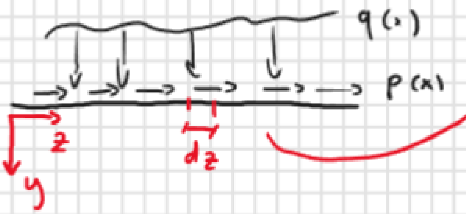
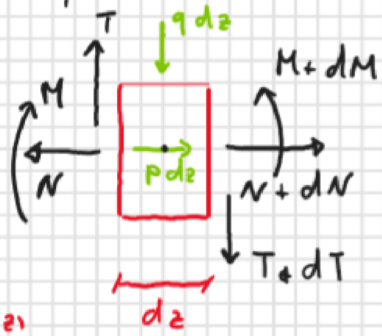
# Equazioni indefinite di equilibrio



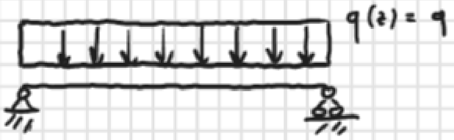




concio infinitesimo

- $\cancel{N} + dN - \cancel{N} + p\,dz = 0$ $\qquad \dfrac{dN(z)}{dz} = -p(z)$
- $\cancel{T} - \cancel{T} - dT - q\,dz = 0$ $\qquad \dfrac{dT(z)}{dz} = -q(z)$
- $-\cancel{M} + \cancel{M} + dM - T\dfrac{dz}{2} - T\dfrac{dz}{2} - dT\dfrac{dz}{2} = 0$ $\qquad \dfrac{dM(z)}{dz} = T(z)$

Trascura

$$\begin{cases} \dfrac{dN(z)}{dz} = -p(z) \\[2mm] \dfrac{dT(z)}{dz} = -q(z) \\[2mm] \dfrac{dM(z)}{dz} = T(z) \end{cases}$$




- $T(z) = \dfrac{qL}{2} - qz$
- $M(z) = \dfrac{qL}{2}z - q\dfrac{z^2}{2}$

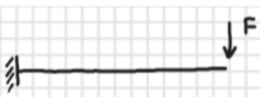
$\Rightarrow$ Verifichiamo le eq. ind. eq

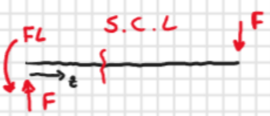
$\dfrac{dT(z)}{dz} = -q$

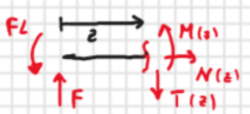
$\dfrac{dM(z)}{dz} = \dfrac{qL}{2} - qz = T(z)$

Rispettano

Es.

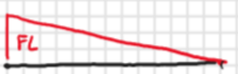



$$\begin{cases} F - T(z) = 0 \Rightarrow T(z) = F \\ M(z) - Fz + FL = 0 \quad M(z) = -FL + Fz \\ N(z) = 0 \end{cases}$$

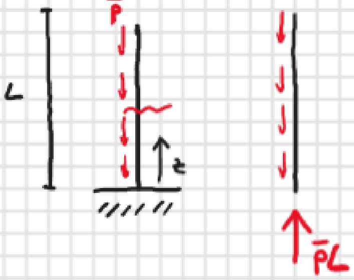
Azioni interne


$$\begin{cases} \dfrac{dN(z)}{dz} = -p(z) \\[2mm] \dfrac{dT(z)}{dz} = -q(z) \\[2mm] \dfrac{dM(z)}{dz} = T(z) \end{cases}$$

$\dfrac{dT(z)}{dz} = -q(z) \longrightarrow q(z) = 0 \qquad \dfrac{dT(z)}{dz} = 0 \rightarrow T(z) = C \qquad T(0) = F = C \qquad T(z) = F$

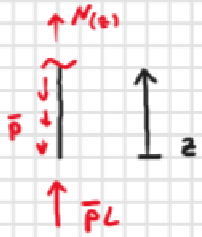
$\dfrac{dM(z)}{dz} = T(z) \longrightarrow T(z) = F \qquad \dfrac{dM(z)}{dz} = F \qquad M(z) = Fz + C \qquad M(0) = C = -FL \qquad M(z) = Fz - FL$




$$\frac{dN(z)}{dz} = -p(z) = \bar{p} \qquad p(z) = -\bar{p}$$

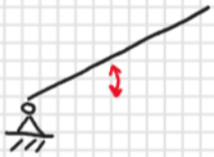
$$\frac{dN}{dz}(z) = \bar{p} \qquad N(z) = \bar{p}z + C \qquad N(0) = C = -\bar{p}L$$

$$N(z) = -\bar{p}L + \bar{p}z$$

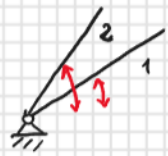



$$\bar{p}L + N(z) - \bar{p}z = 0 \longrightarrow \underline{N(z) = -\bar{p}L + \bar{p}z}$$

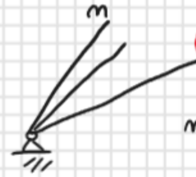
## Vincoli esterni per n aste



$m = 2$



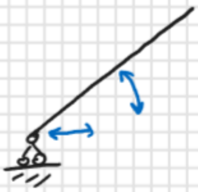

$m = 4$




$\boxed{m = 2 \cdot n}$ cerniera

$n$ = n° corpi rigidi (aste)

$g.l = 3 \cdot n - 2n = n$



$m = 1$

$gl = 2$

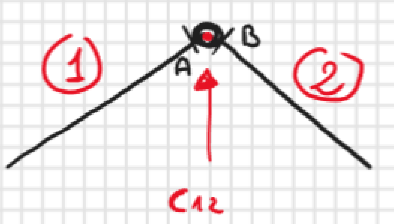

$\boxed{m = 2n - 1}$ carrello

## Vincoli interni

- Incastro


$m = 3$ ⟶


- Cerniera interna

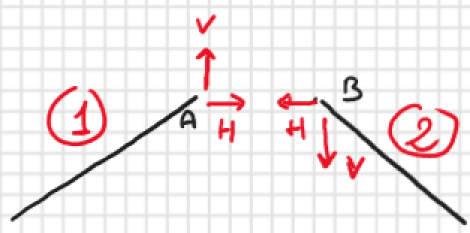


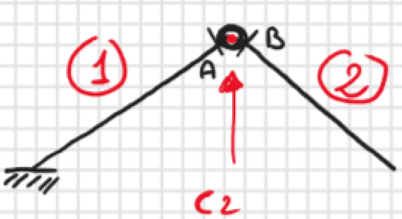

$C_{12}$ centro rotazione relativa tra corpo 1 e 2

$$\vec{u}_A = \vec{u}_B$$

$$\begin{cases} u_{Ax} = u_{Bx} \\ u_{Ay} = u_{By} \end{cases}$$

$m = 2$



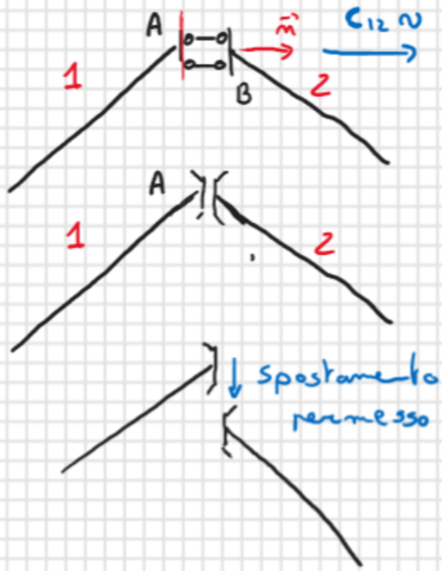

$C_2$ centro rotazione corpo 2

assoluto per 2

$C_{12} \equiv C_2$ se il corpo 1 fosse fisso

Relativo tra 1 e 2

- Doppio pendolo (Pattino)

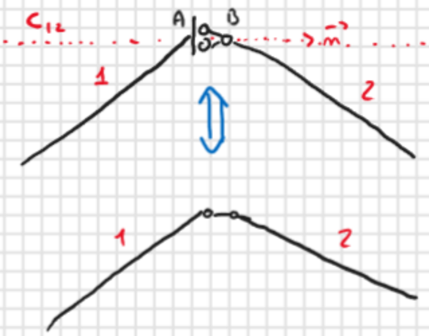



$m = 2$

$$\begin{cases} \theta_A = \theta_B \\ \vec{u}_A \cdot \vec{n} = \vec{u}_B \cdot \vec{n} \end{cases}$$

$C_{12} = ?$


- Carrello (pendolo)

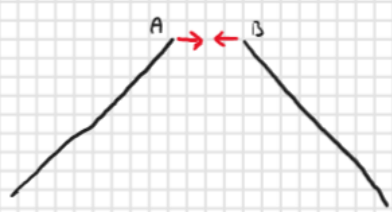


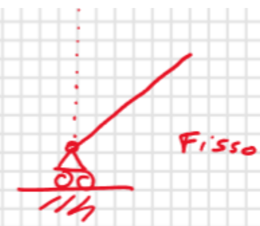

1) $\vec{u}_A \cdot \vec{n} = \vec{u}_B \cdot \vec{n}$

$\Updownarrow$

1) $u_{Ax} = u_{Bx}$

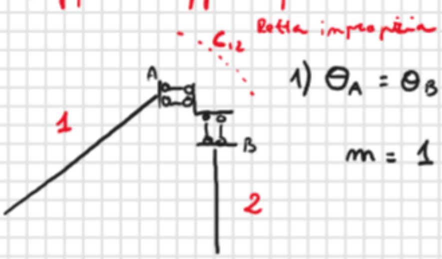
$m = 1$




$C_{12}$ lungo la retta allineata con l'asse del carrello

- Doppio-doppio pendolo




1) $\theta_A = \theta_B$

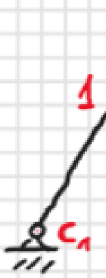
$m = 1$

# Sistemi formati da 2 corpi rigidi (2 aste)

2 corpi rigidi (2 aste)

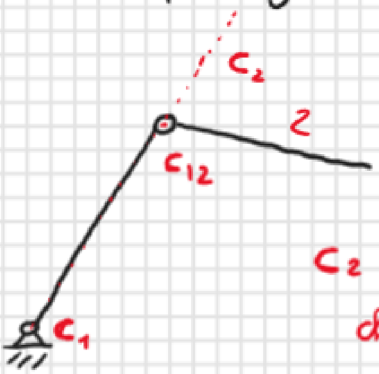
- $C_1$ CIR asta 1
- $C_2$ CIR asta 2 ⟷
- $C_{12}$ CIR relativo tra asta e asta 2

1 corpo rigido

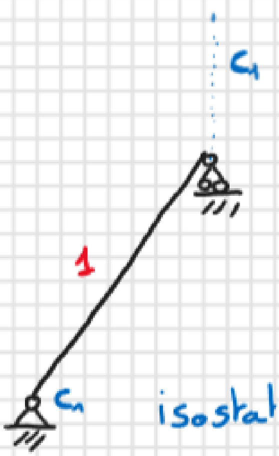



Labili

2 corpi rigidi




$C_2 = ?$ Retta che congiunge $C_1$ e $C_{12}$

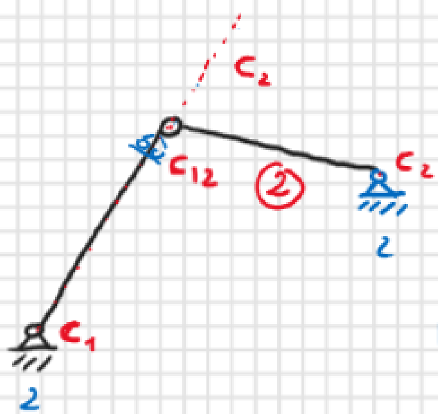



$g = 3$

$m = 2+1 = 3$

$s = 3$

isostatica

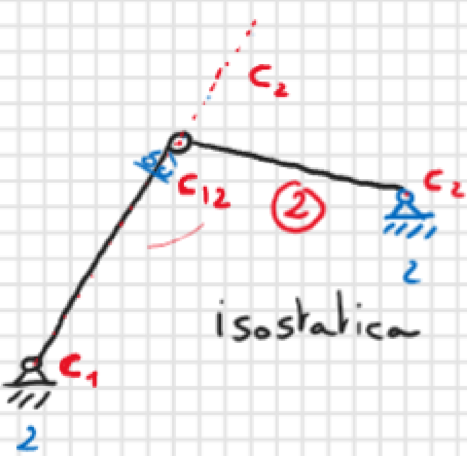



$g = 2 \times 3 = 6$

$m = 2+2+2 = 6$

$s = 6$

isostatica




isostatica

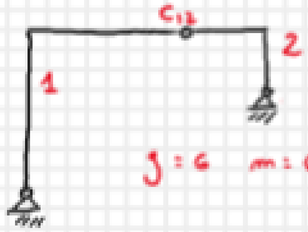
Labile $C_1$, $C_2$ e $C_{12}$ sono allineati


$g = 6$

$m = 6$

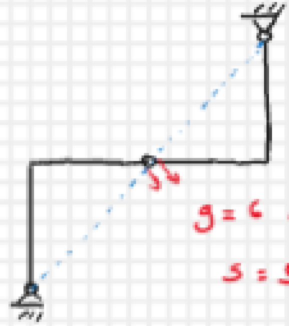
$s = 5$

E5



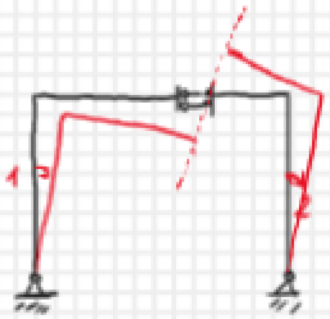

isostatica

g=6 m=6 s=6



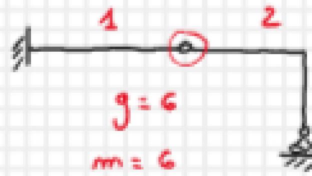
g=6 m=6

s=5




labile

C12 punto improprio in direzione orizzontale

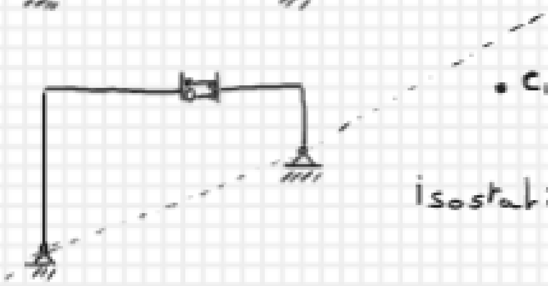



g=6

m=6

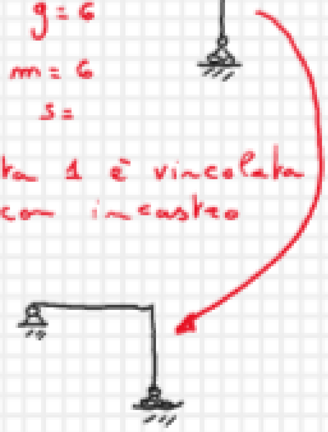
s=

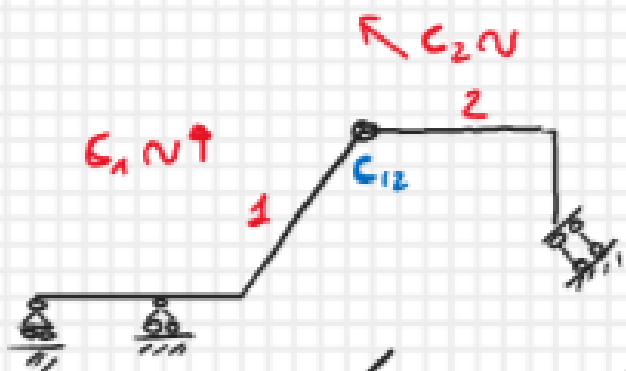
Asta 1 è vincolata con incastro



• C12 infinito orizzontale

isostatica

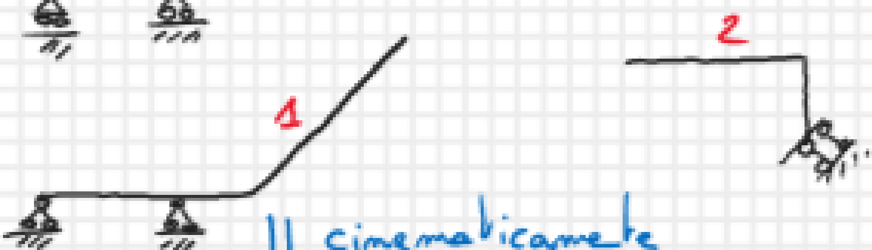





g = 6

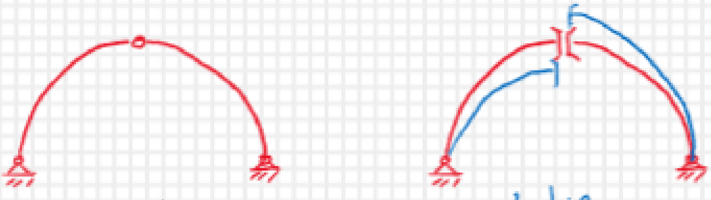
m = 6

C1, C2 e C12 non sono allineati (isostatica)

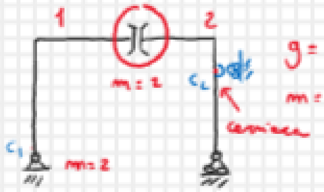



⇓ cinematicamente sostituisco con pattino

· Arco a tre cerniere



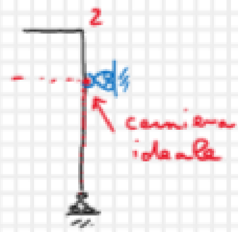

## Arco 3 cerniere generalizzato




$g = 6$

Struttura isostatica

$m: 3+1 = 6$




arco 3 cerniere generalizzato

vincolo doppio relativo

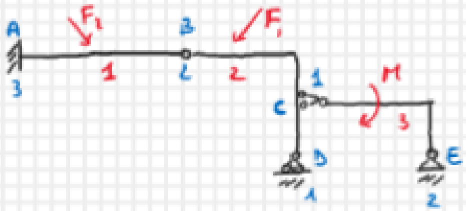
2 vincoli doppi esterni

2 aste

2 vincoli esterni doppi

1 vincolo interno

Calcolo reazioni vincolari (aste ≥ 2)




$g = 9$

$m = 9$

$s = 9$

Risolvo struttura portante 1

↓ isostatica

Risolvo struttura portante 2

↓ isostatica

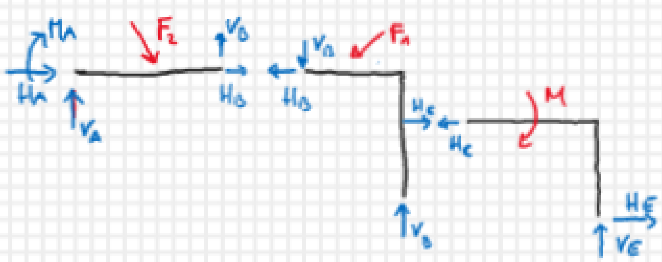
Risolvo struttura 3

↓ isostatica

Tutta la struttura è isostatica

ordine

Sostituisco i vincoli con R.V.

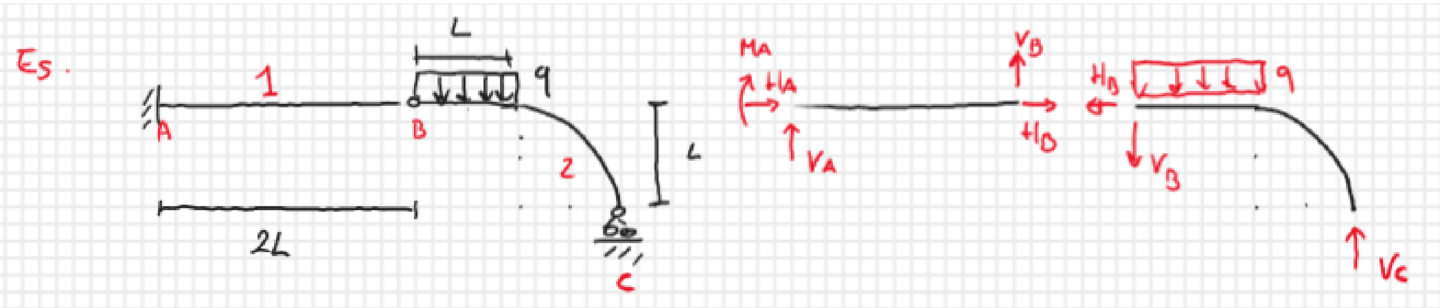



Per ogni asta risolvo

3 eq. c. della statica

9 incognite ($M_A$, $V_A$, $H_A$, $V_B$, $H_B$, ..... )

3 eq. cardi della statica x 3 aste = 9 equazioni




Struttura 1+2 (completa):
- 5 equazioni
- 2 incognite

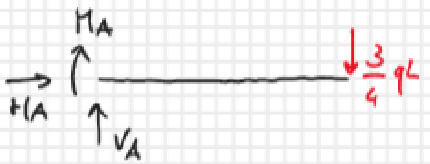
Sotto-struttura 2 (trave curva a destra):
- 3 equazioni
- 3 incognite

Risolvo prima la struttura 2, calcolo le 3 incognite (Hb,Vb e Vc) e sostituisco le incognite trovate nella sottostruttura 1 (Hb e Vb)

Risolvo prima la 2

$H_B = 0$ $\quad V_C = q\frac{L}{4}$ $\quad V_B = -\frac{3}{4}qL$

↓ le sostituisco nella 1




→ Risolvo la 1 $\quad V_A = \frac{3}{4}qL$ $\quad M_A = \frac{3}{2}qL^2$

$H_A = 0$