



Basics

Chapters 2.1, 2.2 and 3.2 of Cormen's book

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Algorithmic Design / Algorithms for Scientific

Computing

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What is an Algorithm?

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Algorithm 1 Linear Search

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2: OUTPUT: An index  $i$  such that  $A[i] = q$ ; or FAIL if no such index exists.
3:  $j \leftarrow 1$ ;
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Problem definition

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Variable initialization: all variables must be initialized to some value.



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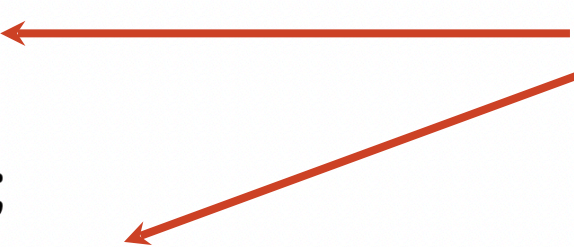
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Return: returns the value and terminates the algorithm. Correct algorithms must **always return something**



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Indentation: the if/else and while conditions only refer to the instructions more indented than them. All instructions on the same vertical line are executed anyway (until the algorithm terminates)

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$q=6, n=5$
 $A[1, \dots, 5] = [3, 6, 5, 4, 9]$



Instance of the problem

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3:  $j \leftarrow 1$ ;
4: while  $j \leq n$  do
5:     if  $A[j] = q$  then
6:         YES! return  $j$ ;
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 $A[3]=6=q!$

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3: $j \leftarrow 1$;
4: **while** $j \leq n$ **do**
5: **if** $A[j] = q$ **then**
6: **return** j ; \longrightarrow OUTPUT: index 2
7: **else**
8: $j \leftarrow j + 1$;
9: **if** $j = n + 1$ **then return** : **FAIL**;

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 $j=2$
 $A[2]=6=q!$

Quiz Time

Please go to www.wooclap.com, use the code **BERNARDINI01** and answer the question (it is anonymous unless you decide to use your name). You do not need to create an account!



Complexity of Algorithms

The **time complexity** of an algorithm is the number of “steps” done by the algorithm as a function of the number of input elements.

To offer **guarantees** to the user on the performance of the algorithm **even with the most unfortunate input**, we carry on a **worst-case analysis**. So we want to compute **upper bounds** on the running time.

Complexity of Algorithms

We are mainly interested in the **order of growth** of an algorithm: the fastest-growing term of the function that expresses the number of steps done by the algorithm in the worst case.

- **Linear** algorithm: $an + b$ steps on the worst-case input of size n
- **Quadratic** algorithm: $cn^2 + dn + e$ steps on the worst-case input of size n
- **Logarithmic** algorithm: $k\log(n) + h$ steps on the worst-case input of size n
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We just don't care about the multiplicative constants and the lower-order terms!

Why Asymptotic Time Matters

Snellius (Amsterdam):
 10^{16} instructions/second



My laptop:
 10^{10} instructions/second



Snellius is 1 million times faster than my laptop!

You would believe that the most stupid algorithm executed by Snellius takes less time than a very efficient algorithm run on my laptop, but...

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Suppose we have two algorithms for the same task:

Algorithm A requires $2n^2$ instructions for an input of size n ;

Algorithm B requires $120n$ instructions for the same input.

Let's take $n=10^{10}$ and run Algo A on Snellius, Algo B on my laptop.

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$$\frac{2 \cdot (10^{10})^2 \text{instructions}}{10^{16} \text{instructions/second}} = 20000 \text{seconds}$$

$$\frac{120 \cdot (10^{10}) \text{instructions}}{10^{10} \text{instructions/second}} = 120 \text{seconds}$$

Counting the Instructions

How many instructions does this algorithm execute in the worst case?

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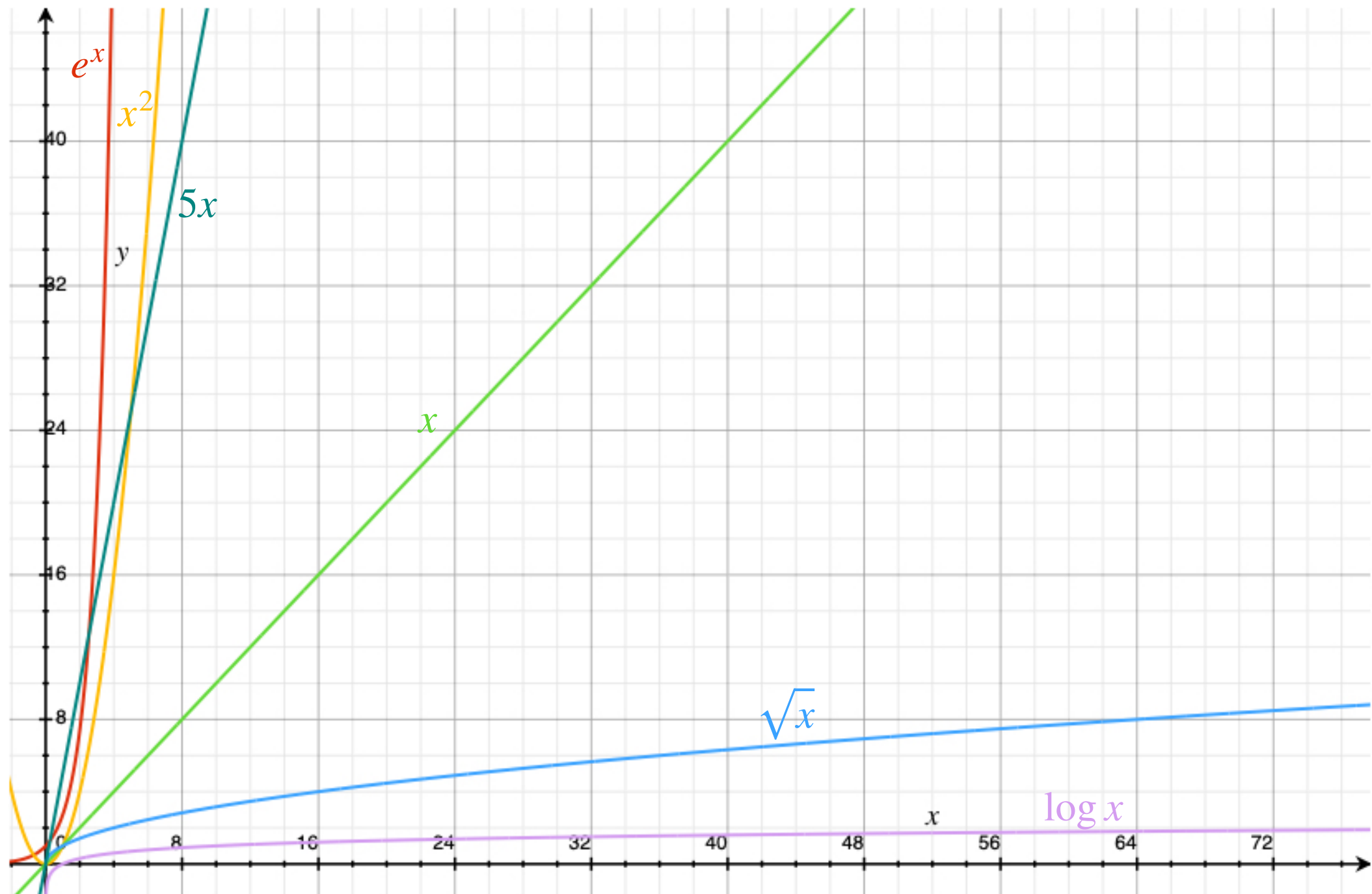


Asymptotic Analysis

Basic idea of asymptotic analysis:

- Ignore the machine-dependent constants
- Look at the growth of the number of instructions as a function of the input size n , when $n \rightarrow \infty$.
- We will express the time complexity of algorithms using the asymptotic notation O , Θ , Ω , o , ω (defined in Cormen, chapter 3.1 and at the blackboard).

Orders of Growth



$$\log x = O(\sqrt{x}) = O(x) = O(x^2) = O(e^x)$$

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The RAM Model

In order to analyse algorithms we need to fix a model of computation.

The Random Access Machine model is the most widely used. It models Random Access Memory of real computers.

In the RAM model we can access, compute and store a constant number of words in constant time.