INTRODUZIONE AL CORSO DI STATICA

28/2/24

RIPASSO GRANDEZZE FISICHE

DIMENSIONI

- scalari : massa [M], lunghezza [L], pressione [FL^{-2}]
- vettoriali : forza [F], spostamento [L]

UNITA' DI MISURA

massa : Kg

lunghezza : m

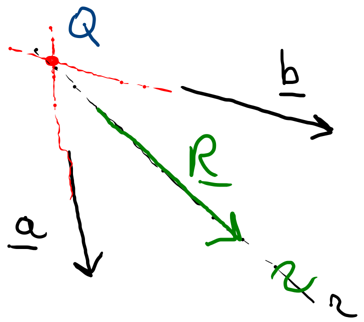
forze : N

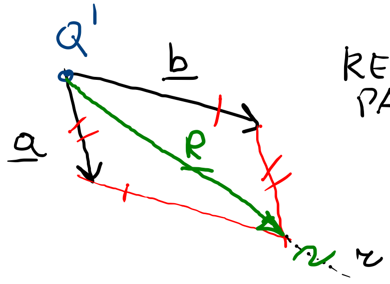
pressione : $Pa = \frac{N}{m^2}$

$MPa = 10^6 \, Pa = \frac{N}{mm^2}$

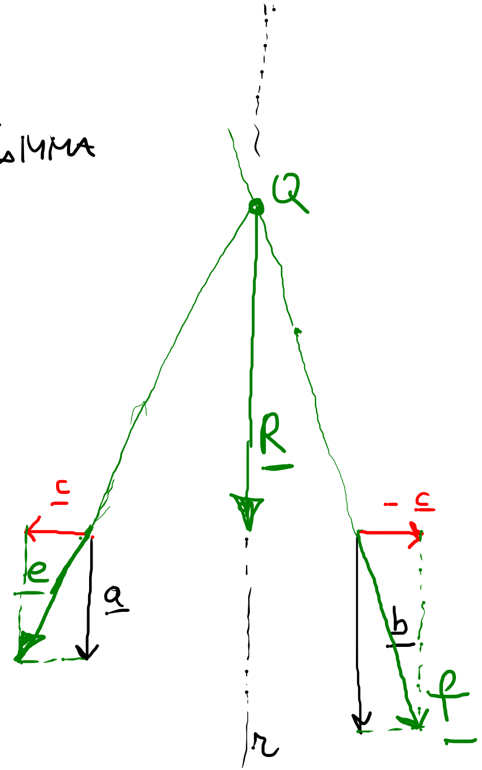
$GPa = 10^9 \, Pa$

QUALCHE CONSIDERAZIONE SUI VETTORI





REGOLA DEL PARALLELOGRAMMA



$$\underline{R} = \underline{a} + \underline{b}$$

VERT VERT VERT

$$\underline{R} = \underline{a} + \underline{b}$$

$|\underline{a}| = 2$; $|\underline{b}| = 3$; $|\underline{R}| = 5$

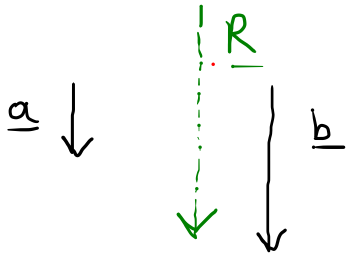
$$\underline{R} = \underbrace{\underline{a} + \underline{c}}_{\underline{e}} + \underbrace{\underline{b} - \underline{c}}_{\underline{f}} = \underline{e} + \underline{f}$$

$\{\underline{c}, -\underline{c}\}$: SIST. DI VETTORI A RISULTANTE NULLA

SIST. DI 2 VETTORI // CONCORDI $\Rightarrow$ RISULTANTE "PASSA" TRA I DUE VETTORI

$|\underline{R}| = |\underline{a}| + |\underline{b}|$

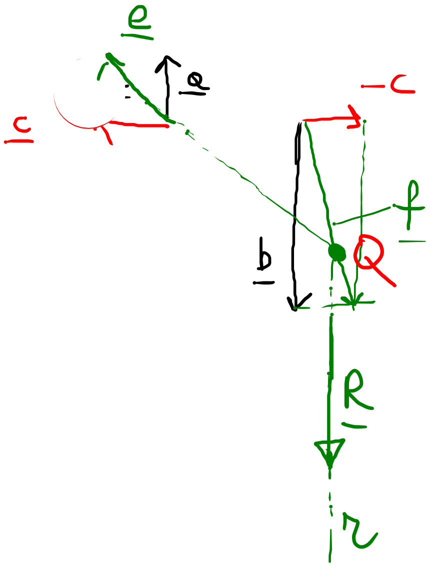
LUNGO UNA RETTA D'AZIONE PIÙ VICINA AL VETTORE DI INTENSITÀ MAGGIORE

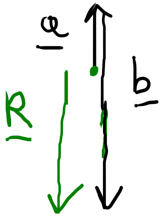


SIST. DI 2 VETTORI // DISCORDI

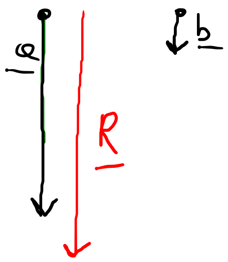
$\underline{R} = \underline{a} + \underline{b}$

$\underline{R} = \underline{e} + \underline{f}$

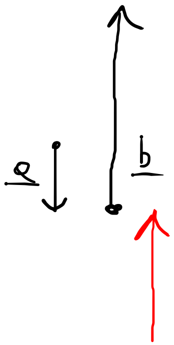




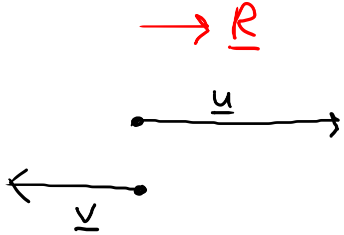
--> RISULTANTE "PASSA" ESTERNAMENTE AL VETTORE DI INTENSITÀ MAGGIORE.



$|R| = 3{,}5$



$|R| = 2$

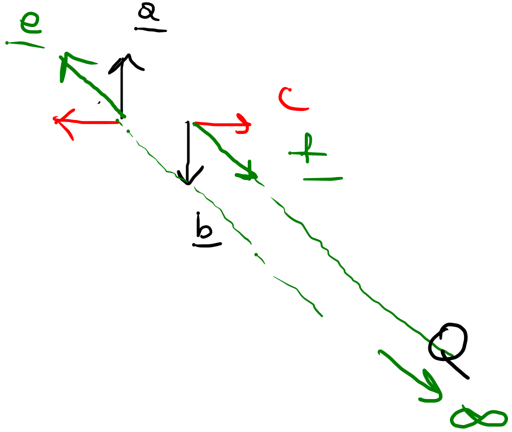


$|R| = 1$

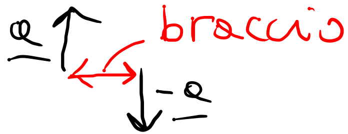
$|a| = |b|$

$|b| \to |a|$

$Q \to \infty$

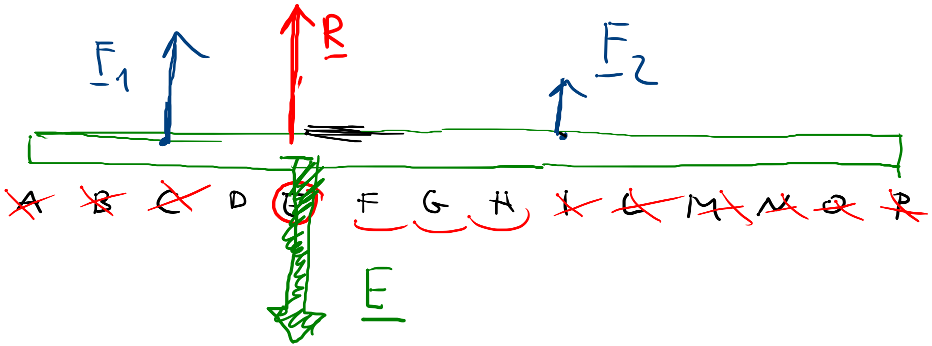


IL SIST. DI DUE VETTORI OPPOSTI CHE NON APPARTENGONO ALLA STESSA RETTA D'AZIONE SI CHIAMA COPPIA $(R = 0)$



COPPIA DI BRACCIO NULLO

ES : RISULTANTE VS EQUILIBRANTE



$|\underline{F}_1| = 2\ kN$

$|\underline{F}_2| = 1\ kN$

$|\underline{R}| = 3\ kN$

$$\underline{E} = -\underline{R}$$

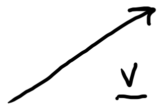
EQUILIBRANTE DI UN SIST. DI VETTORI

=

VETTORE OPPOSTO AL RISULTANTE

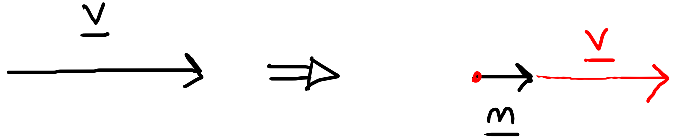
$$\{\underline{F}_1, \underline{F}_2, \underline{E}\} \Rightarrow \tilde{\underline{R}} = \underline{0}$$

VETTORI, LORO RAPPRESENTAZIONE E OPERAZIONI VETTORIALI



$|\underline{v}|$, v : INTENSITÀ O MODULO DEL VETTORE $\underline{v}$

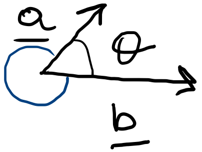
• VERSORE : VETTORE DI MODULO UNITARIO ($\underline{n}$ VERSORE : $|\underline{n}| = 1$)



$\underline{v} = 3\,\underline{n}$ (N)

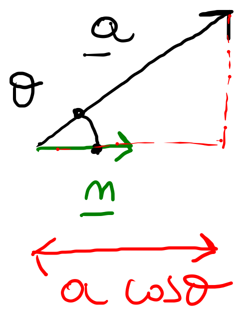
$v = 3$ N

• OPERAZIONI VETTORIALI : 1) PRODOTTO SCALARE : $\underline{a} \cdot \underline{b}$: SCALARE o NUMERO



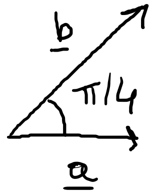
$$\underline{a} \cdot \underline{b} = a\,b\cos\theta \in \mathbb{R} \quad \left.\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \quad \begin{matrix} 0 \leq \theta < \pi/2 \\ \theta = \pi/2 \\ \frac{\pi}{2} < \theta \leq \pi \end{matrix}$$

DUE VETTORI $\perp$ TRA DI LORO HANNO P.S. NULLO.



$$\underline{a}\cdot\underset{\substack{\uparrow\\ \text{VERSORE}}}{\underline{m}} = a\cdot 1\cdot\cos\theta = \underbrace{a\cos\theta}$$: PROIEZIONE DI $\underline{a}$ NELLA DIREZIONE DI $\underline{m}$

ES:

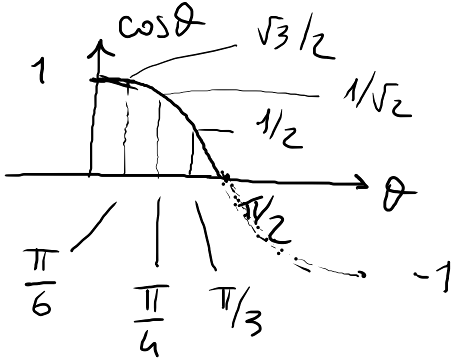


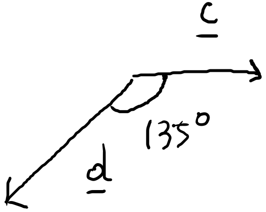
$|\underline{a}| = 2$

$|\underline{b}| = 2\sqrt{2}$

$$\underline{a}\cdot\underline{b} = 2\cdot 2\sqrt{2}\,\cos\frac{\pi}{4}$$

$$= 2\cdot 2\cancel{\sqrt{2}}\cdot\frac{1}{\cancel{\sqrt{2}}} = 4$$





$|\underline{c}| = 2$

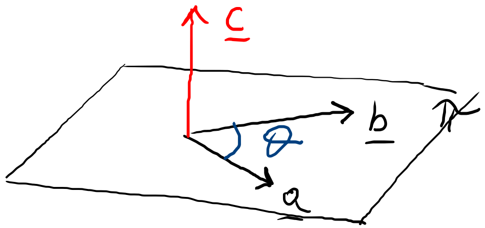
$|\underline{d}| = 2\sqrt{2}$

$$\underline{c}\cdot\underline{d} = 2\cdot 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = -4$$

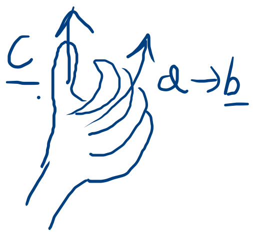
2) PRODOTTO VETTORIALE $\underline{a}, \underline{b} \Rightarrow \underline{a} \times \underline{b} = \underline{c}$ (VETTORE)

$|\underline{c}| = a \cdot b \sin\vartheta$

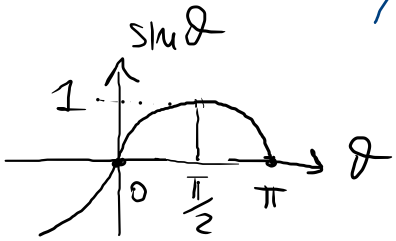
$\underline{c}$ è $\perp \underline{a}$, $\perp \underline{b}$



$\underline{c} \perp \pi$

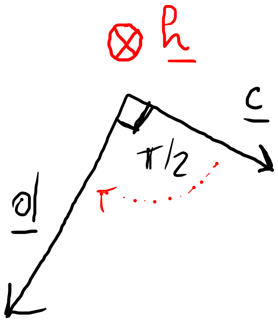


$\sin\vartheta \geq 0 \qquad 0 \leq \vartheta \leq \pi$



se $\vartheta = 0$ $(\vartheta = \pi)$ $\Rightarrow \sin\vartheta = 0$

ES



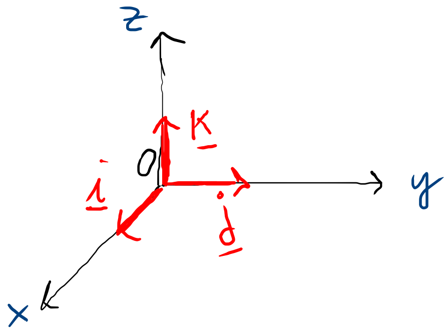
$\underline{h} = \underline{c} \times \underline{d}$

$\underline{h}$ entra o esce dal foglio?

ENTRA NEL FOGLIO

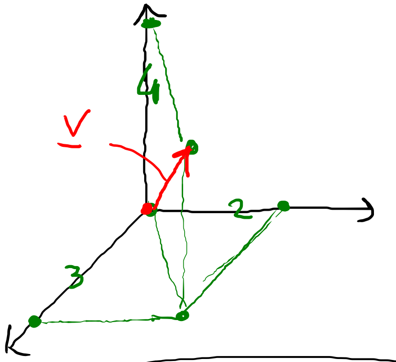
$|\underline{h}| = cd$

RAPPRESENTAZIONE CARTESIANA DEI VETTORI



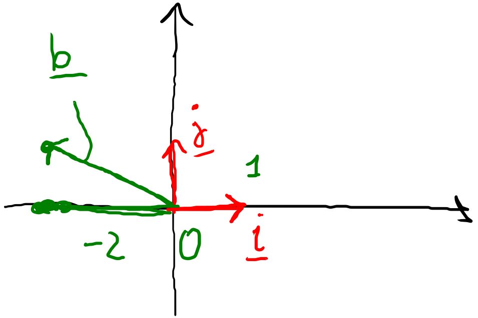
$\{\underline{i}, \underline{j}, \underline{k}\}$ BASE ORTONORMALE DELLO SPAZIO 3-DIMENSION.

$\underline{i}, \underline{j}, \underline{k}$: 3 VERSORI



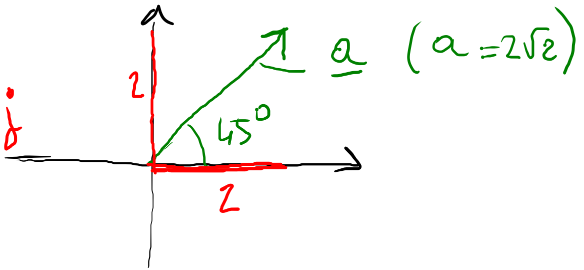
ES: $\underline{v} = 3\underline{i} + 2\underline{j} + 4\underline{k}$ (RAPPRES. CARTESIANA)

ES: $\underline{b} = -2\underline{i} + \underline{j}$



$\underline{a} =$

$= 2\underline{i} + 2\underline{j}$



ES: $\underline{v} \cdot \underline{b}$ UTILIZZANDO LA RAPPRESENT. CARTESIANA

$\underline{i} \cdot \underline{i} = 1$

$\underline{i} \cdot \underline{j} = 0$

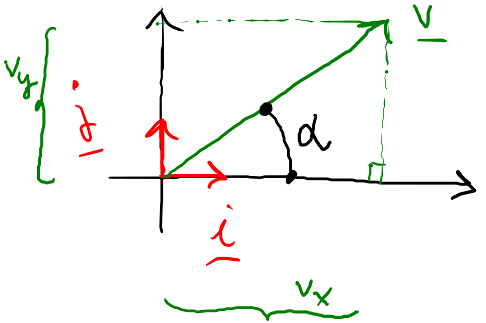
PROPR DISTRIBUTIVA

$$\underline{v} \cdot \underline{b} = (3\underline{i} + 2\underline{j} + 4\underline{k}) \cdot (-2\underline{i} + \underline{j}) \overset{\downarrow}{=} (3\underline{i} \cdot (-2\underline{i})) + \cancel{(3\underline{i} \cdot \underline{j})}$$

$$+ \cancel{2\underline{j} \cdot (-2\underline{i})} + 2\underline{j} \cdot \underline{j} + \cancel{4\underline{k} \cdot (-2\underline{i})} + \cancel{4\underline{k} \cdot \underline{j}}$$

$$= -6 \cdot 1 + 2 = -4$$

MODULO DI UN VETTORE IN RAPPRESENT. CARTESIANA



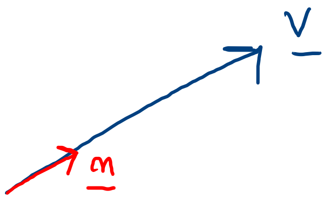
$\sin\alpha = \frac{v_y}{v}$

$\operatorname{tg}\alpha = \frac{v_y}{v_x}$

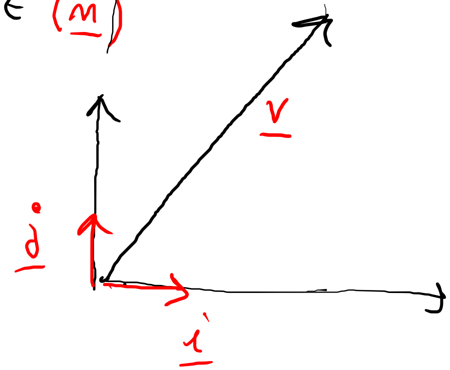
$$\underline{v} = v_x \underline{i} + v_y \underline{j} \quad ; \quad |\underline{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\left(\underline{v} = v_x \underline{i} + v_y \underline{j} + v_z \underline{k} \quad ; \quad |\underline{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \right)$$

PROBLEMA: ASSEGNATO UN VETTORE IN RAPPR. CARTESIANA DETERMINARE LA RAPPR. CARTESIANA DEL VERSORE CORRISPONDENTE ($\underline{n}$)



$$\underline{v} = \underbrace{v}_{\text{MODULO}}\,\underline{n}$$



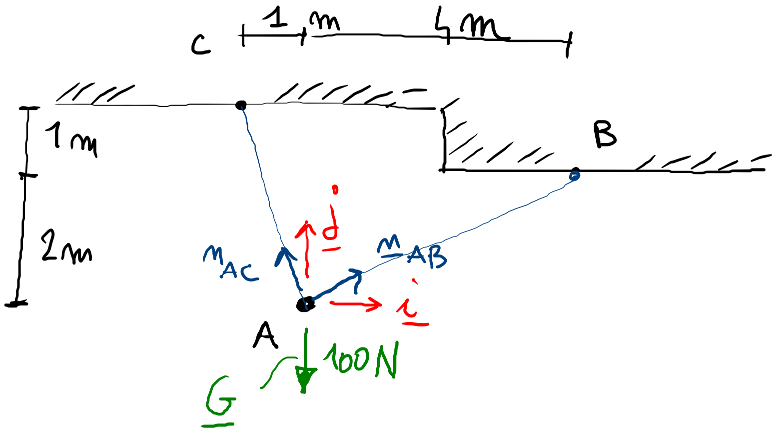
ES: $\underline{v} = 3\underline{i} + 4\underline{j}$; $v = \sqrt{3^2+4^2} = 5$

$$\underline{n} = \underline{v}\,\frac{1}{v} = \frac{1}{5}\left(3\underline{i}+4\underline{j}\right) = \frac{3}{5}\,\underline{i} + \frac{4}{5}\,\underline{j}$$

OK

$$|\underline{n}| \overset{?}{=} 1 \Rightarrow |\underline{n}| = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$$

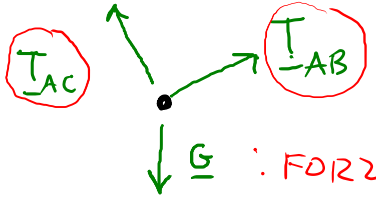
ES: EQUILIBRIO DI UN PUNTO MATERIALE "PESANTE" SOSPESO



IPOTESI VISTE

TESI: CALCOLO TENSIONI NEI FILI AB, AC

STUDIO L'EQUILIBRIO DEL PUNTO A:



: REAZ. VINCOLARI INCOGNITE

: FORZA NOTA

(2 METODI DI SOLUZ.)

1) METODO ANALITICO

$$\boxed{\underline{G} + \underline{T}_{AB} + \underline{T}_{AC} = \underline{0}}$$

EQ. VETTORIALE DI EQUILIBRIO

$$\underline{G} = -100\,\underline{j}$$

$$\underline{T}_{AC} = T_{AC}\,\underline{n}_{AC}$$

$$\underline{T}_{AB} = T_{AB}\,\underline{n}_{AB}$$

INCOGNITE

$$\underline{n}_{AB} = \frac{1}{\sqrt{16+4}}(4\underline{i} + 2\underline{j}) = \frac{1}{\sqrt{20}}(4\underline{i} + 2\underline{j})$$

$$\underline{n}_{AC} = \frac{1}{\sqrt{1+9}}(-\underline{i} + 3\underline{j}) = \frac{1}{\sqrt{10}}(-\underline{i} + 3\underline{j})$$

$$\underline{i} \qquad \underline{j}$$

$$-100\,\underline{j} + T_{AB}\frac{1}{\sqrt{20}}(4\underline{i} + 2\underline{j}) + T_{AC}\frac{1}{\sqrt{10}}(-\underline{i} + 3\underline{j}) = \underline{0}$$

$$\underline{i}: \quad T_{AB}\frac{4}{\sqrt{20}} + T_{AC}\left(-\frac{1}{\sqrt{10}}\right) = 0$$

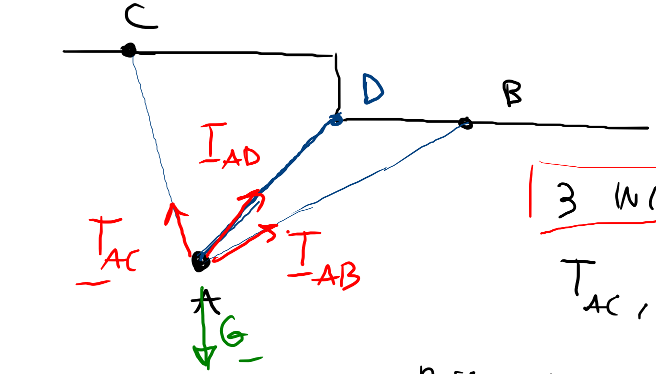
$$\underline{j}: \quad -100 + T_{AB}\frac{2}{\sqrt{20}} + T_{AC}\frac{3}{\sqrt{10}} = 0$$

2 EQ. IN 2 INCOGNITE T_{AB} e T_{AC}

SOLUZ. $T_{AB} = +\frac{100\sqrt{5}}{7}$ N

$T_{AC} = +\frac{200\sqrt{10}}{7}$ N

SEGNO + CONFERMA IL VERSO ARBITRARIO DEI VETTORI $\underline{T}_{AB}$ e $\underline{T}_{AC}$



3 INCOGNITE

T_{AC}, T_{AB}, T_{AD}

POSSO DETERMINARE UNA SOLUZ. UNICA PER LE 3 TENSIONI DALL'EQ.

$$\underline{G} + \underline{T}_{AB} + \underline{T}_{AC} + \underline{T}_{AD} = \underline{0} \quad \to \underline{i}, \ \underline{j}$$

2 EQUAZ. SCALARI

∞ SOLUZIONI