ES 2

$$V(x,y,z) = \frac{2A}{m} \ln R(x,y,z)/a + \frac{B}{2m} R^2(x,y,z) \qquad R(x,y,z) \equiv \sqrt{x^2+y^2+z^2}$$

$A, B > 0$

1) Potenziale inv. per rotaz. $\Rightarrow$ $\bar{M}$ (mom. ang.) si conserva

$\Rightarrow$ $\bar{r}(t)$ e $\dot{\bar{r}}(t)$ $\perp$ $\bar{M}$ cost. $\Rightarrow$ moto su piano $\perp$ $\bar{M}$

2) $\bar{M}$ // asse z $\Rightarrow$ luogo moto $z=0$ e $\dot{z}=0$

$$L = \frac{m}{2}(\dot{x}^2+\dot{y}^2) + \frac{2A}{m} \ln \frac{1}{a}\sqrt{x^2+y^2} - \frac{B}{2m}(x^2+y^2)$$

$x = r\cos\theta \qquad y = r \operatorname{sen}\theta$

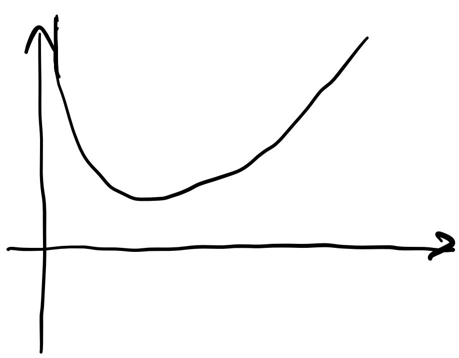
$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{2A}{m} \ln r/a - \frac{B}{2m} r^2$$

Eq. Lag
$$m\ddot{r} - \frac{2A}{mr} + \frac{B}{m} r - mr\dot{\theta}^2 = 0$$
$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} = 0$$

3) θ coord ciclica $\rightarrow$ $\ell = \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$ $\rightsquigarrow$ $\dot{\theta} = \frac{\ell}{mr^2}$

$$L^* = \frac{m}{2}\dot{r}^2 - \frac{\ell^2}{2mr^2} + \frac{2A}{m} \ln r/a - \frac{B}{2m} r^2$$

$$V_{eff}(r) = \frac{\ell^2}{2mr^2} - \frac{2A}{m} \ln \frac{r}{a} + \frac{B}{2m} r^2$$



4) $V_{eff}'(r) = -\frac{\ell^2}{mr^3} - \frac{2A}{mr} + \frac{Br}{m} =$

$= \frac{1}{mr^3}\left(-\ell^2 - 2Ar^2 + Br^4\right)$

zeri: $r^2 = \frac{A}{B} \pm \frac{1}{B}\sqrt{A^2 + B\ell^2} \rightarrow$ Una sola soluz. accett. (positiva)

$r_0 = \frac{A}{B} + \sqrt{\left(\frac{A}{B}\right)^2 + \frac{\ell^2}{B}}$

$V_{eff}''(r) = \frac{3\ell^2}{mr^4} + \frac{2A}{mr^2} + \frac{B}{m}$

$V_{eff}''(r_0) = \frac{3\ell^2}{mr_0^4} + \frac{2A}{mr_0^2} + \frac{B}{m} \equiv K_0 > 0$ STAB.

$r(t) = r_0 \longrightarrow$ CIRCONFERENZA

$mr_0^2\dot{\theta} = \ell \rightarrow \dot{\theta} = \frac{\ell}{mr_0^2} \rightarrow \theta(t) = \theta_0 + \frac{\ell}{mr_0^2}t$

$x(t) = r_0 \cos\left(\frac{\ell}{mr_0^2}t + \theta_0\right)$

$y(t) = r_0 \operatorname{sen}\left(\frac{\ell}{mr_0^2}t + \theta_0\right)$

5) $L_{lin} = \frac{m\dot{\delta r}^2}{2} - \frac{1}{2}\underbrace{V_{eff}''(r_0)}_{}\delta r^2$ $\quad$ ($V_{eff}''(r_0) = K_0$) $\quad$ $\delta r \equiv r - r_0$

$\ddot{\delta r} = -\frac{K_0}{m}\delta r \rightarrow \delta r(t) = A\cos\left(\frac{K_0}{m}t + \varphi\right)$

$\dot{\theta} = \frac{\ell}{m(r_0 + \delta r)^2}$

6) $r(t) = r_0 + A\cos\left(\frac{k_0}{m} t + \varphi\right)$