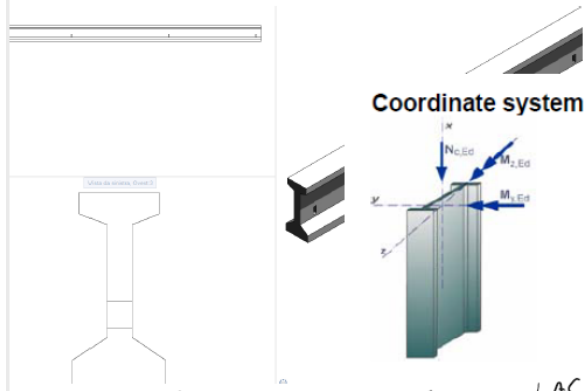
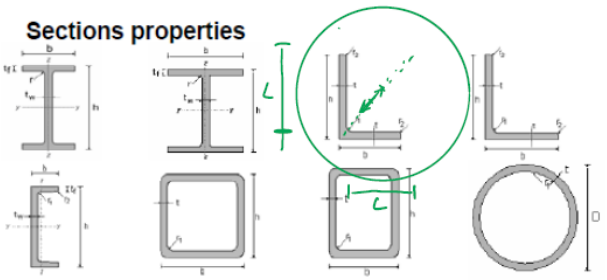
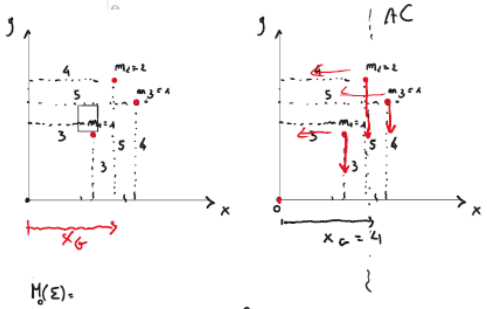


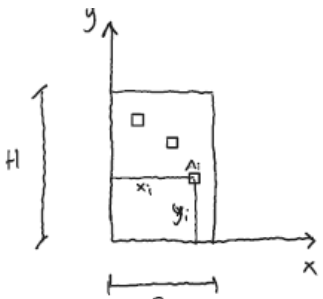



$+\circlearrowright$ $M_0(\Sigma) =$

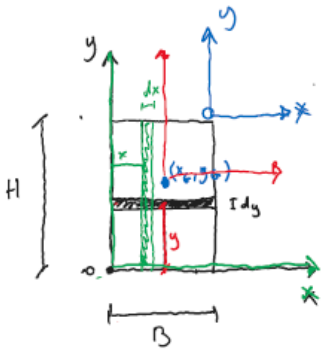
$-(1 \cdot 3 + 2 \cdot 4 + 1 \cdot 5) = -4\, x_G$ $\qquad x_G = \frac{16}{4} = 4$

$$x_G = \frac{\sum m_i x_i}{\sum m_i} \qquad y_G = \frac{\sum m_i y_i}{\sum m_i}$$



$S_y = \sum m_i x_i \Rightarrow \sum A_i x_i \Rightarrow$ M. statico rispetto all'asse y

$$S_x = \sum A_i y_i \Rightarrow S_x = \int_A y\, dA$$



$$S_x = \int_A y\, dA \Rightarrow \int_0^H y\, B\, dy = B\frac{y^2}{2}\Big|_0^H = \frac{BH^2}{2} \quad [m^3]$$

$dA = B \cdot dy$

$$S_y = \int_0^B x \cdot H\, dx = \frac{HB^2}{2}$$

$dA = H\, dx$

$$x_G = \frac{\sum m_i x_i}{\sum m_i} \rightarrow S_y \text{ / } A \qquad y_G = \frac{\sum m_i y_i}{\sum m_i} \rightarrow S_x \text{ / } A$$

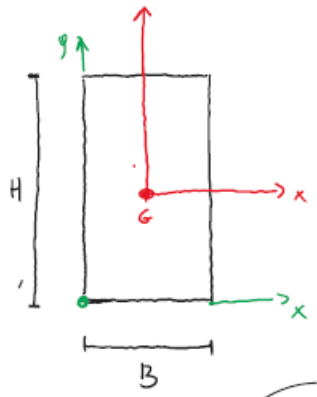
$$\sum A_i = \int_A dA = A \qquad A = B \cdot H$$

$$x_G = \frac{S_y}{A} = \frac{\int_A x\, dA}{A} = \frac{\frac{HB^2}{2}}{BH} = \frac{B}{2}$$

$$y_G = \frac{S_x}{A} = \frac{\int_A y\, dA}{A} = \frac{\frac{BH^2}{2}}{BH} = \frac{H}{2}$$

$$x_G = \frac{S_y}{A} = 0 \Rightarrow S_y = 0$$

$$y_G = \frac{S_x}{A} = 0 \Rightarrow S_x = 0$$




$$S_x = \int_A y\,dA = \int_{-\frac{H}{2}}^{H/2} y\,B\,dy = B\frac{y^2}{2}\Big|_{-H/2}^{H/2} = \varnothing$$

$$dA = B\,dy$$

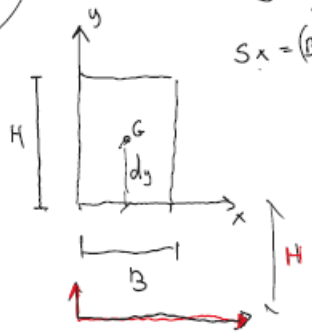
$$y_G = \frac{S_x}{A}$$

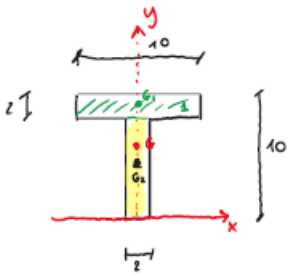
$S_x = \int_A dy$

$$S_x = \frac{BH^2}{2}$$

$$S_x = \overset{A}{(BH)} \cdot \frac{H}{2} = \frac{BH^2}{2}$$

$$S_x = (BH)\,\frac{3}{2}H =$$



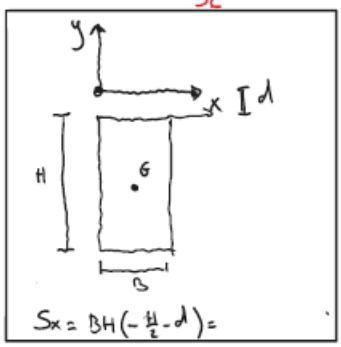

$y_G$ ?

| $A_i$ | $y_i$ | $S_{xi} = A_i y_i$ |
|---|---|---|
| 20 | 9 | 180 |
| 16 | 4 | 64 |
| $A_{TOT}$ 36 | | 244 $S_{x\,TOT}$ |

$$y_G = \frac{S_{x\,TOT}}{A_{TOT}} =$$

$$S_x = A \cdot dy$$

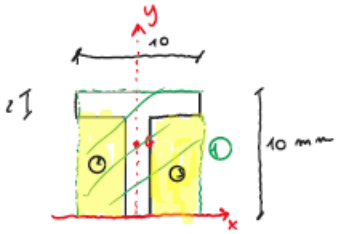
$$y_G = \frac{244}{36} = 6{,}77$$




$S_x = BH\left(-\frac{H}{2} - d\right) =$

$d = 0 \quad S_x = -\frac{BH^2}{2}$

$$y_G = \frac{-\frac{BH^2}{2}}{BH} = -\frac{H}{2}$$




$S'_x = 10 \times 10 \times 5 = 500\ mm^3$

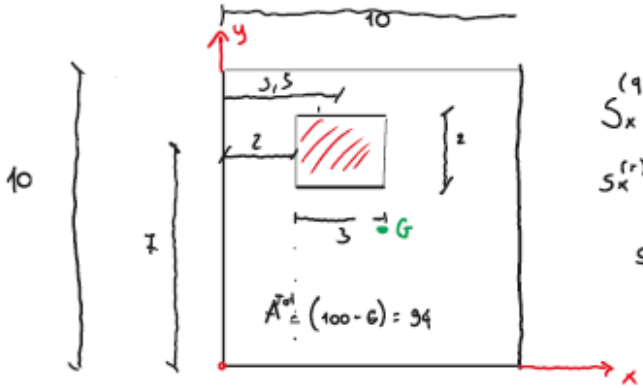
$S^2_x = 8 \times 4 \times 4 = 128\ mm^3$

$S^3_x = 8 \times 4 \times 4 = 128\ mm^3$

$A_{Tot} = 36\ mm^2$

$S_x^{Tot} = S_{x_1} - S_{x_2} - S_{x_3} = 244\ mm^3$

$y_G = \frac{244}{36} \approx 6{,}72\ mm$

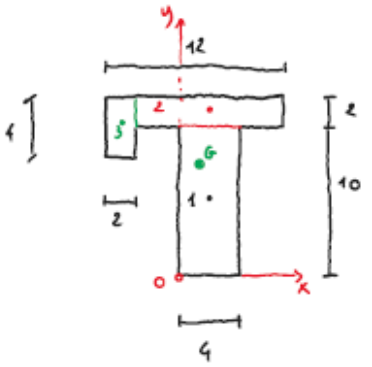



$S_x^{(q)} = 10 \times 10 \times 5 = 500$ $\quad S_y^q = 10 \times 10 \times 5 = 500$

$S_x^{(r)} = 2 \times 3 \times 7 = 42$ $\quad S_y^h = 2 \times 3 \times 3,5 = 21$

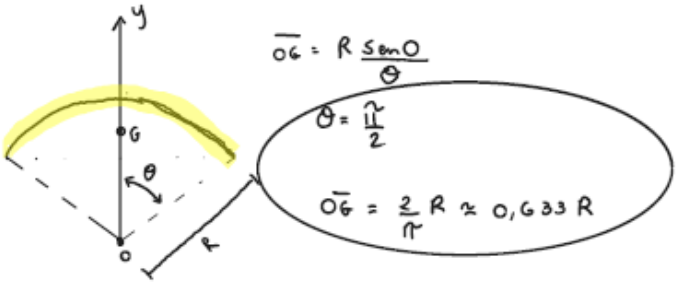
$S_x^{Tot} = 500 - 42 = 458$ $\quad S_y^T = 479$

$y_G^{Tot} = \frac{458}{94} = 4,9$ $\quad x_G^{Tot} = \frac{479}{94} \simeq 5,1$




| | $A_i$ | $x_i$ | $S_{yi}$ | $y_i$ | $S_{xi}$ |
|---|---|---|---|---|---|
| 1) | 40 | 2 | 80 | 3 | 200 |
| 2) | 20 | 2 | 40 | 11 | 220 |
| 3) | 8 | -4 | -32 | 10 | 80 |
| Tot | 68 | | 88 | | 300 |
| | $A_{Tot}$ | | $S_{yTot}$ | | $S_{xTot}$ |

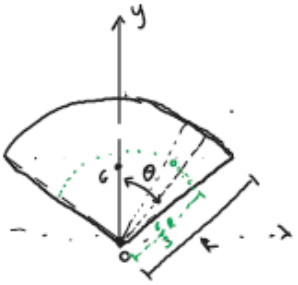
$x_G = \frac{S_y}{A} = \frac{88}{68} = 1,3$ $\quad y_G = \frac{300}{68} = 7,4$




$\overline{OG} = R \frac{\sin\Theta}{\Theta}$

$\Theta = \frac{\pi}{2}$

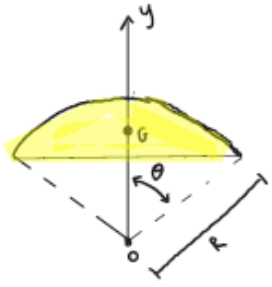
$\overline{OG} = \frac{2}{\pi} R \simeq 0,633\, R$




$OG = \frac{2}{3} R \frac{\sin\Theta}{\Theta}$ $\quad A = R^2\Theta$

$\Theta = \frac{\pi}{2}$ $\quad OG = 0,424\, R$

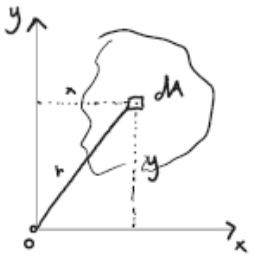
$OG = \frac{2}{3} \frac{1}{\frac{\pi}{2}} R = \frac{4}{3\pi} R = 0,424\, R$




$\overline{OG} = ?$

$S_x = A \cdot \overline{OG}$

# Momenti d'inerzia

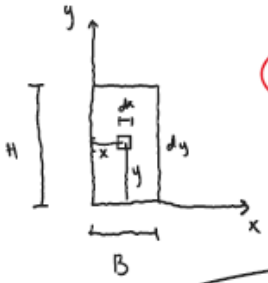



$$S_x = \int_A y\,dA$$

$$S_y = \int_A x\,dA$$

- $I_{xx} = \int_A y^2\,dA$
- $I_{yy} = \int_A x^2\,dA$
- $I_{xy} = \int_A x\,y\,dA$

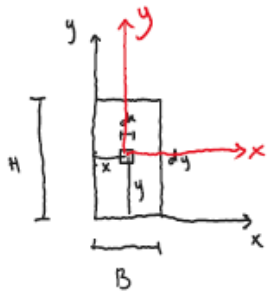
$r^2 = x^2 + y^2$ $\qquad I_p = \int_A r^2\,dA = \int_A (x^2+y^2)\,dA = \int_A x^2\,dA + \int_A y^2\,dA = I_{yy} + I_{xx}$




$$I_{xx} = \int_0^H y^2\,B\,dy = \frac{BH^3}{3}$$

$$I_{yy} = \frac{HB^3}{3}$$

$$I_{xy} = \int_A xy\,dA = \int_0^B \int_0^H xy\,dx\,dy = \int_0^B x\left(\int_0^H y\,dy\right)dx = \int_0^B x\left(\frac{H^2}{2}\right)dx = \frac{B^2H^2}{4}$$




$$I_{xx} = \int_{-\frac{H}{2}}^{\frac{H}{2}} y^2\,B\,dy = \frac{By^3}{3}\Big|_{-H/2}^{H/2} = \frac{BH^3}{12}$$

$$I_{yy} = \frac{HB^3}{12}$$

$$I_{xy} = \int_{-B/2}^{\frac{B}{2}} x\left(\int_{-\frac{H}{2}}^{H/2} y\,dy\right)dx = 0 \qquad \left(\int_{-\frac{H}{2}}^{H/2} y\,dy = 0\right)$$