} \propto \ddot{\mu}$$

$$\mu = \sum_{q_i}$$
 (position of q_i) × (current due to q_i)

Gravitational analogues?...

Mass dipole moment:
$$\mathbf{d} = \sum_{A_i} m_i \mathbf{x}_i$$

But
$$\dot{\mathbf{d}} = \sum_{A_i} m_i \dot{\mathbf{x}}_i \equiv \mathbf{p}$$

Conservation of linear momentum implies no mass dipole radiation







$L_{\rm magnetic\ dipole} \propto \ddot{\mu}$

$$\mu = \sum_{q_i}$$
 (position of q_i) × (current due to q_i)

Gravitational analogues?...

$$\mu = \sum_{A_i} (\mathbf{x}_i) \times (m_i \mathbf{v}_i) \equiv \mathbf{J}$$

Conservation of angular momentum implies no mass dipole radiation







Also, the quadrupole of a **spherically symmetric mass distribution** is zero.

Metric perturbations which are spherically symmetric don't produce gravitational radiation.

Example: binary neutron star system.

$$h_{\mu\nu} = \frac{2G}{c^4 r} \ddot{I}_{\mu\nu}$$

where $I_{\mu\nu}$ is the **reduced quadrupole moment** defined as

$$I_{\mu\nu} = \int \rho(\vec{r}) \left(x_{\mu} x_{\nu} - \frac{1}{3} \delta_{\mu\nu} r^2 \right) dV$$







Consider a binary neutron star system consisting of two stars both of Schwarzschild mass M, in a circular orbit of coordinate radius R and orbital frequency f.

$$I_{xx} = 2MR^2 \left[\cos^2(2\pi ft) - \frac{1}{3} \right]$$

$$I_{yy} = 2MR^2 \left[\sin^2(2\pi f t) - \frac{1}{3} \right]$$

$$I_{xy} = I_{yx} = 2MR^2 \left[\cos(2\pi ft)\sin(2\pi ft)\right]$$







Thus

$$h_{xx} = -h_{yy} = h\cos(4\pi ft)$$

$$h_{xy} = h_{yx} = -h\sin\left(4\pi ft\right)$$

where

$$h = \frac{32\pi^2 GMR^2 f^2}{c^4 r}$$

So the binary system emits gravitational waves at **twice** the orbital frequency of the neutron stars.

Also
$$h = 2.3 \times 10^{-28} \ \frac{R^2 [\text{km}] f^2 [\text{Hz}]}{r [\text{Mpc}]}$$









Gravitational Wave emission: an example

2 identical point masses in circular orbit around their center of mass

- Orbital plane : xOy
- Mass : *M*
- Orbit radius : a
- Orbital frequency : $f_0 = 2\pi \omega_0$

Q: Compute the 2 amplitudes $h_+(t)$ and $h_x(t)$ at a distance r on the z axis (without taking into account the radiation reaction!)

X

Gravitational Wave emission: an example

Positions of the two masses:

$$x_1(t) = a\cos(\omega_0 t)$$

$$x_1(t) = a\cos(\omega_0 t)$$
 $x_2(t) = -a\cos(\omega_0 t)$

$$v(t) = a$$

$$y_1(t) = a\sin(\omega_0 t)$$
 $y_2(t) = -a\sin(\omega_0 t)$

$$y_2(t) = -a\sin(\omega_0 t)$$

So compute the reduced inertia tensor: $Q = ma^2 \sin(2\omega_0 t) \quad ma^2 \left(\frac{1}{3} - \cos(2\omega_0 t)\right) \quad 0$

$$Q = \begin{pmatrix} ma^{2}(\frac{1}{3} + \cos(2\omega_{0}t)) & ma^{2}\sin(2\omega_{0}t) & 0 \\ ma^{2}\sin(2\omega_{0}t) & ma^{2}(\frac{1}{3} - \cos(2\omega_{0}t)) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

After projection on the z direction:

$$h_{+}(t) = -\frac{2G}{rc^{4}} ma^{2} \omega^{2} \cos(\omega(t - r/c))$$

$$h_{\star}(t) = -\frac{2G}{rc^4} ma^2 \omega^2 \sin(\omega(t - r/c))$$

Where $\omega = 2\omega_0$ is **TWICE** the orbital angular frequency

Note that if we look on the x direction:

$$h_{+}(t) = -\frac{G}{rc^{4}} ma^{2} \omega^{2} \cos(\omega(t - r/c))$$

$$h_{\star}(t) = 0$$

Face-on binary => circular polarization Edge-on binary => **linear** polarization

Gravitational Wave emission: Orders of magnitude

Luminøsity (Einstein quadrupole formula):

$$P = \frac{G}{5c^5} \left\langle \ddot{Q}_{\mu\nu} \ddot{Q}^{\mu\nu} \right\rangle$$

 $G/5c^5 \sim 10^{-53} \text{ W}^{-1}$

Factor ridiculously « small »!

	S	ource	;		dis	tance	lh		P (W)
Stee	l bar,	500	Γ, Ø =	2 m	1	m	2x1	0-34	10 -29
L = 20 m, 5 cycles/s									
H	bomb,	1 me	gatonn	e	10	km	2x1) -39	10-11
Asymmetry 10%									
Superno	va 10	M_{\odot} as	ymme	try 3%	10	Мрс	10	-21	10/44
Coalesco	ence 2	black	holes	10 M _☉	10	Мрс	10	-20	1,050
\	\								

Gravitational Wave emission and compact stars

© J. Weber (1974)

Pb: G/c^5 is very « small ». c^5/G would be much better !!!

Source: mass M, size R, period T, asymmetry $a \Rightarrow \ddot{Q} \approx a M R^2 / T^3$

Quadrupole formula becomes:

$$P \approx \frac{G}{c^5} a^2 \frac{M^2 R^4}{T^6}$$

New parameters

- · caracteristic speed v
- Schwarzchild Radius $R_s = 2GM/c^2$

$$P \approx \frac{c^5}{G} a^2 \left(\frac{R_S}{R}\right)^2 \left(\frac{v}{c}\right)^6$$

Huge luminosity if

- $\bullet R \rightarrow R_s$
- $\bullet v \rightarrow c$
- $a \rightarrow 1$

compact stars

Gravitational Wave sources Compact stars

"High frequency" sources (f > 1 Hz)

- supernovae (bursts)
- binary inspirals (chirps)
- black holes ringdowns (damped sine)
- isolated neutron stars, pulsars (periodic sources)
- stochastic background (stochastic)
-

Amplitudes h(t) on Earth? Rate of events?

Gravitational Supernovae

type II SN = gravitational collapse of the core (Fe) of a massive star (> 10 M_{\odot}) after having burned all the H fuel \rightarrow neutron star formation

GW Emission ? Depends on asymmetry (poorly known)

Sources of asymmetry · fast rotation (instabilities)

companion star

Modern models:

h ~ 10⁻²³ @ 10 Mpc
f peaks between 0.3 and 1 kHz
1 SN/ 40 yrs / galaxy

Black hole formation:

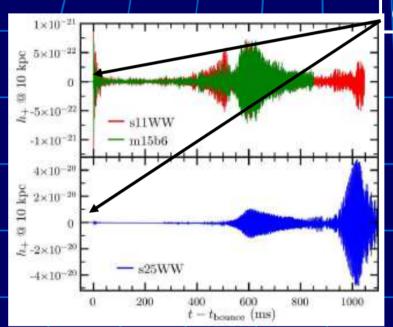
Progenitor too massive > collapse > black hole

 $h \sim 10^{-22}$ @ 10 Mpc f > 1 kHz

+ oscillations...

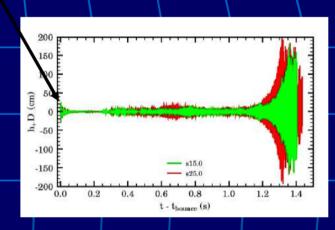
Gravitational Supernovae: GW amplitudes

+ coupling between the proto-neutron star and the envelope (rotation instabilities induced by turbulence and accretion)



Ott and Burrows, 2006.





Marek et al., 2008.

Main conclusions:

- + Waveforms not well predicted
- + weak amplitudes -> only Galactic Supernova detectable?

Binary inspirals: GW amplitudes

System of 2 close compact stars

- Varying quadrupole -> GW emission
- GW emission -> loss of energy and angular momentum
- Loss of (gravitational) energy -> stars become closer
- Finally 2 stars merge (or disrupt)

Spiraling phase (lowest order)

$$h_{+}^{TT}(t) = \frac{4(GM)^{5/3}}{Rc^{4}} \frac{1 + \cos^{2}i}{2} (\pi f(t))^{2/3} \cos \varphi(t)$$

$$h_{\times}^{TT}(t) = \frac{4(GM)^{5/3}}{Rc^{4}} \cos i (\pi f(t))^{2/3} \sin \varphi(t)$$

$$h(t) \propto (t_{c} - t)^{-1/4}$$

$$h_{\times}^{TT}(t) = \frac{4(GM)^{5/3}}{Rc^4} \cos i \left(\pi f(t)\right)^{2/3} \sin \varphi(t)$$

$$h(t) \propto \left(t_c - t\right)^{-1/4}$$

where

• Chirp mass: $M = \mu^{3/5} M_{tot}^{2/5}$

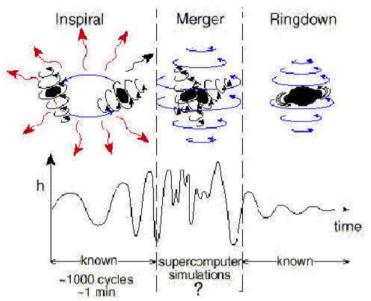
• frequency: $f(t) = \frac{1}{\pi} \left(\frac{256}{5} \frac{(GM)^{5/3}}{c^5} (t_c - t) \right)^{-3/8}$ t_c : coalescence time

• Phase: $\varphi(t) = -2 \left(\frac{G^{5/3}}{c^5} \right)^{-3/8} \left(\frac{t_c - t}{5M} \right)^{5/8} + cste$

Binary inspirals: the chirp signal

 $\times 10^{-21}$

0.15

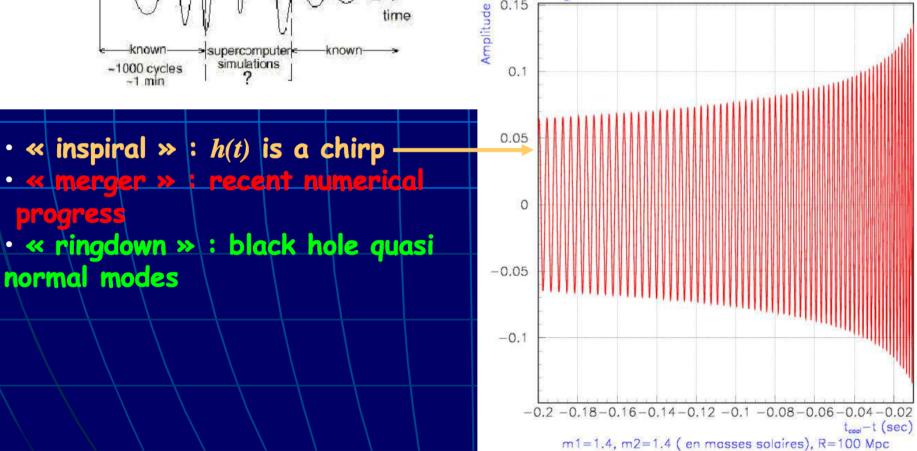


2 neutron stars @ 10Mpc

 $h_{max} \sim 10^{-21}$

 f_{max} (last stable orbit)~ 1 kHz

Signal de coalescence, ordre newtonien



Binary inspirals: rate of events (first generation detectors)

 \sim NS-NS. $1.4M_{\odot} + 1.4M_{\odot}$

(Kalogera et al astro-ph/0111452)

- 0.001 1 / yr ->
 - √ -> 20 Mpc
- \blacksquare /NS-BH: $1.4M_{\Theta} + 10M_{\Theta}$
 - 0.001 1 / yr
 - -> 43 Mpc
- BH-BH: $10M_{\Theta} + 10M_{\Theta}$
 - 0.001-1/yr
- -> 100 Mpc

- Gain Factor 10 on detector sensitivity
 - gain factor 1000 on the event rate

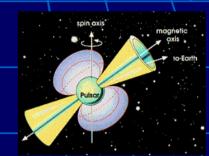
/ Other sources

Pulsars and rotating Neutron Stars

105 pulsars in the Galaxy, several thousands rapidly rotating.

Source of asymmetry?

- rotation instabilities
- magnetic stress
- "mountains" on the solid crust ...



Radio-astronomy observation of pulsar slowdown sets upper limits on GW emission and neutron star asymmetry (if rate of slowdown totally assigned to GW emission)

⇒Expected amplitudes are weak (//<10-24)

$$h \sim 10^{-26} (\frac{10 \text{ kpc}}{\text{distance}}) (\frac{f}{100 \text{Hz}})^2 (\frac{\varepsilon}{10^{-6}})$$

But the signal is periodic! ("simple" Fourier analysis)

Signal to noise ratio $S/N \propto \sqrt{T}$ where T is the observation time

Other sources Stochastic background

A lot of possible ideas

- cosmological backgrounds, phase transitions
- cosmic string vibrations
- superposition of unresolved sources

- |...

GW density:

$$Q_{\text{GW}}(f) = \frac{1}{\rho_{crit}} \frac{d\rho_{GW}}{d\ln f}$$

Theoretical predictions (cosmology): $h_0^2 \Omega_{GW} < 10^{-6}$ rather $\rightarrow 10^{-13}$

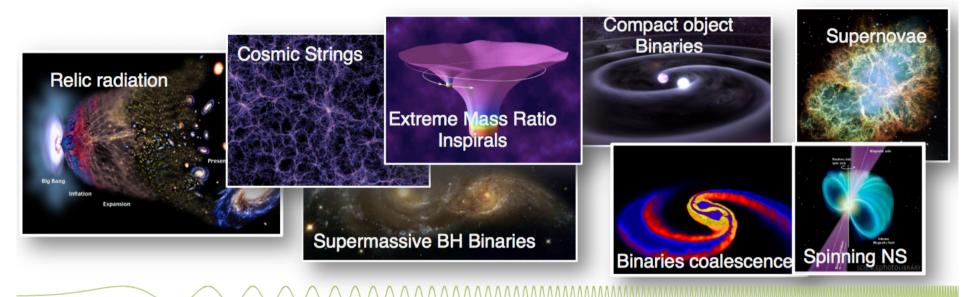
LIGO/Virgo (1 yr integration time) sensitive to

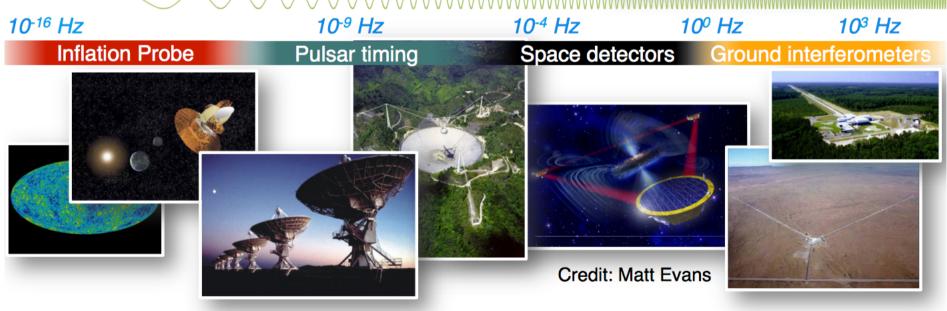
$$h_0^2 \Omega_{GW} \sim 10^{-7}$$

Astrofisica Nucleare e Subnucleare Ground Detectors for GW



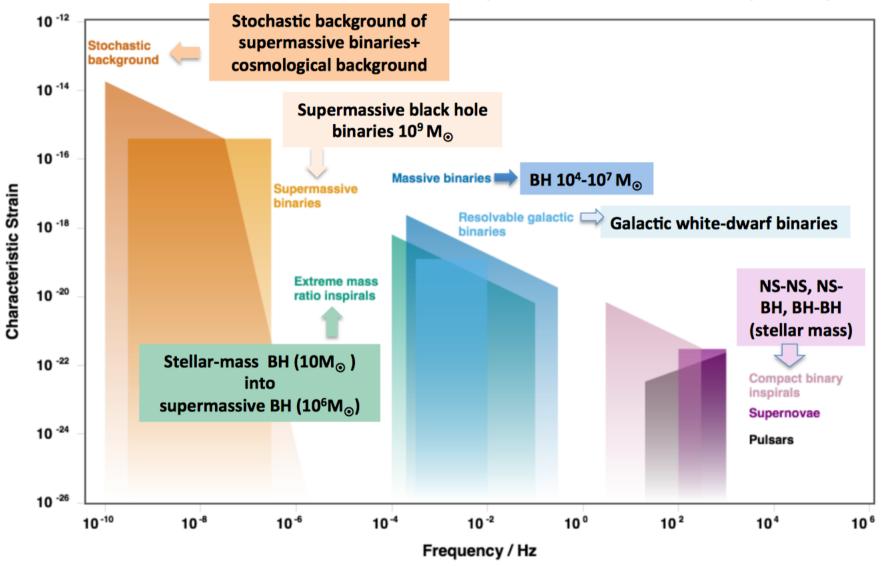
The GW Spectrum





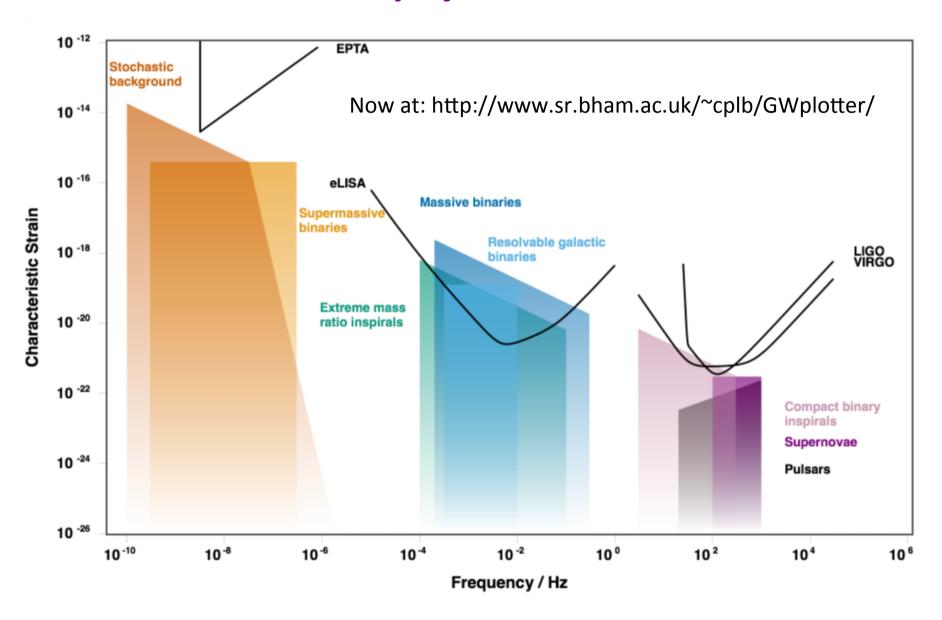
Astrophysical Sources

Now at: http://www.sr.bham.ac.uk/~cplb/GWplotter/



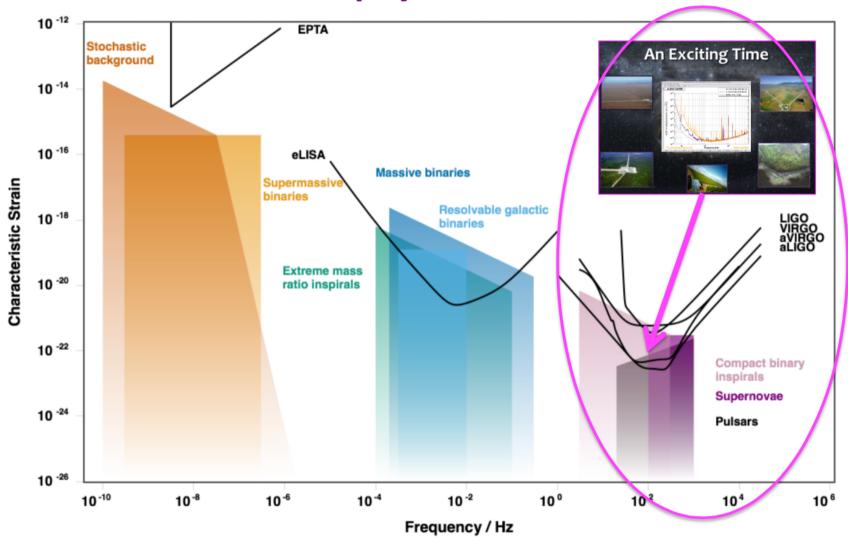
http://rhcole.com/apps/GWplotter/

Astrophysical Sources



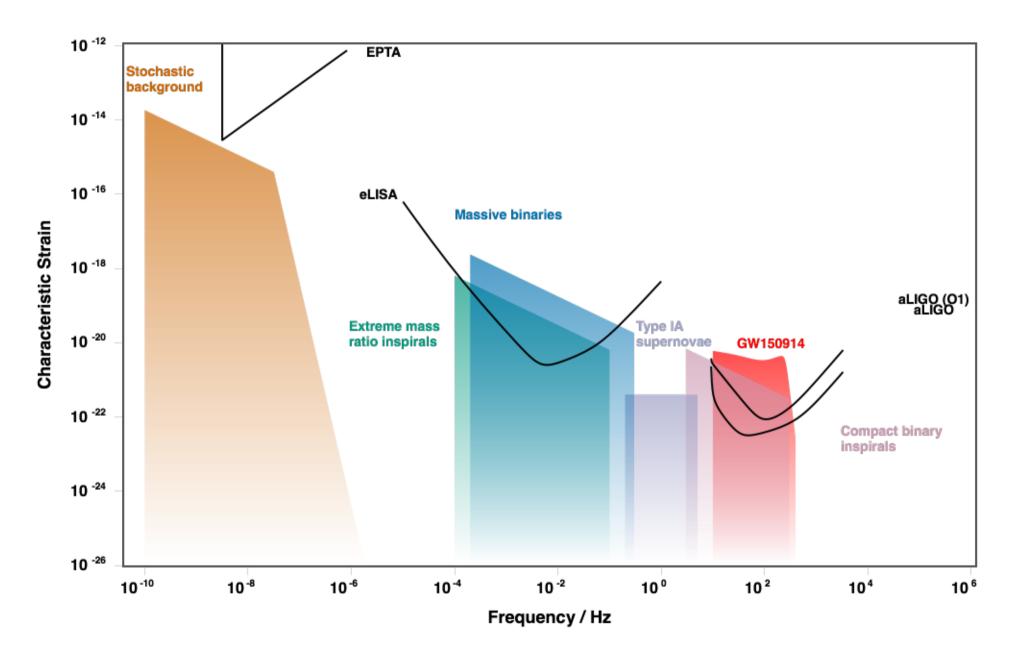
http://rhcole.com/apps/GWplotter/

Astrophysical Sources



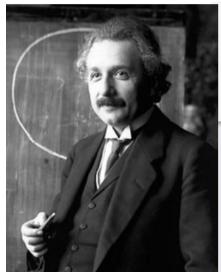
http://rhcole.com/apps/GWplotter/

Now at: http://www.sr.bham.ac.uk/~cplb/GWplotter/



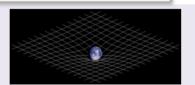
Now at: http://www.sr.bham.ac.uk/~cplb/GWplotter/

To reiterate:

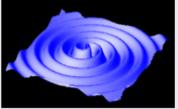


1916 → Einstein's theory of relativity predicts the existence of a new type of wave: the gravitational waves

GWs are perturbations of the space-time metric:



- Generated by mass distributions with time-varying quadrupole moments
- Propagating at the speed of light

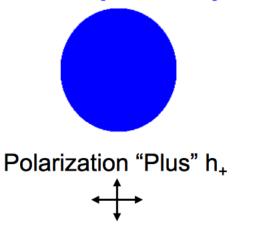


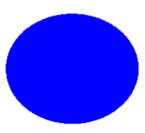
Change in the distance between stationary (inertial) masses

According to GR, GWs have two independent polarization states

Each GW signal can be described as a linear combination of them:

$$h = A_+h_+(t) + A_xh_x(t)$$

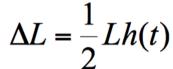




"Cross" h_x

How can GWs be det

A GW deforms space

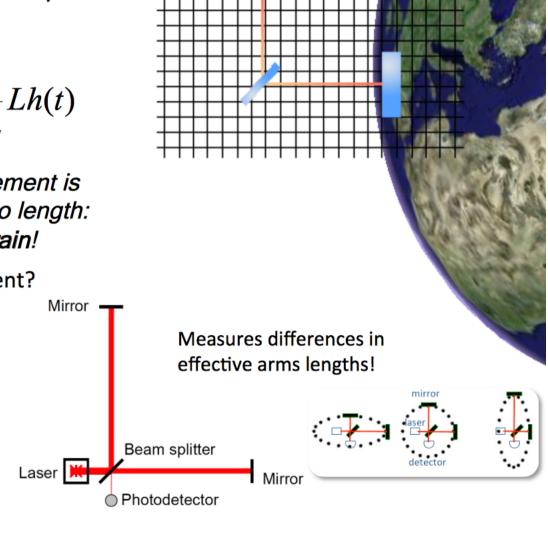


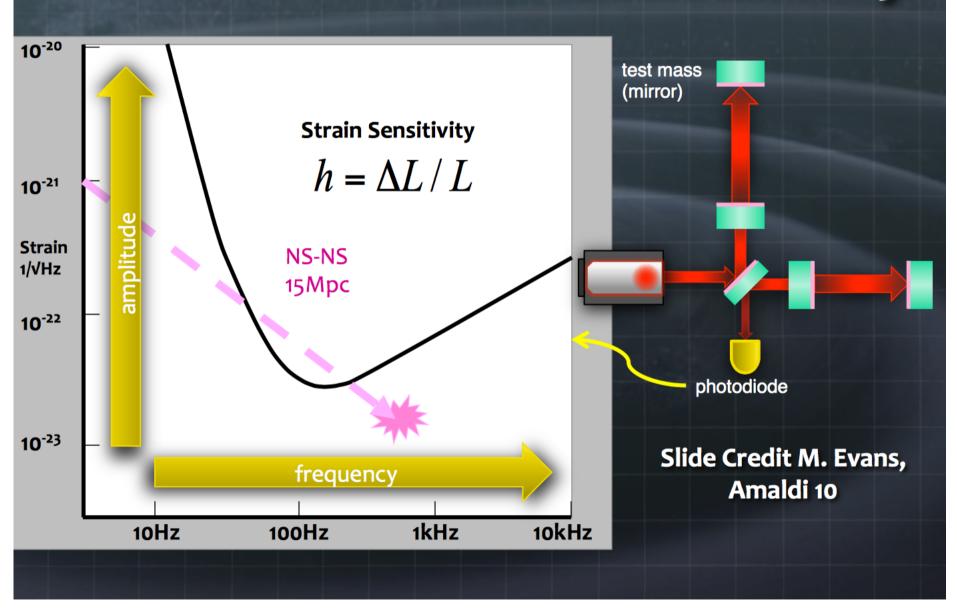
The displacement is proportional to length: it is a strain!

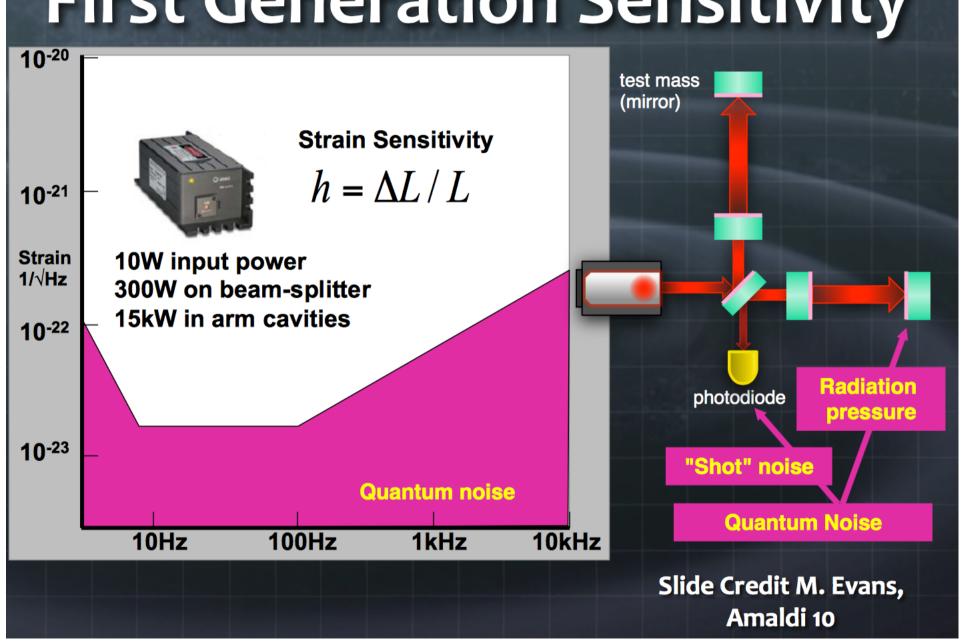
Do we know how to detect displacement?

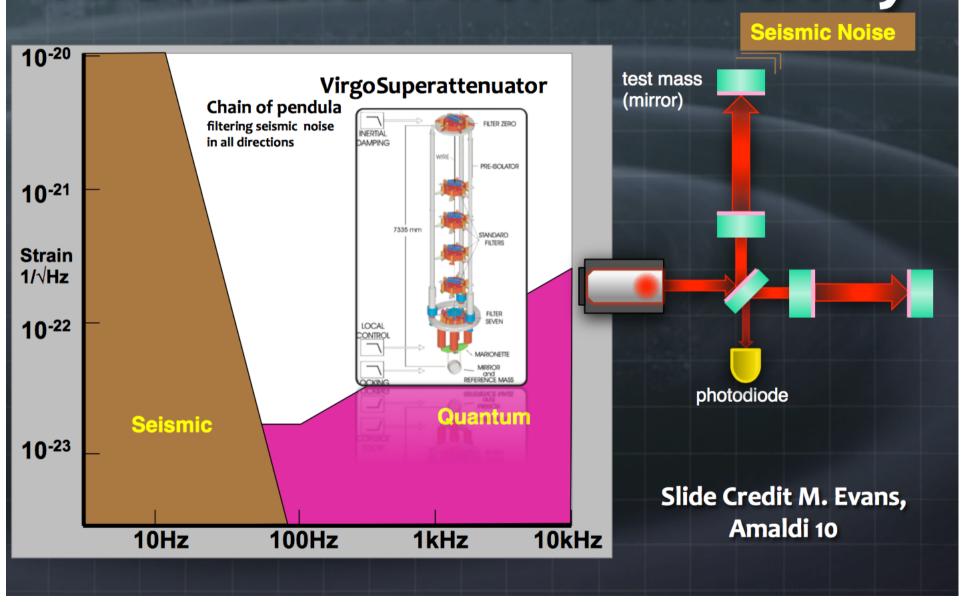


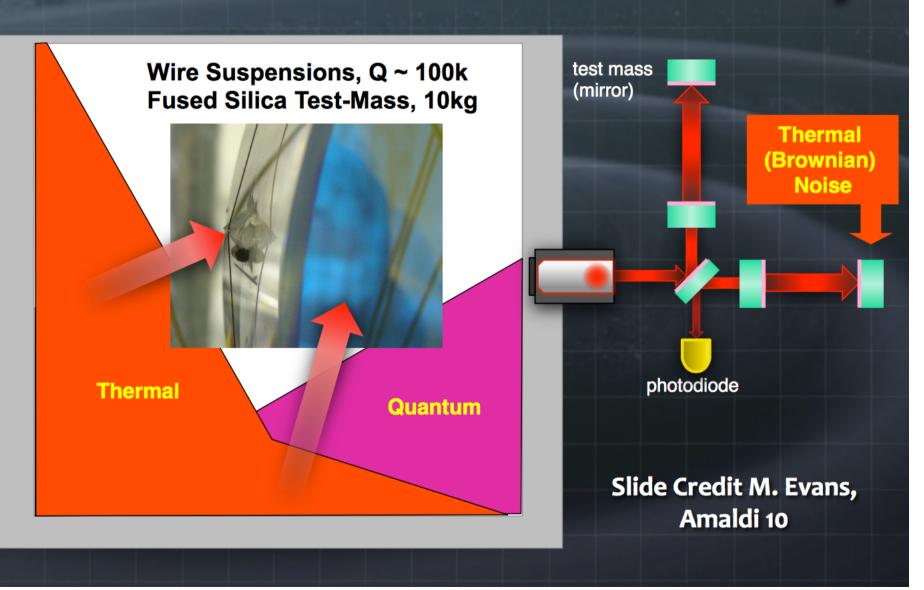
The length change is measured interferometrically by using a laser light beam

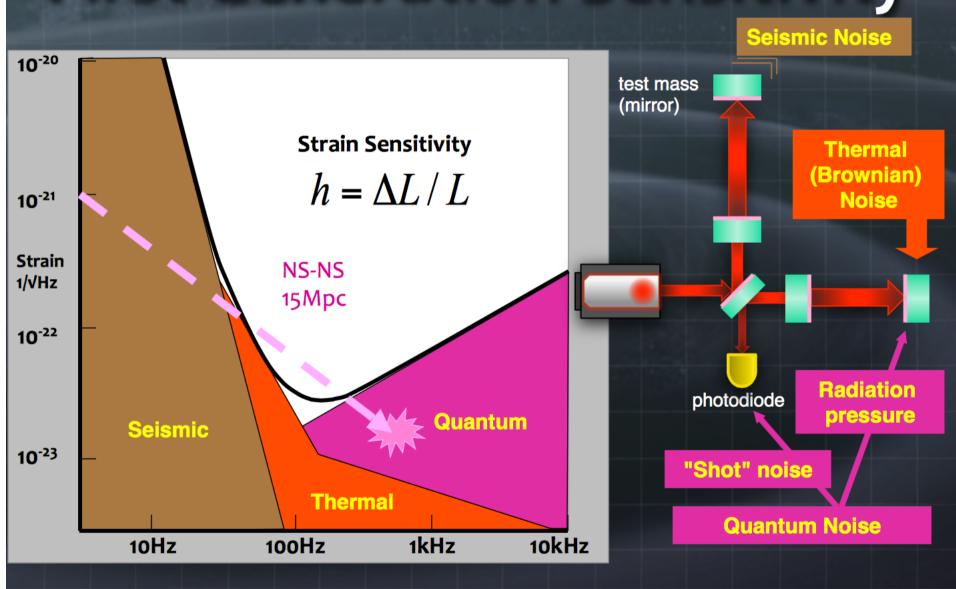




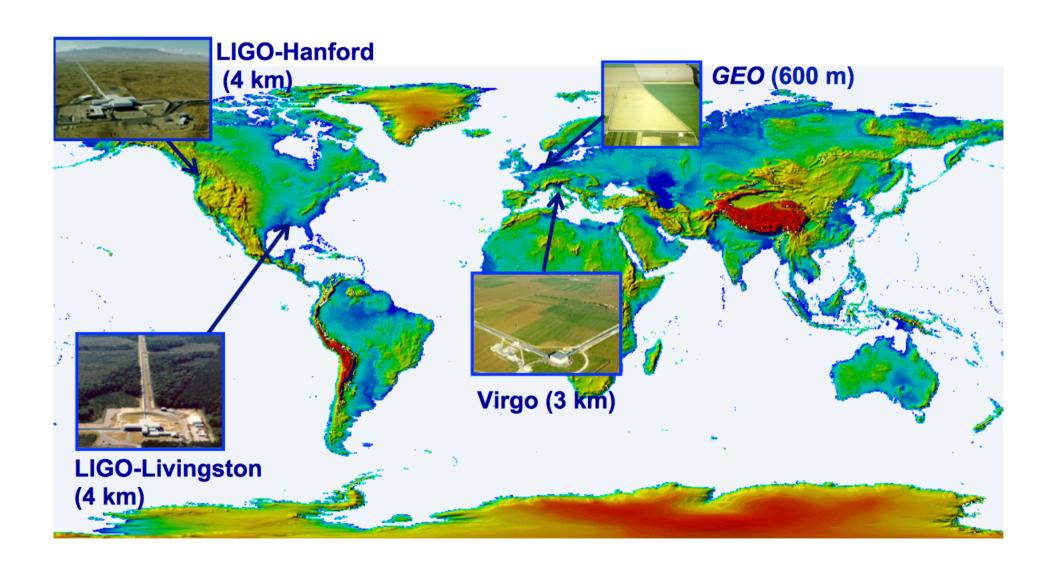




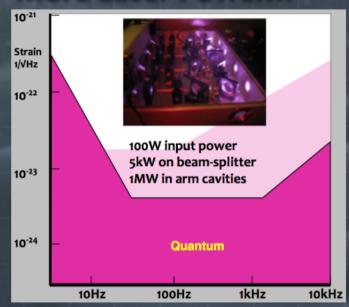


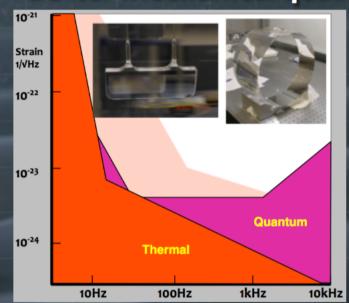


Ground-based Gravitational Wave Detectors



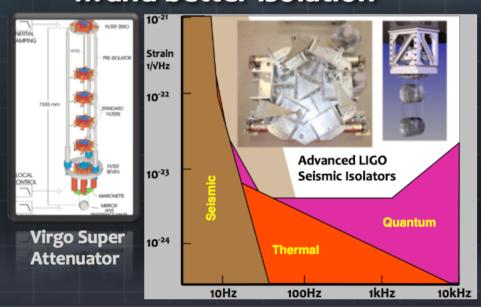
Advanced GW detectors More Laser Power... Better mechanical quality...

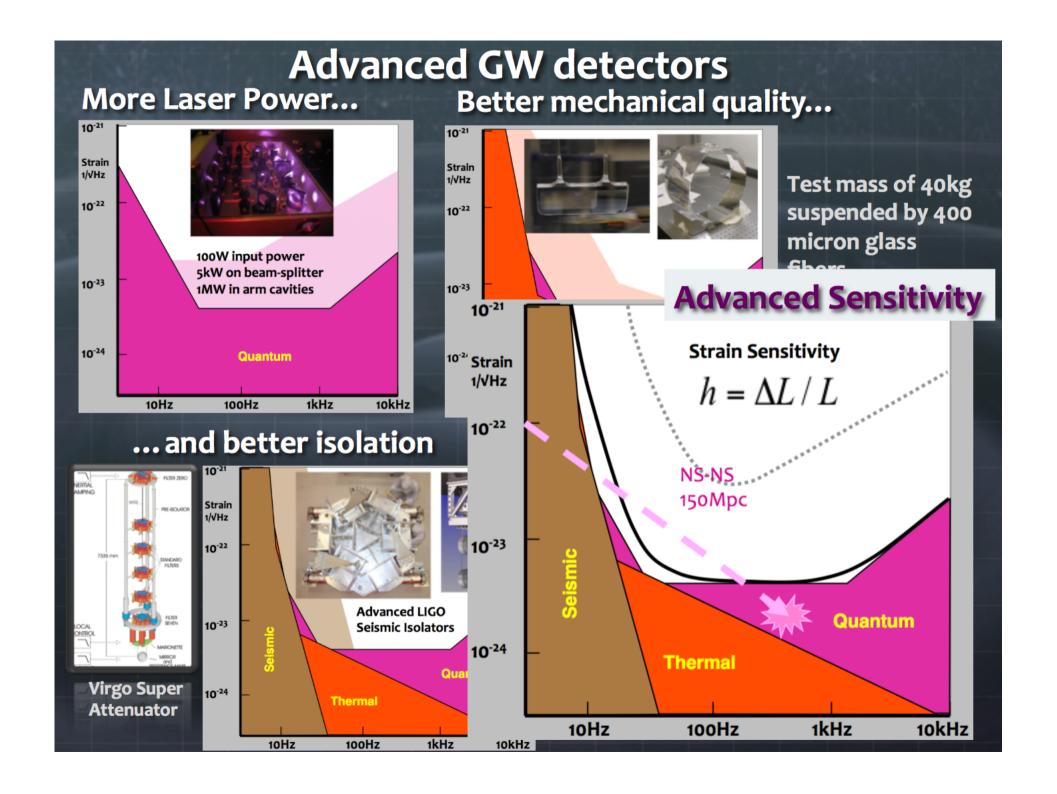




Test mass of 40kg suspended by 400 micron glass fibers...

... and better isolation

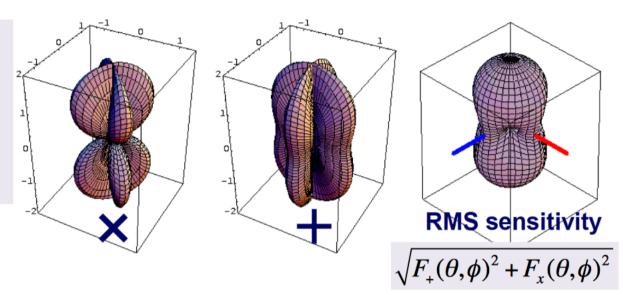


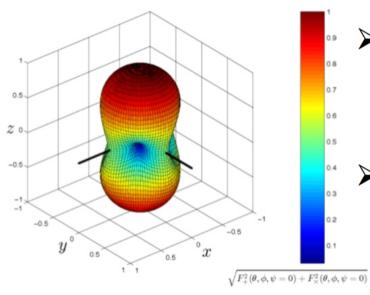


Single GW detector directional sensitivity

$$\frac{\Delta L}{L} = h_{\text{det}}(t) = F_{+}h_{+}(t) + F_{x}h_{x}(t)$$

The **antenna pattern** depends on the polarization in a certain (x,+) basis.

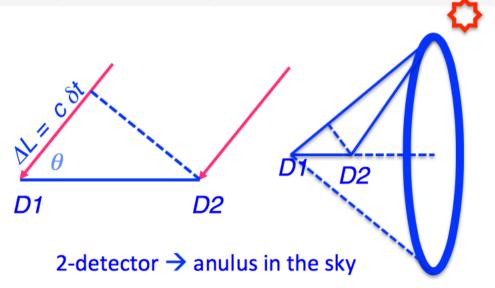


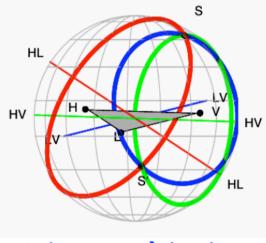


- Single GW detector is a good all-sky monitor, nearly omni-directional (the transparency of Earth to GWs)
- But does not have good directional sensitivity, not a pointing instrument!
 It has a very poor angular resolution (about 100 degrees)

The source localization requires a network of GW detectors

The **sky position** of a GW source is mainly **evaluated by triangulation**, measuring the differences in signal arrival times at the different network detector sites





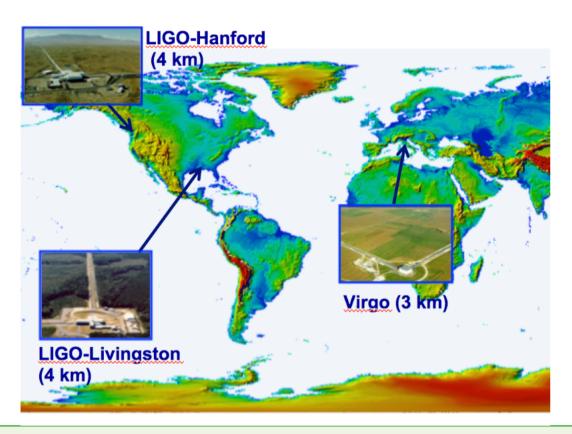
3-detectors → localize

petector $D = 3 \cdot 10^3 km$ baseline $\lambda \sim 3 \cdot 10^2 km$ Angular resolution $\frac{\lambda}{D} \sim 60^\circ$

The GW lengths are comparable to Earth diameter

→ longer baseline and greater number of the sites distributed worldwide significantly improve the sky-localization capabilities!

Other benefits of a network of GW detectors



Improvements:

- Sensitivity
- Observation time, and sky coverage
- In determining the polarization
- Ability to reconstruct the GW source parameters
- False alarm rejection thanks to coincidence

Virgo and the LIGO Scientific Collaborations have signed a MoA for full data exchange and joint data analysis and publication policy

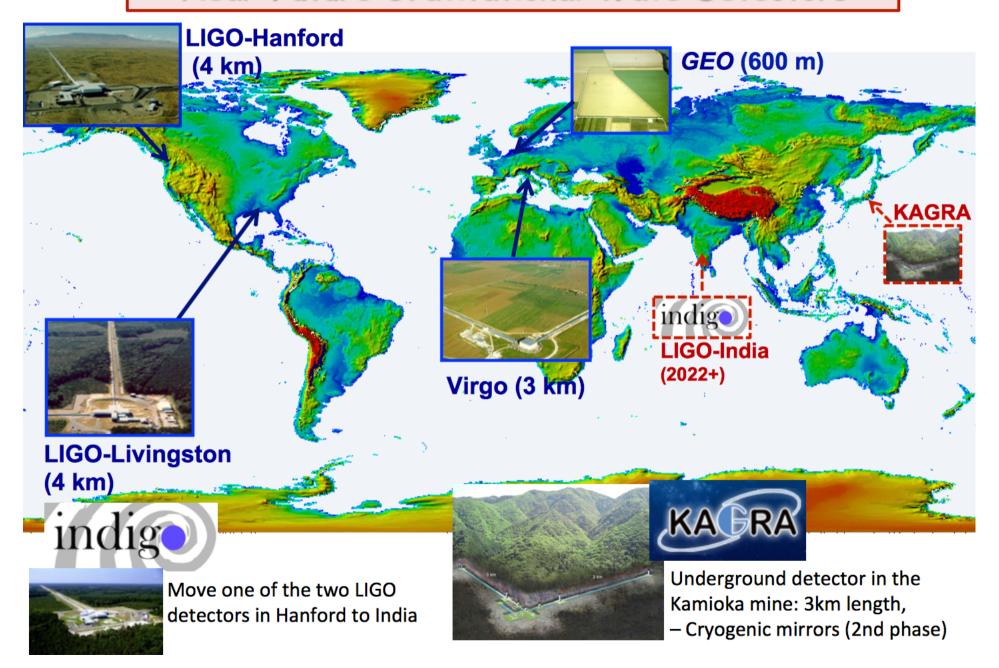
Advanced LIGOs and Virgo will observe the sky (10-1000 Hz) as a single network aiming at the first direct detection of GWs

Example of sky-localization capabilties

NS-NS with SNR=7 in each of the LIGOs and Virgo:

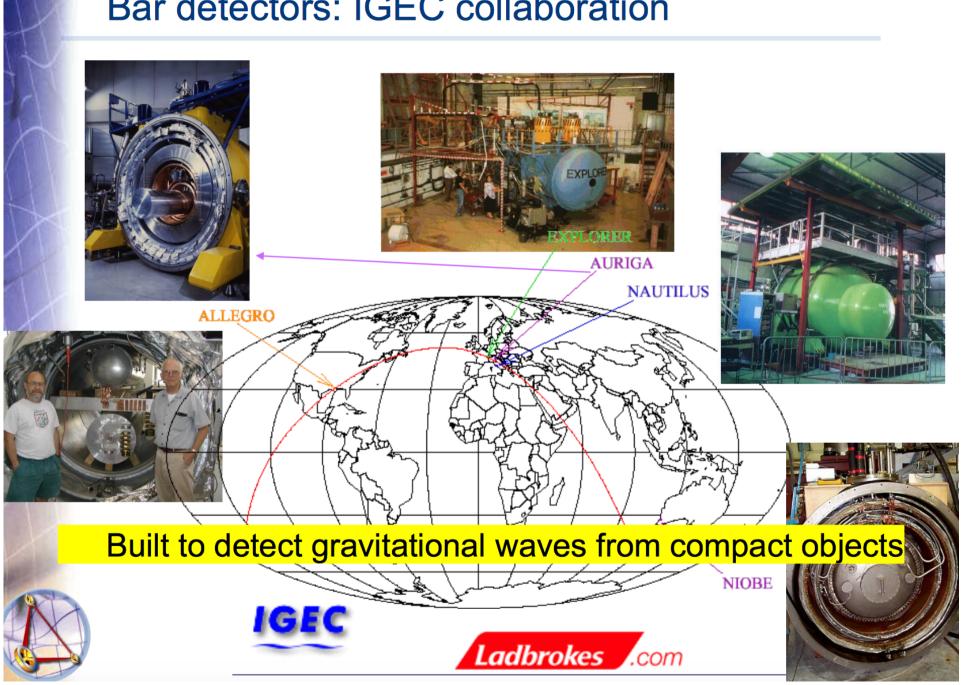
- best case localization of 20 deg²
 (signal is directly over the plane of network)
- median of 40 deg² (Fairhurst 2009)

Near Future Gravitational Wave Detectors

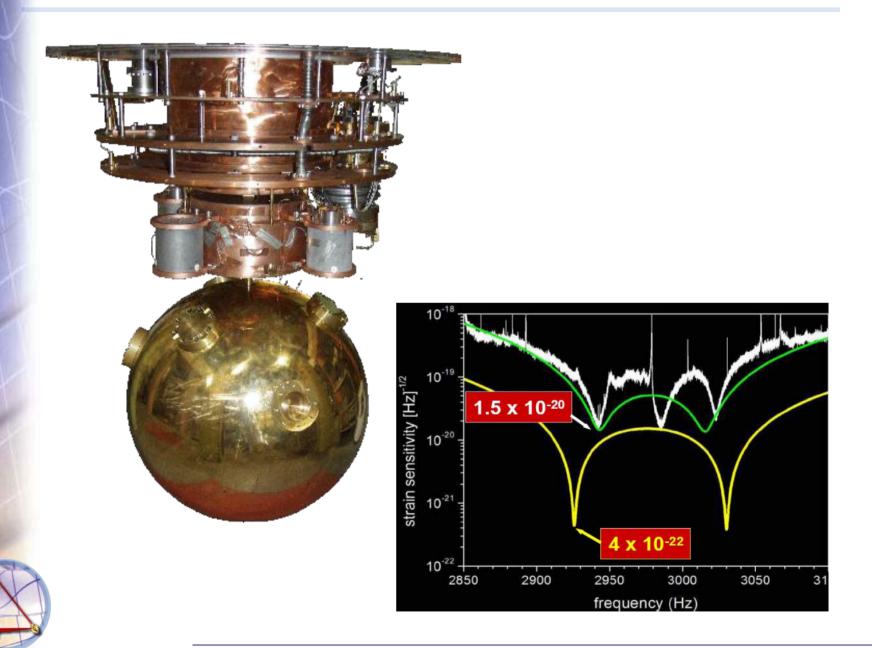


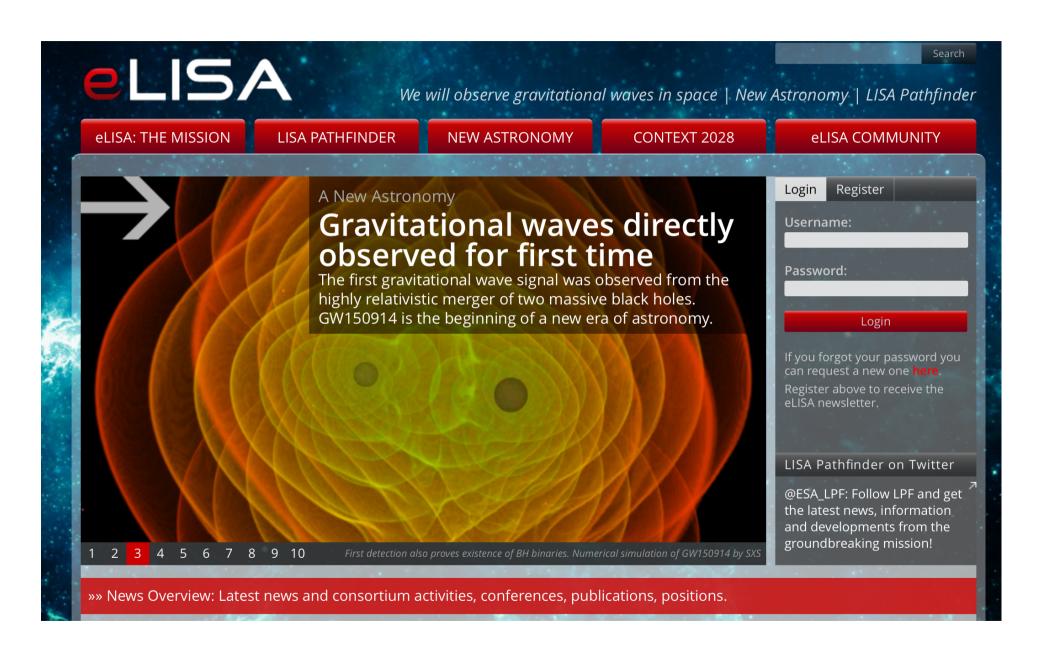
Astrofisica Nucleare e Subnucleare Other Detectors for GW

Bar detectors: IGEC collaboration



Mini-GRAIL: a spherical `bar' in Leiden

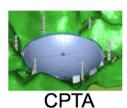








PTAs in 2015













IPTA



Meerkat

Interest in pulsar community Interest in gravitational wave community







