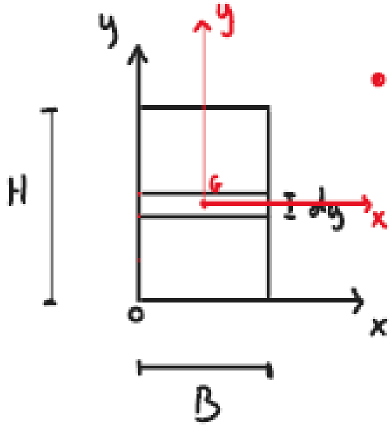


• $I_{xx} = \int_A y^2\, dA = \int_0^H B y^2\, dy = \frac{BH^3}{3}$

$dA = B \cdot dy$

• $I_{yy} = \frac{HB^3}{3}$

• $I_{xy} = \int_A xy\, dA = \frac{B^2H^2}{4}$

1) $\boxed{I_{xx} = \frac{BH^3}{3} \qquad I_{yy} = \frac{HB^3}{3} \qquad I_{xy} = \frac{B^2H^2}{4}}$

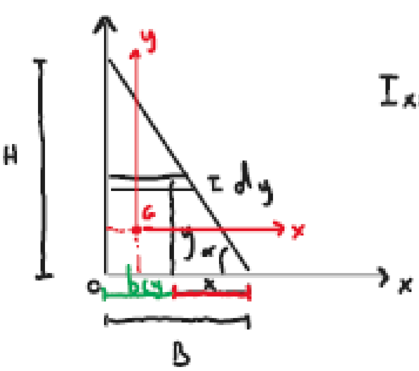
2) $I^G_{xx} = \int_{-\frac{H}{2}}^{\frac{H}{2}} B y^2\, dy = \frac{BH^3}{12}$ $\qquad I^G_{yy} = \frac{HB^3}{12}$ $\qquad I^G_{xy} = 0$

$$\begin{cases} I_{xx} = I^G_{xx} + A d_{y_G}^2 \\ I_{yy} = I^G_{yy} + A d_{x_G}^2 \\ I_{xy} = I^G_{xy} + A d_{x_G} d_{y_G} \end{cases}$$

Verifica

$I_{xx} = \frac{BH^3}{12} + \overbrace{(BH)}^{A} \cdot \frac{H^2}{4} = \frac{BH^3}{3}$

$I_{xy} = 0 + (BH)\,\frac{H}{2}\,\frac{B}{2} = \frac{B^2H^2}{4}$



$I_{xx} = \int_A y^2\, dA = \int_0^H y^2 \left(B - \frac{By}{H}\right) dy = B\frac{y^3}{3} - \frac{B}{H}\frac{y^4}{4}\Big|_0^H = \frac{BH^3}{12}$

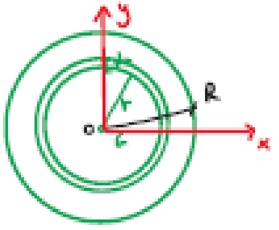
$dA = b(y)\, dy = \left(B - \frac{B}{H} y\right) dy$

$b(y) = \left(B - \frac{B}{H} y\right)$

$tg\alpha = \frac{H}{B} \qquad y = x\, tg\alpha \qquad x = \frac{y}{tg\alpha} = y\frac{B}{H}$

$I^G_{xx} = I_{xx} - A y_G^2 = \frac{BH^3}{12} - \overbrace{\left(\frac{BH}{2}\right)}^{A} \cdot \left(\frac{H}{3}\right)^2 = \frac{BH^3}{12} - \frac{BH^3}{18} = \frac{BH^3}{36}$

Sezione circolare

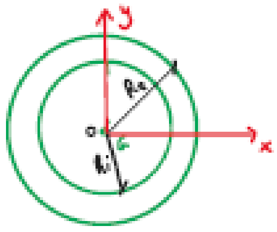


$$I_p = \int_A r^2\, dA = I_{xx} + I_{yy} \longrightarrow$$

$$dA = (2\pi r \cdot dr)$$

$$I_p = \int_0^R r^2 (2\pi r)\, dr = 2\pi \frac{r^4}{4}\Big|_0^R = 2\pi \frac{R^4}{4}$$

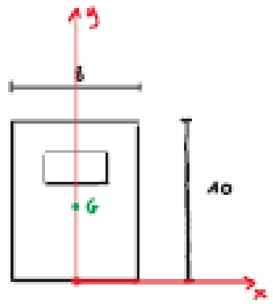
$$I_p = \frac{\pi R^4}{2} = I_{xx} + I_{yy} = 2 I_{xx} \implies I_{xx} = I_{yy} = \frac{\pi R^4}{4}$$



$$I_{xx}^{(est)} = \frac{\pi R^{est\,4}}{4}$$

$$I_{xx}^{(int)} = \frac{\pi R^{int\,4}}{4}$$

$$I_{xx}^{(cava)} = I_{xx}^{(est)} - I_{xx}^{(int)} = \frac{\pi}{4}\left(R^{est\,4} - R^{int\,4}\right)$$

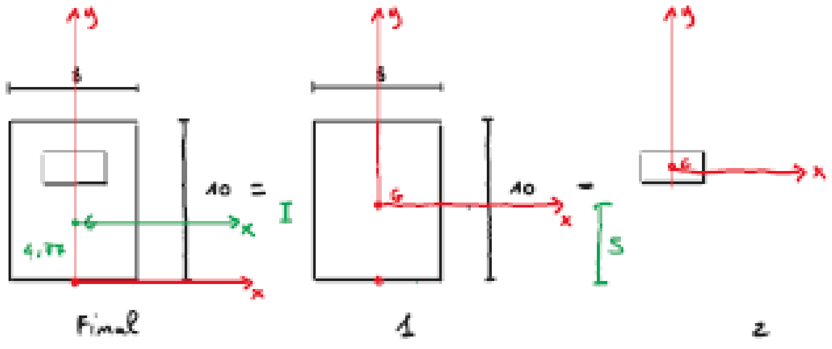


$$S_x = \underbrace{(80 \cdot 5)}_{\text{rett grande}} - \underbrace{(8 \cdot 7)}_{\text{rett piccolo}} = 344$$

$$A = 72$$

$$y_G = \frac{S_x}{A} = 4{,}7$$

$$I_{xxG} = 627{,}8$$



$$I_{xx_G}^{1} = \frac{BH^3}{12} = 666{,}66 = \frac{8 \cdot 10^3}{12}$$

$$\boxed{I_{xx} = I_{xx_G} + A\, d_{y_G}^2}$$

$$\frac{BH^3}{12}$$

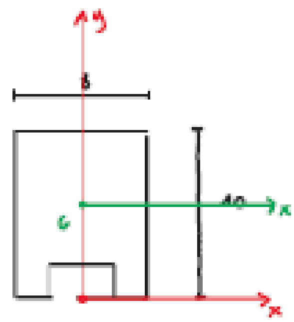
$$I_{xx_G}^{2} = \frac{4 \cdot 2^3}{12} = 2{,}666$$

$$I_{xx_{G_{tot}}}^{1} = 666{,}66 + 80 \cdot (\underbrace{5 - 4{,}77}_{d_{y_G}})^2$$

$$I_{xx_{G_{tot}}}^{2} = 2{,}66 + 8 \cdot (7 - 4{,}77)^2$$

Mi sposto dal sistema baricentrico dei singoli rettangoli a quello baricentro della sezione reale (finale)

$$I_{xx_G}^{finale} = I_{xx_{G_{tot}}}^{1} - I_{xx_{G_{tot}}}^{2} = 627{,}8$$



$G = ?$

$S_x = 80 \times 5 - 8 \times 1 = 332$

$A = 72$

$y_G = \frac{332}{72} = 5{,}444$

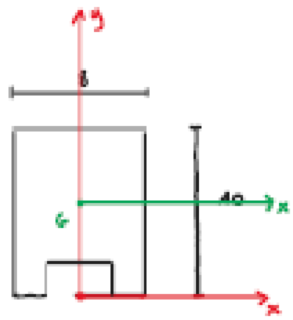
1) Rettangolo grande
2) " piccolo

$I^1_{xx_G} = \frac{8 \cdot 10^3}{12} = 666{,}6$

$I^2_{xx_G} = \frac{4 \cdot 2^3}{12} = 2{,}66$

$$I^{finale}_{xx_G} = \Big[\underbrace{666{,}66 + 80 \cdot (5 - 5{,}44}_{1})^2\Big] - \Big[2{,}66 + 8 \cdot (1 - 5{,}44)^2\Big] = \underline{\underline{521{,}778}}$$

Alternativa



Rispetto asse x rosso

$I^1_{xx} = \frac{BH^3}{3} = \frac{8 \cdot 10^3}{3}$

$I^2_{xx} = \frac{4 \cdot 2^3}{3}$

$I^{finale}_{xx} = I^1_{xx} - I^2_{xx} = 2656$

$I_{xx} = I_{xx_G} + A d_{y_G}^2$

$I^{finale}_{xx_G} = I^{finale}_{xx} - A \cdot y_G^2 = 2656 - 72 \cdot (5{,}44)^2 = 521{,}778$

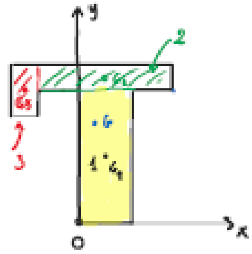
$I^1_{yy} = \frac{HB^3}{12} = \frac{10 \cdot 8^3}{12} = 426{,}6$

$I^2_{yy} = \frac{2 \cdot 4^3}{12} = 10{,}66$

$I^{finale}_{yy} = 416$

$I^{finale}_{xy_G} = 0$

$$\begin{cases} I_{xx} = I_{xx}^{G} + A d_{y_G}^{2} \\ I_{yy} = I_{yy}^{G} + A d_{x_G}^{2} \\ I_{xy} = I_{xy}^{G} + A d_{x} d_{y} \end{cases}$$



	A_i	y_G^i	S_x^i	x_G^i	S_y^i
1)					
2)					
3)					

A_{tot} 68 $\quad S_x^{tot}$ 83 $\quad S_y^{tot}$ 500

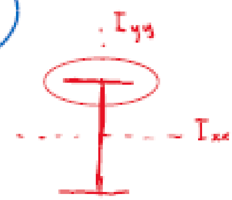
$$x_G = \frac{S_y^{tot}}{A_{tot}} = 1{,}3$$

$$y_G = \frac{S_x^{tot}}{A_{tot}} = 7{,}35$$

1) $I_{xx_G}^{1} = \frac{4 \cdot 10^3}{12} = 333 \quad I_{yy_G}^{1} = \frac{10 \cdot 4^3}{12} = 53{,}3 \quad I_{xy_G}^{1} = 0$

2 $I_{xx_G}^{2} = \frac{10 \cdot 2^3}{12} = 6{,}66 \quad I_{yy}^{2} = \frac{2 \cdot 10^3}{12} = 166{,}6 \quad I_{xy_G}^{2} = 0$

3) $I_{xx_G}^{3} = \frac{2 \cdot 4^3}{12} = 10{,}66 \quad I_{yy_G}^{3} = \frac{4 \cdot 2^3}{12} = 2{,}66 \quad I_{xy_G}^{3} = 0$



$$\begin{cases} I_{xx} = I_{xx}^{G} + A d_{y_G}^{2} \\ I_{yy} = I_{yy}^{G} + A d_{x_G}^{2} \\ I_{xy} = I_{xy}^{G} + A d_{x} d_{y} \end{cases} \Rightarrow$$

$$I_{xx_G} = \underbrace{\left[I_{xx_G}^{1} + A_1 \cdot \overbrace{(y_G^1 - y_G)}^{d_{y_G}}{}^2\right]}_{1} + \underbrace{\left[I_{xx_G}^{2} + A_2 \cdot (y_G^2 - y_G)^2\right]}_{2} + \underbrace{\left[I_{xx_G}^{3} + A_3 \cdot (y_G^3 - y_G)^2\right]}_{3}$$

$$I_{xx}^{G} = 333 + 40 \cdot (5 - 7{,}35)^2 + 6{,}66 + 20(11 - 7{,}35)^2 + 10{,}66 + 8 \cdot (10 - 7{,}35)^2 = 894{,}15$$

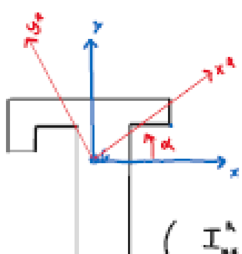
$$I_{yy}^{G} = 53{,}3 + 40(2 - 1{,}3)^2 + 166{,}6 + 20(2 - 1{,}3)^2 + 2{,}66 + 8(-4 - 1{,}3)^2 = 476{,}62$$

$$I_{xy}^{G} = 0 + 40(5 - 7{,}35)(2 - 1{,}3) + 0 + 20(11 - 7{,}35)(2 - 1{,}3) + 0 + 8(10 - 7{,}35)(-4 - 1{,}3) = -127{,}6$$

$$I_{xx_G}^{totale} = \sum_{i=1}^{m} I_{xx_G}^{i} + A_i (y_G^i - y_G)^2$$

$$I_{yy_G}^{totale} = \sum_{i=1}^{m} I_{yy_G}^{i} + A_i (x_G^i - x_G)^2$$

$$I_{xy_G}^{totale} = \sum_{i=1}^{m} I_{xy_G}^{i} + A_i (y_G^i - y_G)(x_G^i - x_G)$$



$\alpha = 0°$	$\alpha = 5°$	$\alpha = 15°$	$\alpha = 15.7169$
$I_{xx}^{G} = 894{,}15$	$I_{xx}^{G} = 913{,}136$	$I_{xx}^{G} = 929{,}9$	$I_{xx}^{G} = 930.057$
$I_{yy}^{G} = 476{,}62$	$I_{yy}^{G} = 457{,}634$	$I_{yy}^{G} = 440{,}8$	$I_{yy}^{G} = 440.713$
$I_{xy}^{G} = -127{,}6$	$I_{xy}^{G} = -89{,}41$	$I_{xy}^{G} = -6.16$	$I_{xy}^{G} = 0$

$$\begin{cases} I_{xx}^{R} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2} \cdot \cos(2\alpha) - I_{xy} \sin(2\alpha) \\ I_{yy}^{R} = \frac{I_{xx} + I_{yy}}{2} - \frac{I_{xx} - I_{yy}}{2} \cdot \cos(2\alpha) + I_{xy} \sin(2\alpha) \\ I_{xy}^{R} = \frac{I_{xx} - I_{yy}}{2} \sin(2\alpha) + I_{xy} \cos(2\alpha) \end{cases}$$

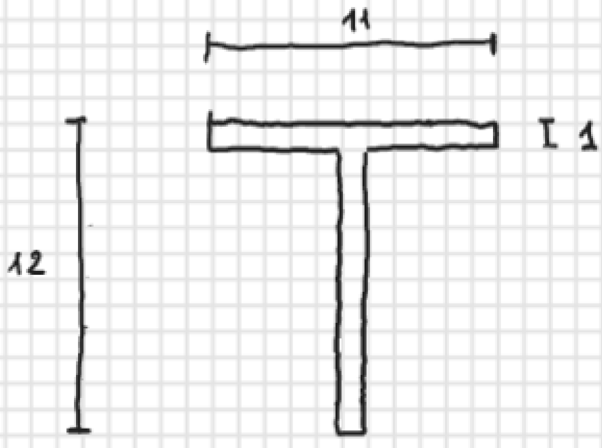
$$I_{xx}^{P} = \frac{I_{xx} + I_{yy}}{2} + \frac{1}{2}\sqrt{(I_{xx} - I_{yy})^2 + 4 I_{xy}^2}$$

$$I_{yy}^{P} = \frac{I_{xx} + I_{yy}}{2} - \frac{1}{2}\sqrt{(I_{xx} - I_{yy})^2 + 4 I_{xy}^2}$$

$$\alpha = \frac{1}{2} \operatorname{arctg}\left(-\frac{2 I_{xy}}{I_{xx} - I_{yy}}\right)$$

Esercizi

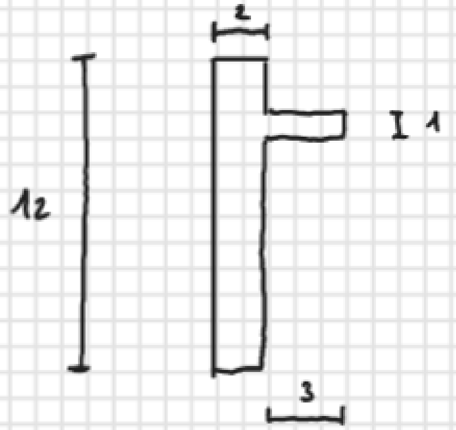
1



$I_{xx}, I_{yy}, I_{xy} = ?$

I^{p}_{xx}, I^{p}_{yy} α

2



$I_{xx}, I_{yy}, I_{xy} = ?$

I^{p}_{xx}, I^{p}_{yy} α

3