Caratteristiche di sollecitazione

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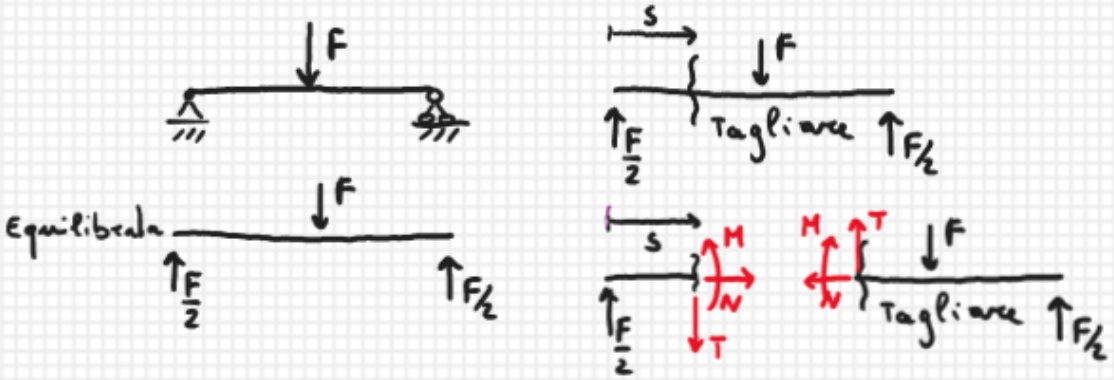
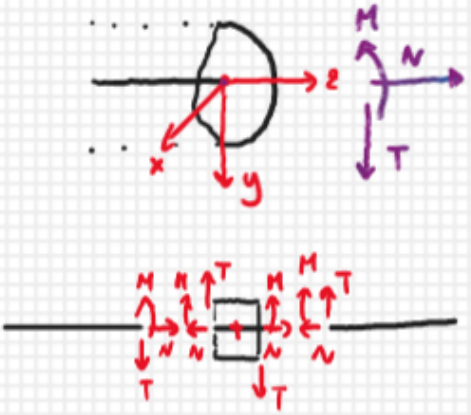
Azioni interne

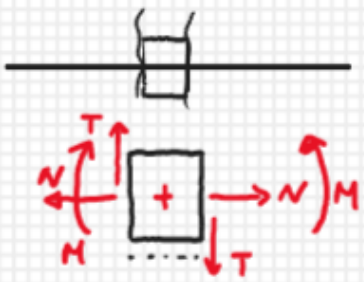


N Forza normale (azione assiale)

T Forza tagliante (taglio)

M Momento flettente



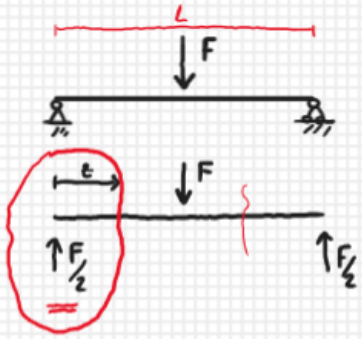


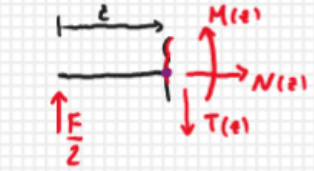
Concio elementare

Tende le fibre inferiori del concio ($y > 0$)

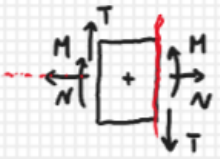
$N, T, M \rightarrow$ funzioni di $s \longrightarrow N(s), T(s), M(s)$

Es





$$\begin{cases} N(z) = 0 \\ \frac{F}{2} - T(z) = 0 \\ -\frac{F}{2} z + M(z) = 0 \end{cases} \Rightarrow \begin{array}{l} N(z) = 0 \\ T(z) = \frac{F}{2} \\ M(z) = \frac{F}{2} z \end{array} \qquad z \in \left[0, \frac{L}{2}\right]$$



Superato il carico concentrato F $\quad z \in \left[\frac{L}{2}, L\right]$

$$\begin{cases} N(z) = 0 \\ \frac{F}{2} - F - T(z) = 0 \\ -\frac{F}{2} z + F\left(z - \frac{L}{2}\right) + M(z) = 0 \end{cases} \Rightarrow \begin{array}{l} N(z) = 0 \\ T(z) = -F/2 \\ M(z) = \frac{F}{2}(L - z) \end{array}$$

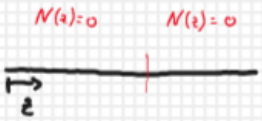
Valutiamo le azioni interne in punti significativi della trave

$z = 0 \quad T(z=0) = F/2 \quad M(z=0) = 0$

$z = \frac{L}{2} \quad T(z=\frac{L}{2}) \text{ discontinua} \quad M(z=\frac{L}{2}) = \frac{FL}{4}$

$z = L \quad T(z=L) = -\frac{F}{2} \quad M(z=L) = 0$

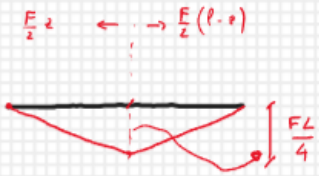
Diagrammi azioni interne



N

salto = F carico applicato

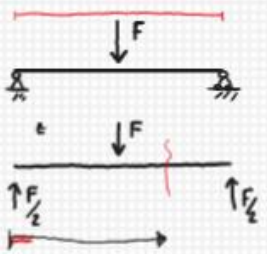
T

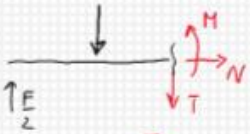


M

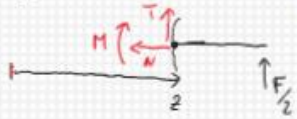
Diagramma di M dalla parte delle fibre tese della struttura

Es

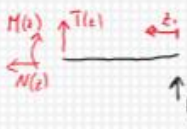




$$T(z) = -F/2 \qquad M(z) = \frac{F}{2}(L - z) \qquad z \in \left[\frac{L}{2}, L\right]$$



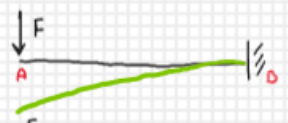
$$\Rightarrow \begin{cases} N(z) = 0 \\ T(z) + F/2 = 0 \\ -M(z) + \frac{F}{2}(L - z) = 0 \end{cases} \Rightarrow \begin{array}{l} T(z) = -F/2 \\ M(z) = \frac{F}{2}(L - z) \end{array}$$



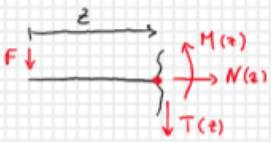
$z_1 \in [0, L/2]$

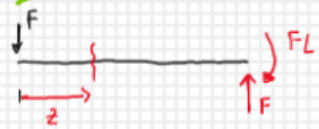
opzione con cambio di coordinata z_1 da destra a sinistra

Es Mensola



3) $FL + M_B = 0 \Rightarrow M_B = -FL$



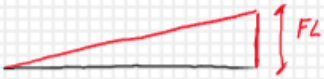


$N(z), T(z), M(z) \Rightarrow ?$

$$\begin{cases} N(z) = 0 \\ -F - T(z) = 0 \\ Fz + M(z) = 0 \end{cases} \Rightarrow \begin{array}{l} N(z) = 0 \\ T(z) = -F \\ M(z) = -Fz \end{array}$$

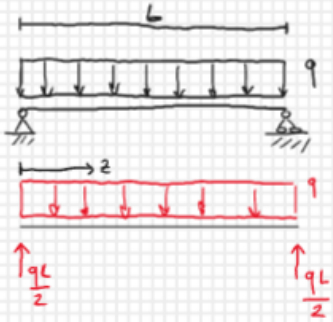
$N(z)$

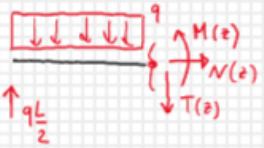
$T(z)$



$M(z)$

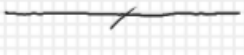
Es





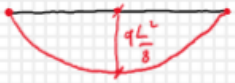
$$\begin{cases} N(z) = 0 \\ -T(z) + q\frac{L}{2} - qz = 0 \\ M(z) - q\frac{L}{2}z + q\frac{z^2}{2} = 0 \end{cases} \Rightarrow \begin{array}{l} N(z) = 0 \\ T(z) = q\frac{L}{2} - qz \\ M(z) = q\frac{L}{2}z - q\frac{z^2}{2} \end{array}$$

$z = 0$	$T(0) = q\frac{L}{2}$	$M(0) = 0$
$z = \frac{L}{2}$	$T(\frac{L}{2}) = 0$	$M(\frac{L}{2}) = \frac{qL^2}{8}$
$z = L$	$T(L) = -q\frac{L}{2}$	$M(L) = 0$



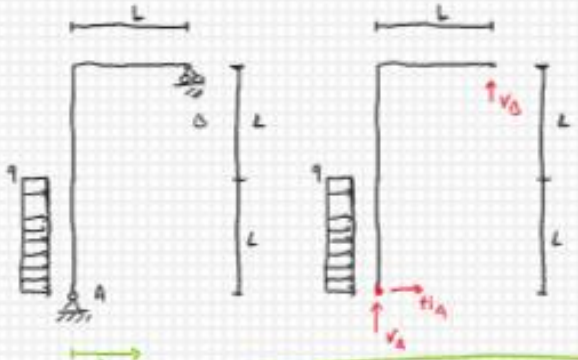
N

T



$M_{max} = q\frac{L^2}{8}$

Es

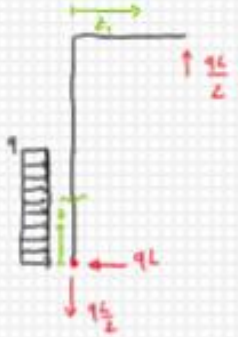


3 E.C.S

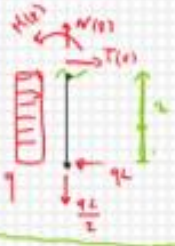
1) $H_A + qL = 0$ $\quad H_A = -qL$

2) $V_A + V_B = 0$ $\quad \Rightarrow \quad V_A = -\frac{qL}{2}$

3) $-A \quad V_B L - q\frac{L^2}{2} = 0$ $\quad V_B = q\frac{L}{2}$



- Tratto 1 $\quad z \in [0, L]$



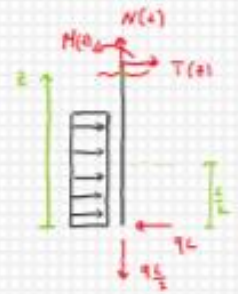
$-q\frac{L}{2} + N(z) = 0$ $\quad$ Asse sulla trave

$qL - T(z) - qz = 0$ $\quad$ Taglio sulla sezione

$-qLz + q\frac{z^2}{2} + M(z) = 0$

$N(z) = q\frac{L}{2}$ $\quad T(z) = qL - qz$ $\quad M(z) = qLz - q\frac{z^2}{2}$

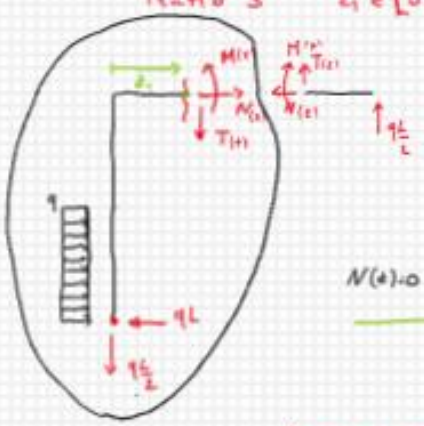
- Tratto 2 $\quad z \in [L, 2L]$



- $N(z) - q\frac{L}{2} = 0$
- $qL - T(z) - qL = 0$
- $M(z) - qLz + qL\left(z - \frac{L}{2}\right) = 0$

$N(z) = q\frac{L}{2}$ $\quad T(z) = 0$ $\quad M(z) = q\frac{L^2}{2}$

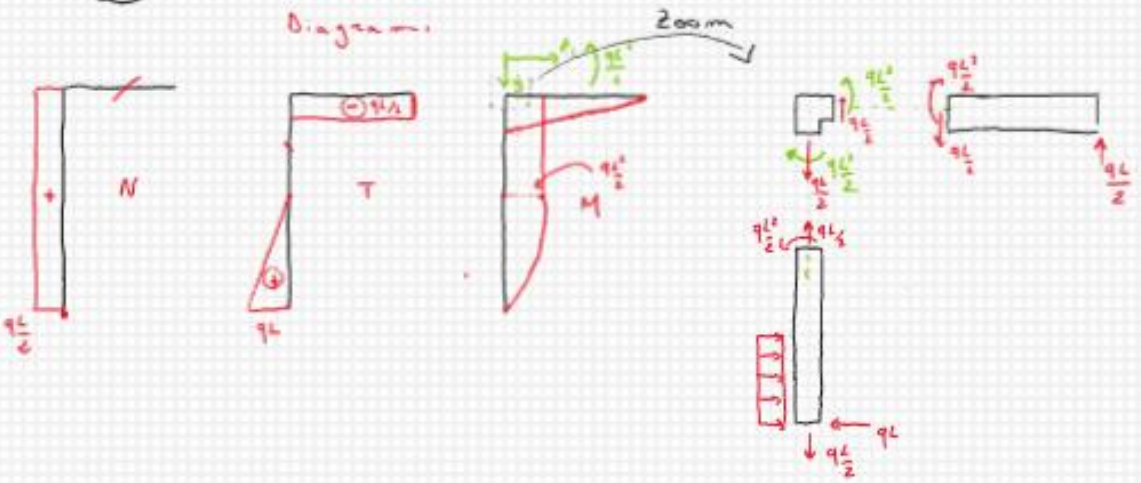
Tratto 3 $\quad z_1 \in [0, L]$



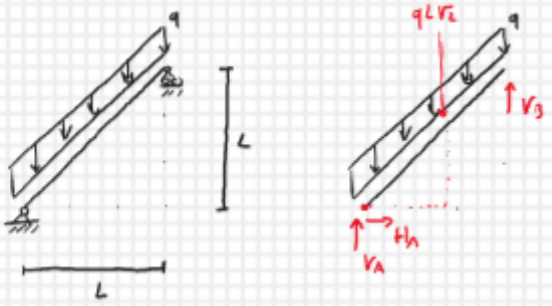
- $N(z) - qL + qL = 0$
- $-T(z) - q\frac{L}{2}$
- $M(z) - 2qL^2 + \frac{1}{2}qL^2 + q\frac{L}{2}z_1 = 0$

$N(z) = 0$ $\quad T(z) = -q\frac{L}{2}$ $\quad M(z) = q\frac{L^2}{2} - q\frac{L}{2}z_1$

Diagrami:



Es

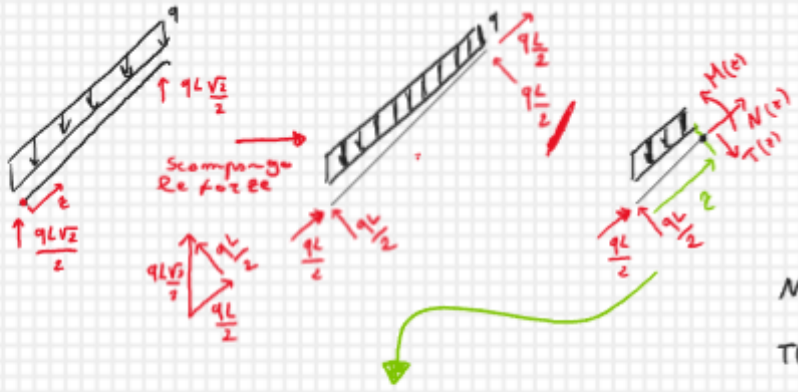


3 ECS

$H_A = 0$ $\quad H_A = 0$

$V_A + V_B - qL\sqrt{2} = 0$ $\quad V_A = qL\frac{\sqrt{2}}{2}$

$V_B L - qL\sqrt{2}\cdot\left(\frac{L}{2}\right) = 0$ $\quad V_B = qL\frac{\sqrt{2}}{2}$



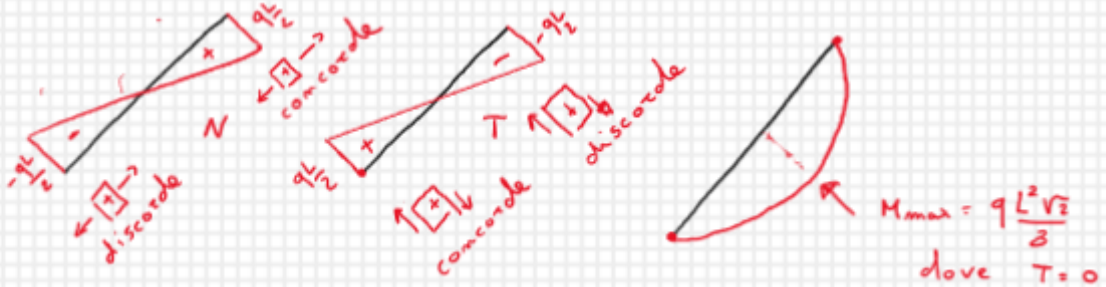
componente carico lungo asse trave

- $qL\frac{\sqrt{2}}{2} + N(z) - qL\frac{\sqrt{2}}{2}z = 0$
- $qL\frac{\sqrt{2}}{2} - T(z) - qL\frac{\sqrt{2}}{2}z = 0$
- $-qL\frac{\sqrt{2}}{2}z + M(z) + qL\frac{\sqrt{2}}{2}\frac{z^2}{2} = 0$

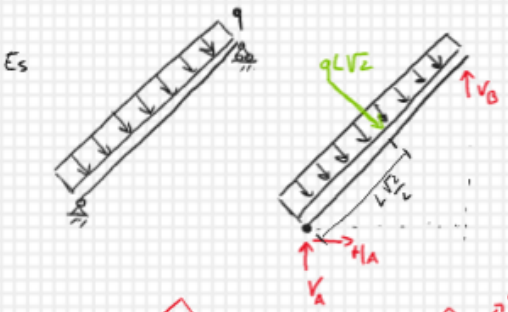
$N(z) = -qL\frac{\sqrt{2}}{2} + qL\frac{\sqrt{2}}{2}z$

$T(z) = qL\frac{\sqrt{2}}{2} - qL\frac{\sqrt{2}}{2}z$

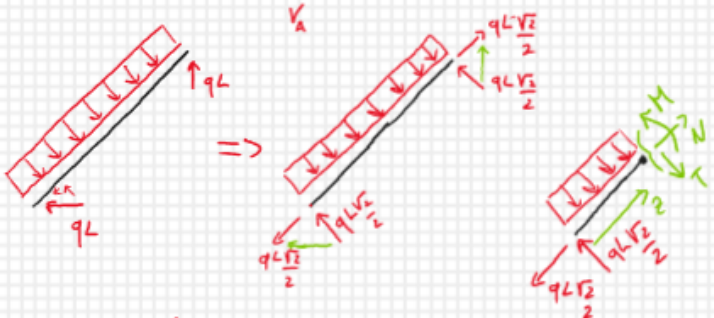
$M(z) = qL\frac{\sqrt{2}}{2}z - qL\frac{\sqrt{2}}{4}z^2$



Es



- $H_A + qL = 0$ $\quad H_A = -qL$
- $V_A + V_B - qL = 0$ $\quad \Rightarrow \quad V_B = qL$
- $V_B L - qL\sqrt{2}\left(L\frac{\sqrt{2}}{2}\right) = 0$ $\quad V_A = 0$



- $N(z) - qL\frac{\sqrt{2}}{2} = 0$
- $qL\frac{\sqrt{2}}{2} - qz - T(z) = 0$
- $-qL\frac{\sqrt{2}}{2}z + M(z) + q\frac{z^2}{2} = 0$

$N(z) = qL\frac{\sqrt{2}}{2}$

$T(z) = qL\frac{\sqrt{2}}{2} - qz$

$M(z) = qL\frac{\sqrt{2}}{2}z - q\frac{z^2}{2}$