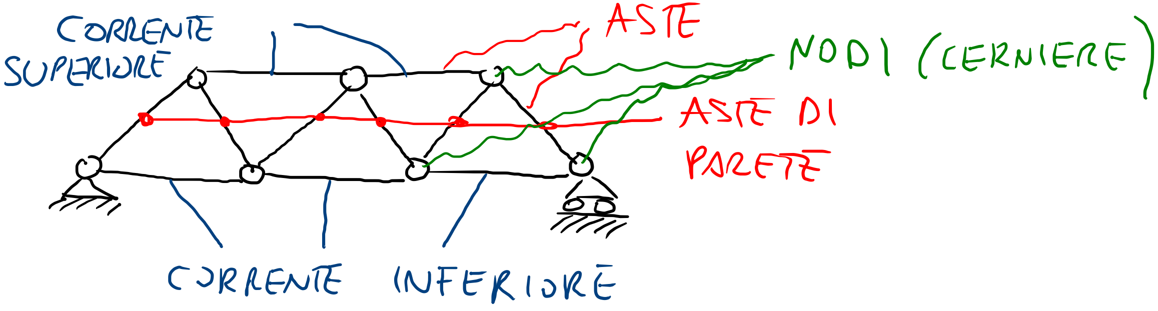
STRUTTURE RETICOLARI

15/5/24

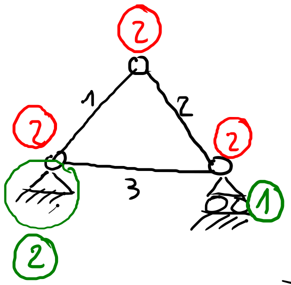


Modello di calcolo delle strutture reticolari:

- nodi incernierati
- aste rettilinee
- forze applicate ai nodi

⇓

Aste sollecitate solo a forza normale ($N \neq 0$), $T = M = 0$

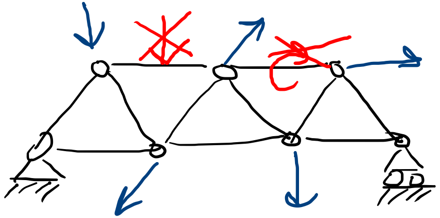


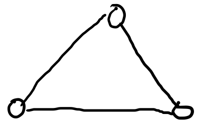
$g = 9$ (GDL)

$v = 2 + 2 + 2 + 2 + 1 = 9$

$s = 9$ (n° di gdl effettivamente sottratti dai vincoli)

ISOSTATICA

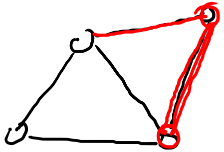




TRIANGOLO "ISOSTATICO": HA 3 GDL NEL PIANO (È COME UN CORPO RIGIDO)

3 C.RIGIDI $\longrightarrow$ 9 G.D.L.

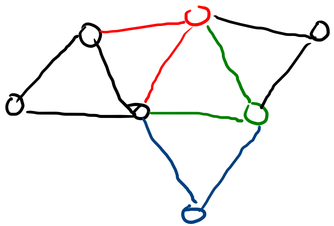
3 CERNIERE INT. $\longrightarrow$ 6 GRADI DI VINCOLO $\Longrightarrow$ $9-6=3$ G.D.L. "RESIDUI"



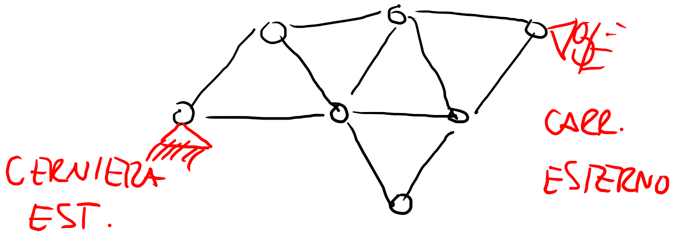
AGGIUNGO UN "NODO" E DUE ASTE

"NODO" $\downarrow$ + 2 GDL

DUE ASTE $\downarrow$ + 2 GRADI DI VINCOLO $\Longrightarrow$ 3 G.D.L. "RESIDUI"

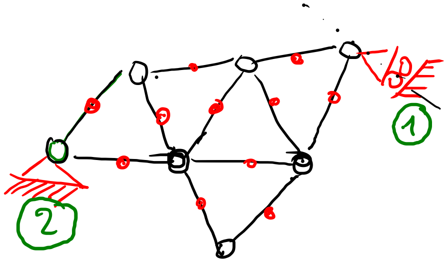


: C.RIGIDO CON 3 G.D.L. "RESIDUI"



ISOSTATICA

COSTRUITO UNA STR RETICOLARE ISOSTATICA CON IL METODO DELLA "TRIANGOLAZIONE"



n_{NODI} (n_N) : 7 $\Rightarrow$ n° GDL DEI NODI $\Rightarrow$ $2n_N = 14$

n_{ASTE} (n_A) : 11 (VINCOLI INTERNI : ELIMINANO n_A GDL)

$v_{esterno}$ (v_e) 3

$2n_N = 14$

$n_A + v_e = 14$)) STR. ISOSTATICA

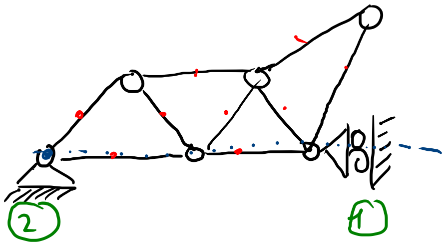
CONDIZIONE NECESSARIA PER L'ISOSTATICITÀ DI UNA STR. RETICOLARE :

$$\boxed{2n_N = n_A + v_e}$$

REGOLA DI MAXWELL

(ATTENZIONE : L'ASSE DEL CARRELLO NON DEVE PASSARE PER IL PUNTO CERNIERA ESTERNA)

ESEMPI DI CATTIVA DISPOSIZ DI ASTE E VINCOLI ESTERNI

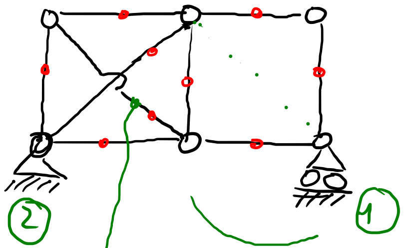


$$2m_N \overset{?}{=} m_A + v_e$$

$$12 = 9 + 3$$

OK

STR LABILE
PERCHÈ L'ASSE DEL CARRELLO "PASSA" IL PUNTO CERNIERA ESTERNA

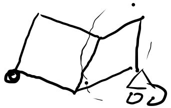


$$2m_N \overset{?}{=} m_A + v_e$$

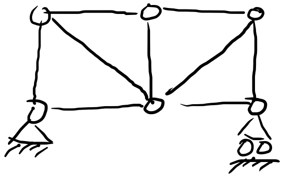
$$12 = 9 + 3$$

OK

QUADRILATERO ARTICOLATO (LABILE)



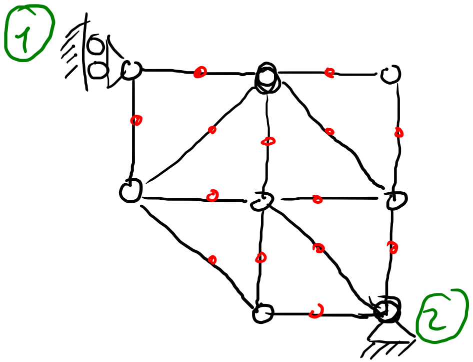
ASTA SOVRABBONDANTE



ISOSTATICA !

(SI PUÒ OTTENERE PER TRIANGOLAZIONE)

ESEMPI DI STRUTTURE RETICOLARI IPERSTATICHE

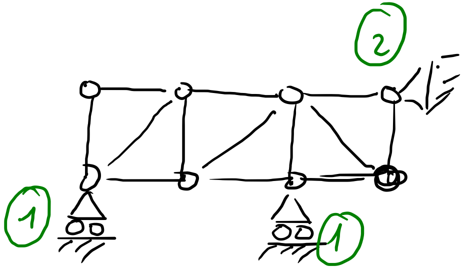


$$2m_N \overset{?}{=} m_A + v_e$$

$$16 \neq 14 + 3 = 17$$

NO

È PRESENTE UN'ASTA IN PIÙ CHE RENDE LA STRUTTURE IPERSTATICA INTERNAMENTE.



$v_e = 4$

TRALICCIO ISOSTATICO PERÒ CON $v_e > 3$: ESTERNAMENTE IPERSTATICA.

LE STRUTTURE RETICOLARI ISOSTATICHE FORMATE DA UN "TRALICCIO" SONO VINCOLATE A TERRA DA UN GRADO DI VINCOLO $v_e = 3$:

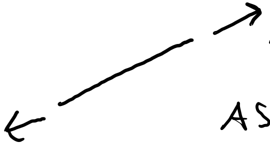
- CERNIERA + CARRELLO
- 3 CARRELLI

IPOTESI PER IL CALCOLO DEGLI "SFORZI" INTERNI NELLE ASTE.

- NODI INCERNIERATI e ASTE RETTILINEE
- FORZE APPLICATE AI NODI

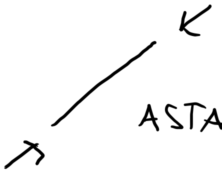
$\Rightarrow$ $N \neq 0$ NELLE ASTE

$T, M = 0$ IN TUTTE LE ASTE



$N > 0$

ASTA TESA: TIRANTE "TIE-ROD"



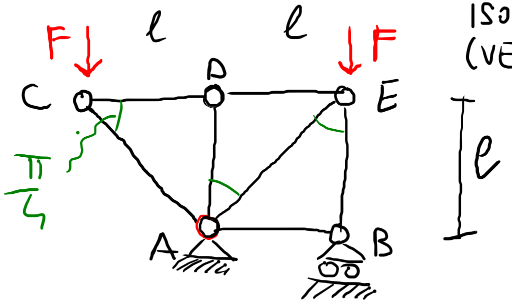
$N < 0$

ASTA COMPRESSA: PUNTONE "STRUT"

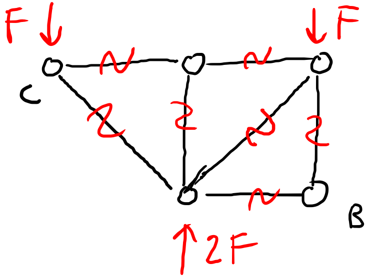
IL PROBLEMA STRUTTURALE CONSISTE NEL CALCOLO DELLE REAZIONI ESTERNE E NEL CALCOLO DELLE FORZE N_i DELLE ASTE INTERNE.

2 METODI:
- METODO DEI NODI
- METODO DELLE SEZIONI (O DI RITTER)

ES: METODO DEI NODI



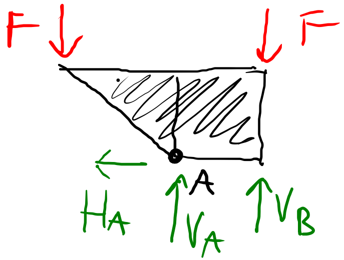
ISOSTATICA (VERIFICARE)



S.C.L. EQUILIBRATO

"PARTO" SEMPRE DA UN NODO CON AL PIÙ 2 ASTE INCOGNITE.

CALCOLO REAZ. ESTERNE

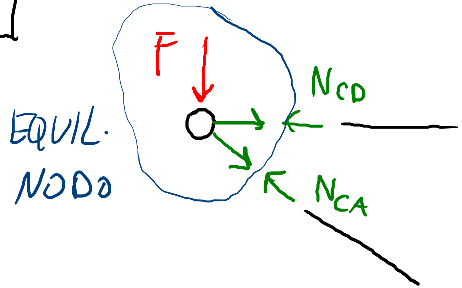


EQ. EQUILIBRIO:

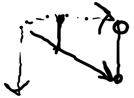
$\xrightarrow{+}: \boxed{-H_A = 0}$; $\uparrow +: V_A + V_B - 2F = 0 \Rightarrow \boxed{V_A = 2F}$

$\circlearrowleft A^{+}: +\cancel{F\ell} - \cancel{F\ell} + V_B \ell = 0 \Rightarrow \boxed{V_B = 0}$

NODO C



N_{CD}, N_{CA} DI TRAZIONE NELLO SCHEMA PRELIMINARE



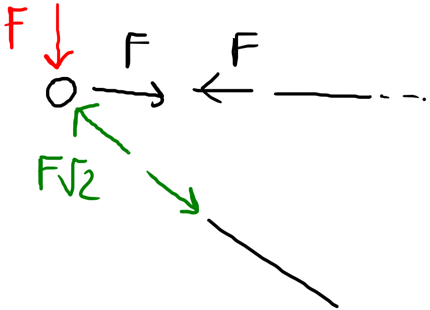
$$\left.\begin{aligned} &\xrightarrow{+}: +N_{CD} + N_{CA}\cos 45^\circ = 0 \\ &+\uparrow: -F - N_{CA}\sin 45^\circ = 0 \end{aligned}\right\}$$

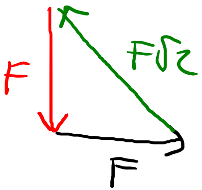
$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\left.\begin{aligned} &+N_{CD}+N_{CA}\frac{1}{\sqrt{2}}=0 \\ &-F-N_{CA}\frac{1}{\sqrt{2}}=0 \end{aligned}\right\}$$

$N_{CD}-F=0 \Rightarrow \boxed{N_{CD}=+F}$ "TESA"

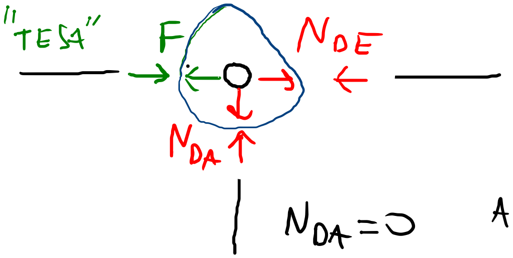
$\boxed{N_{CA}=-F\sqrt{2}}$ "COMPRESSA"





TRIANGOLO FORZE CHIUSO

NODO D



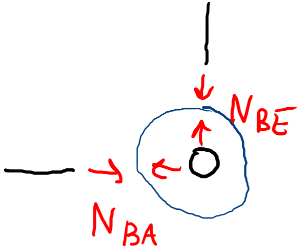
$N_{DA}=0$ ASTA SCARICA

$+\uparrow: \; -N_{DA}=0$

$\overset{+}{\rightarrow}: \; -F+N_{DE}=0$

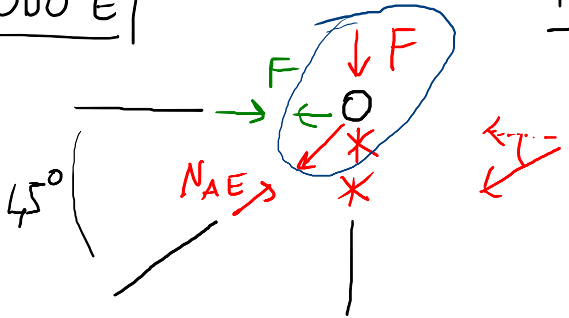
$\boxed{N_{DE}=+F}$ "TESA"

NODO B



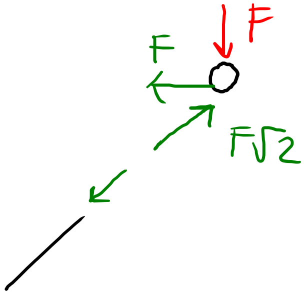
$$\left.\begin{aligned} &+\uparrow: \; +N_{BE}=0 \\ &\overset{+}{\rightarrow}: \; -N_{BA}=0 \end{aligned}\right\} \text{ASTE SCARICHE}$$

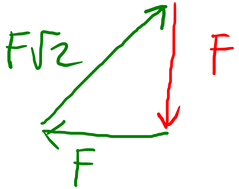
NODO E



$$\xrightarrow{+}: \; -F - N_{AE}\underbrace{\frac{1}{\sqrt{2}}}_{\cos 45^\circ} = 0 \;\Rightarrow\; \boxed{N_{AE} = -F\sqrt{2}}$$

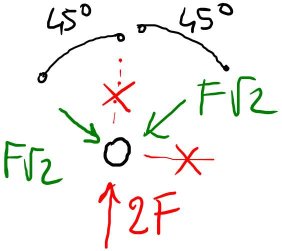
"COMPRESSA"





TRIANGOLO DELLE FORZE CHIUSO.

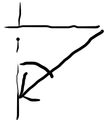
VERIFICA DEL NODO A



$$\xrightarrow{+}: \; +F\cancel{\sqrt{2}}\frac{1}{\cancel{\sqrt{2}}} - F\cancel{\sqrt{2}}\frac{1}{\cancel{\sqrt{2}}} \overset{?}{=} 0$$

OK

NODO IN EQUILIBRIO



$$+\uparrow: \; 2F - F\cancel{\sqrt{2}}\frac{1}{\cancel{\sqrt{2}}} - F\cancel{\sqrt{2}}\frac{1}{\cancel{\sqrt{2}}} \overset{?}{=} 0$$

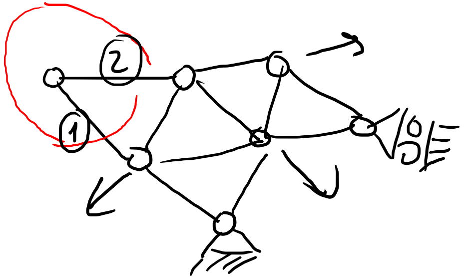
OK

ASTA	AB	AC	AD	AE	BE	CD	DE
N_i	0	$-F\sqrt{2}$	0	$-F\sqrt{2}$	0	$+F$	$+F$

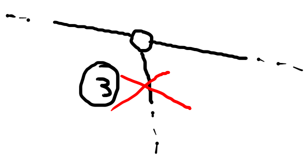
NELLA TABELLA (−) : PUNTONE

(+) : TIRANTE

CRITERI PER IL RICONOSCIMENTO DI ASTE SCARICHE

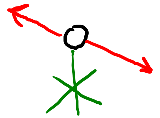


2)



NODO SCARICO A CUI CONVERGONO 2 ASTE ALLINEATE E UNA TERZA NON ALLINEATA

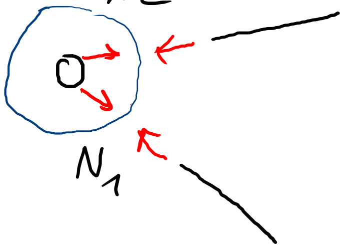
$\Rightarrow N_3 = 0$



1) NODO SCARICO A CUI CONVERGONO DUE ASTE NON ALLINEATE

$\Downarrow$

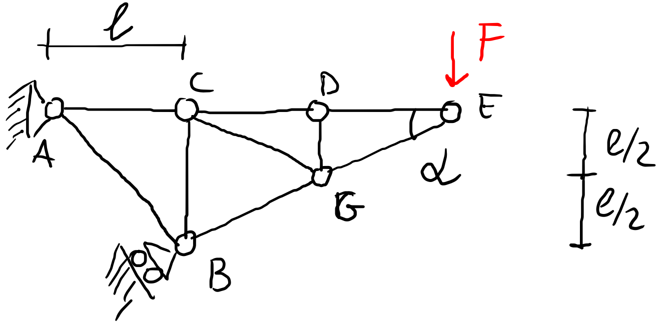
$N_1 = N_2 = 0$



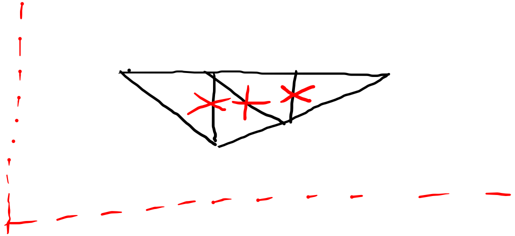
2 EQUAZIONI DI EQUILIBRIO DOVE È ASSENTE IL TERMINE NOTO

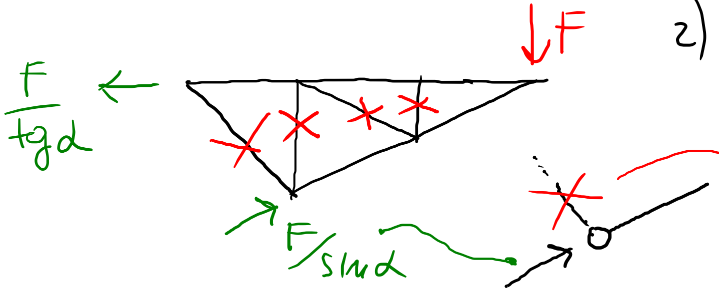
$$\begin{cases} \alpha N_1 + \beta N_2 = 0 \\ \gamma N_1 + \varepsilon N_2 = 0 \end{cases} \Rightarrow N_1 = N_2 = 0$$

ES.



$\alpha: \ \tan\alpha = \frac{1}{2}$





2) CRITERI ASTE SCARICHE

SCARICA PER ESTENSIONE DEL CRITERIO 2

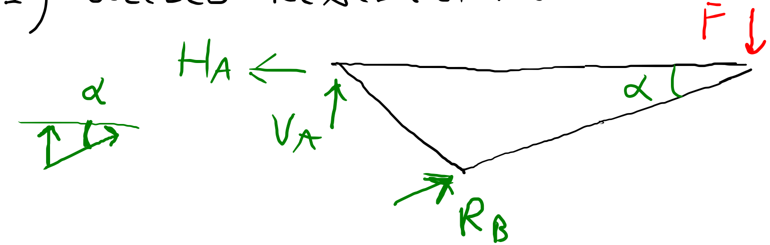
0) STR ISO?

$$2n_N \overset{?}{=} n_A + v_e$$

$$12 = 9 + 3$$

OK

1) CALCOLO REAZ. ESTERNE

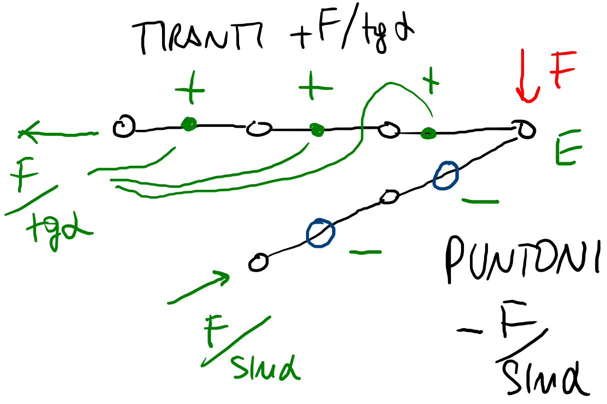


$$\left.\begin{aligned} +\uparrow:&\ +V_A - F + R_B \sin\alpha = 0 \\ \overset{+}{\rightarrow}:&\ -H_A + R_B \cos\alpha = 0 \\ +\circlearrowleft E:&\ -V_A\, 3\ell = 0 \end{aligned}\right\}$$

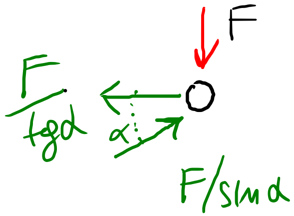
$$R_B = +F/\sin\alpha$$

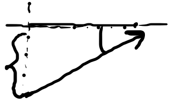
$$H_A = R_B\cos\alpha = \underset{+}{F/\tan\alpha}$$

$$V_A = 0$$



VERIFICA NODO E





$$\overset{+}{\rightarrow}: \; -\frac{F}{tg\alpha} + \frac{F}{\sin\alpha}\cos\alpha \overset{?}{=} 0 \qquad -\frac{F}{tg\alpha} + \frac{F}{tg\alpha} = 0$$

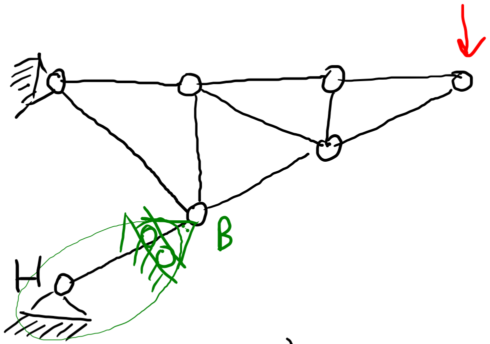
OK

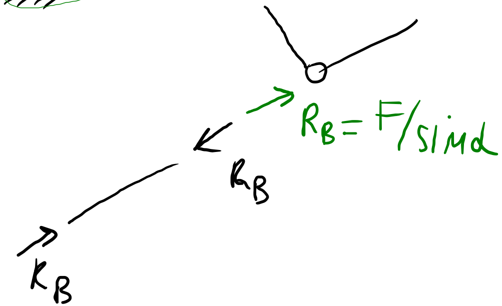
$$+\uparrow: \; -F + \frac{F}{\cancel{\sin\alpha}}\cancel{\sin\alpha} \overset{?}{=} 0$$

OK

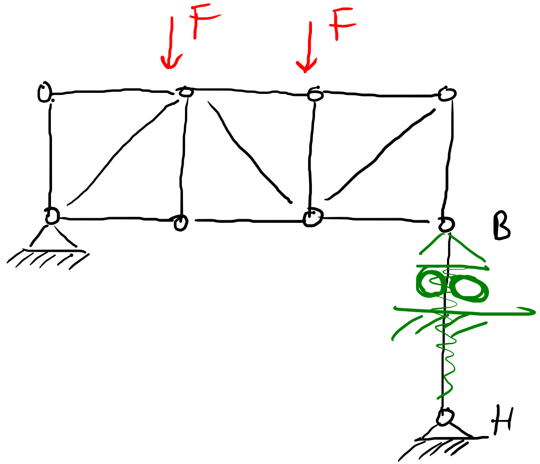
ASTE	N_i	
AC − CD − DE	$+F/tg\alpha$	(TIRANTI)
BG − EG	$-F/\sin\alpha$	(PUNTONI)
AB − BC − CG − DG	0	

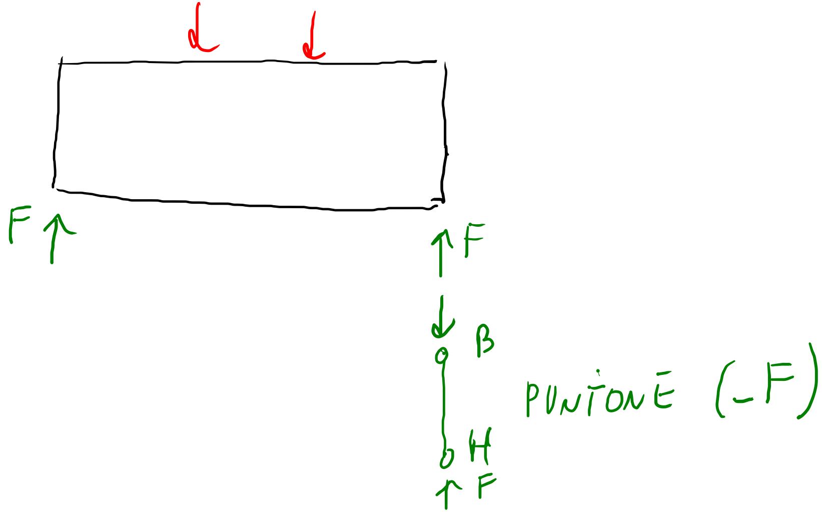
OSSERVAZIONE SUI VINCOLI ESTERNI



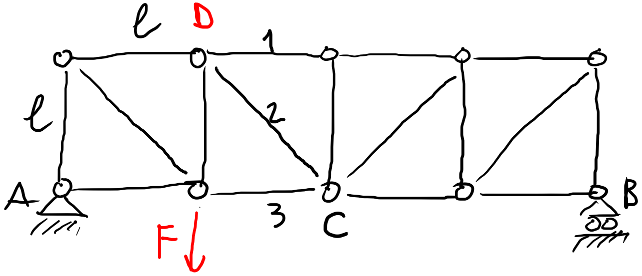


$$BH: \; -\frac{F}{\sin\alpha} = N_{BH} \quad (\text{PUNTONE})$$

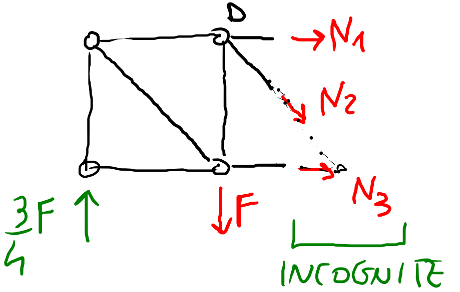




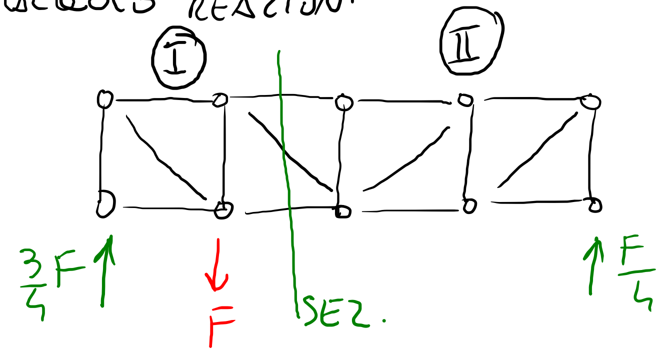
ES: METODO DELLE SEZIONI (O DI RITTER)



$$\left.\begin{aligned} 2m_N &= m_A + v_e \\ 20 &= 17 + 3 \end{aligned}\right) \text{ ok: ISO}$$



1) CALCOLO REAZIONI



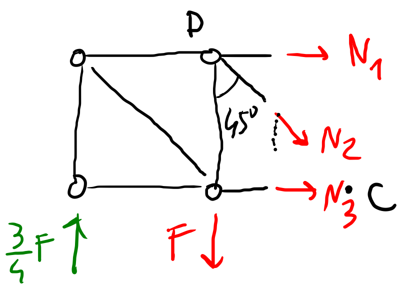
I: IN EQUILIBRIO GRAZIE AL CONTRIBUTO DI N_1, N_2, N_3: POSSONO SCRIVERE 3 EQUAZ. DI EQUIL. DI I E DET. N_1, N_2, N_3

PERCHÉ NON PROVARE A SCRIVERE UNA EQUAZIONE PER DETERMINARE UNA INCOGNITA ALLA VOLTA?

N_1? $\boxed{\circlearrowleft C^{+}}$ I

N_3? $\boxed{\circlearrowleft D^{+}}$ I

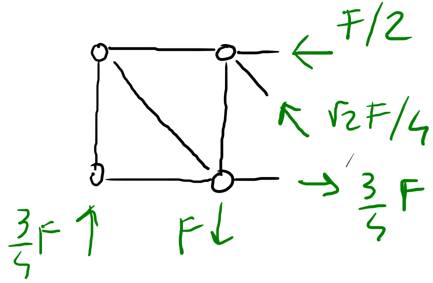
N_2? $\boxed{\uparrow +}$ I



$$\boxed{N_1}: \quad \overset{\curvearrowleft +}{C}: -N_1\cancel{\ell} + F\cancel{\ell} - \frac{3}{4}F\cdot 2\cancel{\ell} = 0 \quad ; \quad +N_1 = F - \frac{3}{2}F = \underbrace{-\frac{F}{2}}_{\text{PUNTONE}}$$

$$\boxed{N_2}: \quad +\uparrow: -N_2\frac{1}{\sqrt{2}} - F + \frac{3}{4}F = 0 \quad ; \quad N_2 = \sqrt{2}\left(-F + \frac{3}{4}F\right) = \underbrace{-\sqrt{2}\frac{F}{4}}_{\text{PUNTONE}}$$

$$\boxed{N_3}: \quad \overset{\curvearrowleft +}{D}: +N_3\cancel{\ell} - \frac{3}{4}F\cancel{\ell} = 0 \quad ; \quad N_3 = \underbrace{+\frac{3}{4}F}_{\text{TIRANTE}}$$



$\leftarrow 2/4$

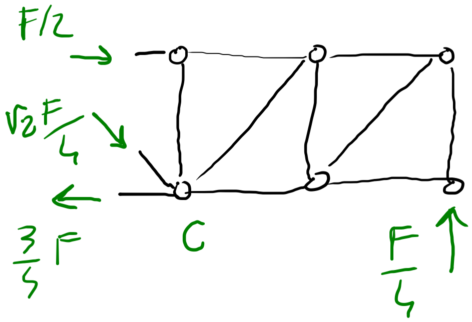
$\leftarrow 1/4$

$\rightarrow 3/4$

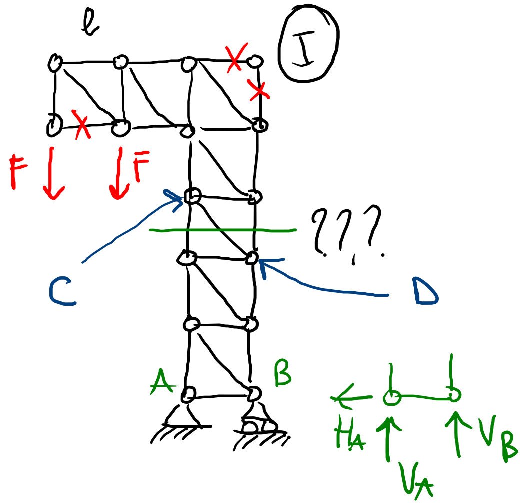
VERIFICA EQUIL. II

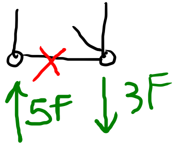
$$\overset{\curvearrowleft +}{C}: -\frac{F}{2}\ell + \frac{F}{\cancel{4}_2}\cancel{2}\ell \overset{?}{=} 0$$

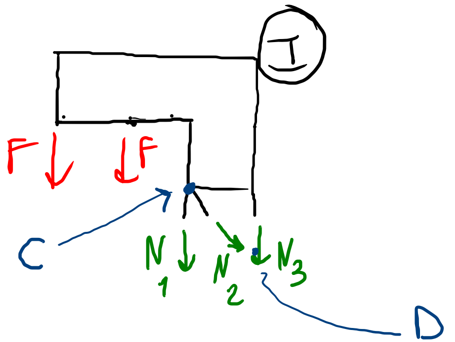
OK



ES







$\boxed{N_1 = -5F}$

$\boxed{N_1}$ $\circlearrowleft^{+} D$: $N_1\cancel{\ell} + F2\cancel{\ell} + F3\cancel{\ell} = 0$

$\boxed{N_2}$ $\xrightarrow{+}$: $+N_2 \frac{1}{\sqrt{2}} = 0$ $\boxed{N_2 = 0}$

$\boxed{N_3}$ $\circlearrowleft^{+} C$: $-N_3\cancel{\ell} + F\cancel{\ell} + F2\cancel{\ell} = 0$

$\boxed{N_3 = +3F}$

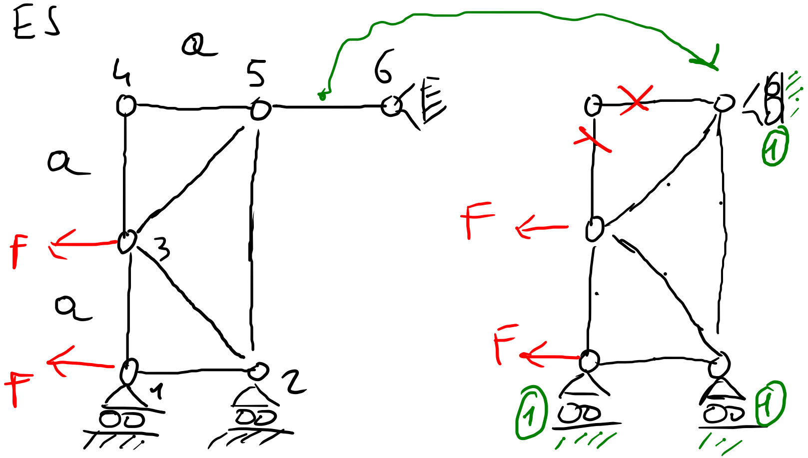
1) REAZ. VINCOLARI

$\xrightarrow{+}$: $-H_A = 0$ $H_A = 0$

$+\uparrow$: $-2F + V_A + V_B = 0$ $V_A = 2F - V_B = 5F$

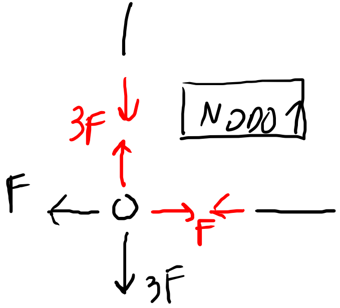
$\circlearrowleft^{+} A$: $+F2\cancel{\ell} + F\cancel{\ell} + V_B\cancel{\ell} = 0$; $V_B = -3F$

ES

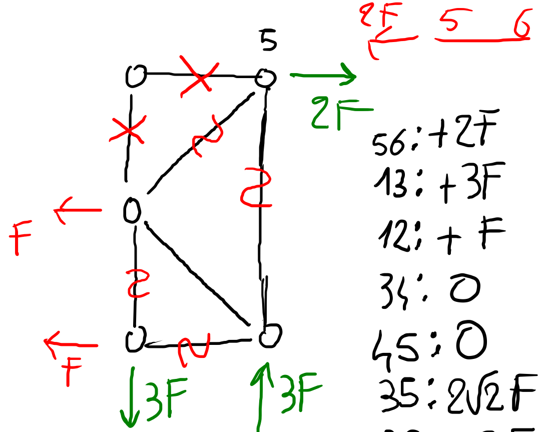


$$2 m_N = m_A + v_e$$

$$10 = 7 + 3$$

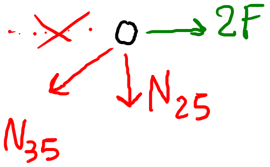


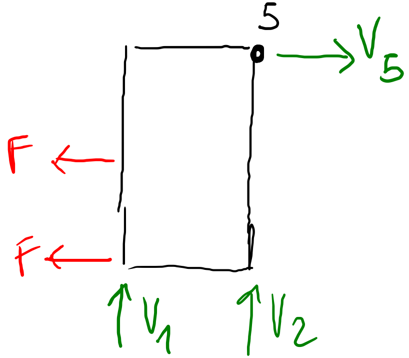
NODO 1



56: $+2F$
13: $+3F$
12: $+F$
34: 0
45: 0
35: $2\sqrt{2}F$
25: $-2F$
23: $-\sqrt{2}F$

NODO 5





$$+\uparrow: V_1 + V_2 = 0$$

$$\overset{+}{\rightarrow}: -2F + V_5 = 0$$

$$\circlearrowleft 5^{+}: -V_1 \cancel{a} - F\,2\cancel{a} - F\cancel{a} = 0$$

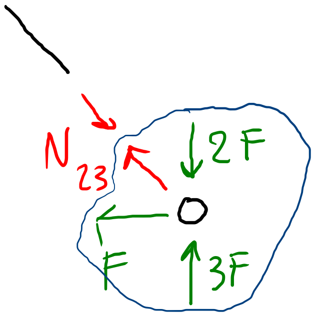
$$V_5 = +2F \qquad V_2 = -V_1 = +3F$$

$$V_1 = -3F$$

$$\begin{cases} \overset{+}{\rightarrow}: 2F - N_{35}\frac{1}{\sqrt{2}} = 0 \;;\; N_{35} = 2\sqrt{2}F \\ +\uparrow: -N_{25} - N_{35}\frac{1}{\sqrt{2}} = 0 \;;\; N_{25} = -2F \end{cases}$$

REAZ. VINCOLARI?

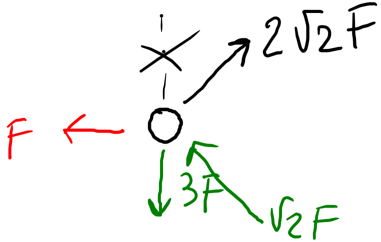
NODO 2



$$\xrightarrow{+} : -F - N_{23}\frac{1}{\sqrt{2}} = 0 \quad ; \quad N_{23} = -\sqrt{2}F$$

NODO 3

VERIFICA FINALE



$$\xrightarrow{+} : -F - \cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} + 2\cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} \overset{?}{=} 0$$ OK

$$+\uparrow : -3F + \cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} + 2\cancel{\sqrt{2}}F\frac{1}{\cancel{\sqrt{2}}} \overset{?}{=} 0$$ OK