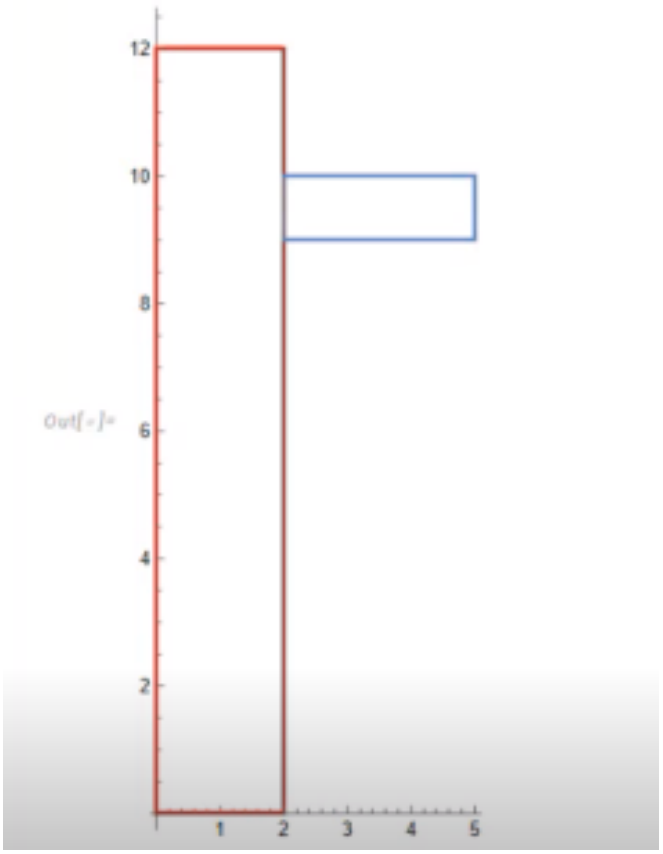
Esempio geometria aree



In[ ]= **Momenti statici e baricentro**

Out[ ]= baricentro e Momenti statici

```
In[ ]= A1 := 12 * 2
       xg1 := 1
       yg1 := 6
       Sx1 := A1 * yg1
       Sy1 := A1 * xg1
       A2 := 3 * 1.
       xg2 := 2 + 3 / 2
       yg2 := 9 + 1 / 2
       Sx2 := A2 * yg2
       Sy2 := A2 * xg2

In[ ]= Atot = A1 + A2
       Sxtot = Sx1 + Sx2
       Sytot = Sy1 + Sy2
       xg = Sytot / Atot
       yg = Sxtot / Atot
Out[ ]= 27.
Out[ ]= 172.5
Out[ ]= 34.5
Out[ ]= 1.27778
Out[ ]= 6.38889
```

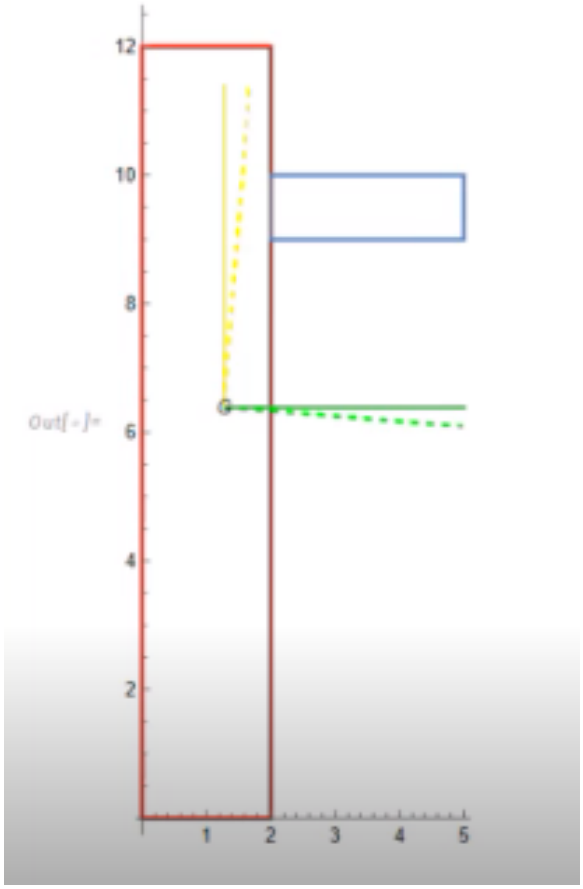
In[ ]= **Momenti d'inerzia rettangoli**

Out[ ]= inerzia Momenti rettangoli d'

```
In[ ]= Ixxg1 = 1 / 12 * 2. * 12^3
       Iyyg1 = 1 / 12 * 12. * 2^3
       Ixxg2 = 1 / 12 * 3. * 1^3
       Iyyg2 = 1 / 12 * 1. * 3^3
Out[ ]= 288.
Out[ ]= 8.
Out[ ]= 0.25
Out[ ]= 2.25
```

Momenti d'inerzia sezione intera

```
In[ ]= Ixxg = Ixxg1 + A1 * (yg1 - yg) ^2 + Ixxg2 + A2 * (yg2 - yg) ^2
       Iyyg = Iyyg1 + A1 * (xg1 - xg) ^2 + Iyyg2 + A2 * (xg2 - xg) ^2
       Ixyg = A1 * (yg1 - yg) * (xg1 - xg) + A2 * (yg2 - yg) * (xg2 - xg)
Out[ ]= 320.917
Out[ ]= 26.9167
Out[ ]= 23.3333
```



In[ ]:= **Momenti d'inerzia principali**

Out[ ]= inerzia Momenti principali d'

In[ ]:= $\text{IxxPrinc} = \frac{\text{Ixxg} + \text{Iyyg}}{2} + \frac{1}{2} * \sqrt{(\text{Ixxg} - \text{Iyyg})^2 + 4 * \text{Ixyg}^2}$

Out[ ]= 322.757

In[ ]:= $\text{IyyPrinc} = \frac{\text{Ixxg} + \text{Iyyg}}{2} - \frac{1}{2} * \sqrt{(\text{Ixxg} - \text{Iyyg})^2 + 4 * \text{Ixyg}^2}$

Out[ ]= 25.0763

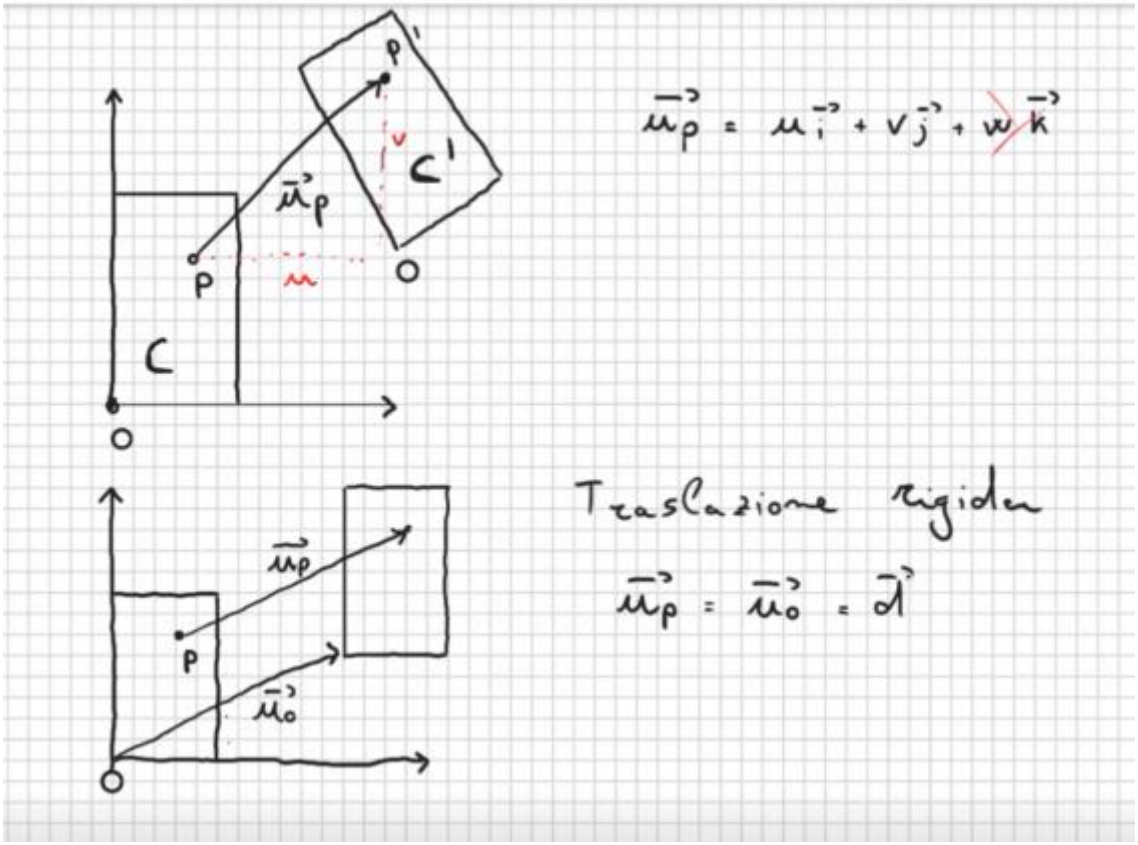
In[ ]:= $\text{RotazionePrinc} = \frac{1}{2} * \text{ArcTan}\left[-\frac{2 * \text{Ixyg}}{\text{Ixxg} - \text{Iyyg}}\right]$

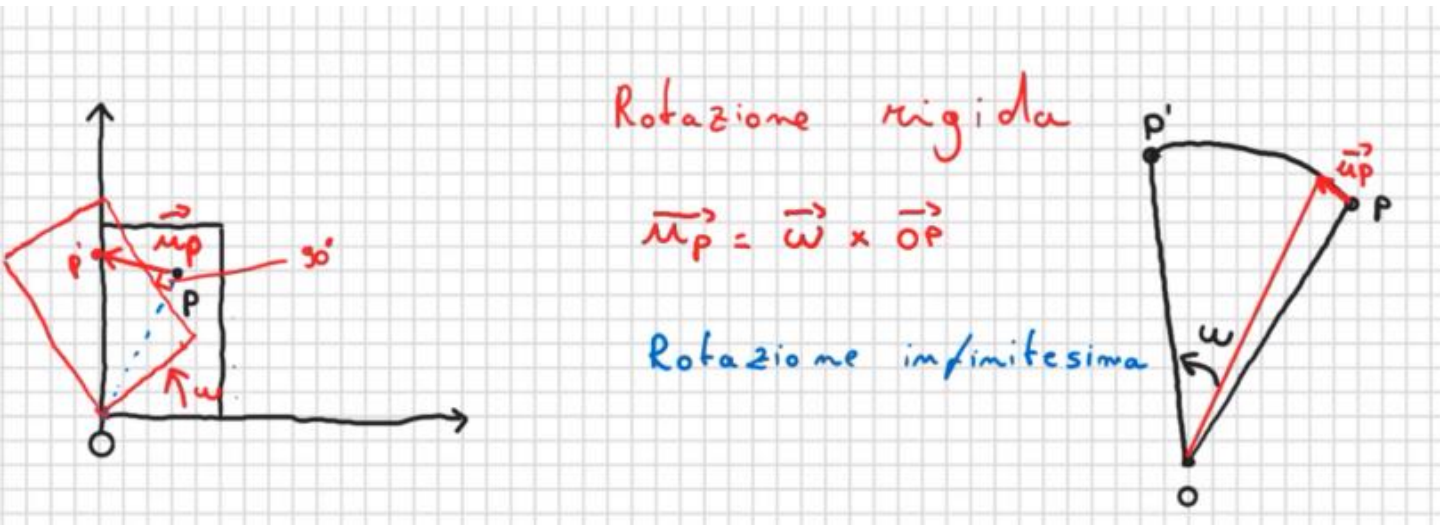
RotazionePrincGradi = RotazionePrinc * 180 / Pi

Out[ ]= -0.0787084

Out[ ]= -4.50966

Cinematica corpo rigido





Roto - traslazione

$$\vec{u}_P = \vec{u}_0 + \vec{\omega} \times \vec{OP}$$

$$\begin{bmatrix} u_P \\ v_P \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} + \begin{bmatrix} -\omega \cdot y \\ \omega \cdot x \end{bmatrix}$$

$$\vec{u}_P = (u_0 - \omega y)\,\vec{i} + (v_0 + \omega x)\,\vec{j}$$

$$\begin{bmatrix} -\omega \cdot y \\ \omega \cdot x \end{bmatrix} \Rightarrow \vec{\omega} \times \vec{OP} = \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix} \begin{matrix} \\ \rightarrow \text{componenti di } \vec{\omega} \\ \rightarrow \text{coordinate di } P \end{matrix}$$

$$= (\omega_y \cdot z - \omega_z y)\,\vec{i} - (\omega_x \cdot z - \omega_z \cdot x)\,\vec{j} + (\omega_x \cdot y - \omega_y \cdot x)\,\vec{k} \qquad 3\Delta$$

$$\Downarrow \quad 2\Delta \qquad \omega_x = 0 \qquad \omega_y = 0 \qquad \omega_z = \omega$$

$$= \vec{\omega} \times \vec{OP} = -\omega \cdot y\,\vec{i} + \omega \cdot x\,\vec{j}$$

$$\vec{u}_P = (u_0 - \omega y)\,\vec{i} + (v_0 + \omega x)\,\vec{j}$$

C_R ? $\Rightarrow$ $\vec{u}_{C_R} = 0$

$$u_0 - \omega y_{C_R} = 0 \qquad \Rightarrow \qquad y_{C_R} = \frac{u_0}{\omega}$$

$$v_0 + \omega \cdot x_{C_R} = 0 \qquad x_{C_R} = -\frac{v_0}{\omega}$$

$$u_P = (\omega y_{C_R} - \omega y) = \omega (y_{C_R} - y)$$

$$v_P = -\omega x_{C_R} + \omega x = \omega (x - x_{C_R})$$

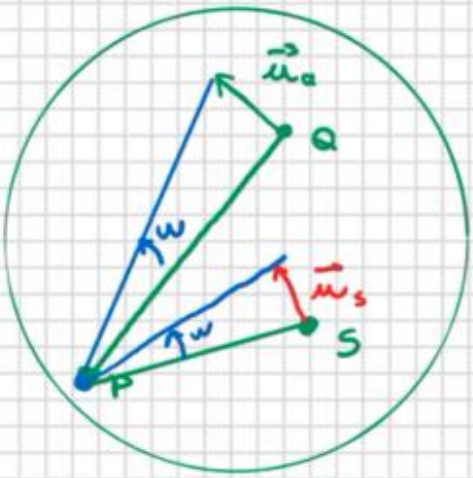
Traslazione rigida $\omega = 0$ $\vec{v}_P = \vec{v}_O + \omega \times \vec{OP}$

$y_{CR} \to \infty$

$x_{CR} \to \infty$

$C_R(\infty, \infty)$

1)

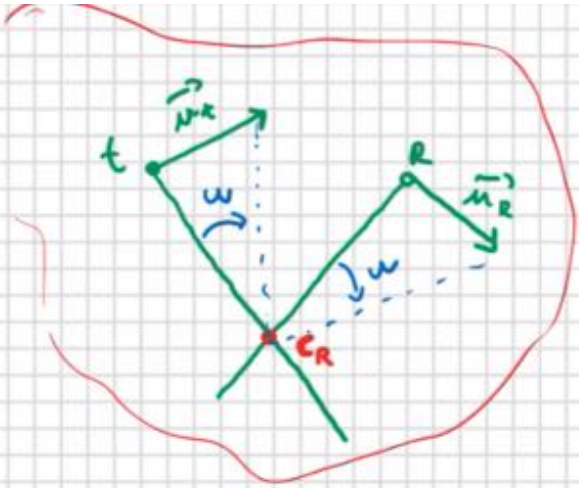


$C_R \equiv P$ $\vec{v}_a$ Noti

$\Downarrow$

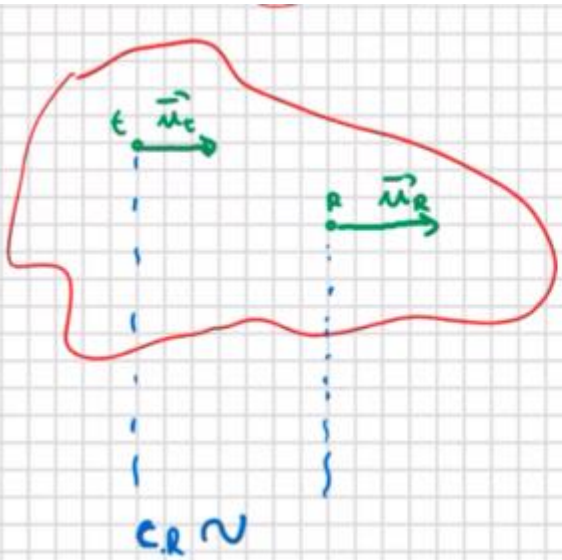
$\vec{v}_s$

2)



$\vec{v}_t$ e $\vec{v}_R$ Noti

$\Downarrow$

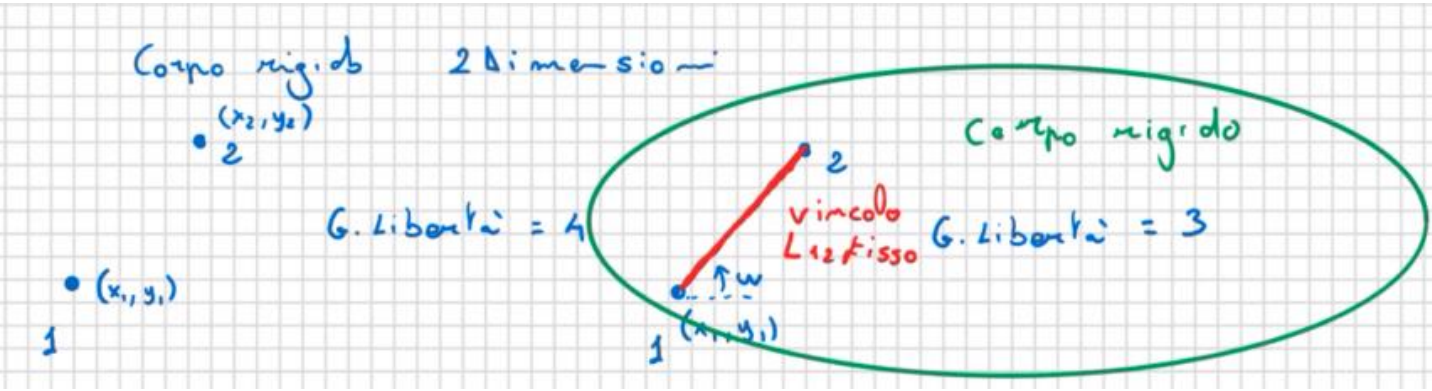


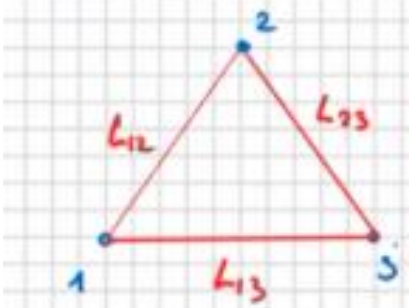
Traslazione rigida

- Gradi di libertà (G)

Punto 3G spazio 2G piano

Per n punti 3n o 2n





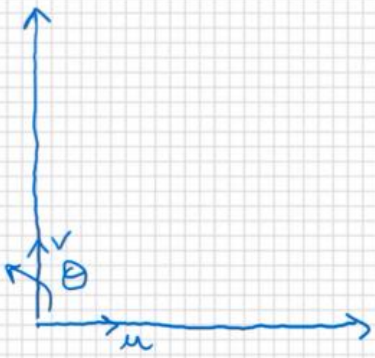
GL = G - GL dei punti

GV : 3 = G di vincolo

3 GL del sistema

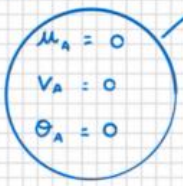
Trave ⟶ 3 GL

Vincoli



- Incastro

GL = 3



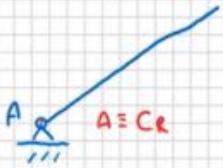
molteplicità vincolo = 3

GL sistema = 3 - 3 = 0

C_R non esiste

- Cerniera

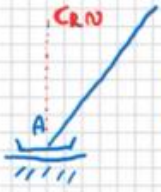
molteplicità 2



GL sistema = 3 - 2 = 1

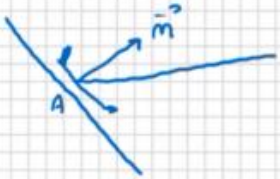
$$\begin{cases} u_A = 0 \\ V_A = 0 \end{cases}$$

- Pattino molteplicità 2



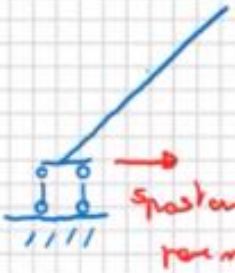
$$\begin{cases} V_A = 0 \\ \Theta_A = 0 \end{cases}$$

$$\begin{cases} u_A = 0 \\ \Theta_A = 0 \end{cases}$$



$$\begin{cases} \vec{u}_A \cdot \vec{n} = 0 \\ \Theta_A = 0 \end{cases}$$

$u_A \cos\alpha + V_A \operatorname{sen}\alpha = 0$



Doppio pendolo molteplicità 2

spostamento permesso

$\begin{cases} v_A = 0 \\ \Theta_A = 0 \end{cases}$

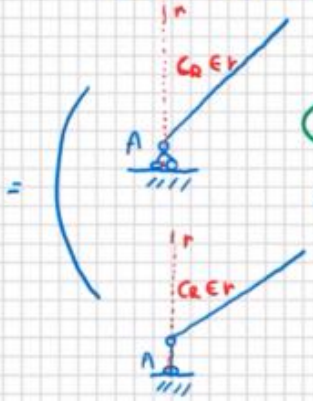
spostamento permesso

$\begin{cases} v_A = 0 \\ \Theta_A = 0 \end{cases}$

- Manicotto molteplicità 2

$\begin{cases} v_A = 0 \\ \Theta_A = 0 \end{cases}$

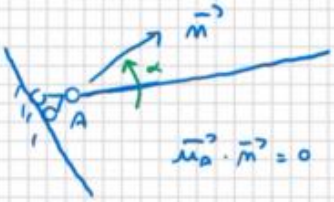
- Vincoli semplici



Carrello molteplicità 1

$v_A = 0$

GL sistema = 3 - 1 = 2



$\vec{u}_A \cdot \vec{n} = 0$

- $u_A \cos\alpha + v_A \sin\alpha = 0$

Pendolo semplice molteplicità 1

$v_A = 0$

- 

Pattino-manicotto molteplicità 1

$\Theta_A = 0$

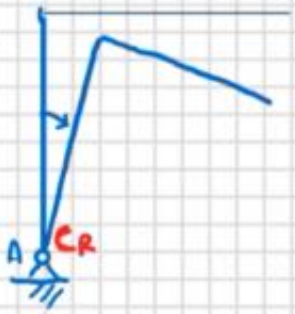
Combinazione dei vincoli

$m = 3$

$GL_{sistema} = 3 - 3 = 0$

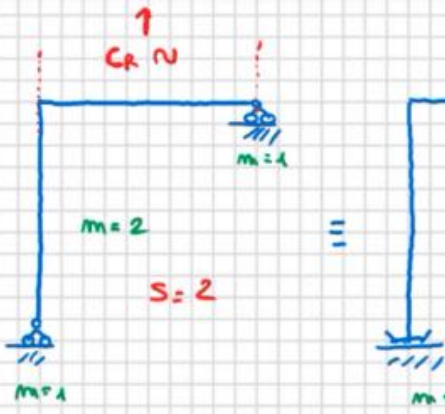
S. vincoli effettivamente sottratti al sistema

$s = 3$



$GL_{sistema} = 3 - 2 = 1$ (Residuo)

$s = 2$



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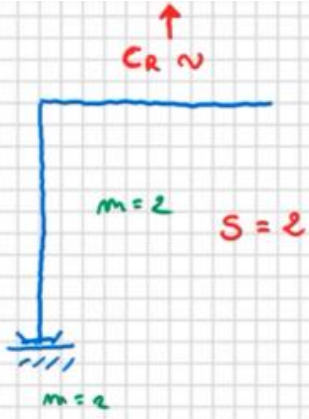
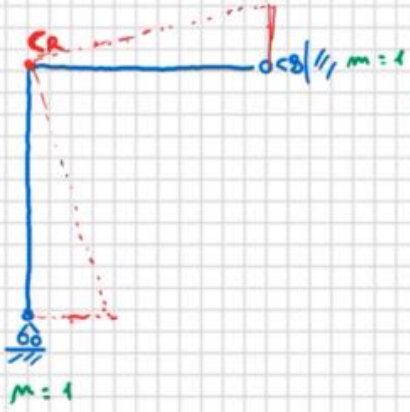
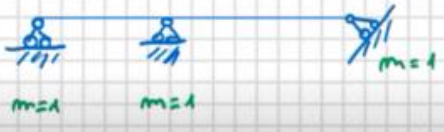




$g = 3$

$m = 2$

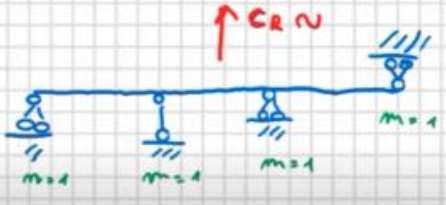
$s = 2$



$g = 3$

$m = 3$

$s = ?\ 3$



$g = 3$

$m = 4$

$s = 2$

$g = 3$

$m = 3$

$s = 2$

- Labili
- isostatiche
- iperstatiche

- Labili $s < g$

1)

$g = 3$

$m = 2$

$s = 2$

2)

$g = 3$

$m = 3$

$s = 2$

isostatiche = $g = m = s$

$g = 3$

$m = 3$

$s = 3$

$g = 3$

$m = 3$

$s = 3$

- iperstatiche $s = g < m$

$g = 3$

$m = 5$

$s = 3$

$g = 3$

$m = 4$

$s = 3$

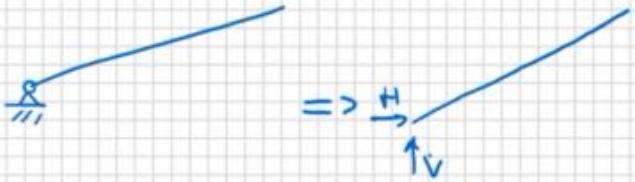
Reazioni vincolari

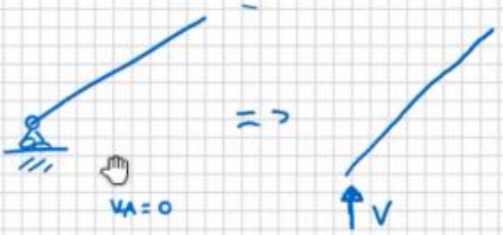
$u_A = 0$

$v_A = 0$

$\Theta_A = 0$

H
V = incognite del problema
M





$v_A = 0$