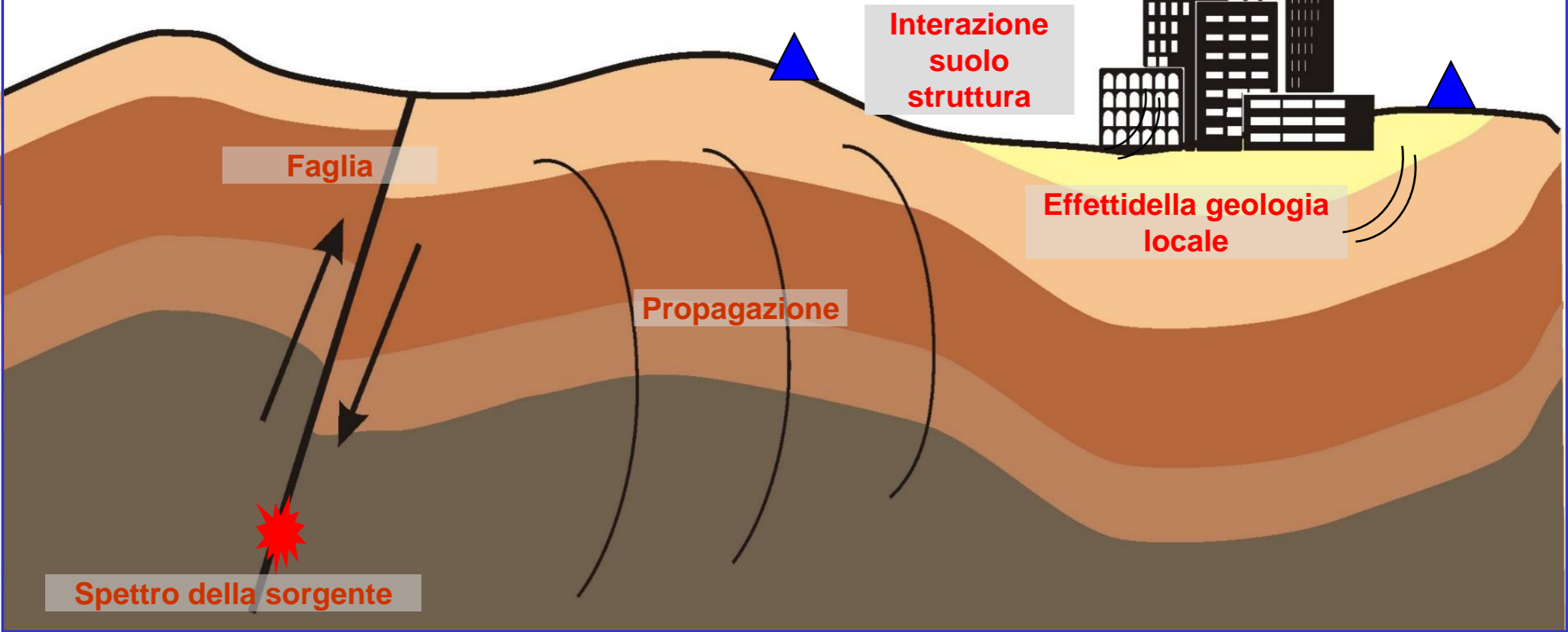
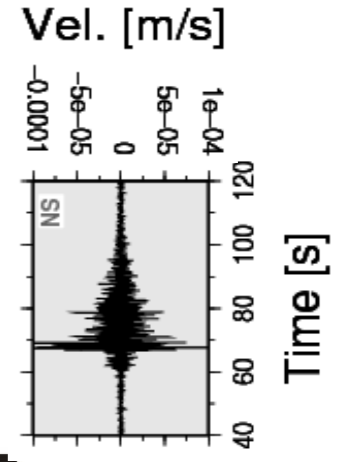
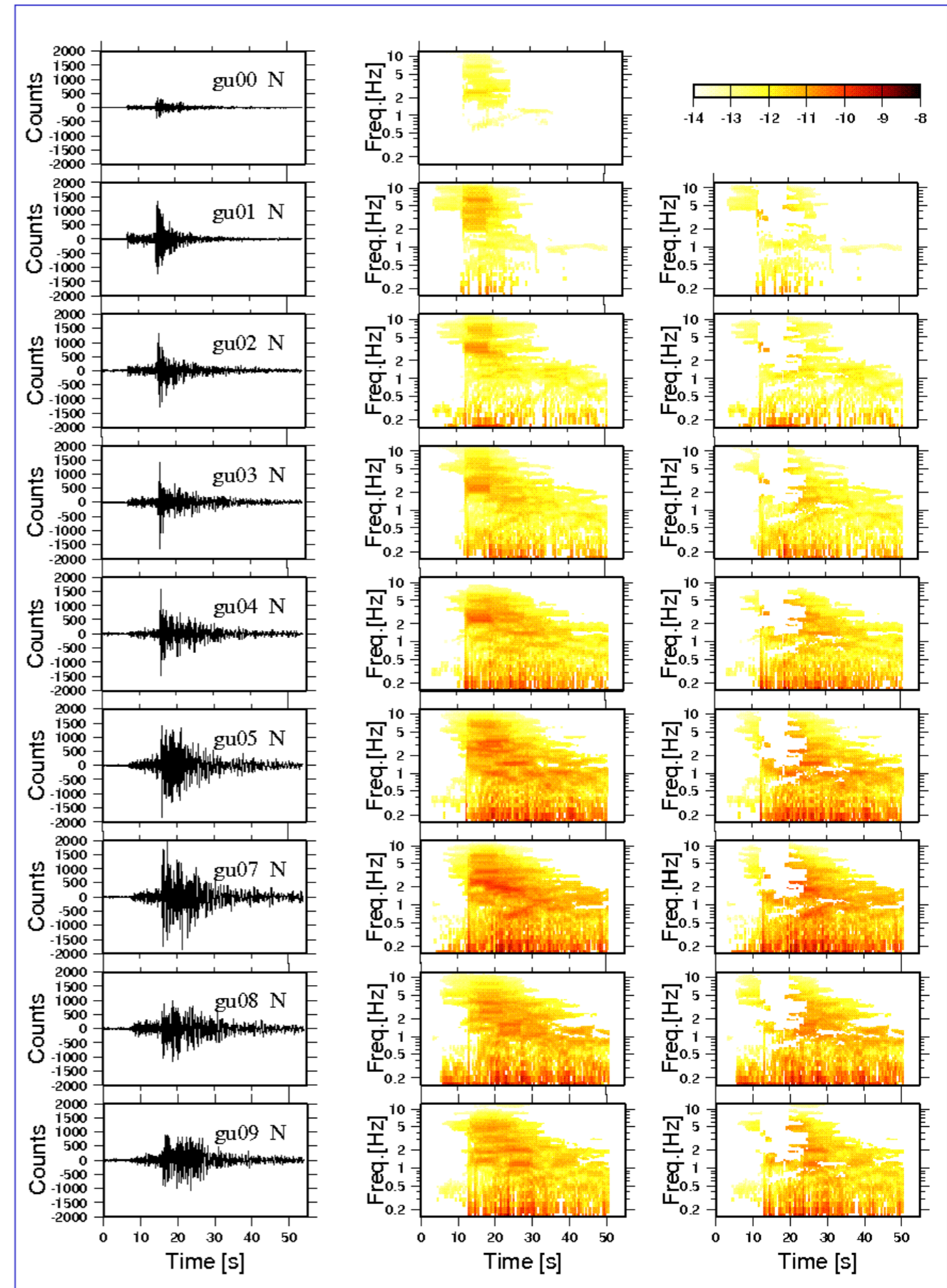
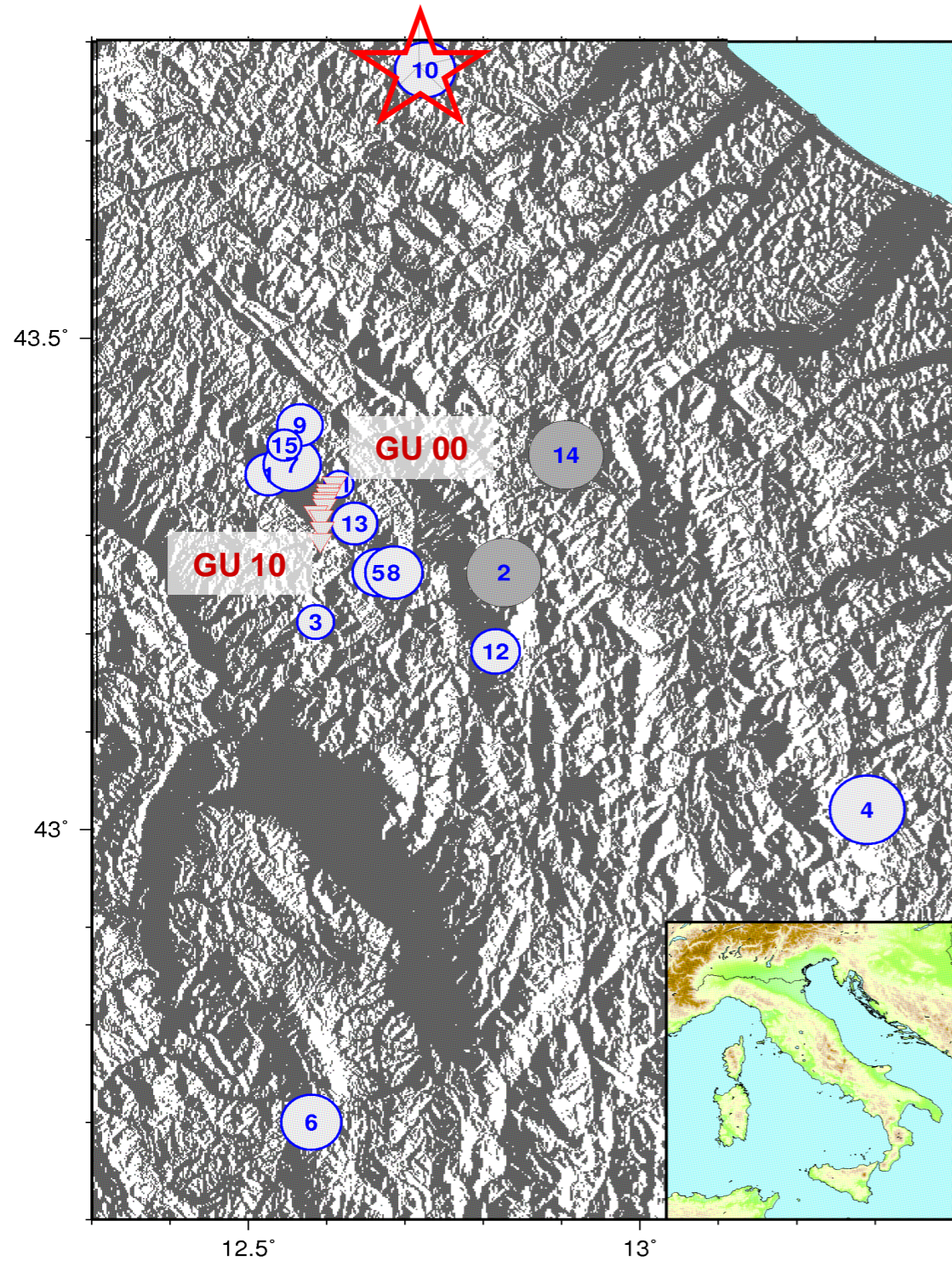


Frequenza di risonanza degli edifici



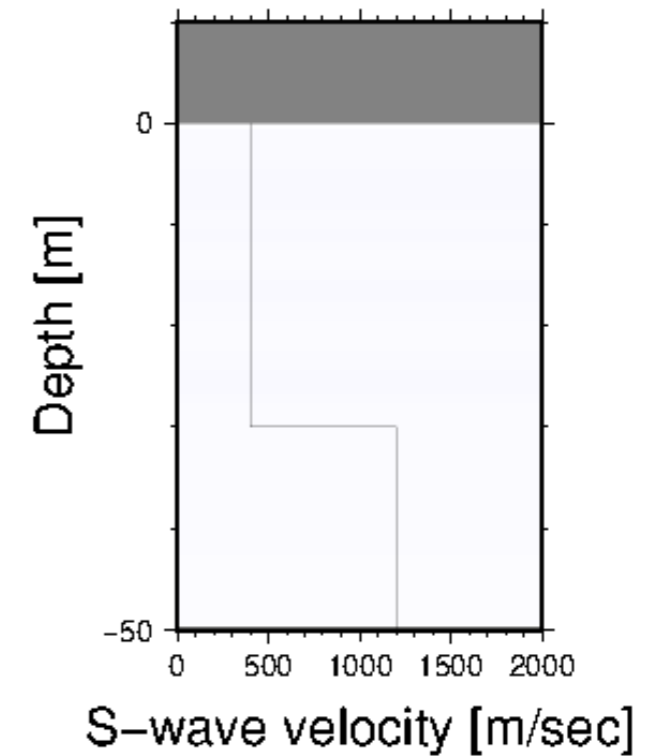
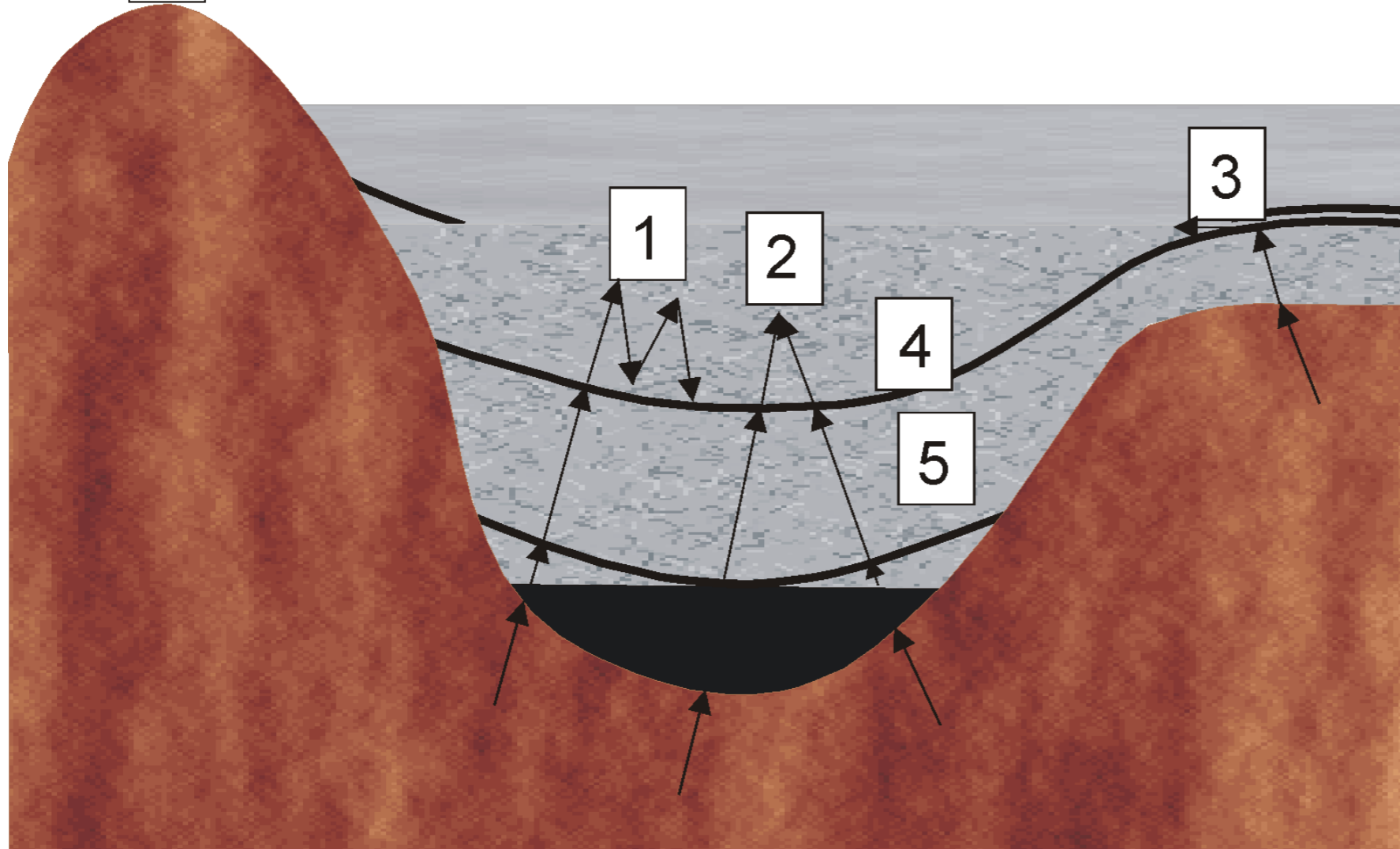
Effetti di sito: Valle di Gubbio

L'ampiezza dello scuotimento AUMENTA!
all'aumentare della distanza dalla sorgente!



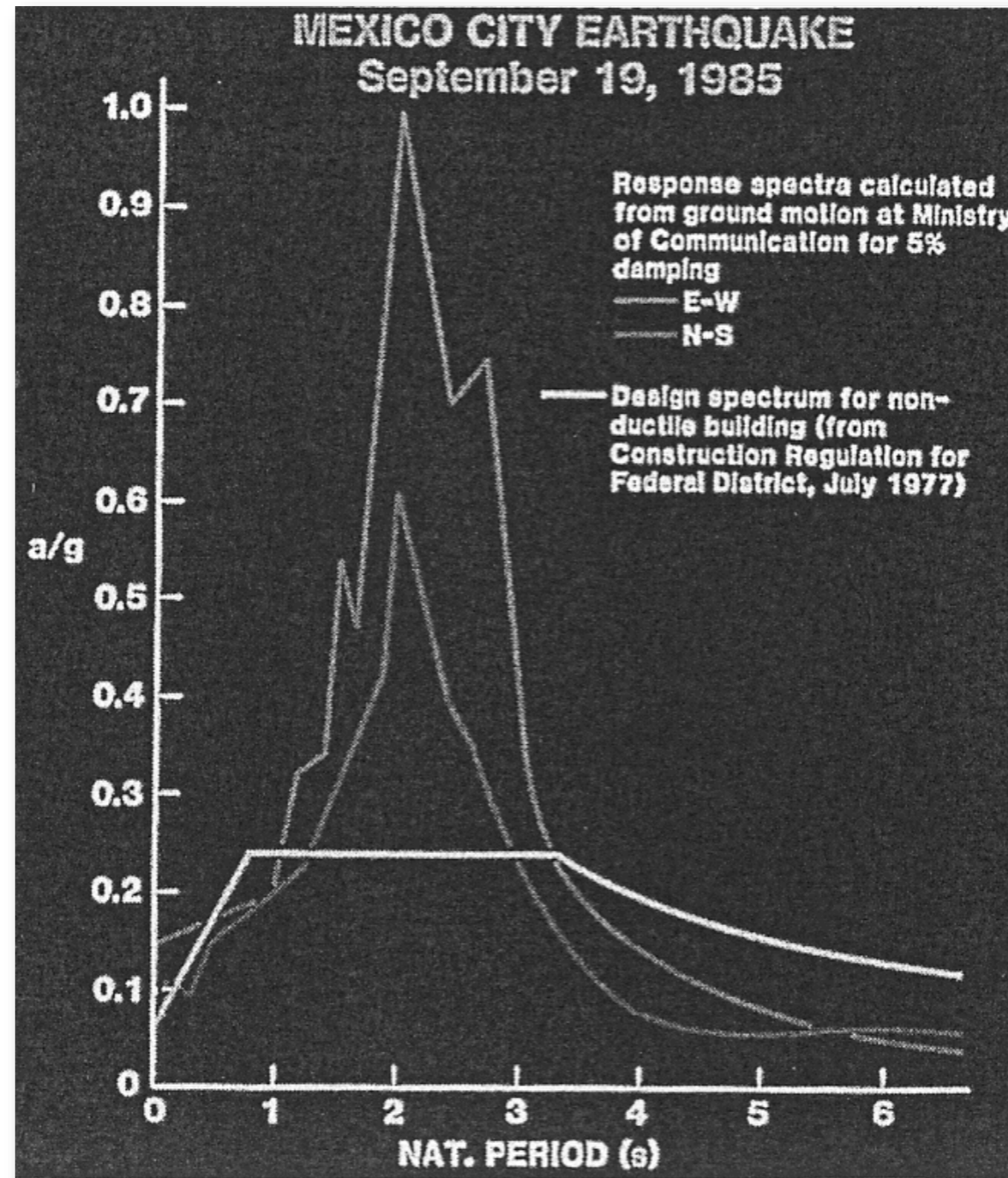
La banda di frequenza amplificata dipende dalla struttura del sottosuolo

6

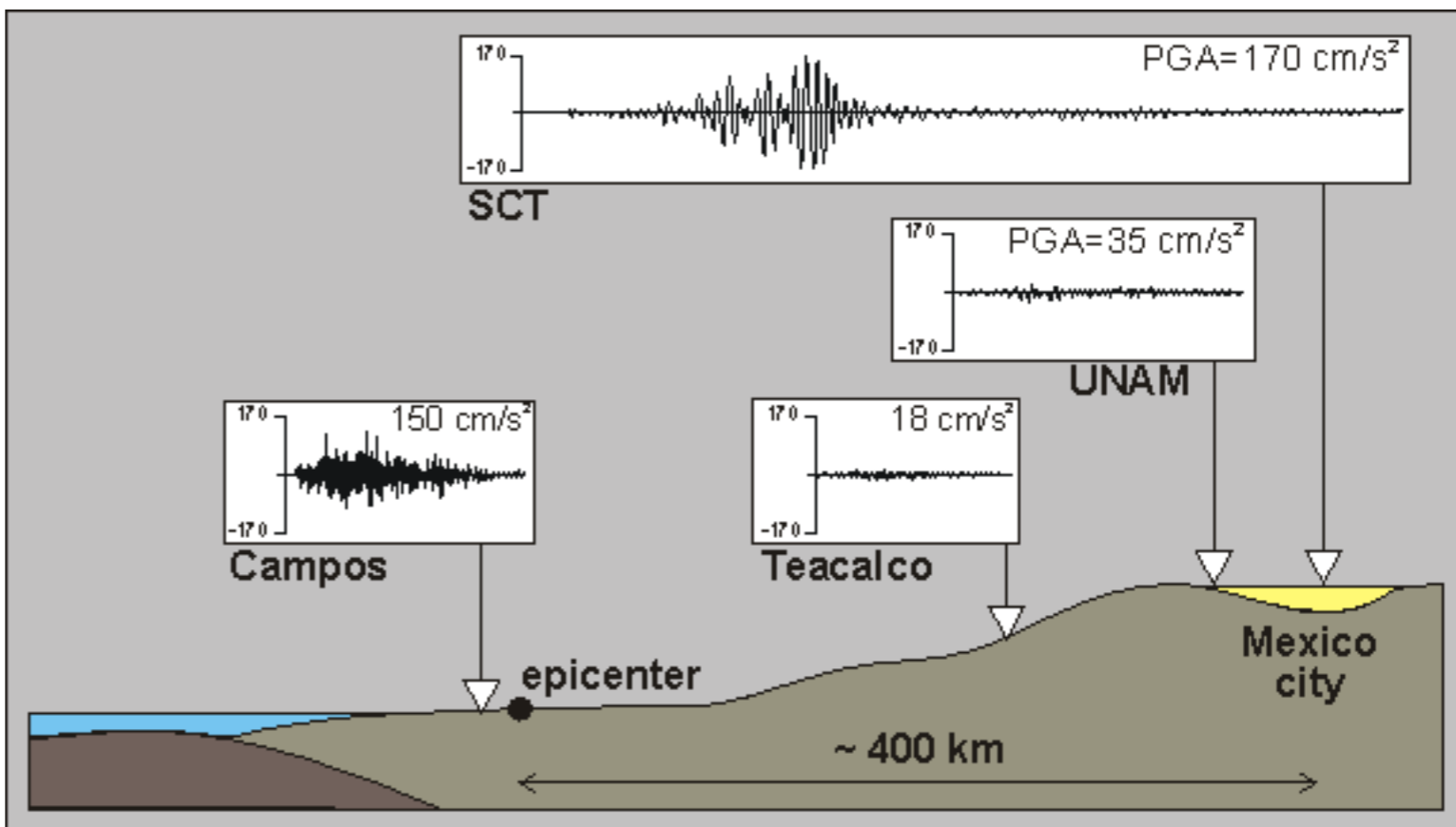
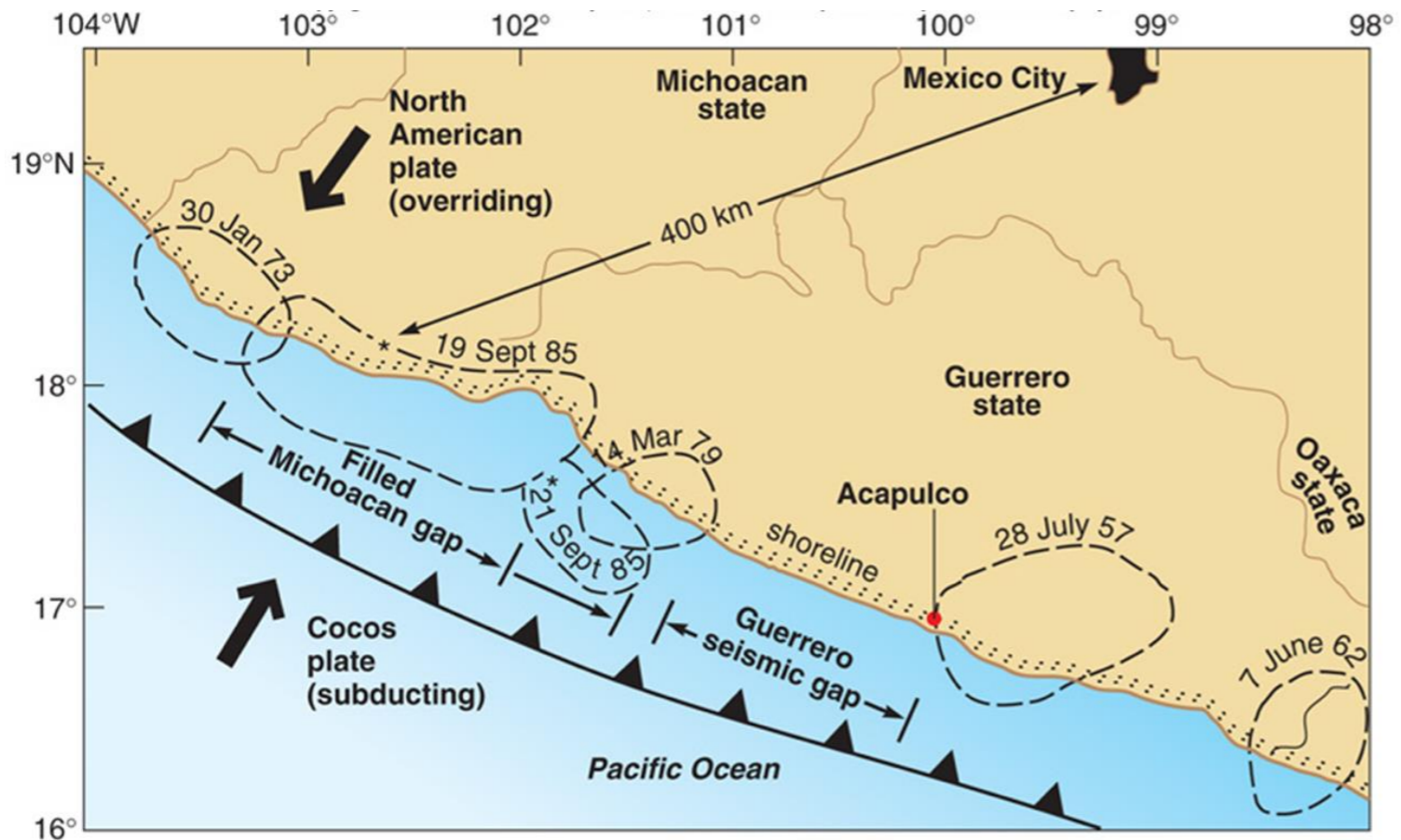


1 - Resonance due to impedance contrasts, 2 - Focusing due to subsurface topography, 3 - Body waves converted to surface waves, 4 - Water content, 5 - Randomness of the medium and 6 - Surface topography

MZS - Engineering Seismology



Michoacan 1985 event: way to DF...



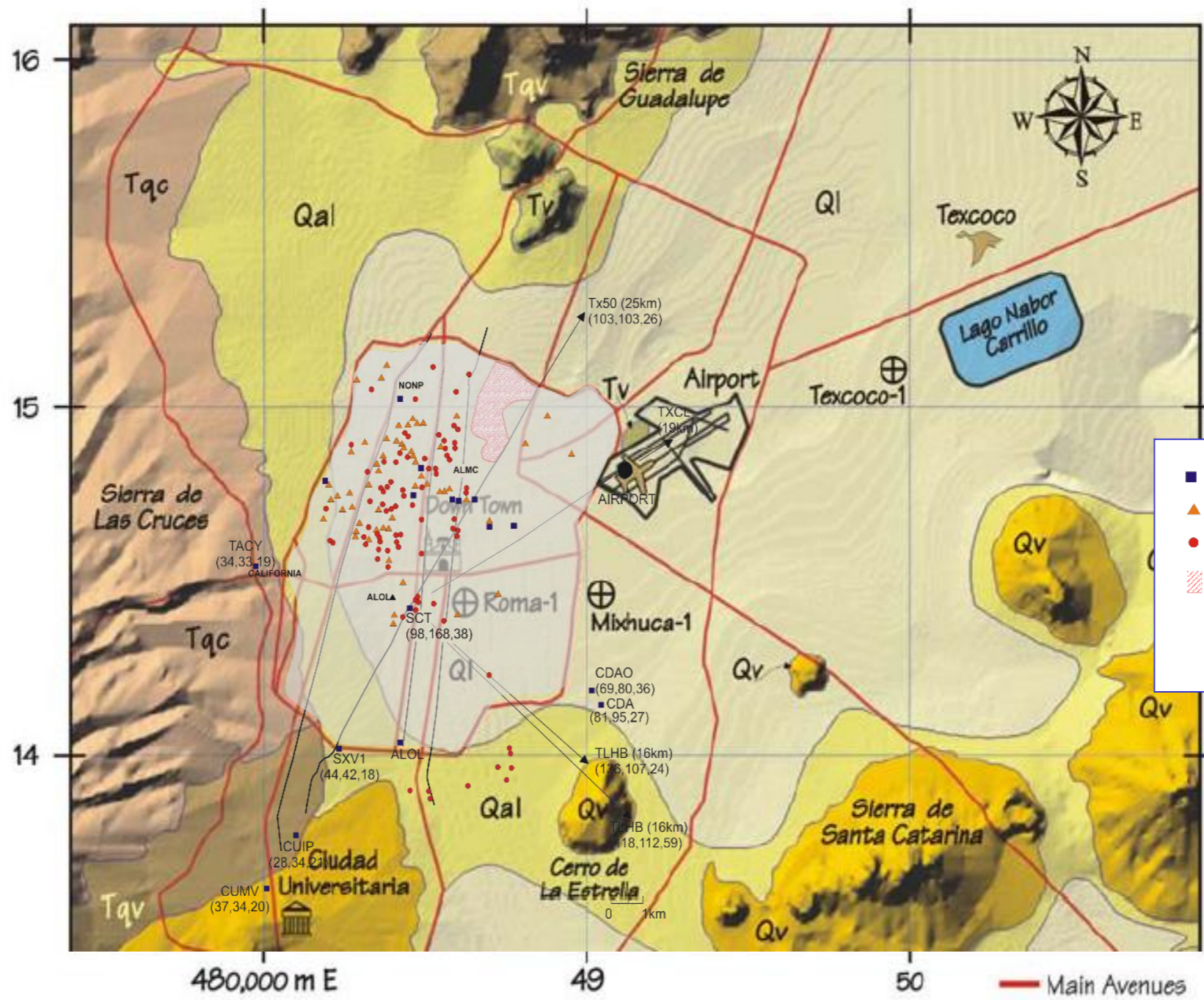
Tenochtitlan and Mexico City (DF)



La ciudad de Tenochtitlan y su entorno en el siglo XVI Pintura de Miguel Covarrubias, Museo Nacional de Antropología, México DF



The actual boundaries of the World Heritage Property follows the boundaries of the Historical Monuments Zones, according to the limits of the city in the 19th century (perimeter A), and a buffer zone (perimeter B)

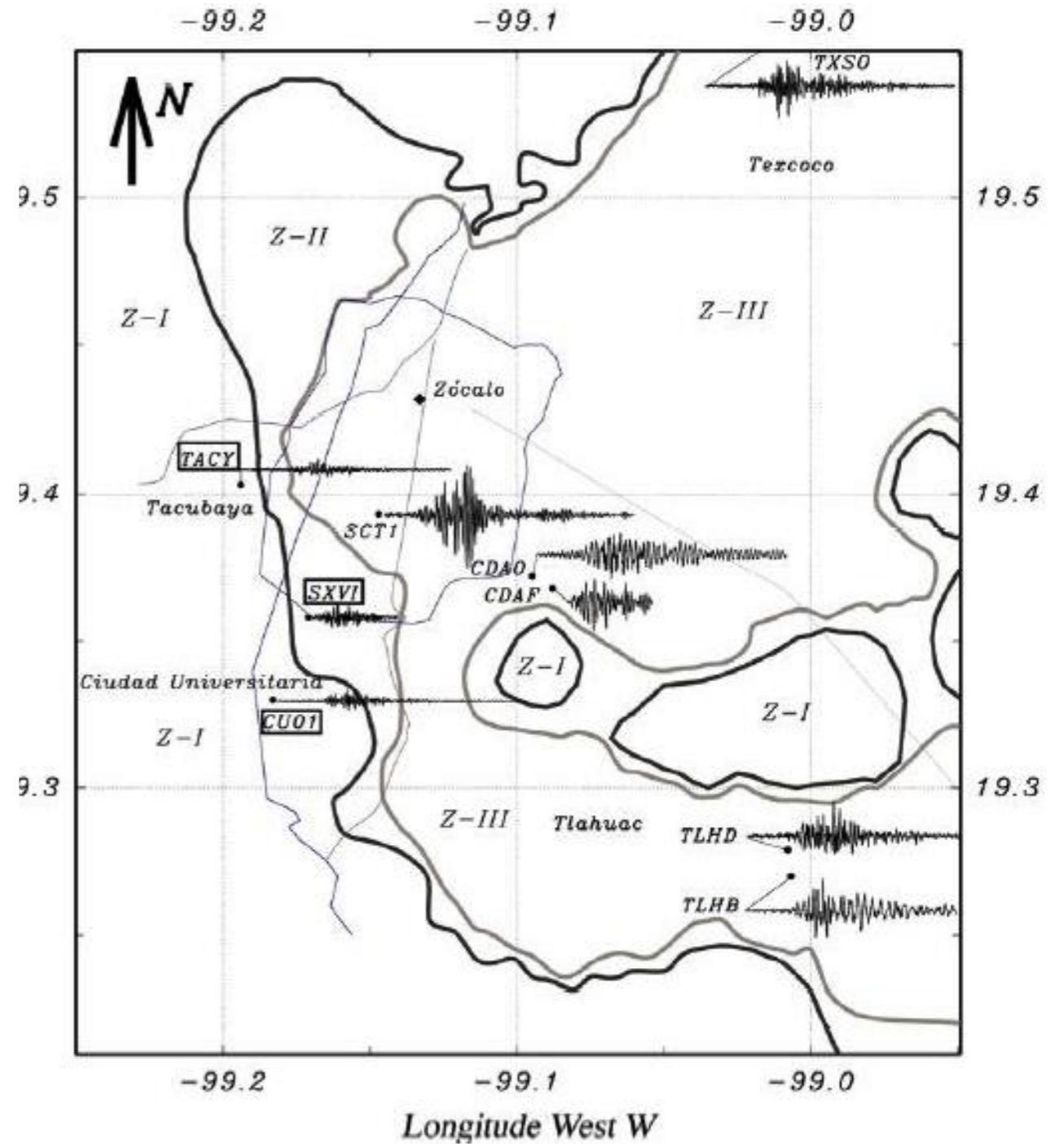
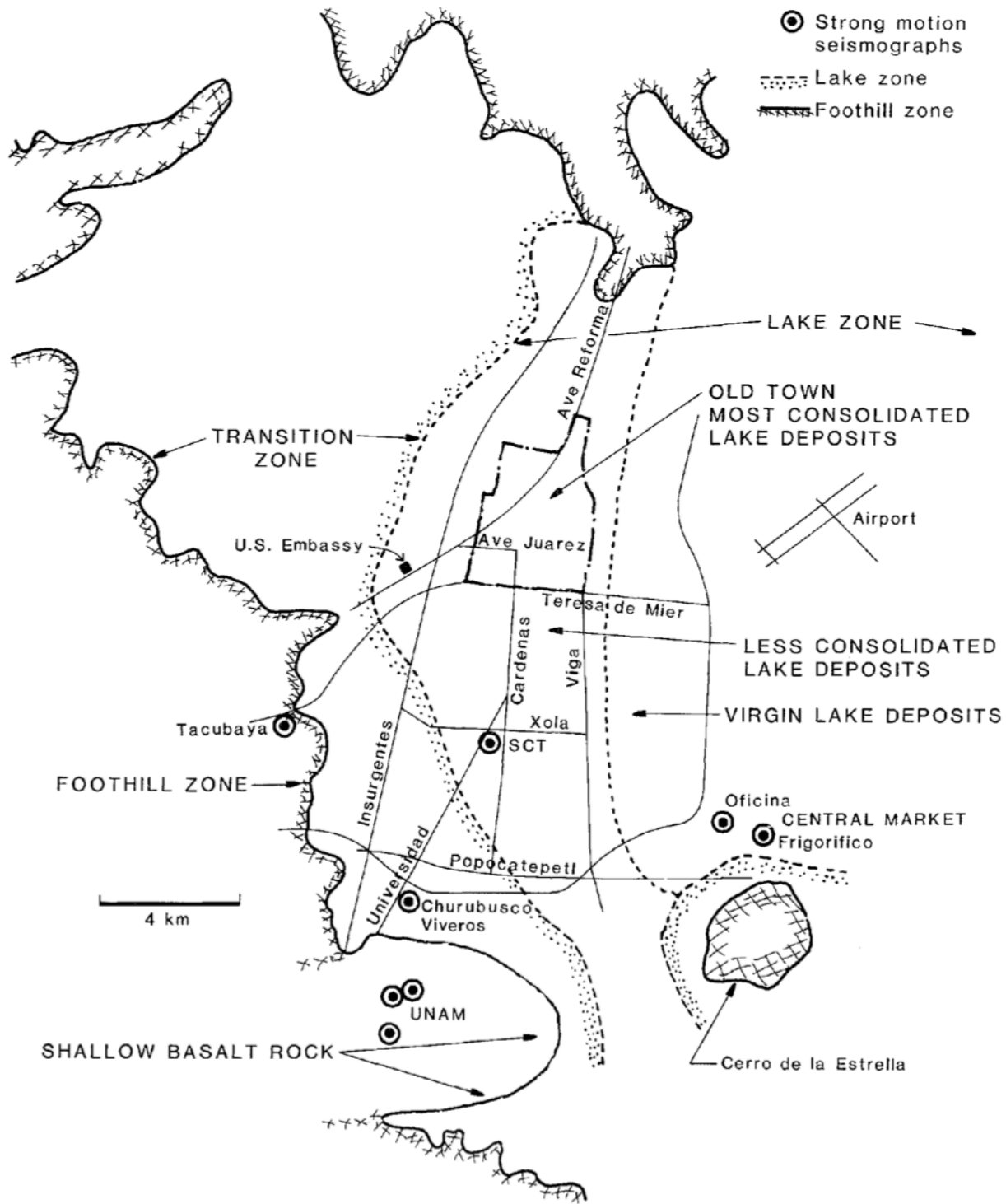


- ACCELEROGRAPH
- ▲ SEVERELY DAMAGED BUILDING
- COLLAPSED BUILDING
- ▨ ZONE WITH MANY COLLAPSED 1 AND 2 STORY HOUSES (BRICK AND ADOBE)



Modified Flores-Estrella et al. (2007) and Singh et al. (1989)

Michoacan 1985 event: GM in DF



Michoacan 1985 event: damage in DF



Wreckage of a twenty-one-story building in
Conjunto Pino Suarez Complex



Totally destroyed office building in the foreground,
while the 44-floor Torre Latinoamericana office
building, in the background on the right, stands

Response spectra

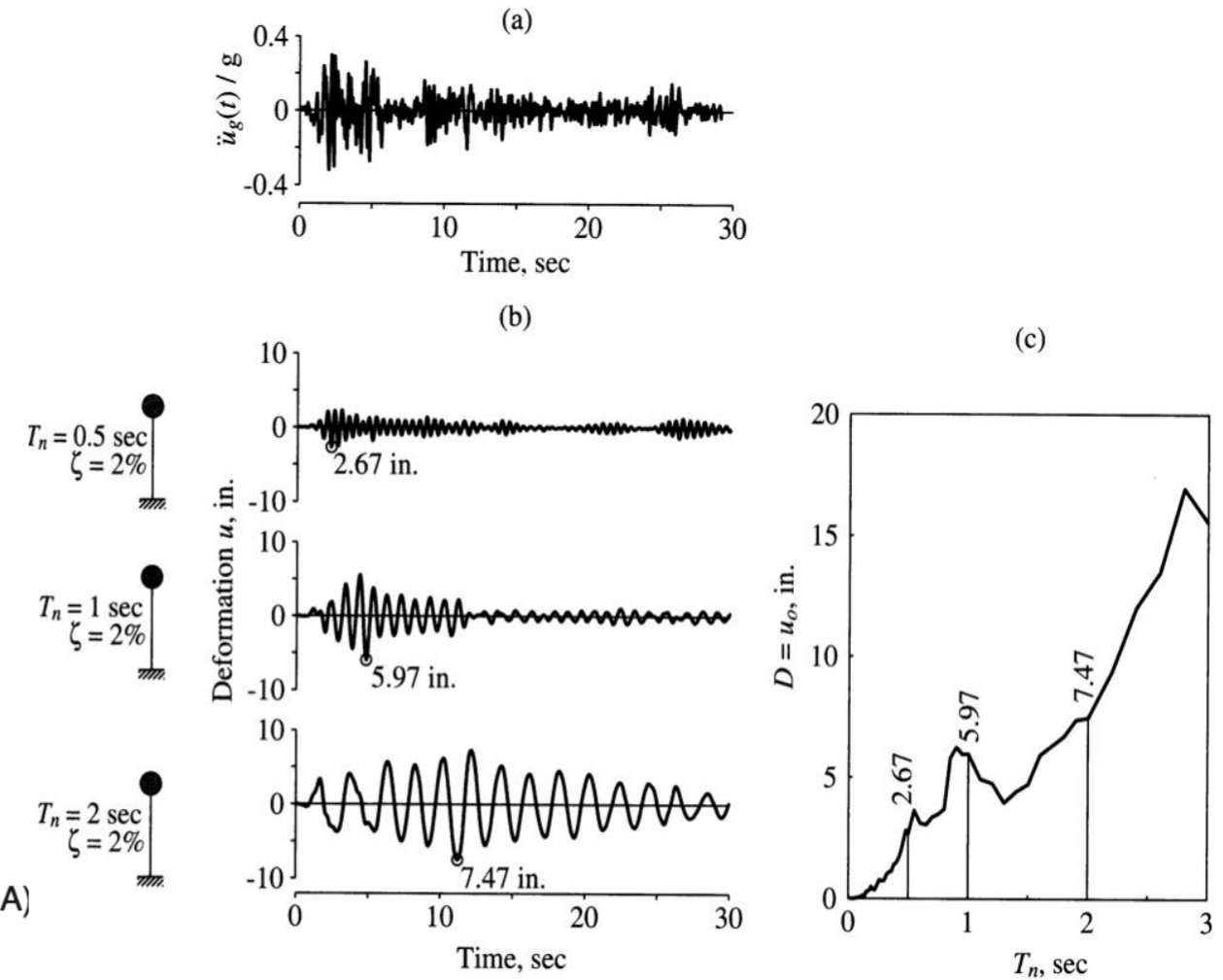
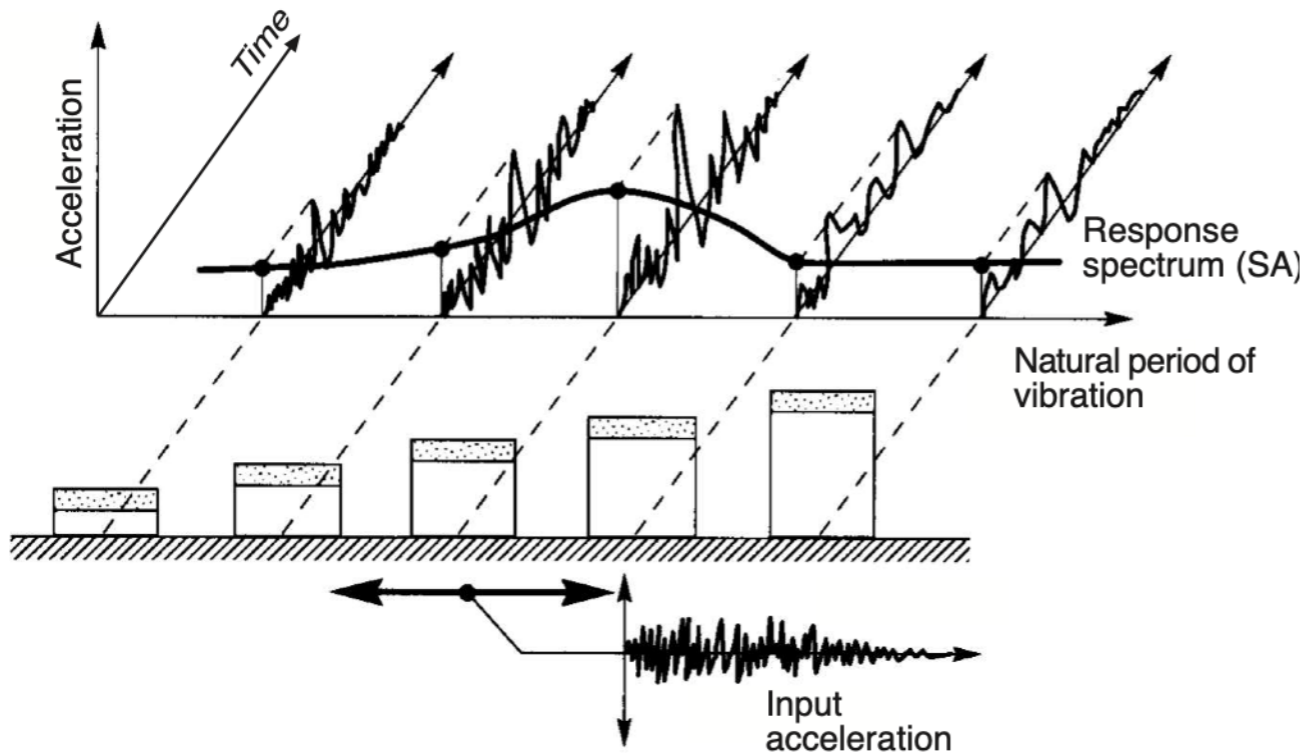
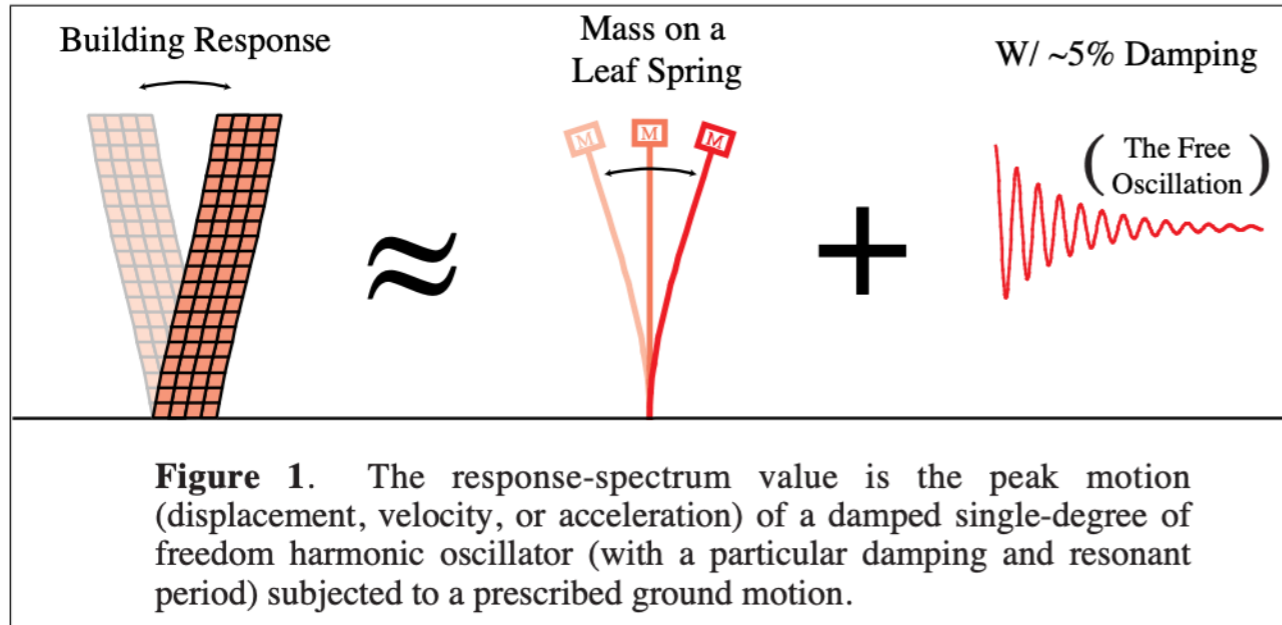
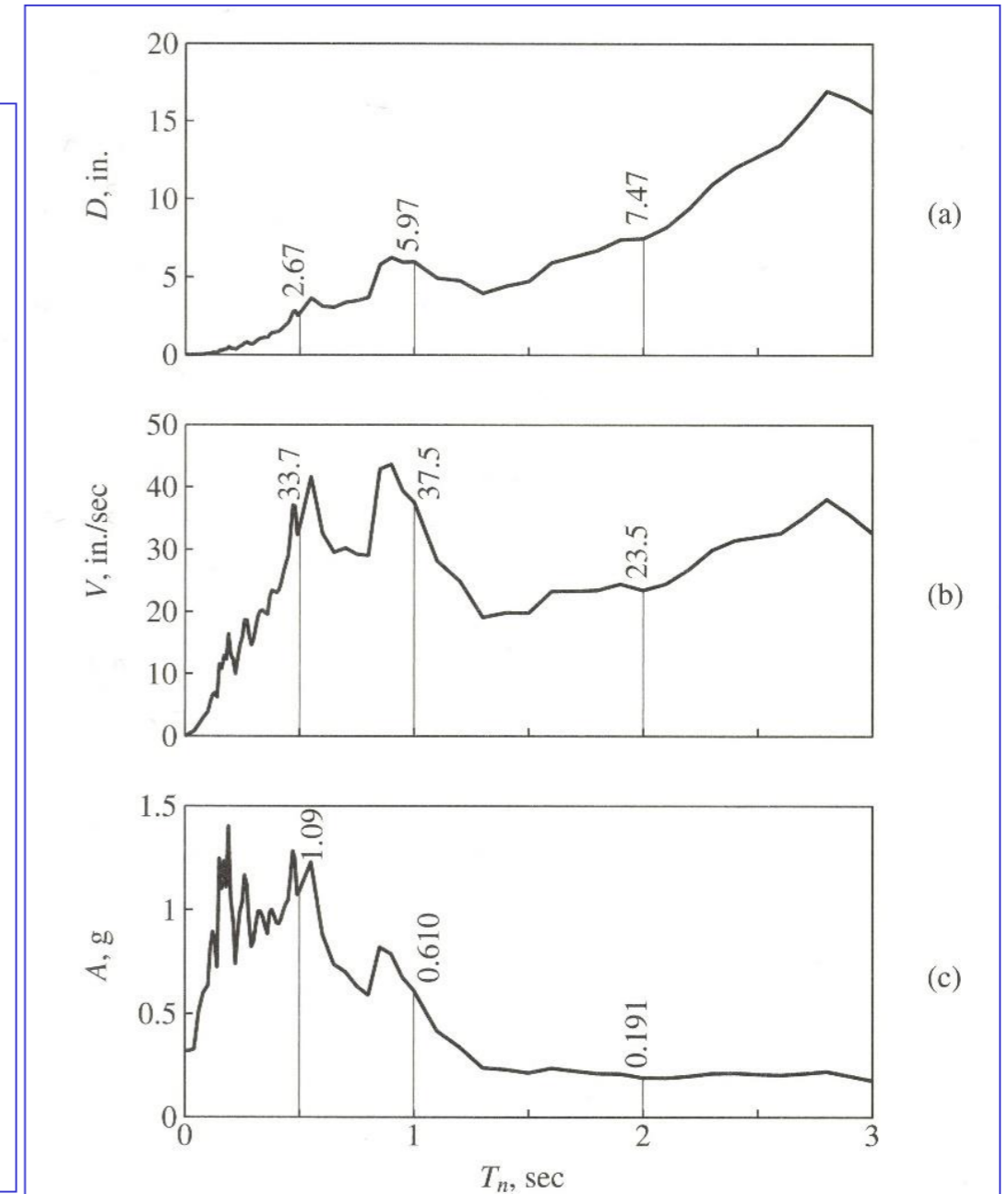
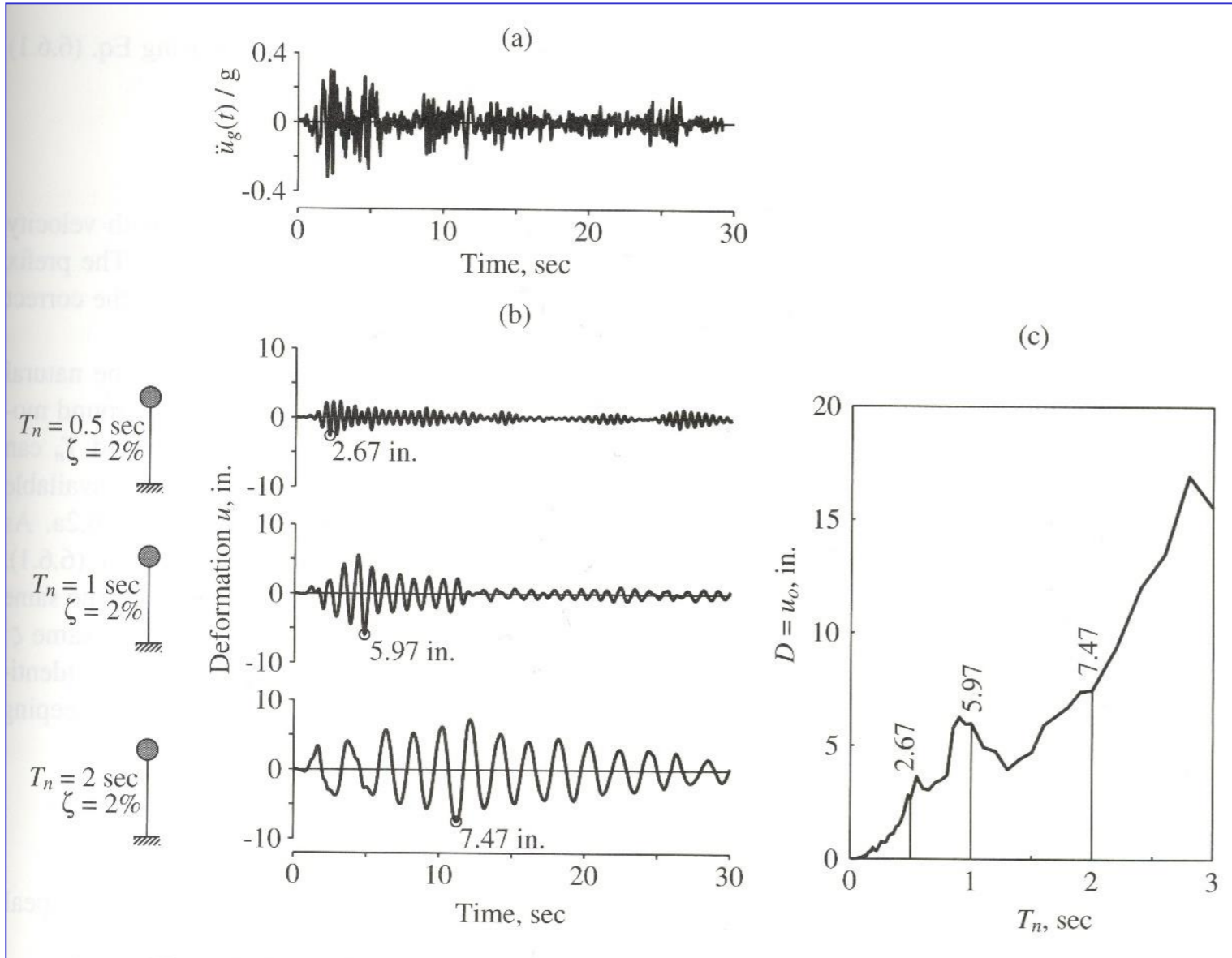


Figure 6.6.1 (a) Ground acceleration; (b) deformation response of three SDF systems with $\zeta = 2\%$ and $T_n = 0.5, 1,$ and 2 sec; (c) deformation response spectrum for $\zeta = 2\%$.

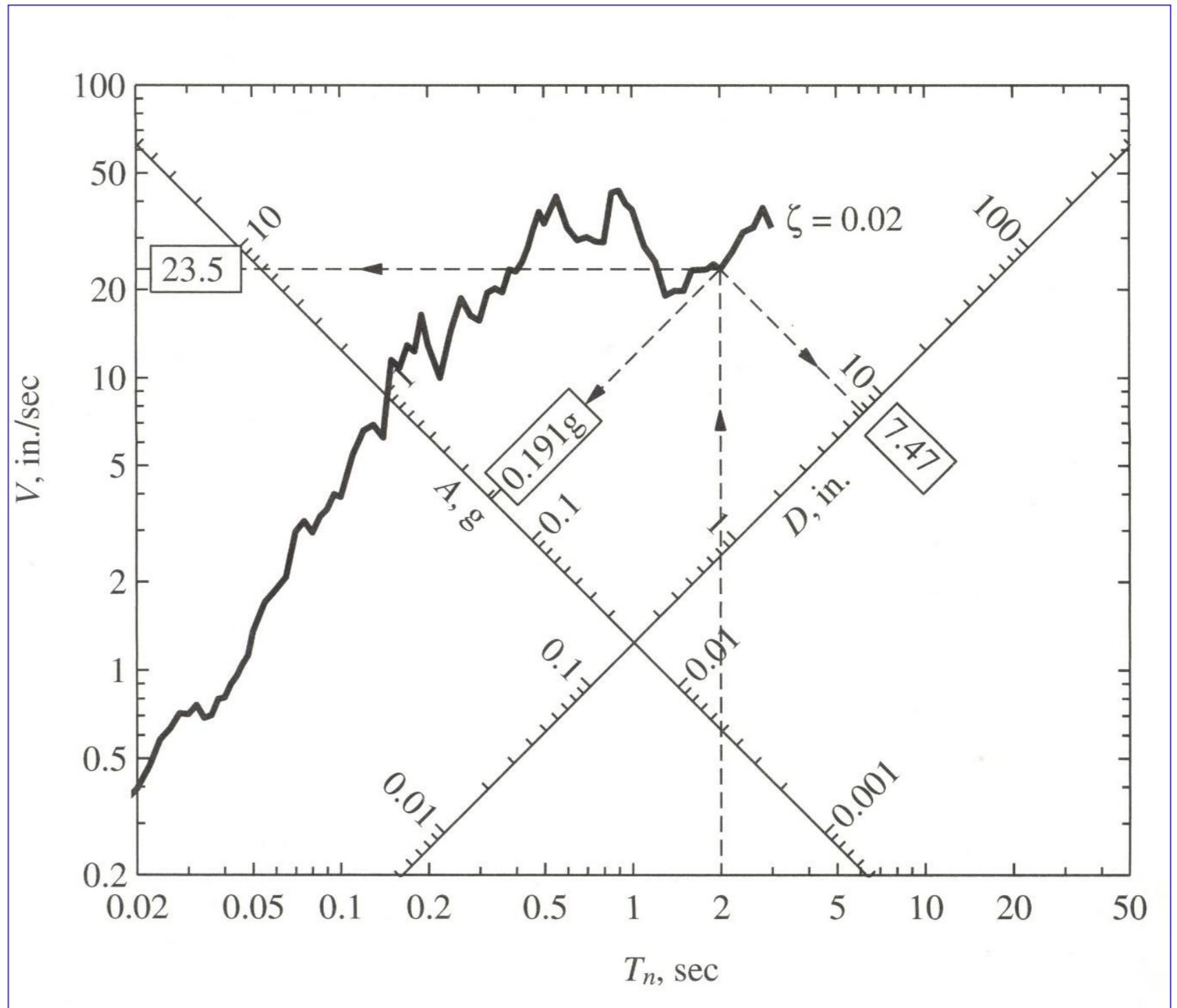
Relative displacement



Pseudo Velocity and Pseudo accelerations

Since PSV and PSA are obtained by SD simply multiplying for a factor \square

The 3 spectra can be displayed on the same plot

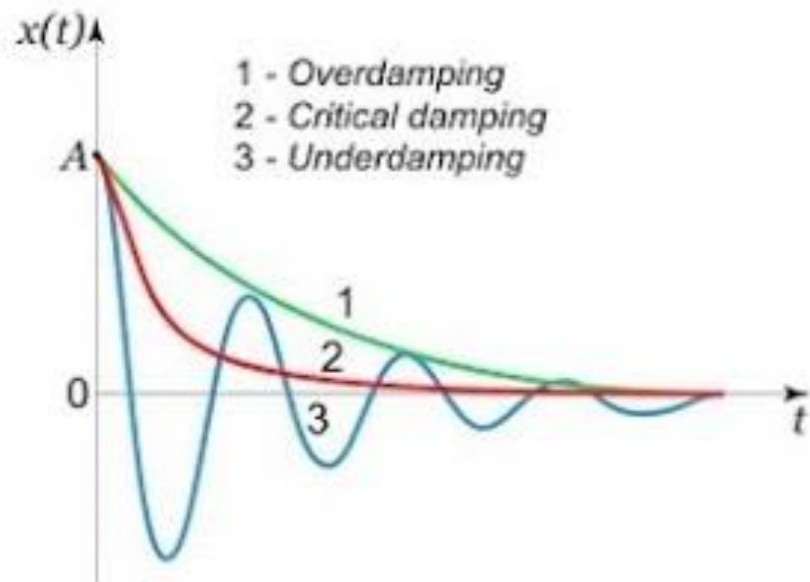


Response spectra

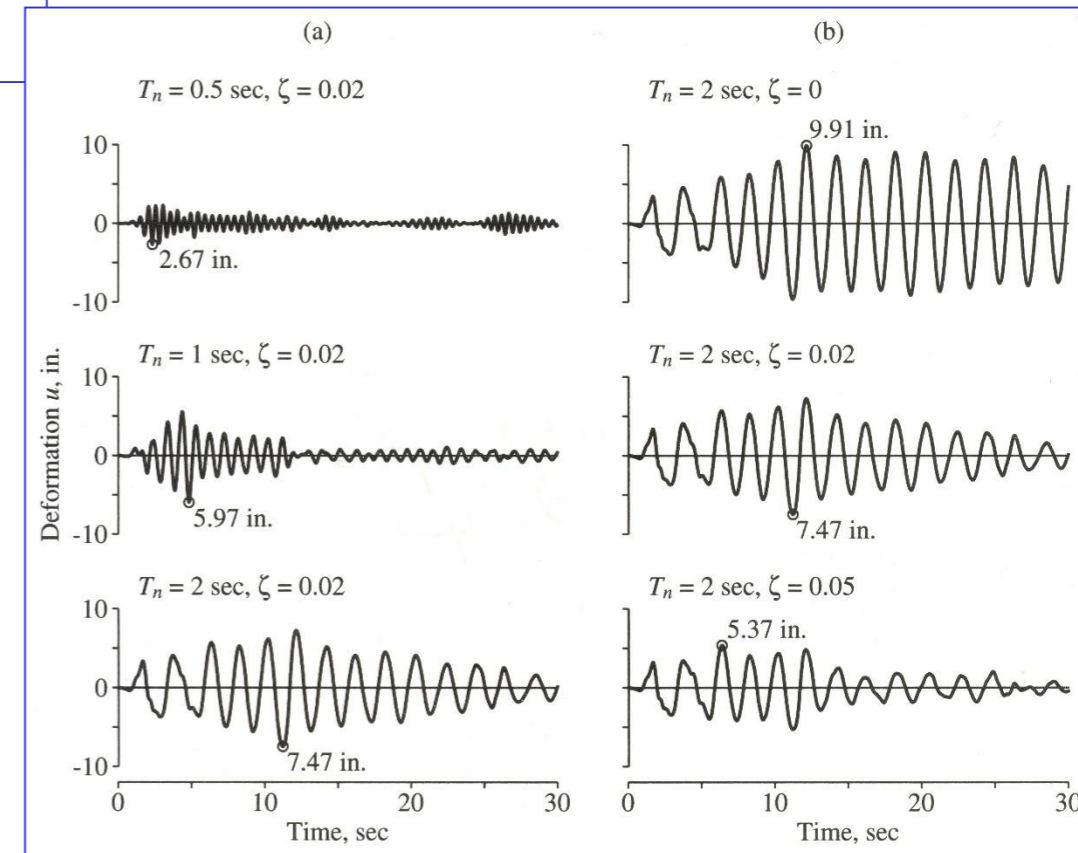
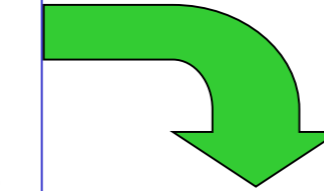
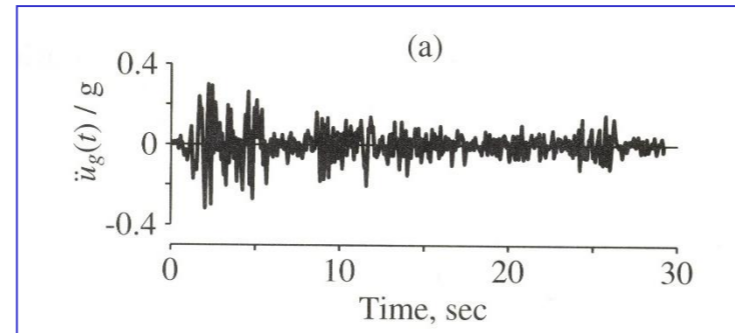
Critically damped system ($\xi = 1$).

2. Overdamped system ($\xi > 1$).

3. Underdamped system ($\xi < 1$)



La risposta dell'oscillatore dipende dalla sua frequenza e dallo smorzamento!





**Born: 21 March
1768 in Auxerre,
Bourgogne,
France**

**Died: 16 May
1830 in Paris,
France**

$$\frac{1}{2\pi} \sum |A(\omega)| e^{i[\omega t + \phi(\omega)]} \Delta\omega$$

=
f(t)

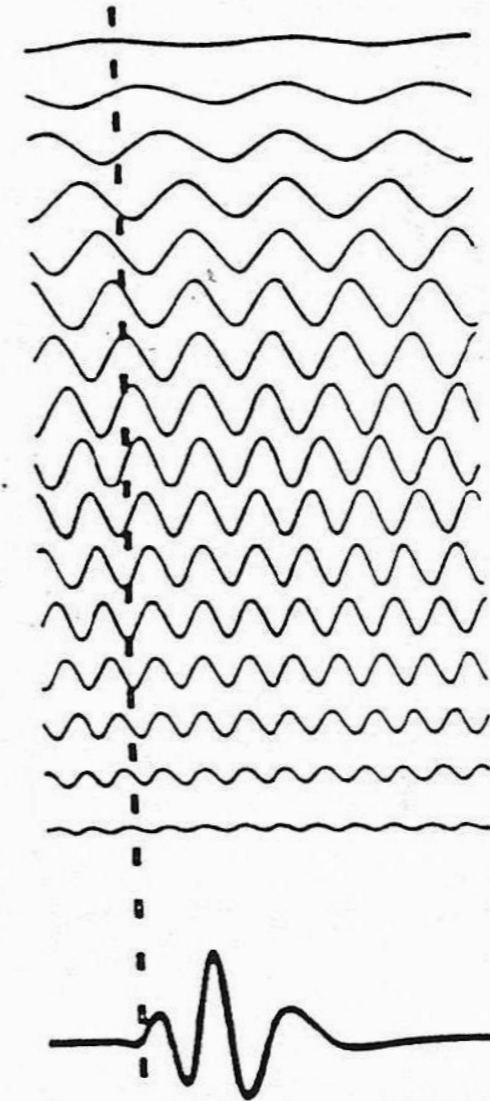


FIGURE 5.B1.2 A discretized version of Eq. (5.1.1), showing how a sum of harmonic terms can equal an arbitrary function. The amplitudes of each harmonic term vary, being prescribed by the amplitude spectrum. The shift of the phase of each harmonic term is given by the phase spectrum.


Fourier spectrum: $G(\omega) = \int_{-\infty}^{\infty} g(t) \exp(i\omega t) dt$

- In what way are there two numbers at each frequency? From basic complex number theory:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

- Using this, the definition can be rewritten as:

$$G(\omega) = \int_{-\infty}^{\infty} g(t) [\cos(\omega t) + i \sin(\omega t)] dt$$

- Thus, the definition can be rewritten as: 

$$a(\omega) = \int_{-\infty}^{\infty} g(t) \cos(\omega t) dt$$

$\cos(\square t)$ even function

- The two numbers at each frequency are $a(\omega)$ and $b(\omega)$ (for $g(t)$ real).

$$b(\omega) = \int_{-\infty}^{\infty} g(t) \sin(\omega t) dt$$

$\sin(\square t)$ odd function

$$G(\omega) = a(\omega) + ib(\omega)$$

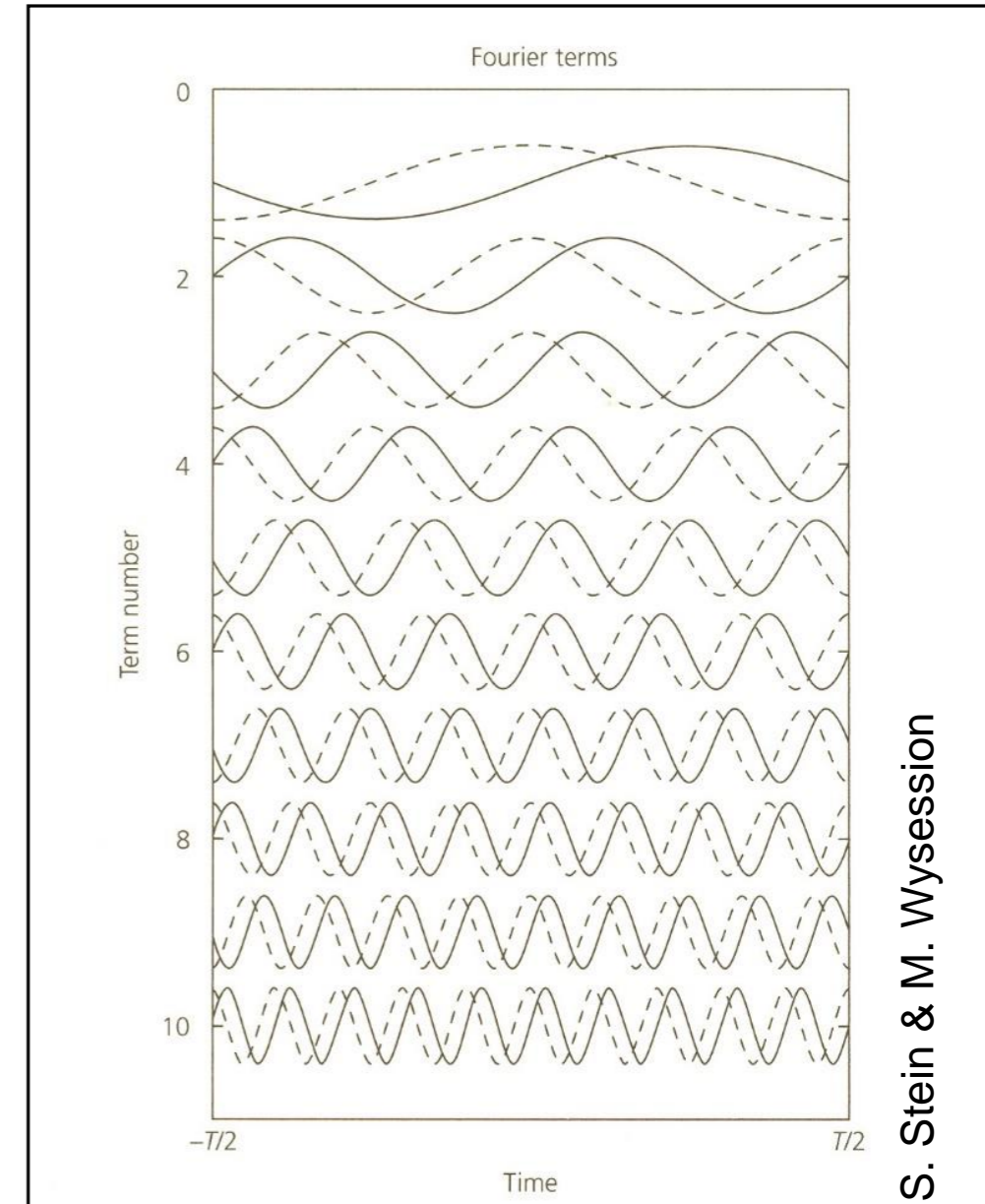
Modified from D. Boore, 2004

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n2\pi t}{T}\right) + b_n \sin\left(\frac{n2\pi t}{T}\right) \right]$$

$$a_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) \cos\left(\frac{n2\pi t}{T}\right) dt$$

$$b_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) \sin\left(\frac{n2\pi t}{T}\right) dt$$

with
n positive integer



S. Stein & M. Wyession

Fourier spectrum: $G(\omega) = \int_{-\infty}^{\infty} g(t) \exp(i\omega t) dt$

$$a(\omega) = \int_{-\infty}^{\infty} g(t) \cos(\omega t) dt$$

$$b(\omega) = \int_{-\infty}^{\infty} g(t) \sin(\omega t) dt$$

$$G(\omega) = a(\omega) + ib(\omega)$$

It is very useful to define the Fourier Amplitude Spectrum:

$$|G(\omega)| = \sqrt{a^2(\omega) + b^2(\omega)}$$

and the Fourier Phase Spectrum:

$$\phi(\omega) = \tan^{-1} \frac{b(\omega)}{a(\omega)}$$

Modified from D. Boore, 2004

Some properties of the Fourier Transform \mathfrak{F}

-Linearity: $\mathfrak{F}[a_1 f_1(t) + a_2 f_2(t)] = a_1 \mathfrak{F}f_1(\omega) + a_2 \mathfrak{F}f_2(\omega)$

-Derivative: $\mathfrak{F}[f^{(n)}(t)] = (i\omega)^n \mathfrak{F}f(\omega)$

-Shift: $\mathfrak{F}[f(t - a)] = e^{-i\omega a} \mathfrak{F}f(\omega)$

-Convolution: $\mathfrak{F}[f_1(t) * f_2(t)] = \mathfrak{F} \int_0^t f_1(\tau) f_2(t - \tau) d\tau = \mathfrak{F}f_1(\omega) \mathfrak{F}f_2(\omega)$

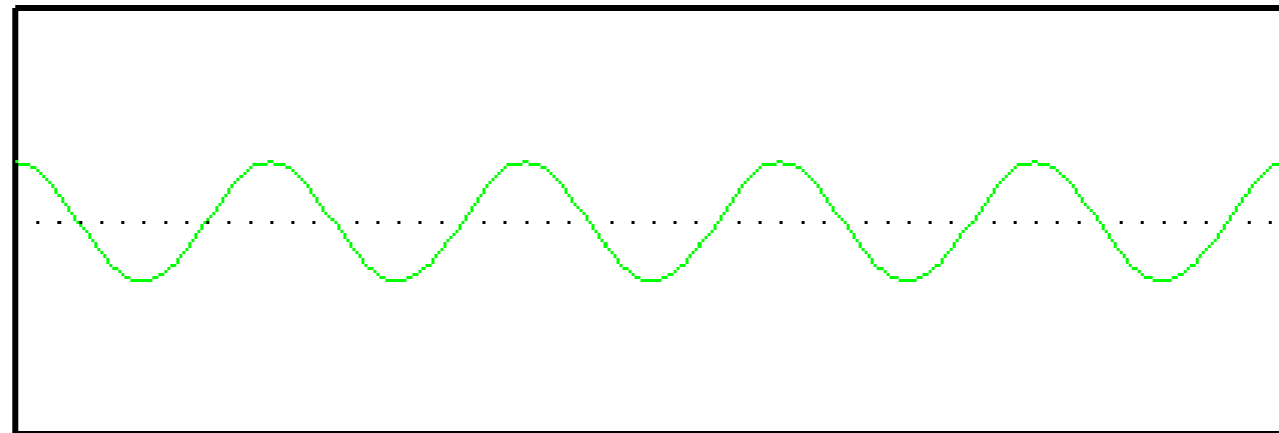
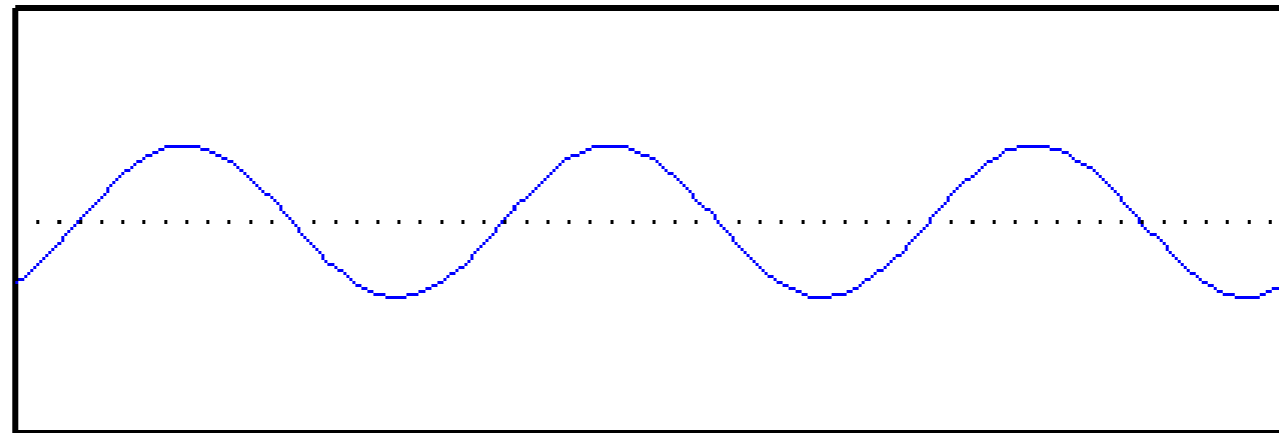
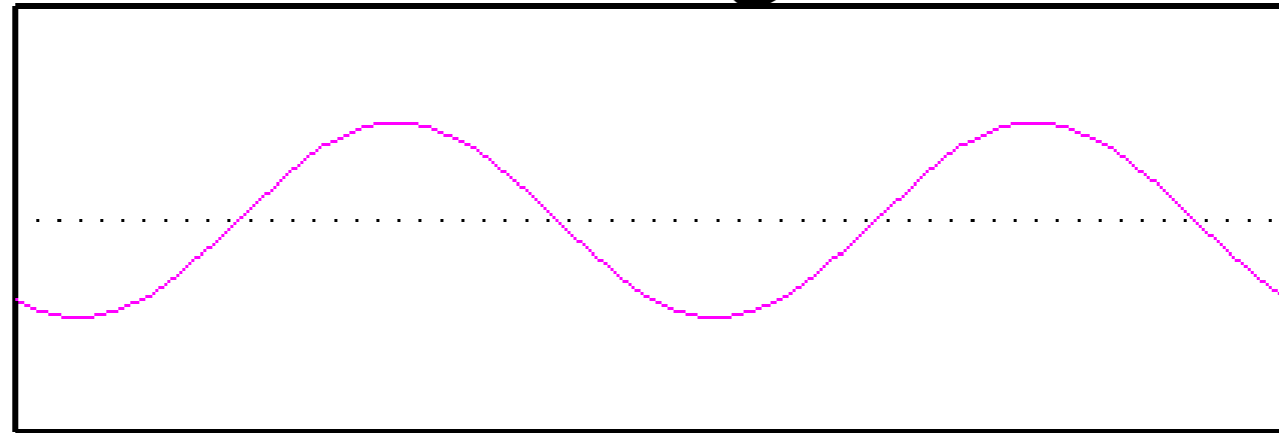
Applications: linear system (source*path*site*instrument), time-delay of propagation (e.g. array analysis), solving differential equations, etc...

Parseval identity
(sum of the square values)

$$\|f(t)\|_2 = \|\mathfrak{F}f(\omega)\|_2$$

Fourier spectrum

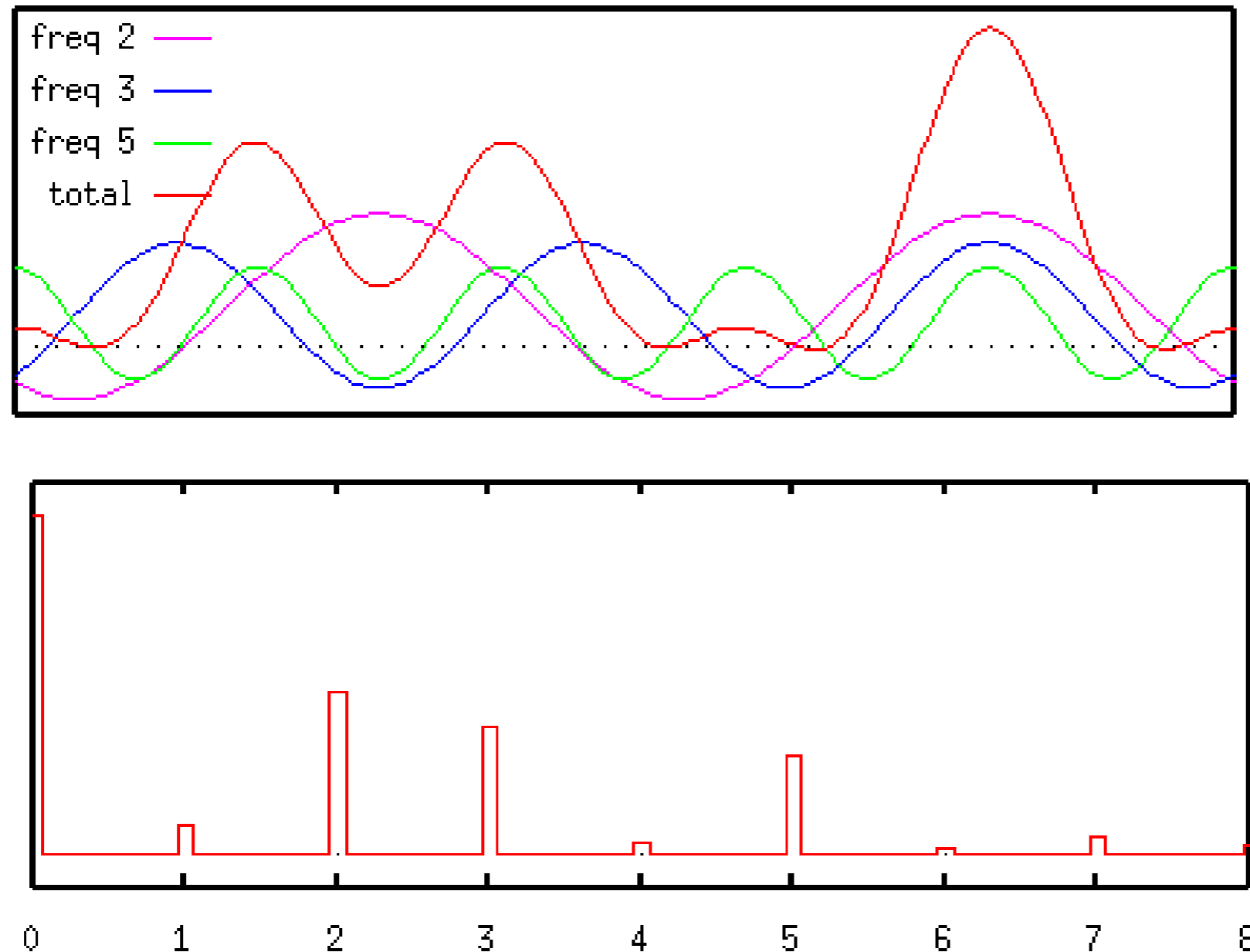
Add the following 3 sinusoids:



Courtesy of D. Boore

Fourier spectrum

This shows the summation of the 3 sinusoids to produce the total signal (top) and the Fourier amplitude (bottom)



Courtesy of D. Boore

Some properties of the Fourier Transform \mathfrak{F}

-Linearity: $\mathfrak{F}[a_1 f_1(t) + a_2 f_2(t)] = a_1 \mathfrak{F}f_1(\omega) + a_2 \mathfrak{F}f_2(\omega)$

-Derivative: $\mathfrak{F}[f^{(n)}(t)] = (i\omega)^n \mathfrak{F}f(\omega)$

-Shift: $\mathfrak{F}[f(t - a)] = e^{-i\omega a} \mathfrak{F}f(\omega)$

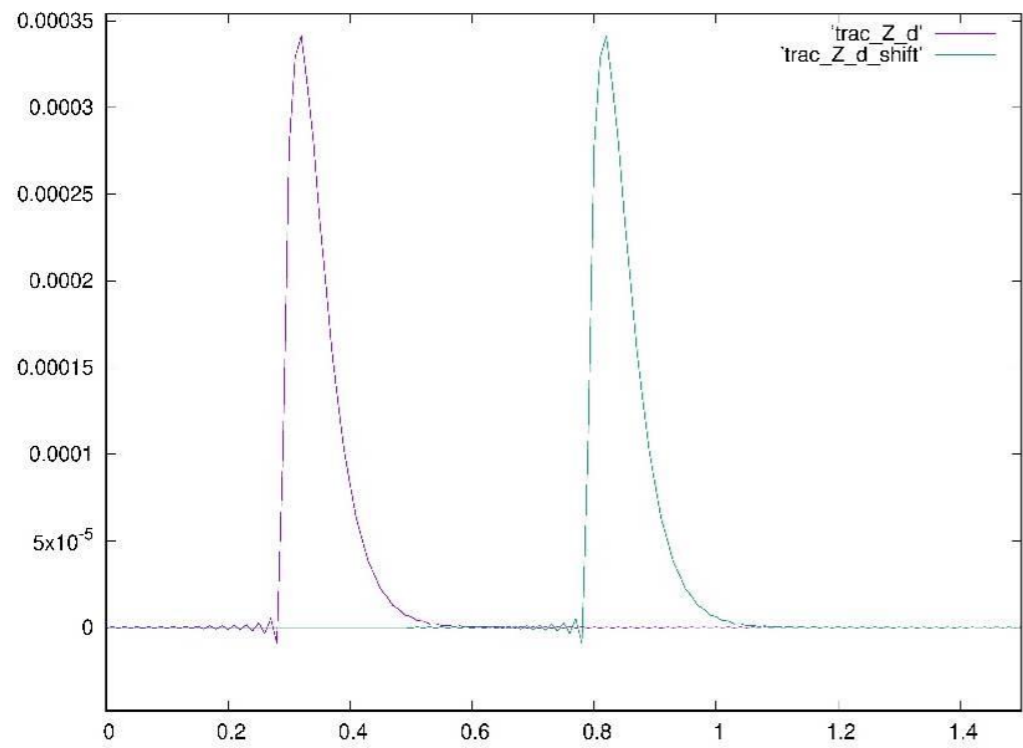
-Convolution: $\mathfrak{F}[f_1(t) * f_2(t)] = \mathfrak{F} \int_0^t f_1(\tau) f_2(t - \tau) d\tau = \mathfrak{F}f_1(\omega) \mathfrak{F}f_2(\omega)$

Applications: linear system (source*path*site*instrument), time-delay of propagation (e.g. array analysis), solving differential equations, etc...

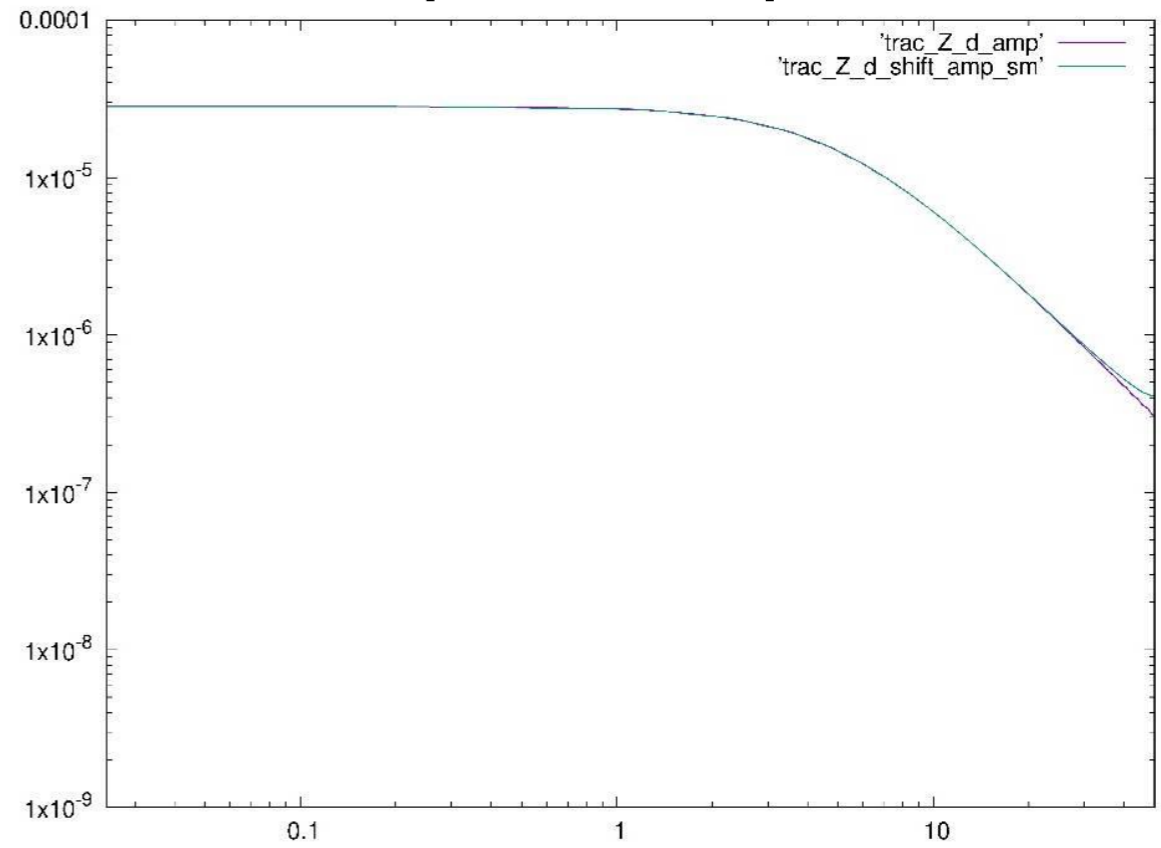
Parseval identity
(sum of the square values)

$$\|f(t)\|_2 = \|\mathfrak{F}f(\omega)\|_2$$

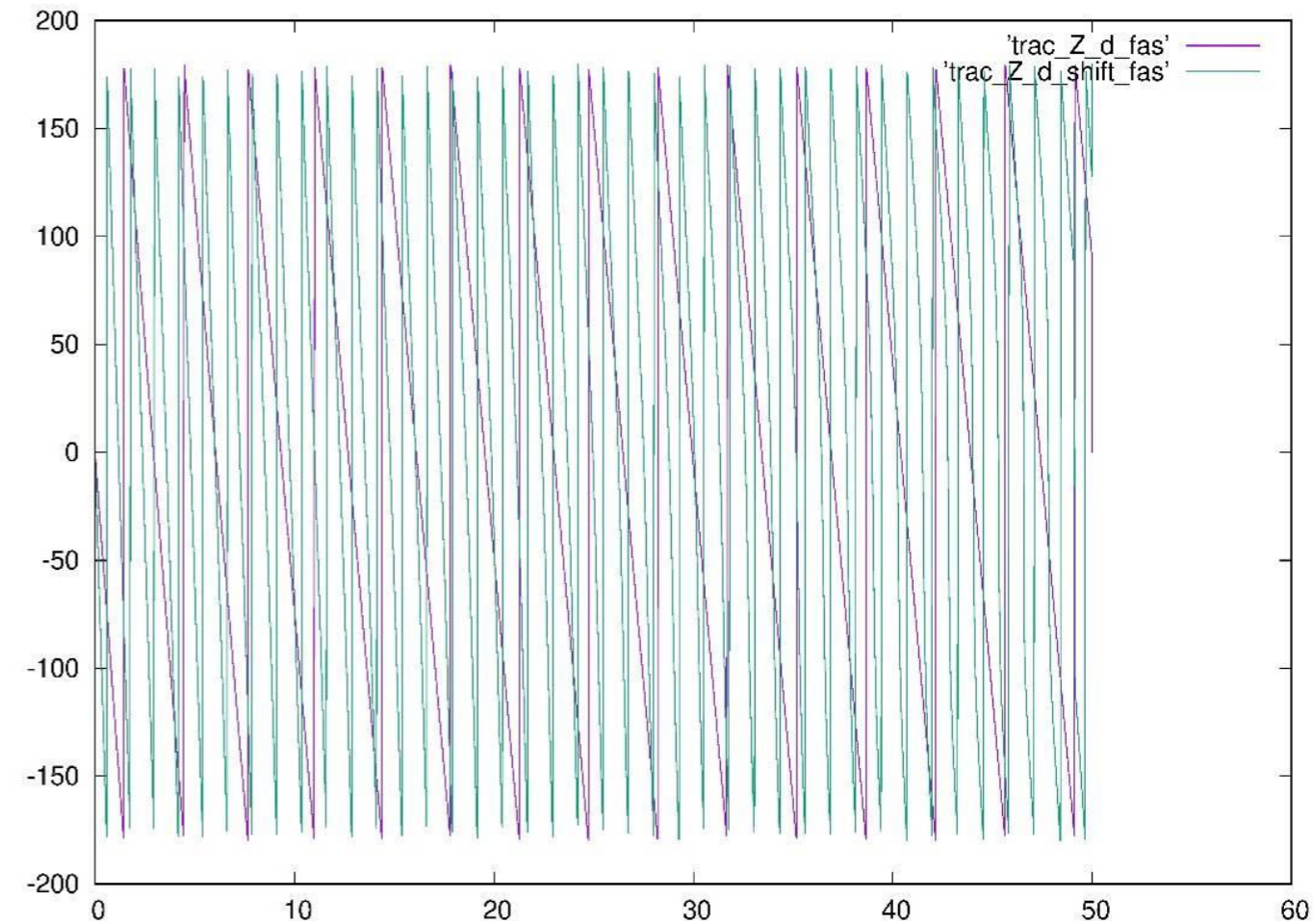
Time Series

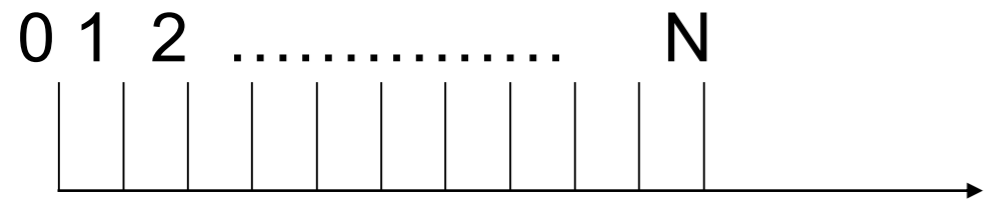


Amplitude spectra



Phase spectra



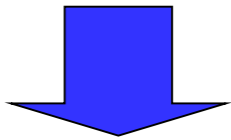


sampling

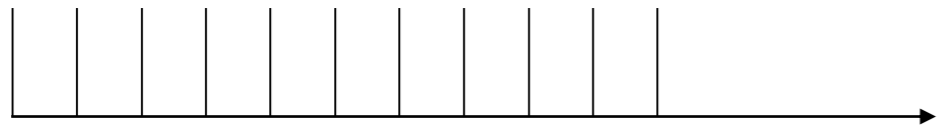
0 dt 2dt Ndt=T

Time t [s]

N intervals with spacing dt
Duration T of the whole signal
 $T=N*dt$



Sampling



N intervals of width df

Nyquist frequency: $f_N=1/(2*dt)$

$-f_N - f_N +df.....$

f_N

Frequency [Hz]

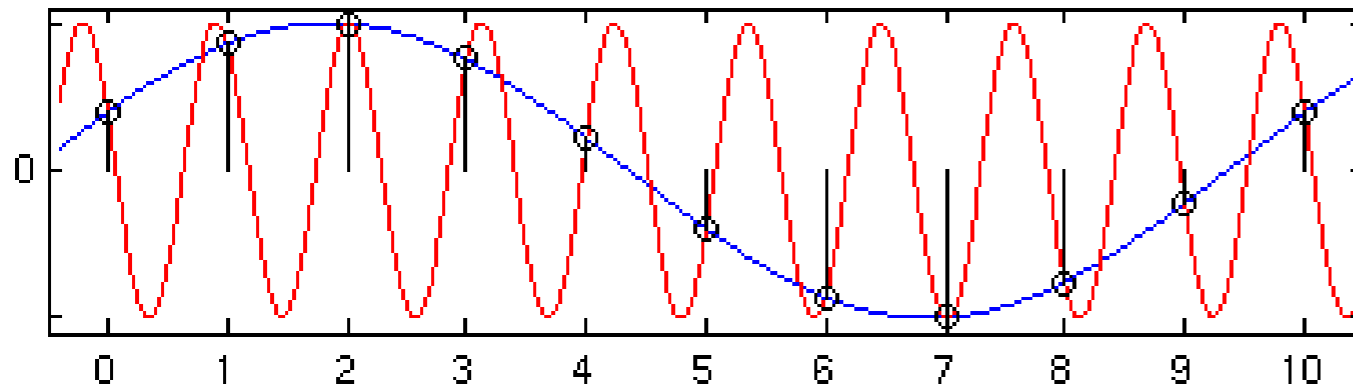
$$N*df=2f_N \quad \square$$

$$df=2f_N/N=1/Ndt=1/T$$

$$df=1/T$$

df depends on the window length

$$df dt=1/N$$



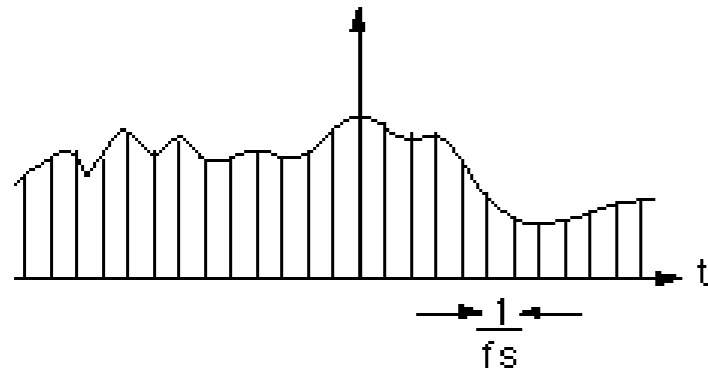
$dt=1s;$
 $f_N=0.5\text{Hz}$
 $f>0.5\text{Hz}$ cannot be retrieved en
 (Periods $T < 2\text{ s}$)

The red curve has a period of $T=1.1\text{s}$
 red: $\text{freq} > \text{Nyquist}$
 Look at the blue curve!!!

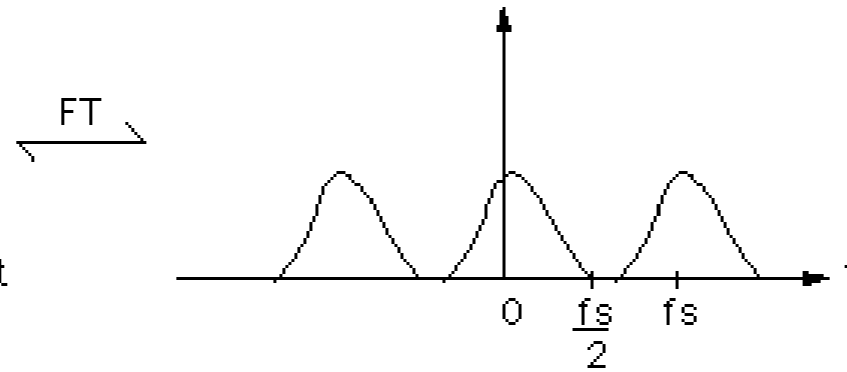
Due to Aliasing the data must be low-pass filtered before the analog to digital conversion (anti-alias filter). The corner frequency of the filter $0.8f_N$.

Input: Time signal

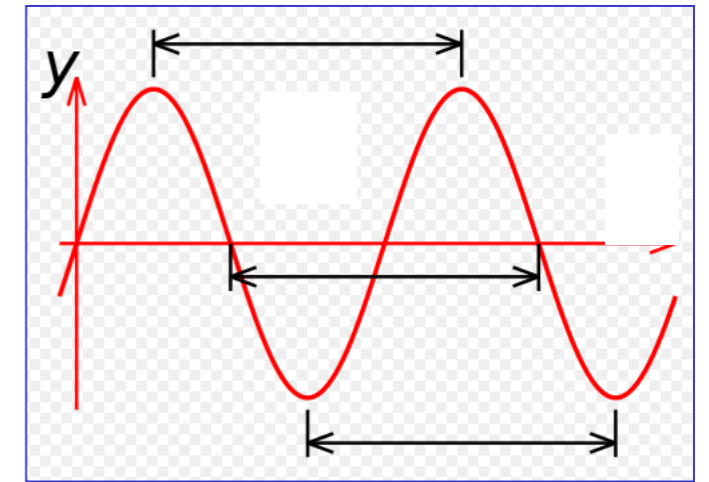
signal $y(t_n)$



Fourierspectrum $Y(\omega)$



Sinus form **Period T**



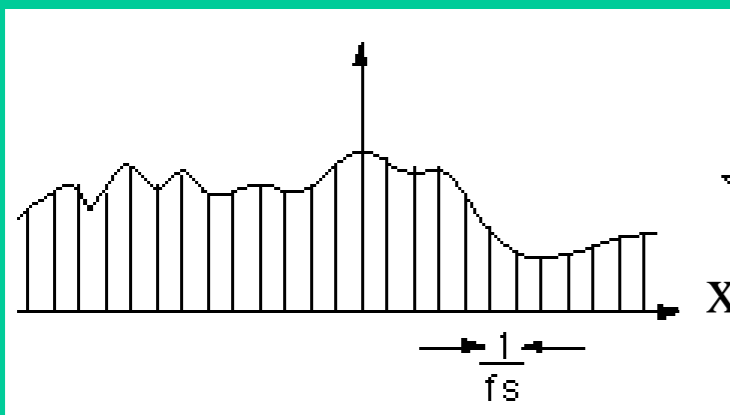
$$f_s = 1/dt$$

$$f_{\text{Nyquist}} = f_s/2$$

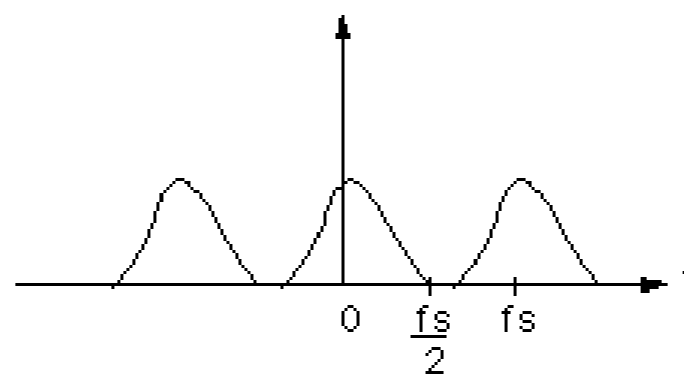
$$\omega = 2\pi f = 2\pi/T \text{ (angular frequency [rad/s])}$$

(time) Aliasing

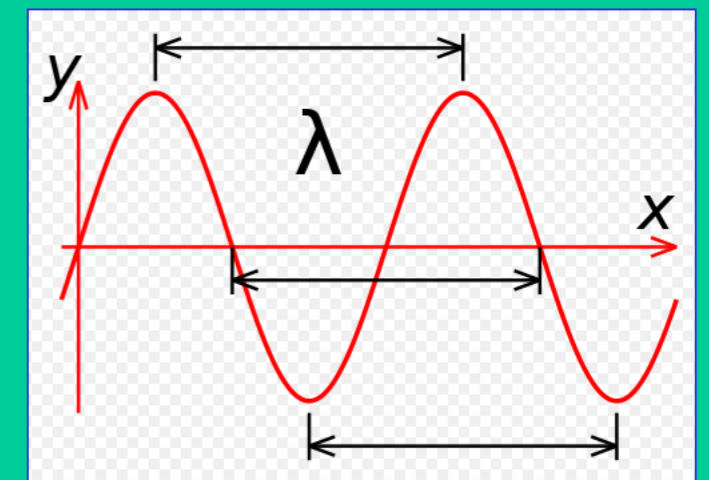
Signal $y(x_n)$



Fourierspectrum $Y(\kappa)$



Sinus form **wavelength** λ



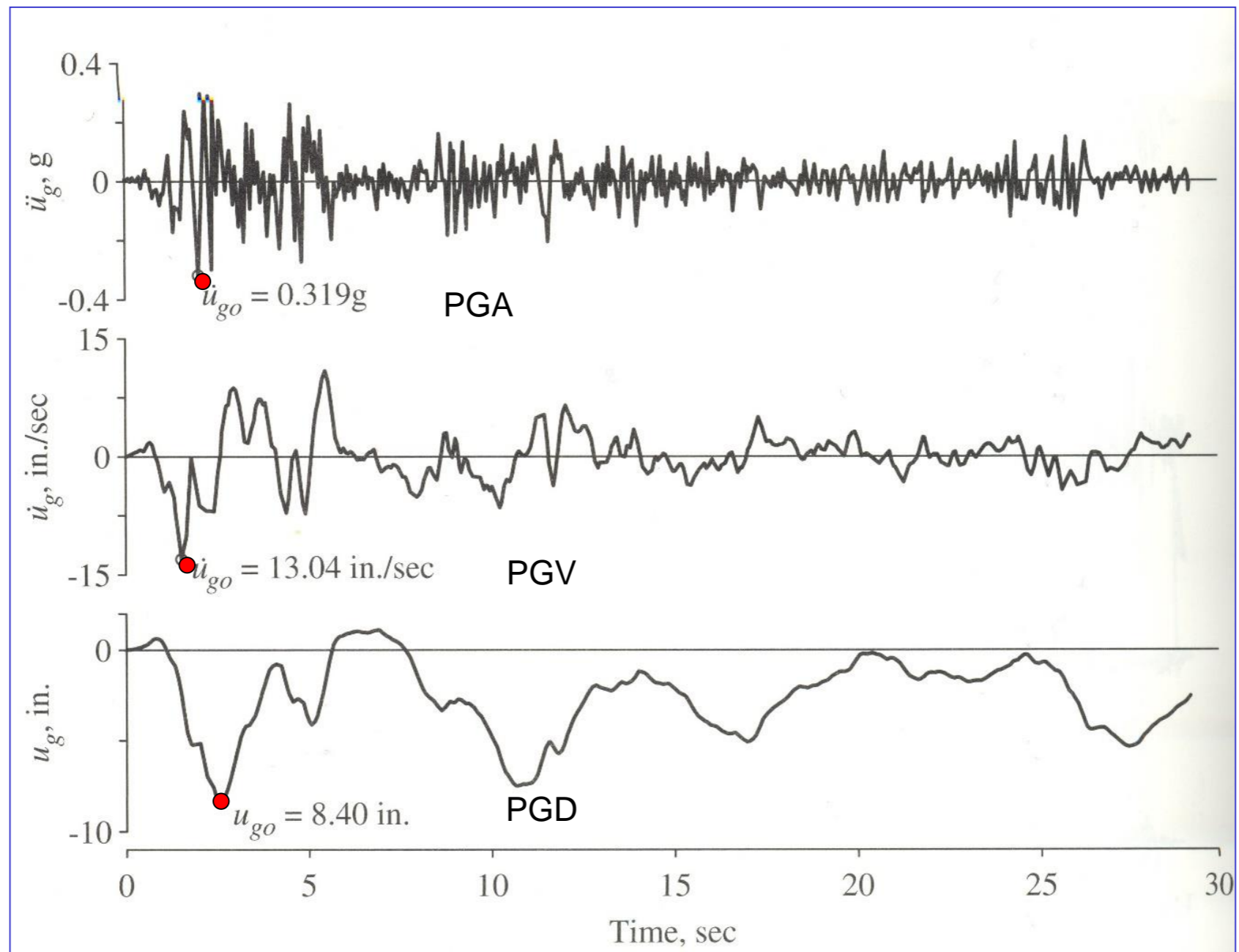
$$f_s = 1/dx$$

$$f_{\text{Nyquist}} = f_s/2$$

$$\kappa = 2\pi f = 2\pi/\lambda$$

(Wavenumber [rad/m])

(spatial) Aliasing



Some properties of the Fourier Transform \mathfrak{F}

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-Derivative: $\mathfrak{F}[f^{(n)}(t)] = (i\omega)^n \mathfrak{F}f(\omega)$

-Shift: $\mathfrak{F}[f(t - a)] = e^{-i\omega a} \mathfrak{F}f(\omega)$

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Applications: linear system (source*path*site*instrument), time-delay of propagation (e.g. array analysis), solving differential equations, etc...

Parseval identity
(sum of the square values)

$$\|f(t)\|_2 = \|\mathfrak{F}f(\omega)\|_2$$

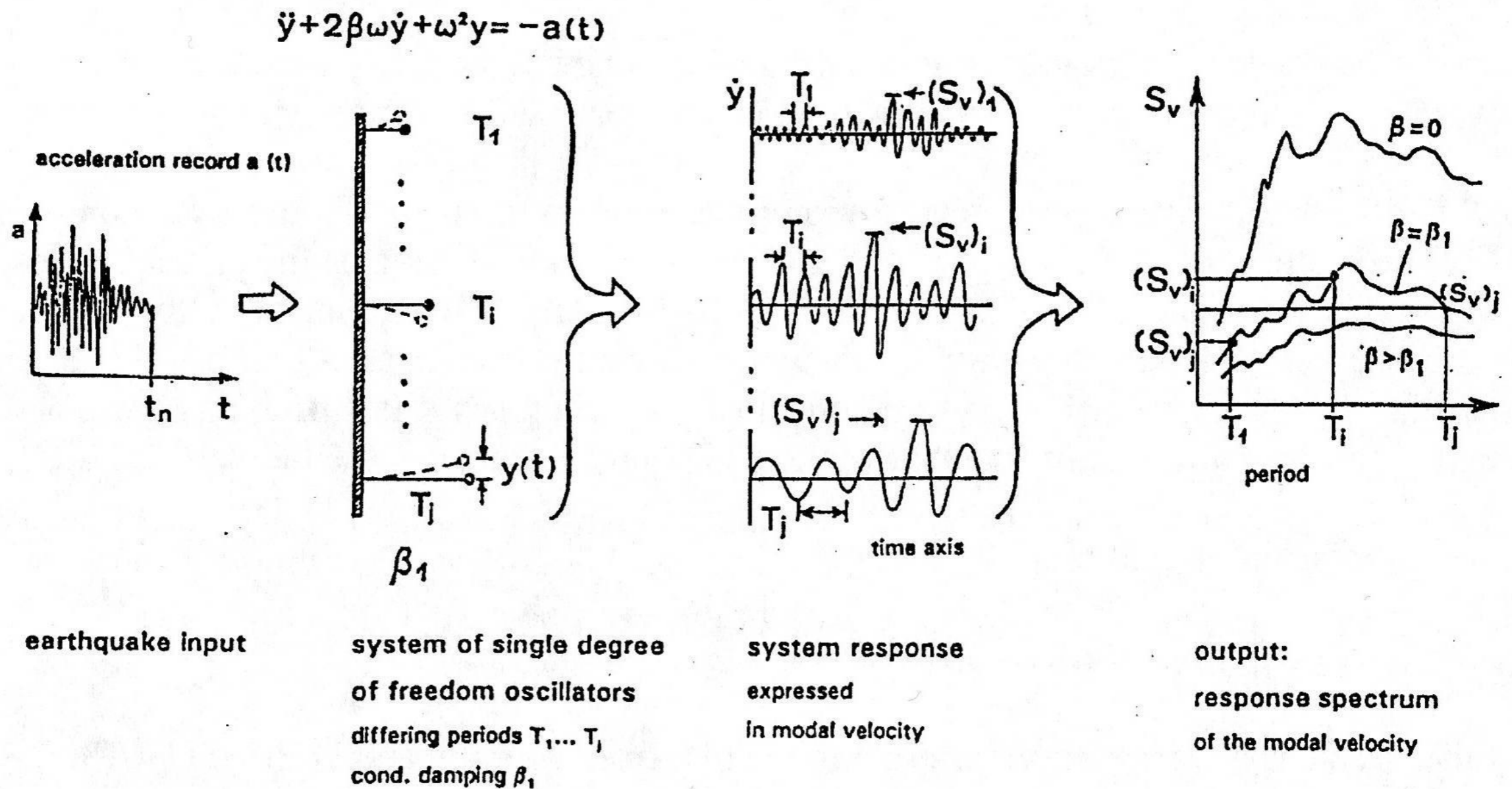
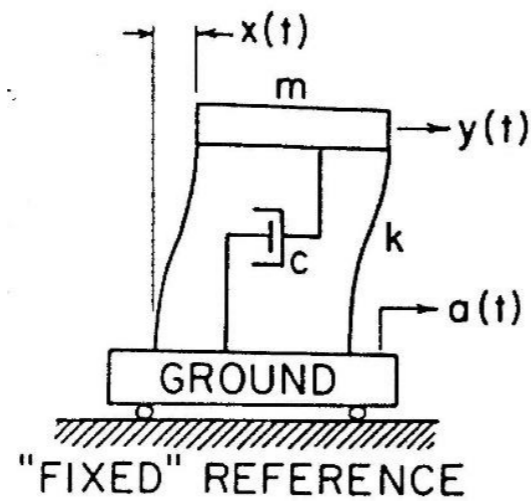


Fig.7: Schematic presentation of the construction of response spectra.

Courtesy of G. Grünthal



$x(t)$ = RELATIVE MOTION OF m WITH RESPECT TO GROUND

$y(t)$ = ABSOLUTE MOTION OF m WITH RESPECT TO "FIXED" REFERENCE

$a(t)$ = ABSOLUTE ACCELERATION OF GROUND WITH RESPECT TO "FIXED" REFERENCE

EQUATION OF MOTION OF m : $\ddot{x} + 2\omega_n\zeta\dot{x} + \omega_n^2x = -a(t)$

WHERE: $\omega_n = \sqrt{k/m}$ = NATURAL FREQUENCY; $T = \frac{2\pi}{\omega_n}$ = PERIOD

$\zeta = c/2m\omega_n$ = FRACTION CRITICAL DAMPING

GENERAL SOLUTION:

$$x(t) = -\frac{1}{\omega_n\sqrt{1-\zeta^2}} \int_0^t a(\tau)e^{-\omega_n\zeta(t-\tau)} \sin \omega_n\sqrt{1-\zeta^2}(t-\tau) d\tau$$

DEFINITION OF RESPONSE SPECTRA:

SD = $|x(t)|_{\max}$ = RELATIVE DISPLACEMENT RESPONSE SPECTRUM

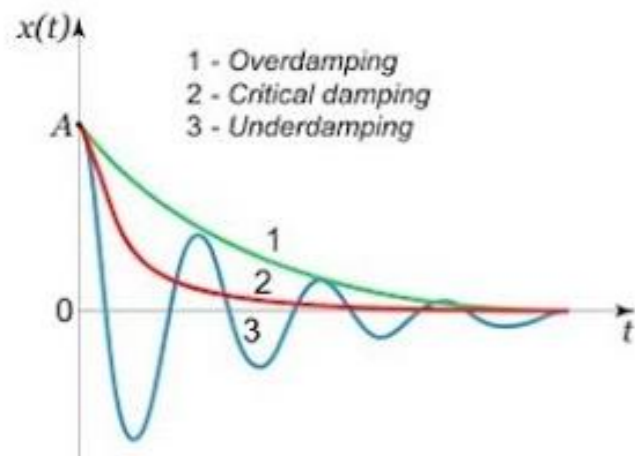
SV = $|\dot{x}(t)|_{\max}$ = RELATIVE VELOCITY RESPONSE SPECTRUM

SA = $|\ddot{y}(t)|_{\max}$ = ABSOLUTE ACCELERATION RESPONSE SPECTRUM

PSV = $\omega_n SD = \frac{2\pi}{T} SD$ = PSEUDOVELOCITY SPECTRUM

PSA = $\omega_n^2 SD = \left(\frac{2\pi}{T}\right)^2 SD$ = PSEUDOACCELERATION SPECTRUM

- 1. Critically damped system ($\zeta = 1$).
- 2. Overdamped system ($\zeta > 1$).
- 3. Underdamped system ($\zeta < 1$)



after Bolt

Some terminology

Hazard: A dangerous phenomenon, substance, human activity or condition that may cause loss of life injury, or other health impacts, property damage, loss of livelihoods and services, social and economic disruption, or environmental damage (UNISDR, 2009).

Exposure: People, property, systems or other elements present in hazard zones that are thereby subject to potential losses (UNISDR, 2009).

Vulnerability: Characteristics and circumstances of a community, system or asset that make it susceptible to the damaging effects of a hazard (UNISDR, 2009).

Risk: The combination of the consequences of an event (hazard) and the associated likelihood/probability of its occurrence (ISO 31010, 2009).



Search...

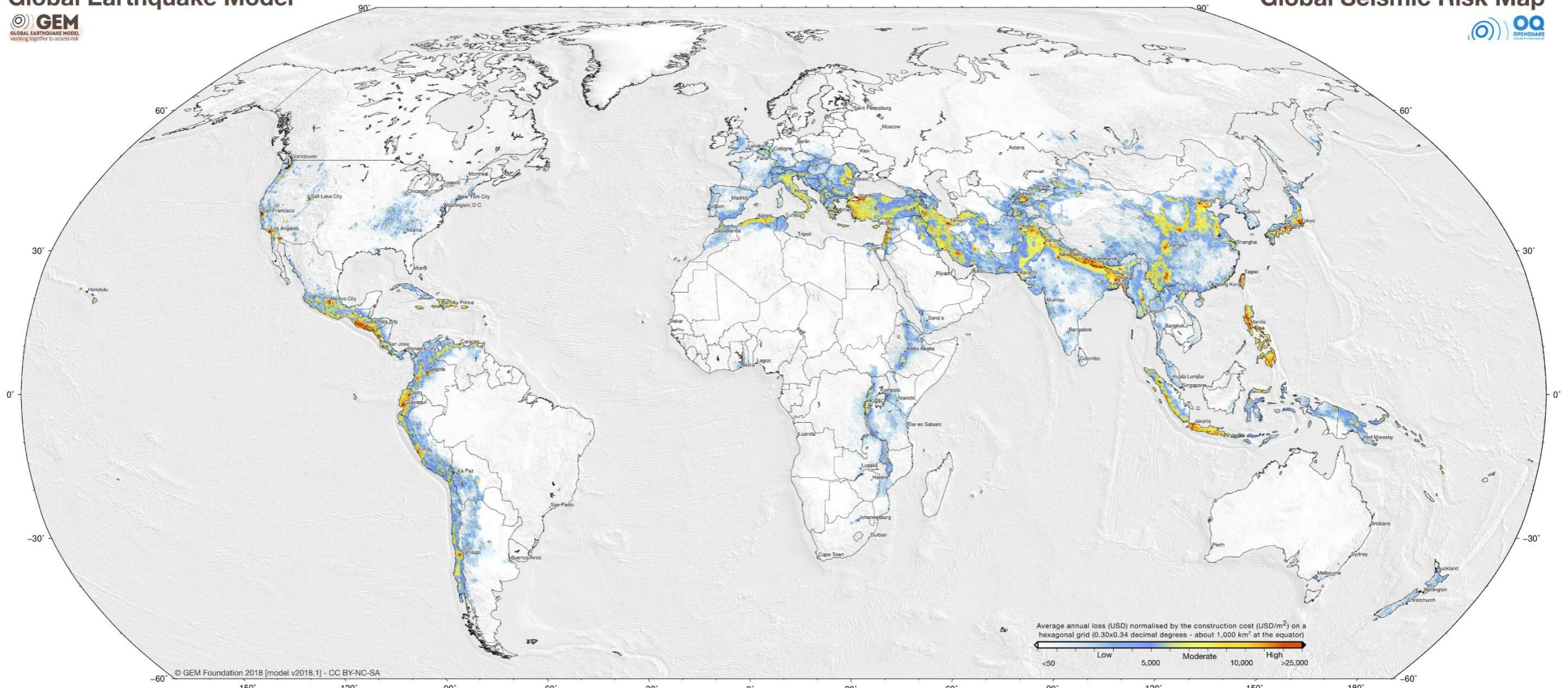


Layers



3000 km
2000 mi

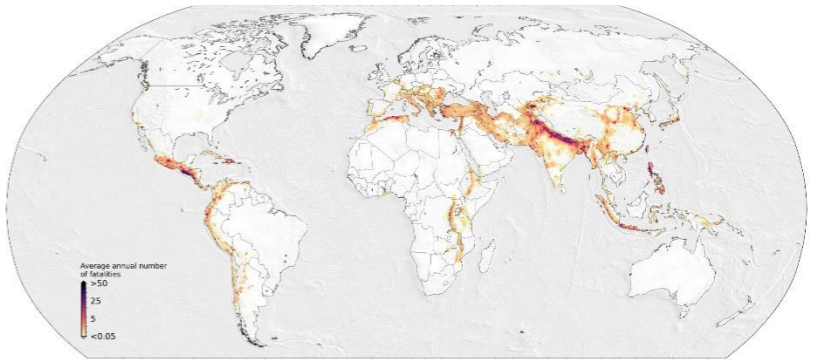
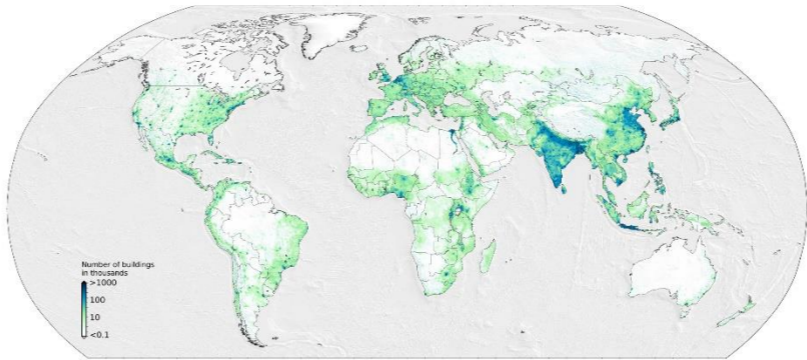
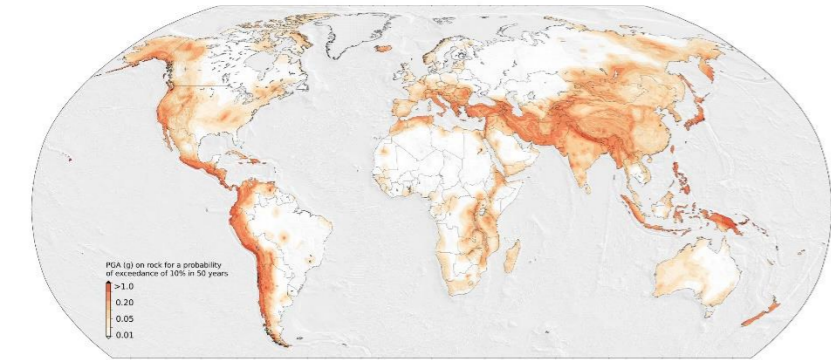
Legend



Global Seismic Hazard Map

Global Exposure Map

Global Seismic Fatalities Map



Global Earthquake Model (GEM) Global Seismic Risk Map
 The Global Seismic Risk Map (v2018.1) comprises four global maps. The main map presents the geographic distribution of average annual loss (USD) normalised by the average construction costs of the respective country (USD/m²) due to ground shaking in the residential, commercial and industrial building stock, considering contents, structural and non-structural components. The normalised metric allows a direct comparison of the risk between countries with widely different construction costs. It does not consider the effects of tsunamis, liquefaction, landslides, and fires following earthquakes. The loss estimates are from direct physical damage to buildings due to shaking, and thus damage to infrastructure or indirect losses due to business interruption are not included. The Global Seismic Hazard Map depicts the geographic distribution of the Peak Ground Acceleration (PGA) with a 10% probability of being exceeded in 50 years, computed for reference rock conditions (shear wave velocity of 760-800 m/s). The Global Exposure Map depicts the geographic distribution of residential, commercial and industrial buildings. The Global Seismic Fatalities Map depicts an estimate of average annual human losses due to earthquake-induced structural collapse of buildings. The results for

human losses do not consider indirect fatalities such as those from post-earthquake epidemics. The average annual losses and number of buildings are presented on a hexagonal grid, with a spacing of 0.30 x 0.34 decimal degrees (approximately 1,000 km² at the equator). The average annual losses were computed using the event-based calculator of the OpenQuake engine, an open-source software for seismic hazard and risk analysis developed by the GEM Foundation. The seismic hazard, exposure and vulnerability models employed in these calculations were provided by national institutions, or developed within the scope of regional programs or bilateral collaborations. These global maps and the underlying databases are based on best available and publicly accessible datasets and models. Due to possible model limitations, regions portrayed with low risk may still experience potentially damaging earthquakes. The GEM Risk Map is intended to be a dynamic product, such that it may be updated when new datasets and models become available. Releases of updated versions of the seismic risk map are anticipated on a regular basis. Additional hazard and risk metrics for each country can be explored at globalquakemodel.org/gem.

The Global Earthquake Model (GEM) Foundation
 The Earthquake Risk Map 2018 is a product of the GEM Foundation, initiated by the Organisation for Economic Co-operation and Development (OECD) Global Science Forum in 2006. GEM was formed in 2009 as a non-profit foundation in Pavia (Italy), funded through a public-private sponsorship with the vision to create a world that is resilient to earthquakes. Participants represent national research or disaster management institutions, the private sector and international organisations. GEM expands the assessment of seismic hazard at the global scale initially started by the Global Seismic Hazard Assessment Program (GSHAP) in support of the UN International Decade of Natural Disaster Reduction in 1999 to the consideration of direct economic and human losses. Observing its core values of collaboration, transparency, openness, credibility, and serving the public good, GEM goes beyond GSHAP by extending the scope of work to the risk domain, providing an institutional framework for continuous updates, and fostering direct applications to risk reduction and prevention projects. GEM's collaborative network comprises more than 70 public and private institutions organised under more than 25 regional, national and multilateral projects.

GEM's OpenQuake platform (platform.openquake.org) provides access to all data, models, tools and software behind the maps. GEM's open-source OpenQuake engine enables probabilistic hazard and risk calculations worldwide and at all scales, from global down to regional, national, local, and site-specific in a single software package. The Sendai Framework for Disaster Risk Reduction (SFDRR) calls for "decision-making on disaster risk reduction to be based on solid and openly accessible scientific work". GEM supports the SFDRR goals by contributing openly accessible products for hazard and risk assessment and capacity development in risk reduction projects. GEM also serves as a baseline or exemplar for the development of a broader multi-hazard framework for risk assessment in support of a holistic and comprehensive approach to disaster risk reduction. Technical details on the development and compilation of the hazard and risk maps, underlying models and the list of contributors can be found at globalquakemodel.org/gem.

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Acknowledgements
 This map is the result of a collaborative effort and extensively relies on the enthusiasm and commitment of various organisations to openly share and collaborate. The creation of this map would not have been possible without the support provided by several public and private organisations during GEM's second working programme (2014-2018). None of this would have been possible without the extensive support of all GEM Secretariat staff. These key contributions are profoundly acknowledged. A complete list of the contributors can be found at globalquakemodel.org/gem.

Legal statements
 This map is an informational product created by the GEM Foundation for public dissemination purposes. The information included in this map must not be used for the design of earthquake-resistant structures or to support any important decisions involving human life, capital and movable and immovable properties. The values of seismic hazard and risk in this map do not constitute an alternative nor do they replace building actions defined in national building codes or earthquake risk estimates derived nationally. Readers seeking this information should contact the national authorities tasked with seismic hazard and risk assessment. The seismic risk map results from an integration process that is solely the responsibility of the GEM Foundation.

Contacts
 GEM (Global Earthquake Model) Foundation
 Via Ferrata, 1 - 27100, Pavia, Italy
info@globalquakemodel.org

Sponsors and major contributors



SHA dualism: P & D

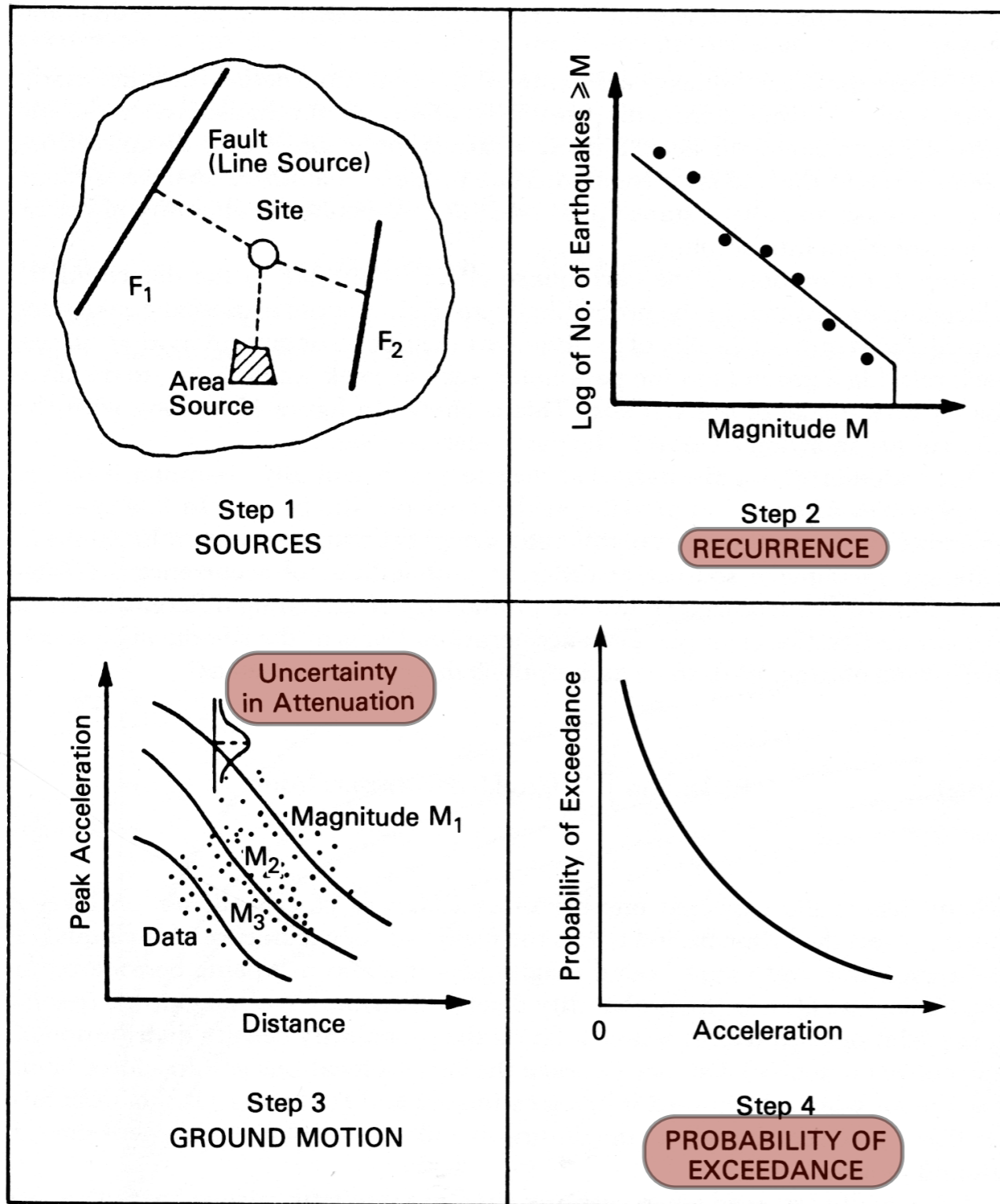


FIGURE 10.2 Basic steps of probabilistic seismic hazard analysis (after TERA Corporation 1978).

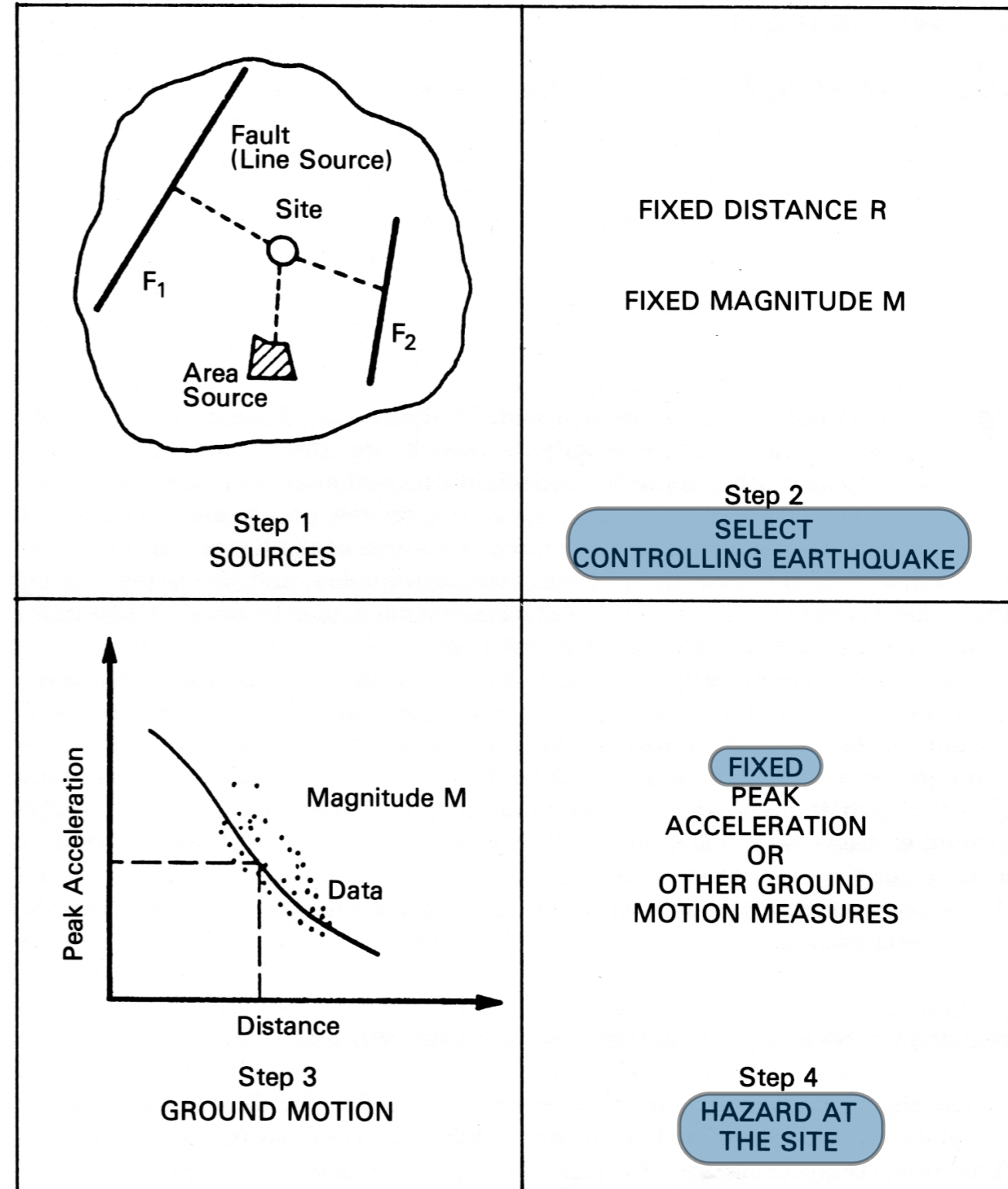
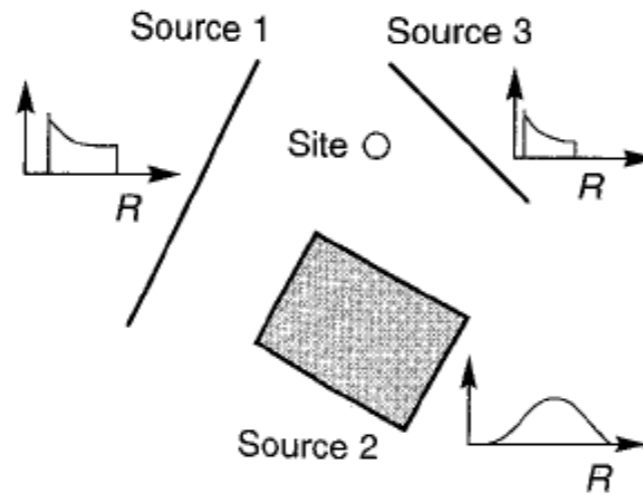


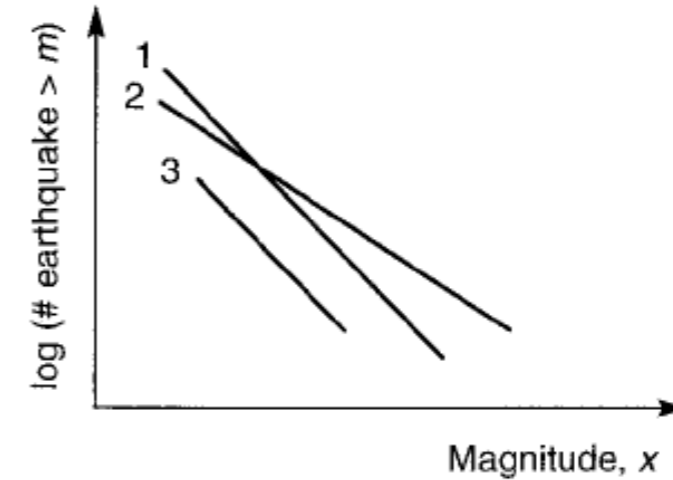
FIGURE 4.1 Basic steps of deterministic seismic hazard analysis (after TERA Corporation 1978).

“Earthquake Hazard Analysis”, Reiter, 1990

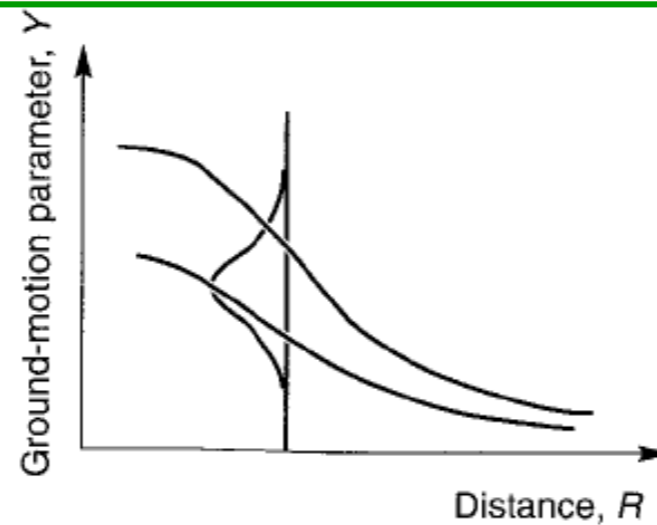
Seismic Zonation and Catalogs



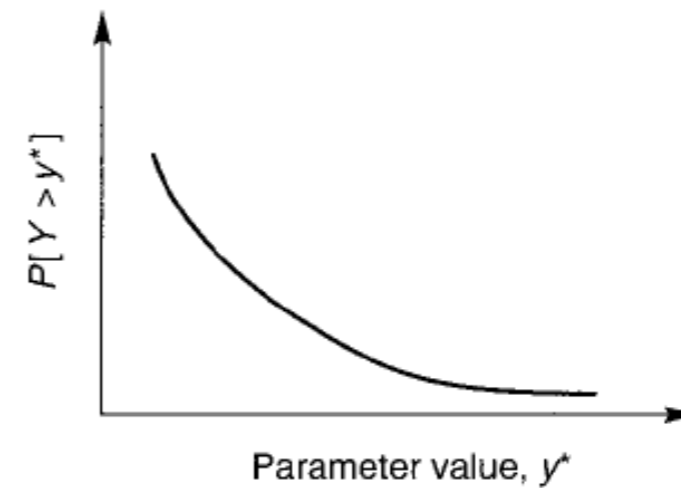
STEP 1



STEP 2



STEP 3



STEP 4

GMPEs and Probability of exceedance

Issues regarding magnitude

The **advantages** of magnitude scales are

They can be determined directly from the seismogram, without the need for sophisticated processing.

The units of order 1 are intuitively attractive.

to 2.9	Minor	Generally not felt but recorded	1000/day
3 to 3.9	Minor	Often felt, but rarely cause damage.	49000/year
4 to 4.9	Light	Noticeable shaking, damage unlikely.	6200/year
5 to 5.9	Moderate	Can cause damage to poor quality buildings	800/year
6 to 6.9	Strong	Destructive in areas up to <i>ca.</i> 160 km.	120/year
7 to 7.9	Major	Serious damage over larger areas.	18/year
8 to 8.9	Great	Serious damage over areas of 100's km.	1/year
9 to 9.9	Great	Serious damage over areas of 1000' s km.	1/20 years

Earthquake distribution

AS noted, the numbers of earthquakes of a given size decreases by about an order of magnitude per magnitude unit increase.

Quantified by the ***Gutenberg-Richter*** relation:

$$\log N = a_1 - bM$$

where N is the number of earthquakes with a magnitude greater than M occurring in a given time (cumulative) or

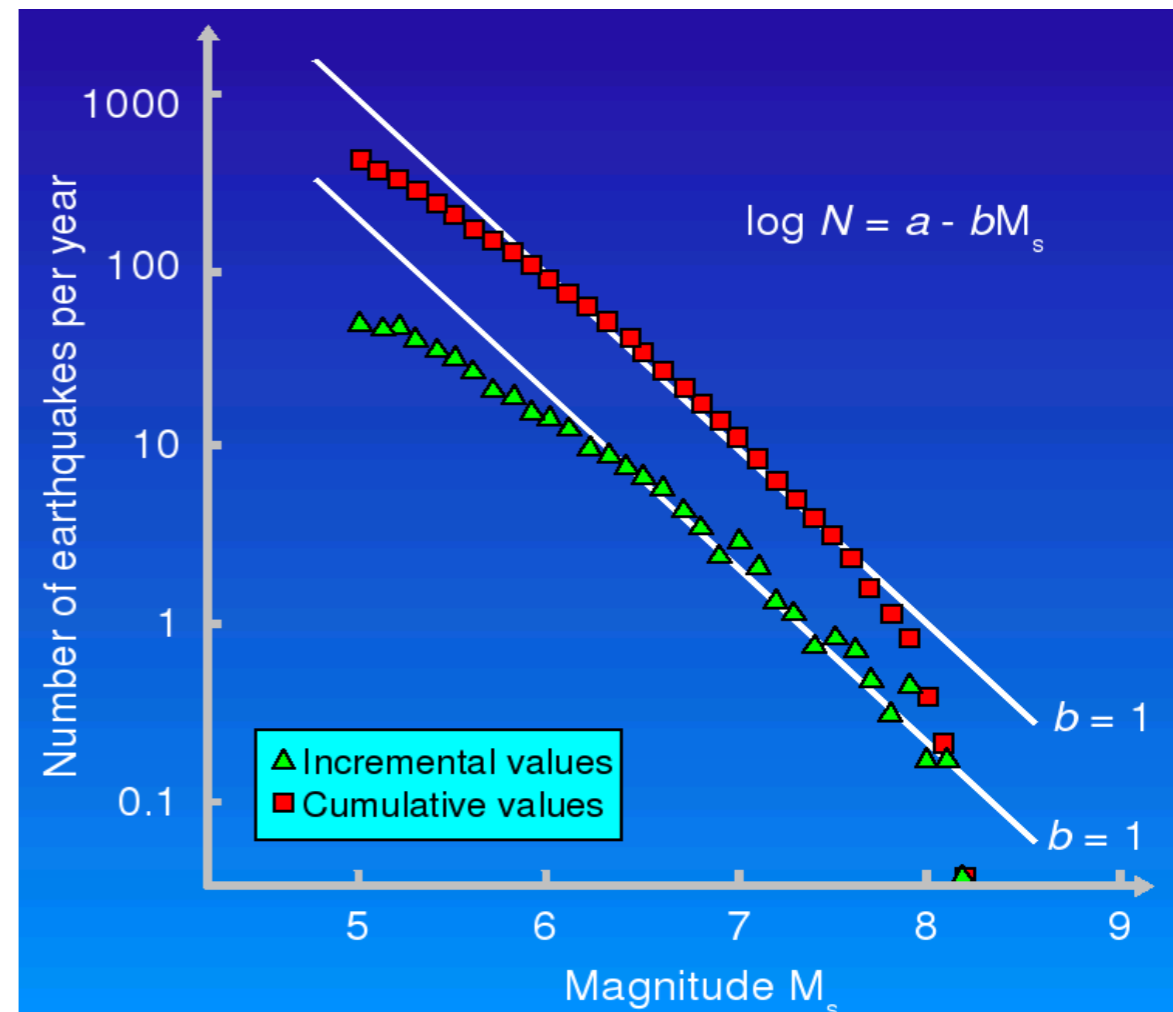
$$\log n = a_2 - bM$$

n is (dN/dM) , and a_1 , a_2 and b are constants for a given

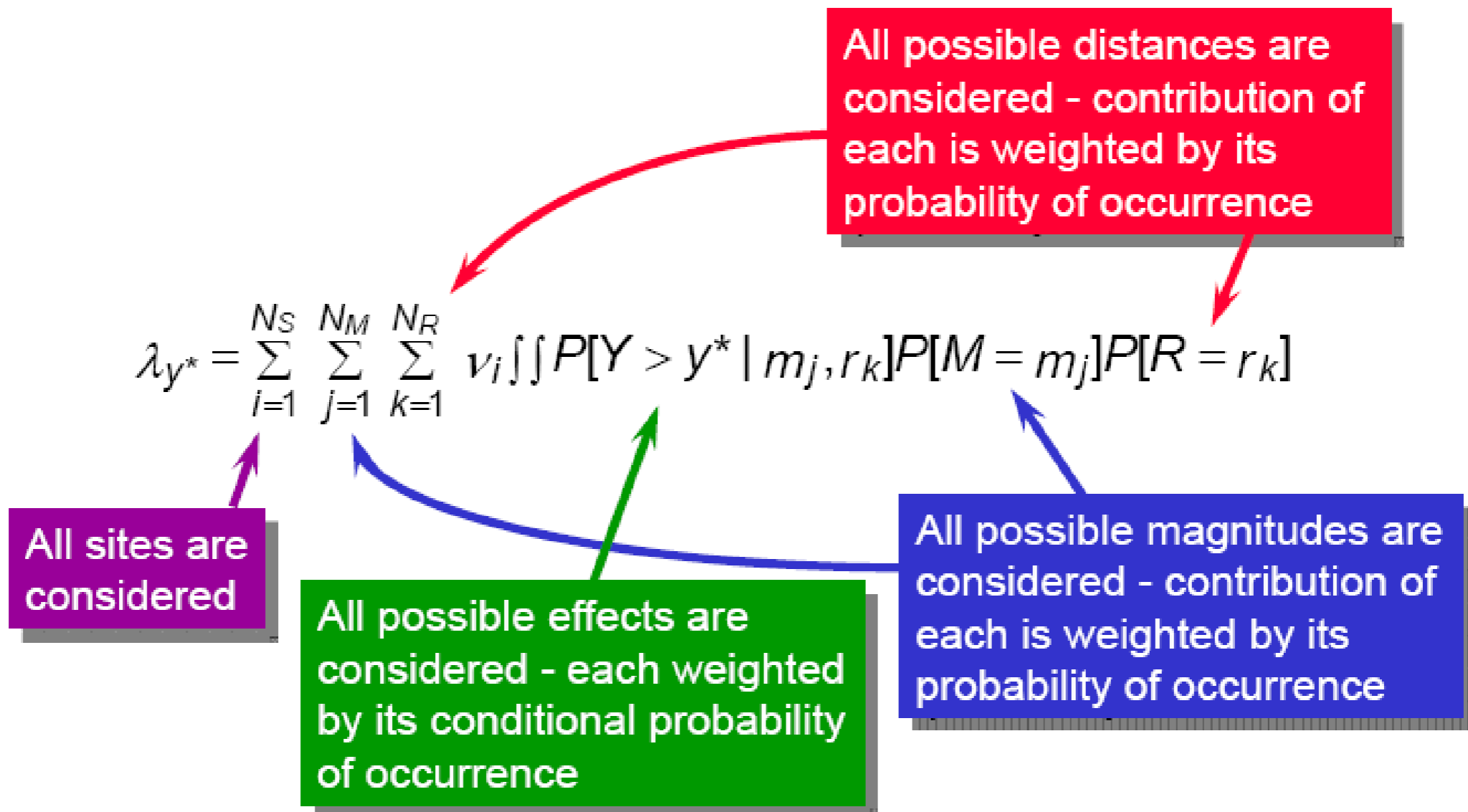
Right: Frequency-magnitude for all earthquakes $M > 5$ between 1968-97.

The value of b is generally about 1, even for individual seismic areas.

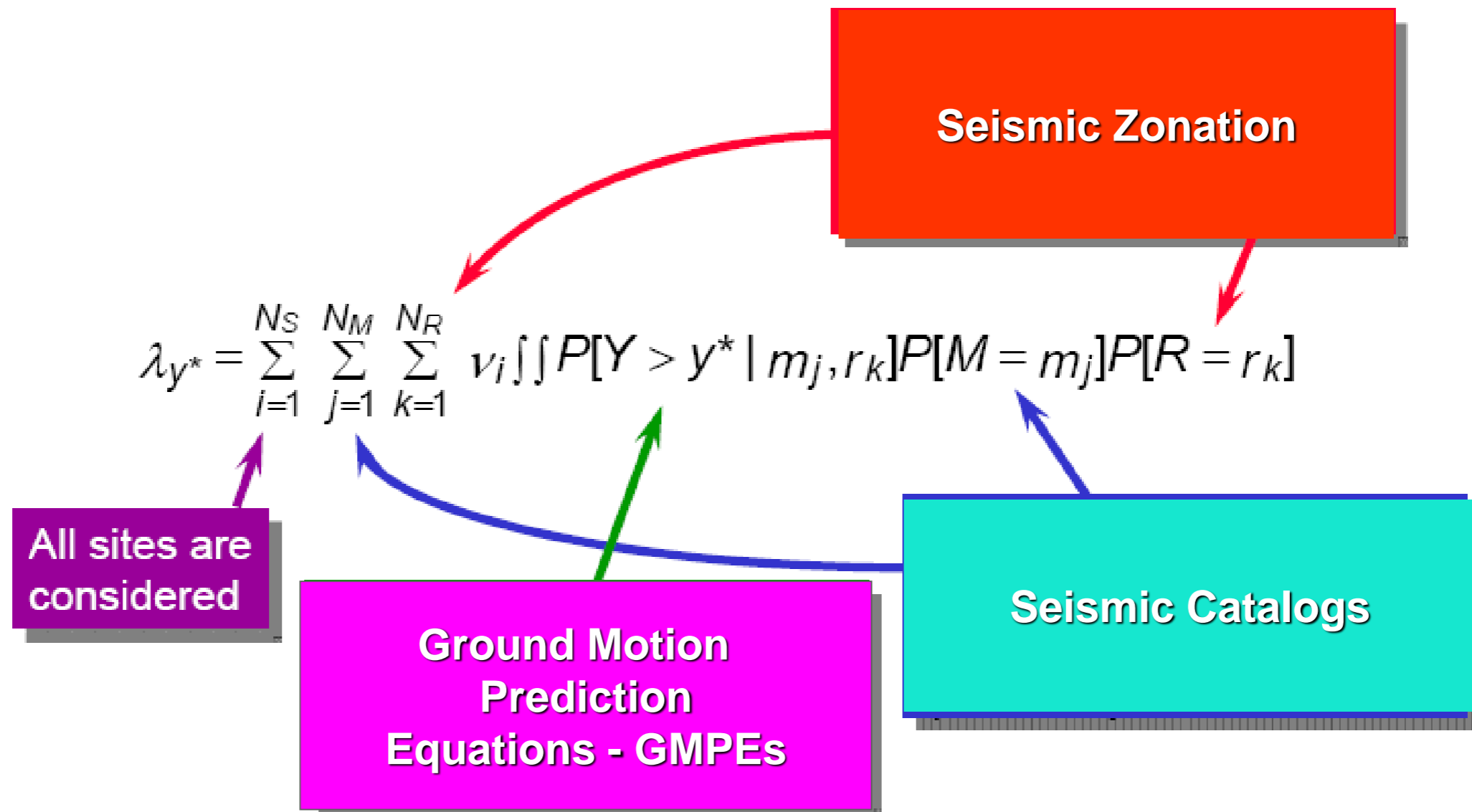
(*Stein & Wyssession, 2003*)



Probabilistic Seismic Hazard Analysis



Probabilistic Seismic Hazard Analysis



Random events

Sample Space

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E_1 = \{1, 3, 5\} \quad E_2 = \{4, 5, 6\}$$

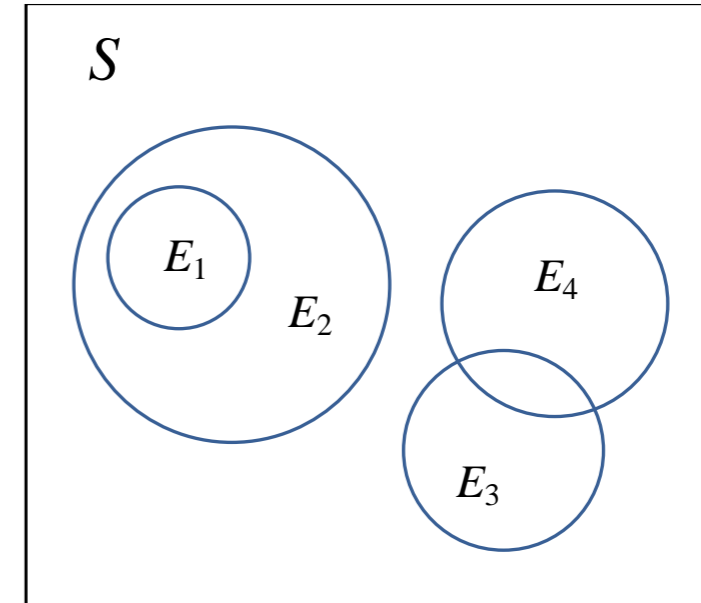
Union

$$E_1 \cup E_2 = \{1, 3, 4, 5, 6\}$$

Intersection

$$E_1 \cap E_2 = \{5\}$$

Or $E_1 E_2$



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1. Events E_1 and E_2 are *mutually exclusive* when they have no common outcomes (i.e., $E_1 E_2 = \phi$, where ϕ is the *null event*).
2. Events E_1, E_2, \dots, E_n are *collectively exhaustive* when their union contains every possible outcome of the random event (i.e., $E_1 \cup E_2 \cup \dots \cup E_n = S$).
3. The *complementary event*, $\overline{E_1}$, of an event E_1 , contains all outcomes in the sample space that are not in event E_1 . By this definition, $\overline{E_1} \cup E_1 = S$ and $\overline{E_1} E_1 = \phi$. That is, $\overline{E_1}$ and E_1 are mutually exclusive and collectively exhaustive.

Random events

We will be interested in the probabilities of occurrence of various events. These probabilities must follow three axioms of probability:

$$0 \leq P(E) \leq 1, \quad (\text{A.1})$$

$$P(S) = 1, \quad (\text{A.2})$$

and, for mutually exclusive events E_1 and E_2 ,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2). \quad (\text{A.3})$$

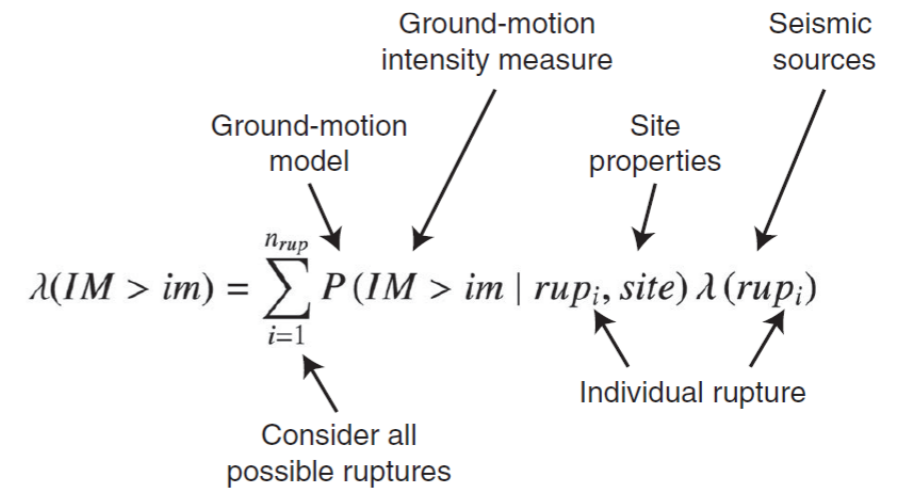
$$P(\bar{E}) = 1 - P(E) \quad (\text{A.4})$$

$$P(\phi) = 0 \quad (\text{A.5})$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 E_2). \quad (\text{A.6})$$

Conditional Probability

$$P(E_1|E_2) = \begin{cases} \frac{P(E_1 E_2)}{P(E_2)} & \text{if } P(E_2) > 0 \\ 0 & \text{if } P(E_2) = 0. \end{cases}$$



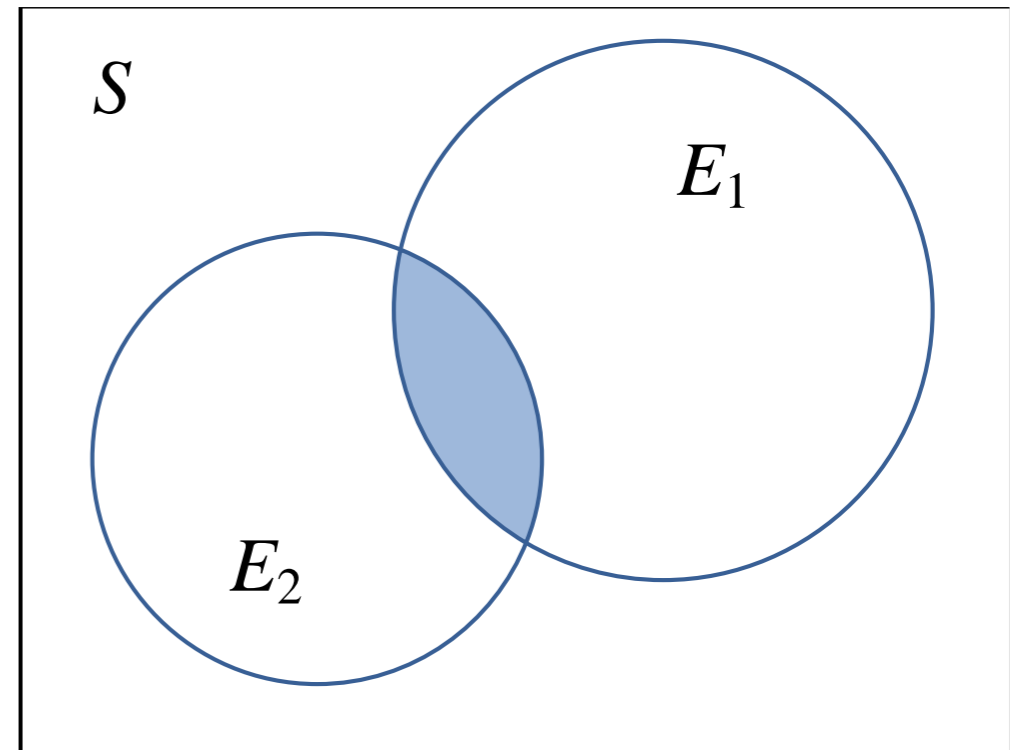
for the nontrivial case of $P(E_2) > 0$, gives

$$P(E_1 E_2) = P(E_1|E_2)P(E_2).$$

Independence

$$P(E_1|E_2) = P(E_1). \quad (\text{A.9})$$

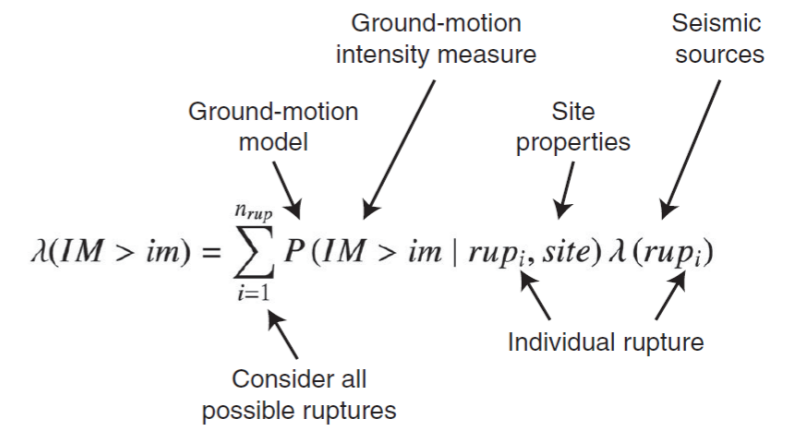
$$P(E_1 E_2) = P(E_1)P(E_2),$$



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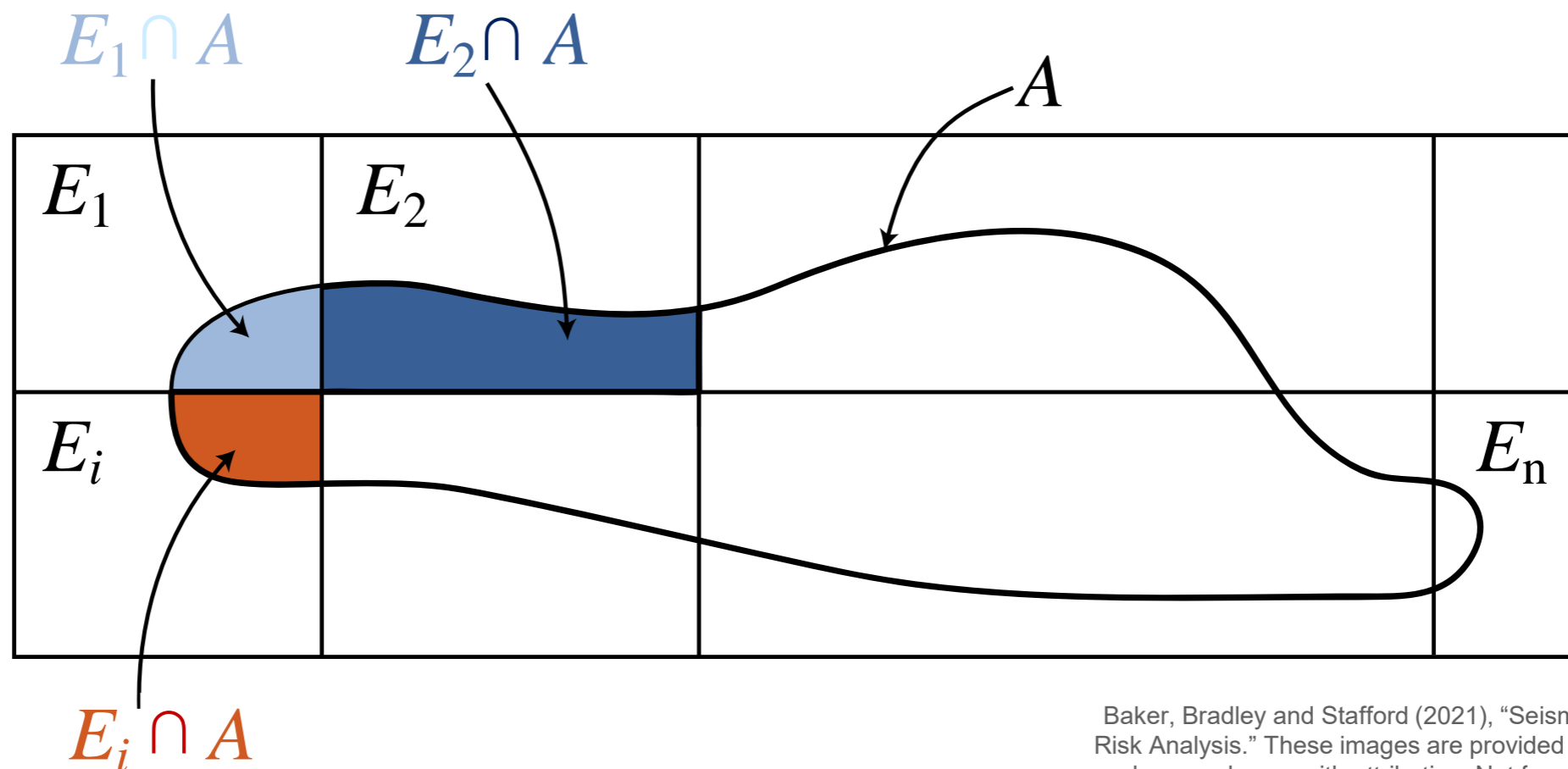
Conditional Probability

Total Probability Theorem



Consider an event A and a set of mutually exclusive and collectively exhaustive events E_1, E_2, \dots, E_n . The Total Probability Theorem states that

$$P(A) = \sum_{i=1}^n P(A|E_i)P(E_i). \quad (\text{A.11})$$



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Conditional Probability

Bayes' Rule

$$P(E_1|E_2) = P(E_1). \quad (\text{A.9})$$

Consider an event A and a set of mutually exclusive and collectively exhaustive events E_1, E_2, \dots, E_n . Using Equation A.9, we can write

$$P(AE_j) = P(E_j|A)P(A) = P(A|E_j)P(E_j). \quad (\text{A.12})$$

Rearranging the last two terms gives

$$P(E_j|A) = \frac{P(A|E_j)P(E_j)}{P(A)}. \quad (\text{A.13})$$

Random Variables

A **random variable** is a numerical variable whose specific value cannot be predicted with certainty before the occurrence of an event

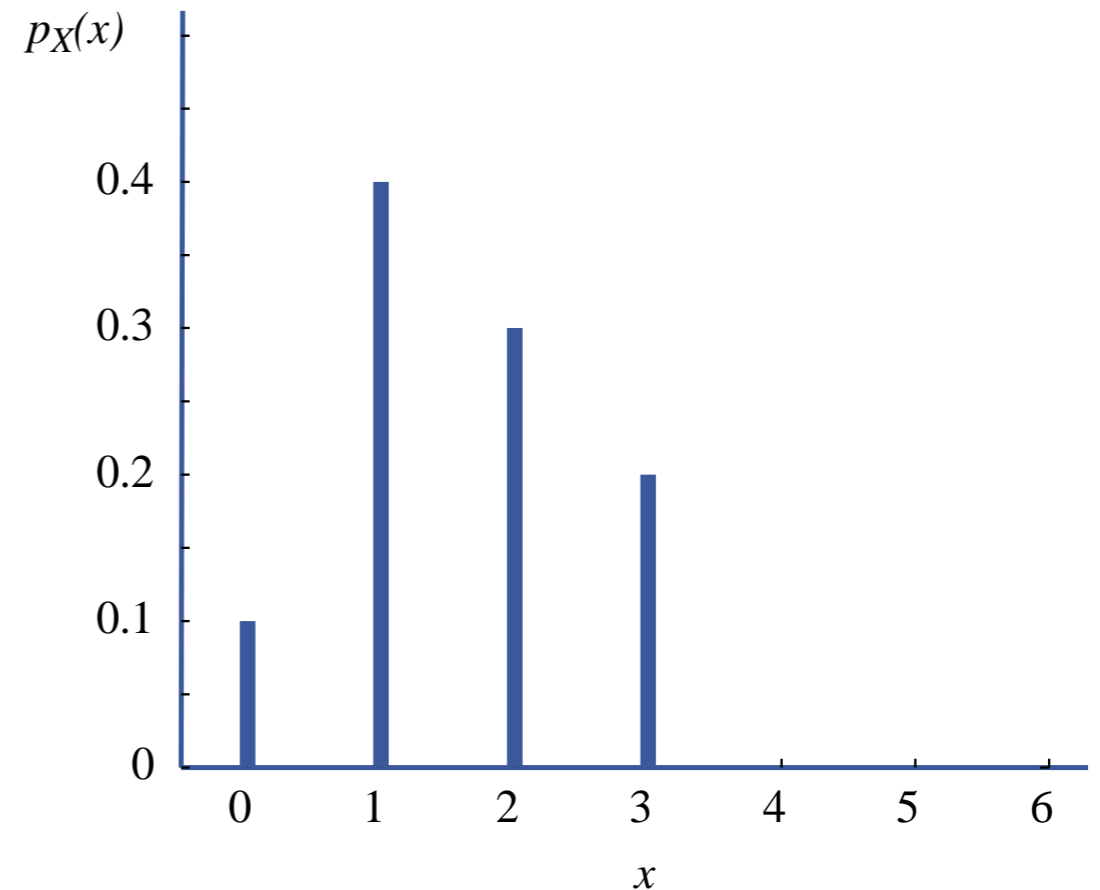
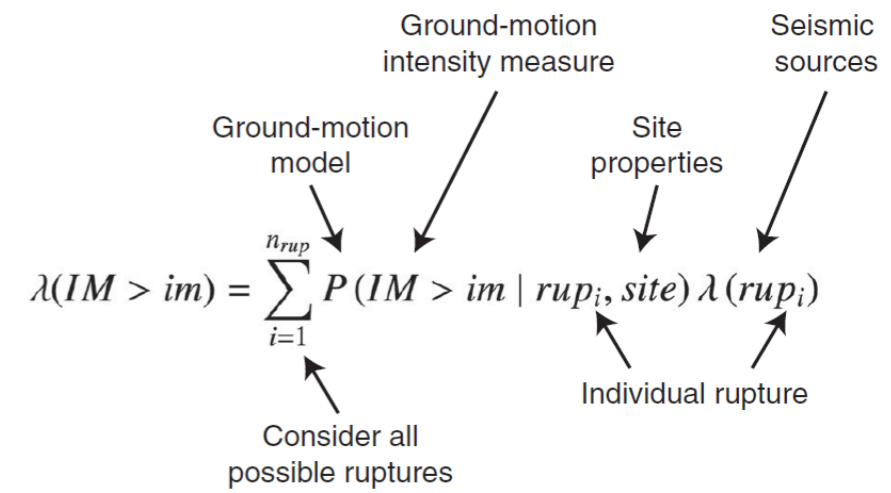
x_1, x_2, x_3 ...denote possible outcome of X

$P(X=x_1)$ is the probability of X of assuming the value x_1

Random variable can be **discrete** (e.g. number of earthquakes occurring in a region in a certain amount of time) or **continuous**

The probability distribution of a discrete random variable is quantified by the **probability mass function (PMF)**:

$$p_X(x) = P(X = x).$$



(a)

Random Variables

The **cumulative distribution function (CDF)** is defined as the probability of the event that the random variable takes a value less than or equal to the value of the argument:

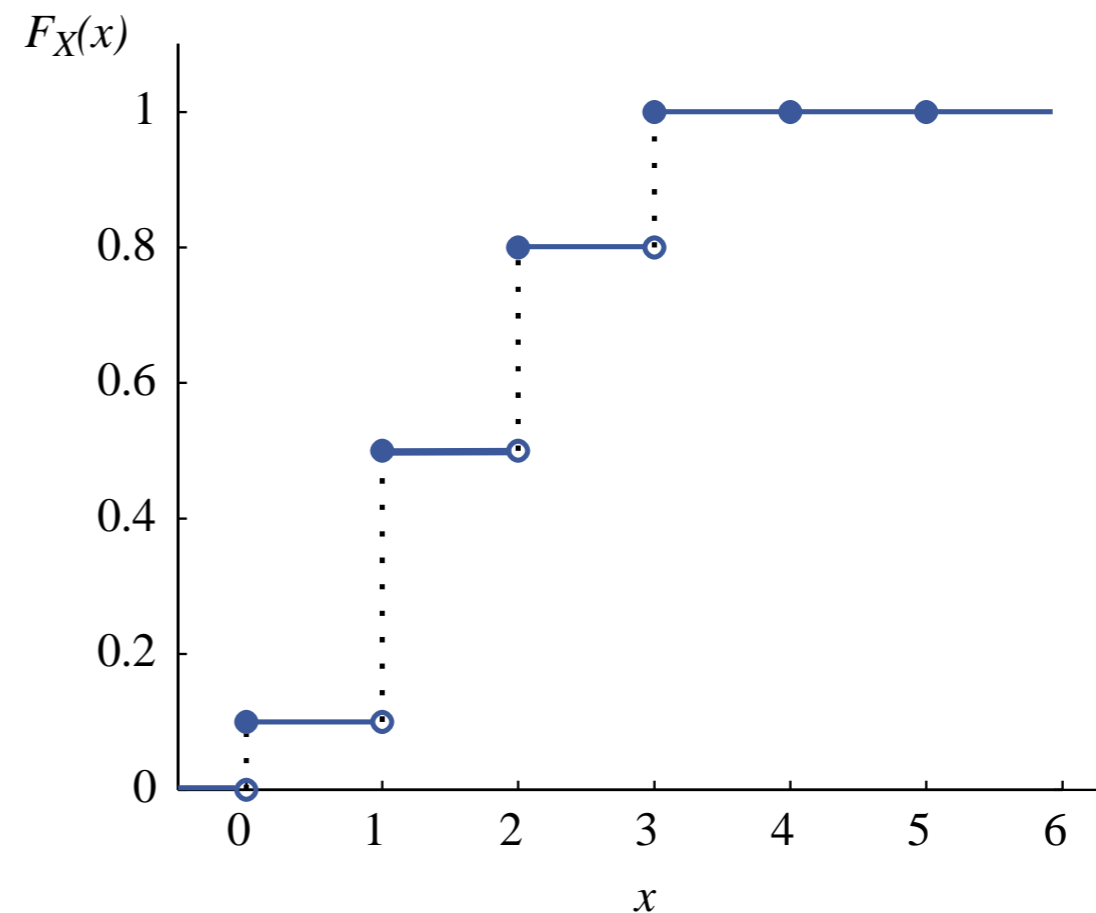
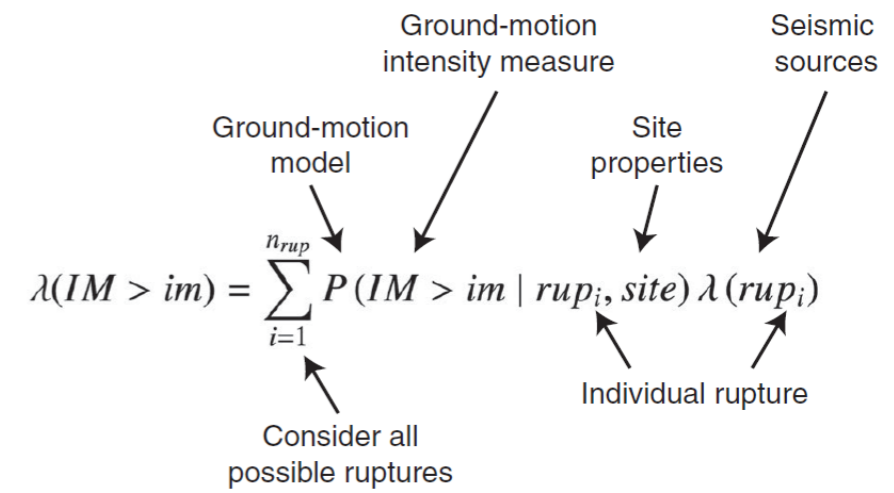
$$F_X(x) = P(X \leq x).$$

PMF and **CDF** are related by:

$$F_X(a) = \sum_{\text{all } x_i \leq a} p_X(x_i).$$

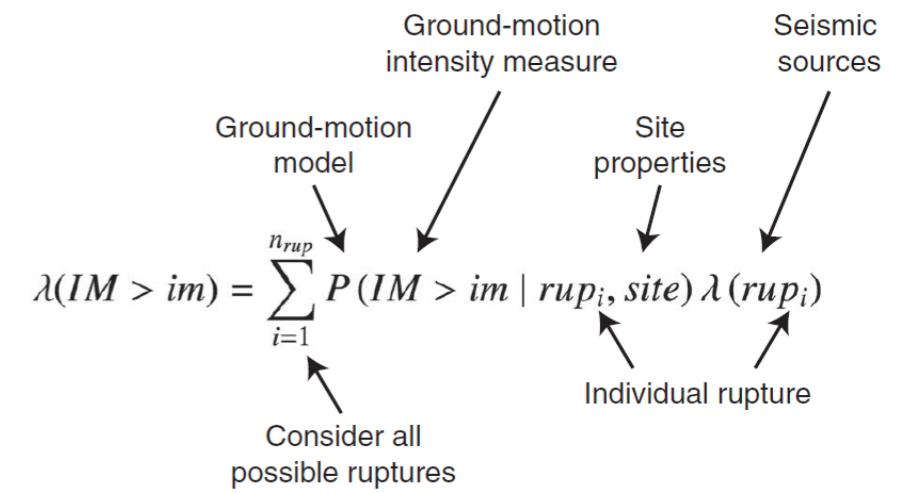
In many cases (see equation on the top right) we are interested in the probability of $X \geq x$:

$$P(X > x) = 1 - P(X \leq x)$$



(b)

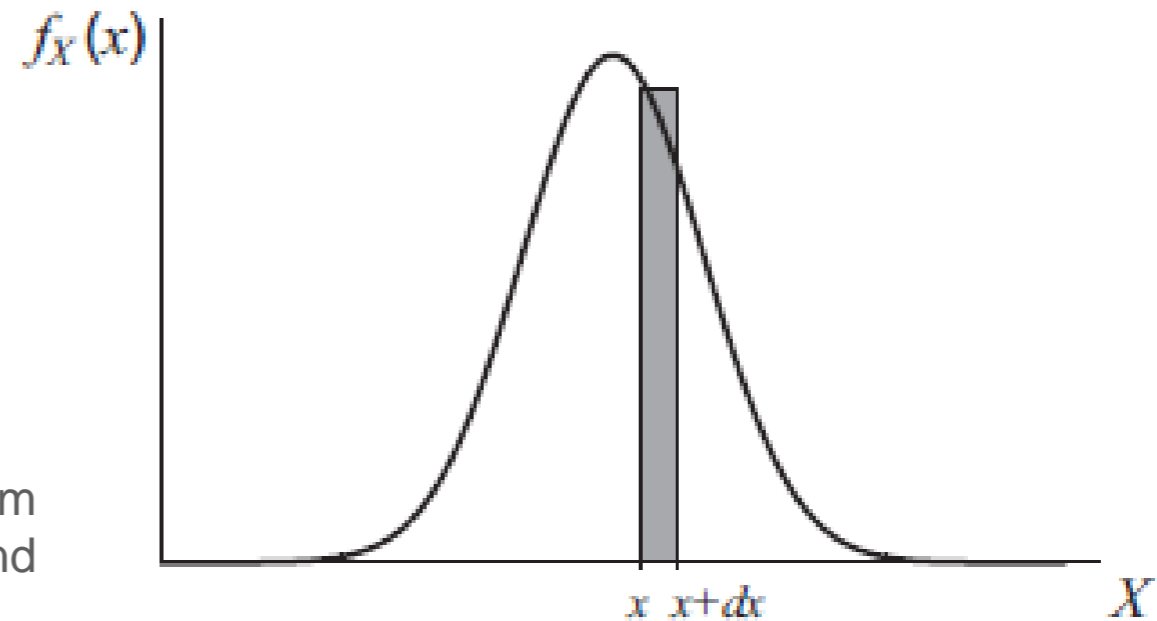
Random Variables



In the case of a **continuous** variable the **Probability Density Function (PDF)** is defined:

$$f_X(x) dx = P(x < X \leq x + dx)$$

$f_X(x) dx$ represents the probability of the random variable X taking values between x and $x+dx$



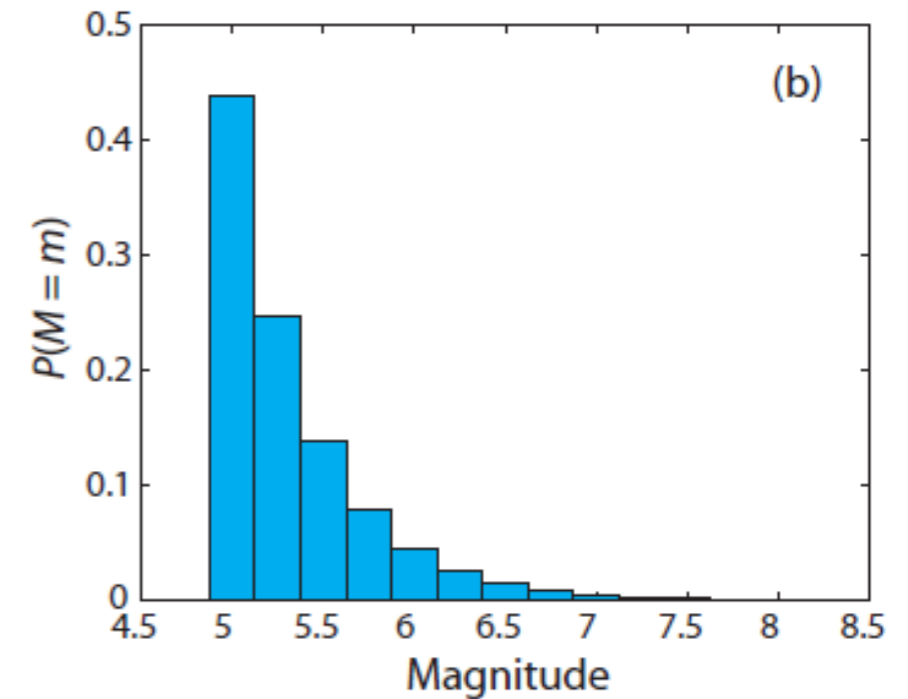
Random Variables

Probability that the outcome of X is in the interval between a and b

$$P(a < X \leq b) = \int_a^b f_X(x) dx.$$

For discrete random variables

$$p_{\tilde{X}}(x) = f_X(x) \Delta x = P(x < X \leq x + \Delta x)$$



Relation between PDF and CDF

CDF $F_X(x) = P(X \leq x).$

CDF $F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(u) du$

PDF $f_X(x) = \frac{d}{dx} F_X(x).$

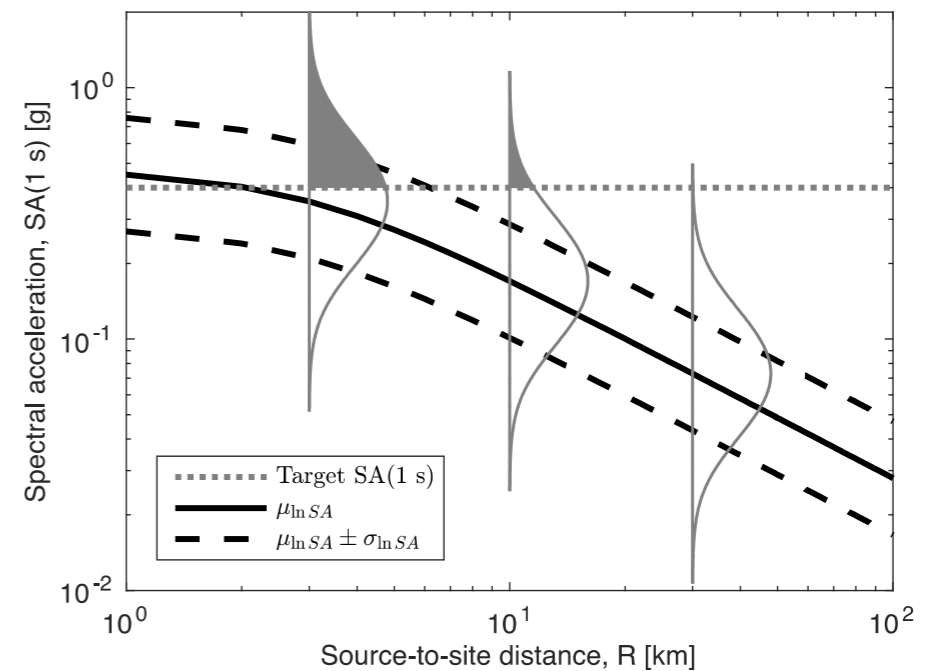
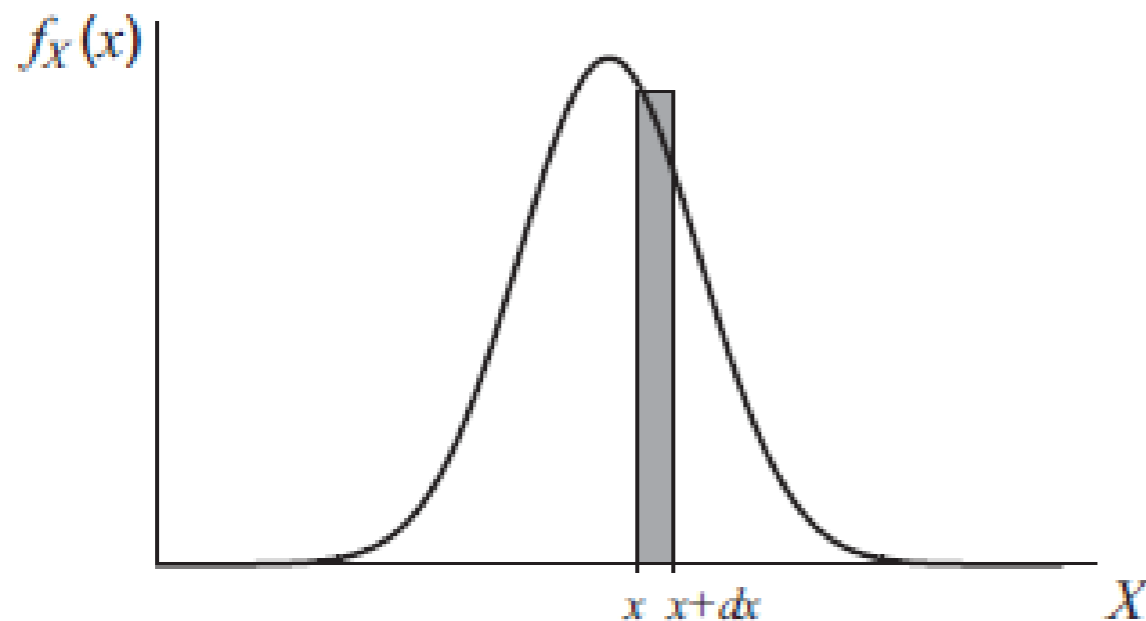
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Common Probability Distributions

Normal Distribution

PDF
$$f_X(x) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{x - \mu_X}{\sigma_X}\right)^2\right) \quad -\infty \leq x \leq \infty$$

where μ_X and σ_X denote the mean value and standard deviation, respectively, of X .



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Common Probability Distributions

Normal Distribution

A normal random variable, X , can be transformed into a standard normal random variable as

$$U = \frac{X - \mu_X}{\sigma_X} \quad (\text{A.51})$$

where U is a standard normal random variable.

The CDF for general normal random variable can be written as:

$$P(X \leq x) = \Phi \left(\frac{X - \mu_X}{\sigma_X} \right)$$

Common Probability Distributions

Bivariate Normal Distribution

Normal distribution
of 2 random
variables

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho_{X,Y}^2}} \exp\left\{-\frac{z}{2(1-\rho_{X,Y}^2)}\right\} \quad -\infty \leq x, y \leq \infty$$

where $\rho_{X,Y}$ is the correlation coefficient between X and Y , and

$$z = \frac{(x - \mu_X)^2}{\sigma_X^2} - \frac{2\rho_{X,Y}(x - \mu_X)(y - \mu_Y)}{\sigma_X\sigma_Y} + \frac{(y - \mu_Y)^2}{\sigma_Y^2}.$$

A useful property of random variables having this distribution is that if X and Y are jointly normal, then their marginal distributions ($f_X(x)$ and $f_Y(y)$) are normal, and their conditional distributions are also normal. Specifically, the distribution of X given $Y = y$ has conditional mean

$$\mu_{X|Y=y} = \mu_X + \rho_{X,Y} \sigma_X \left(\frac{y - \mu_Y}{\sigma_Y}\right) \quad (\text{A.55})$$

and conditional standard deviation

$$\sigma_{X|Y=y} = \sigma_X \sqrt{1 - \rho_{X,Y}^2}. \quad (\text{A.56})$$

These properties are convenient when computing joint distributions of ground-motion parameters.

Common Probability Distributions

Lognormal Distribution

A random variable Y has a lognormal distribution if its logarithm, $X = \ln Y$ has a normal distribution

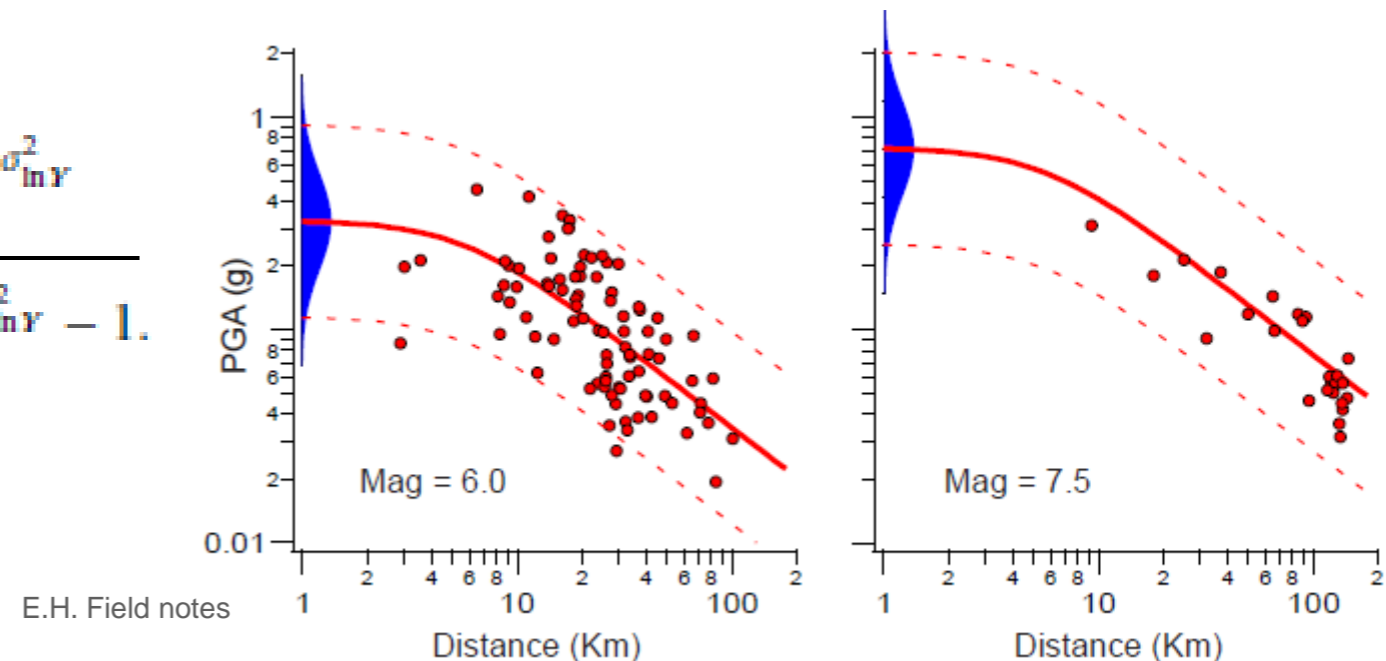
Relation to the mean and standard deviation of Y

$$f_Y(y) = \frac{1}{y\sigma_{\ln Y}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln y - \mu_{\ln Y}}{\sigma_{\ln Y}}\right)^2\right) \quad 0 \leq y \leq \infty$$

$$F_Y(y) = \Phi\left(\frac{\ln y - \mu_{\ln Y}}{\sigma_{\ln Y}}\right) \quad 0 \leq y \leq \infty$$

$$\mu_Y = e^{\mu_{\ln Y} + \frac{1}{2}\sigma_{\ln Y}^2}$$

$$\sigma_Y = \mu_Y \sqrt{e^{\sigma_{\ln Y}^2} - 1}$$



The relationship between the median of Y , y_{50} , and $\mu_{\ln Y}$ can be determined by setting the CDF of Equation A.69 equal to 0.5 when y equals the median, y_{50} :

$$0.5 = \Phi\left(\frac{\ln y_{50} - \mu_{\ln Y}}{\sigma_{\ln Y}}\right) \rightarrow y_{50} = e^{\mu_{\ln Y}} \quad (\text{A.72})$$

The equivalence of $\ln y_{50}$ and $\mu_{\ln Y}$ can be stated in words as “the log of the median is equal to the logarithmic mean.”

Common Probability Distributions

The poisson process

A *Poisson process* is a sequence of discrete events having the following properties:

1. **Stationarity:** the probability of an event in a short interval from time t to $t + h$ is approximately λh , for any t .
2. **Nonmultiplicity:** the probability of two or more events in a short time interval is negligible compared with λh .
3. **Independence:** the number of events in any interval of time is independent of the number of events in any other (nonoverlapping) interval of time.

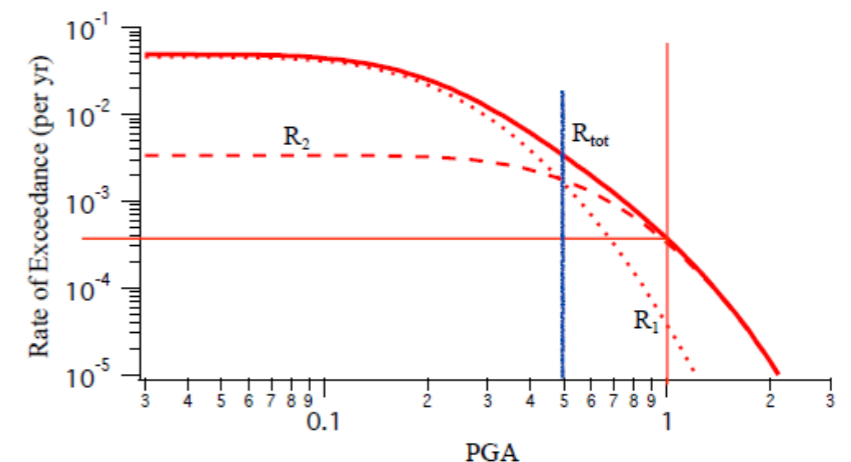
The number of events observed in time t from a poisson process has a Poisson distribution.

X is the number of success in time t
The process has a mean rate of events λ

Poisson PMF
$$p_X(x) = \frac{(\lambda t)^x}{x!} \exp(-\lambda t), \quad x = 0, 1, 2, \dots$$

Mean
$$\mu_X = \lambda t$$

Standard deviation
$$\sigma_X = \sqrt{\lambda t}.$$



E.H. Field notes

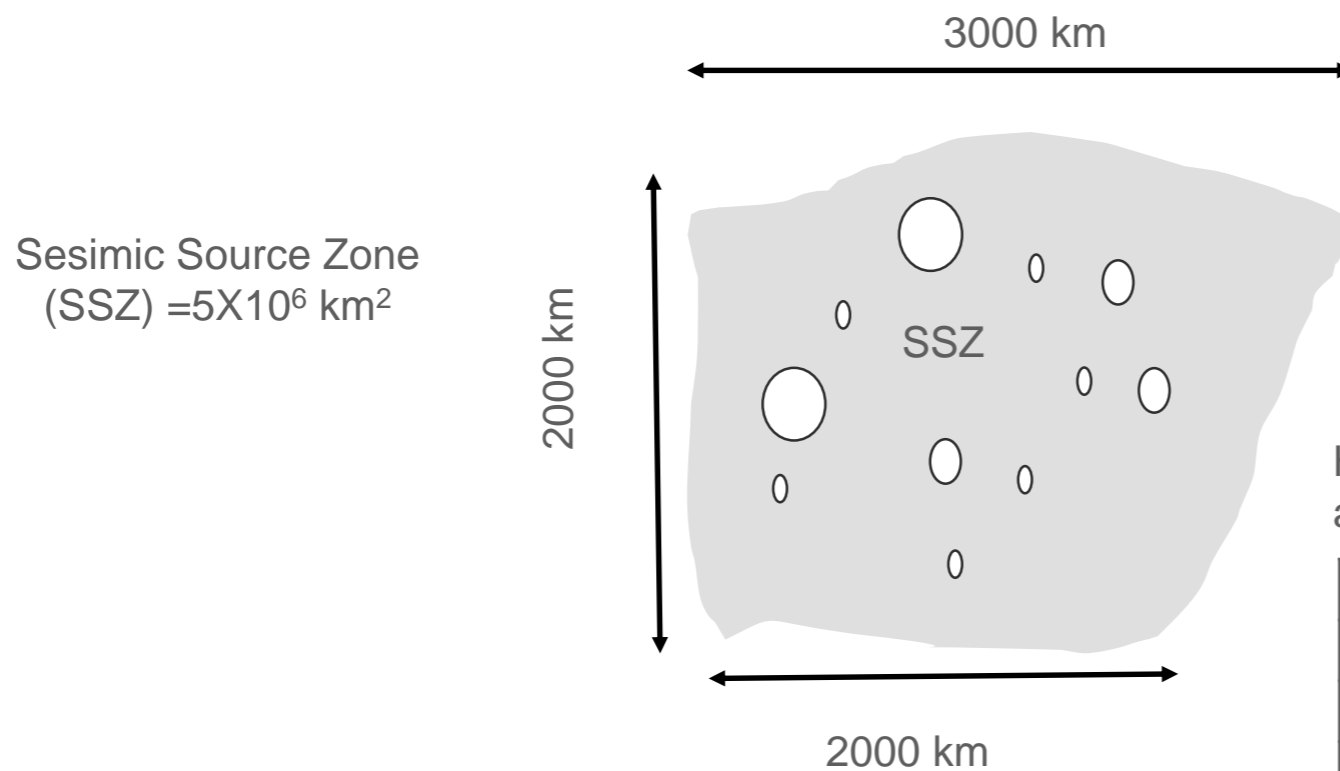
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Poissonian probability of exceeding each ground motion level in the next T years from the annual rate

Basic of Probabilistic Seismic Hazard Assessment (1)

Hazard is the mean rate of exceedence of a certain ground motion measure (PGA, SA, PGV) etc UNIT is (years)⁻¹

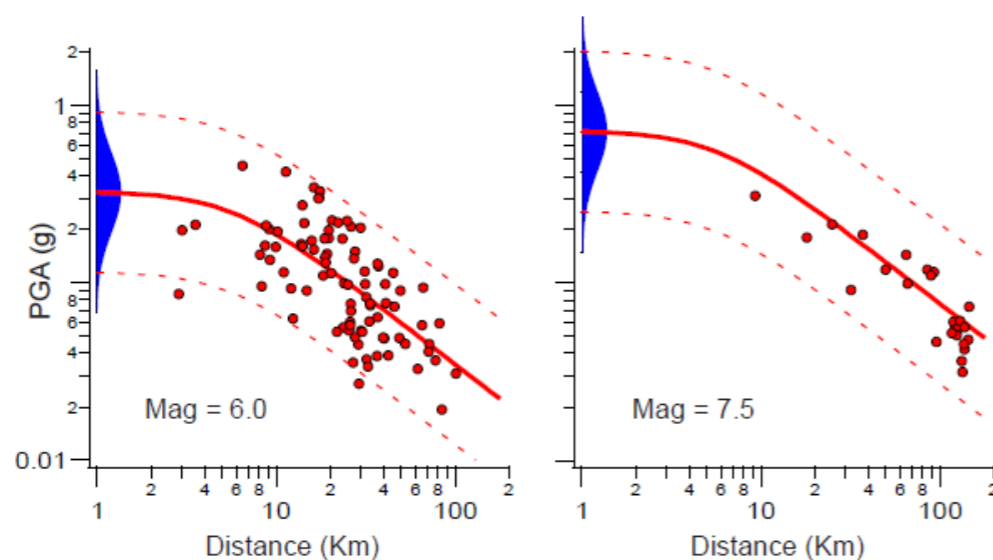
Risk is the mean annual loss (dollars, properties, lives) UNITS dollars/years lives/years



One M=5 per year
 One M=6 per decade
 One M=7 per century

Horizontal distance (km) within which the given pga's are achieved or exceeded for the given magnitudes

	M=5	M=6	M=7
0.1 g	14	25	41
0.2 g	3.2	12	22
0.4 g	0	0	10



Mean rate of exceedance (MROES) x 10⁻⁴ per year, for given pga's for the given magnitudes

	M=5	M=6	M=7	Σ	Σ^σ
0.1 g	1.23	0.39	0.11	1.73	1.47
0.2 g	0.06	0.09	0.03	0.18	0.41
0.4 g	0	0	0.006	0.006	0.034

Basic of Probabilistic Seismic Hazard Assessment (1)

Example for MROE

M=5

Pga=0.1

The PGA will be greater than or equal to the given value of PGA within each distance R

That is where exceedance comes!

Likelihood that the place of interest will be affected by the level of pga or higher

$$\text{MROE} = \left(\frac{\pi 14^2}{5 \times 10^6} \right) \text{ km}^2 / \text{km}^2 \times 1/\text{year} = 1.23 \times 10^{-4}$$

Occurrence rate of each Magnitude

One M=5 per year

One M=6 per decade

One M=7 per century

Horizontal distance R (km) within which the given pga's are achieved or exceeded for the given magnitudes

	M=5	M=6	M=7
0.1 g	14	25	41
0.2 g	3.2	12	22
0.4 g	0	0	10

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0.1 g	1.23	0.39	0.11	1.73	1.47
0.2 g	0.06	0.09	0.03	0.18	0.41
0.4 g	0	0	0.006	0.006	0.034

Numerical integration with ΔM=1

Basic of Probabilistic Seismic Hazard Assessment (1)

Example for MROE
M=5
Pga=0.1

The mean rate in the order of 10^{-4} /year does not mean that we need data for 10.000 year.

The small value is not due to the seismicity rate but to the ratio of the area!

The earthquakes are occurring at the rate of 1/year for M=5 and 10^{-2} year for M=7

One M=5 per year
 One M=6 per decade
 One M=7 per century

Horizontal distance R (km) within which the given pga's are achieved or exceeded for the given magnitudes

	M=5	M=6	M=7
0.1 g	14	25	41
0.2 g	3.2	12	22
0.4 g	0	0	10

Mean rate of exceedance (MROES) x 10^{-4} per year, for given pga's for the given magnitudes

	M=5	M=6	M=7	Σ	Σ^σ
0.1 g	1.23	0.39	0.11	1.73	1.47
0.2 g	0.06	0.09	0.03	0.18	0.41
0.4 g	0	0	0.006	0.006	0.034

Numerical integration with $\Delta M=1$

Basic of Probabilistic Seismic Hazard Assessment (2)

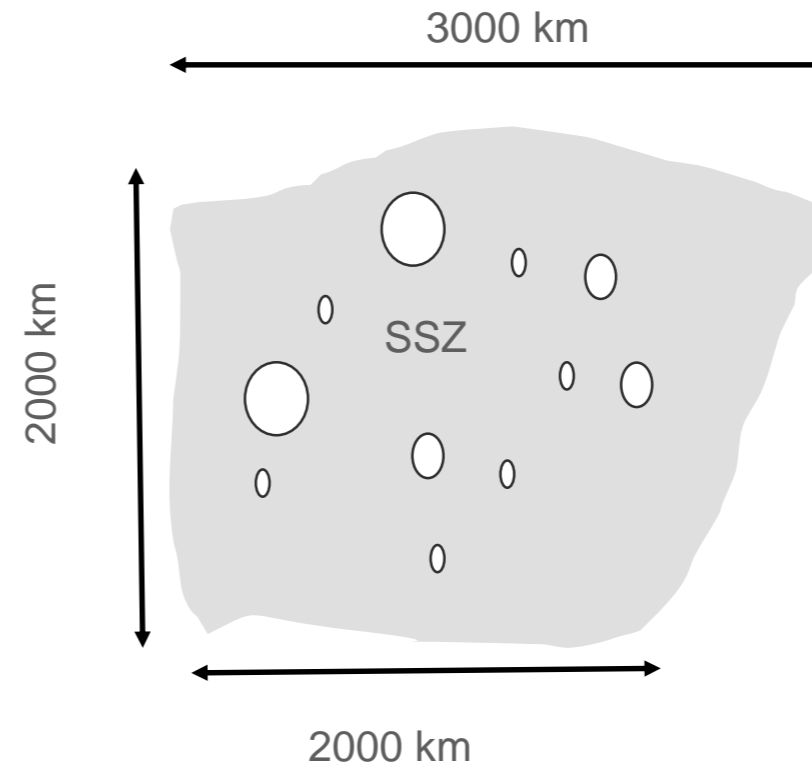
Consider that the source model is made by N earthquake scenarios E_n , each one with its magnitude (m_n), location (L_n) and rate (r_n)

$$E_n = E(m_n, L_n, r_n).$$

r_n represents the annual rate of the earthquake scenario

The probability of the scenario over some specified time period should be given; this would allow the implementation of time-dependent models.

Time dependent models are usually implemented by converting the conditional probability into an equivalent Poissonian time-dependent rate



Example

An average repeat time of an earthquake on a fault is 147 years $\rightarrow r=0.007$ events per year

The Poissonian probability of having more than one event over T years is:

$$P_{\text{pois}} = 1 - \exp(-rT)$$

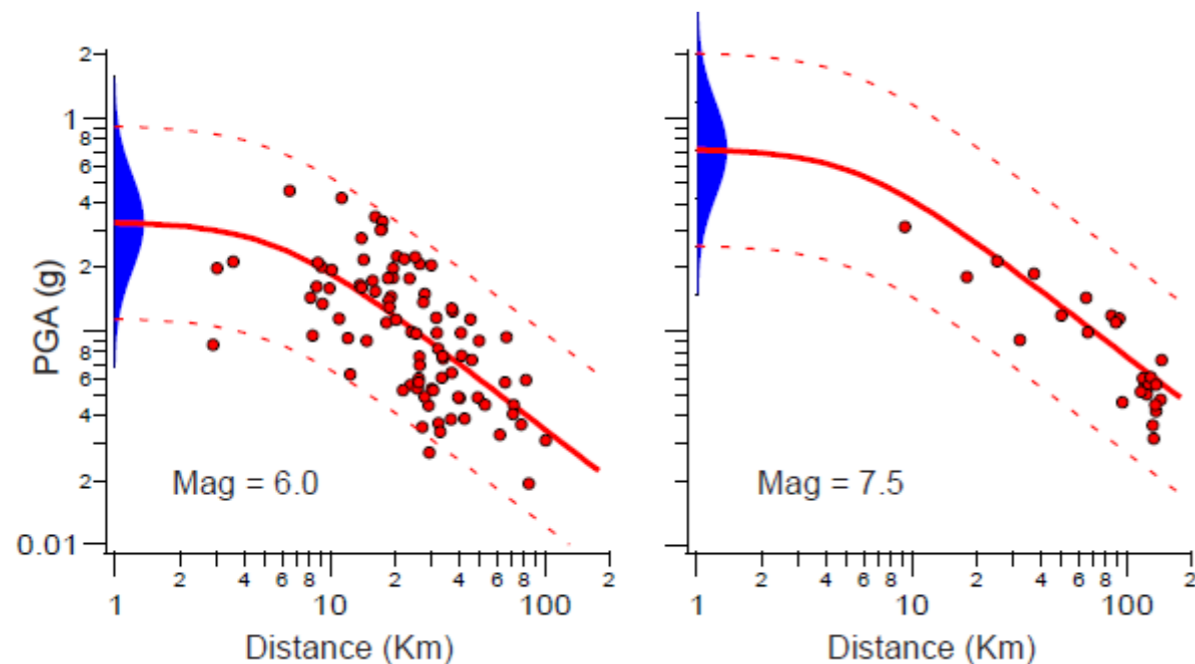
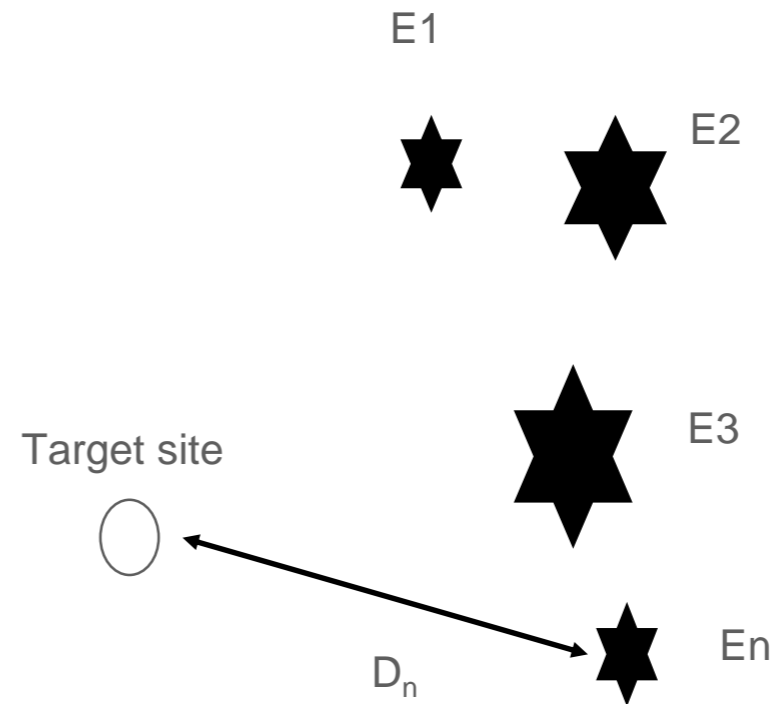
The Poissonian probability for an event in the next 30 years is 19%

Basic of Probabilistic Seismic Hazard Assessment (2)

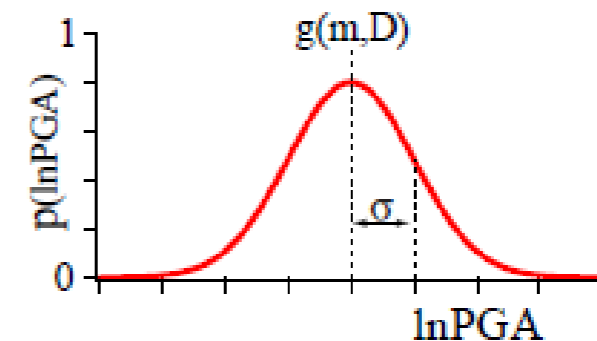
The target is to calculate the PSHA at a certain site
 The Seismic source model provide the N earthquake scenarios E_n , each one with its magnitude (m_n) location (L_n) and rate (r_n)

From the scenario L_n we can calculate the distance D_n to the target site.

Given m_n and D_n and using a Ground Motion Prediction equation.



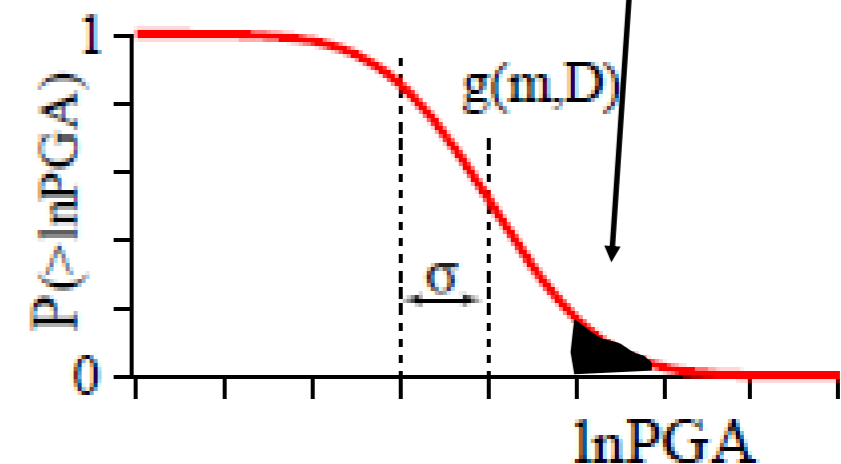
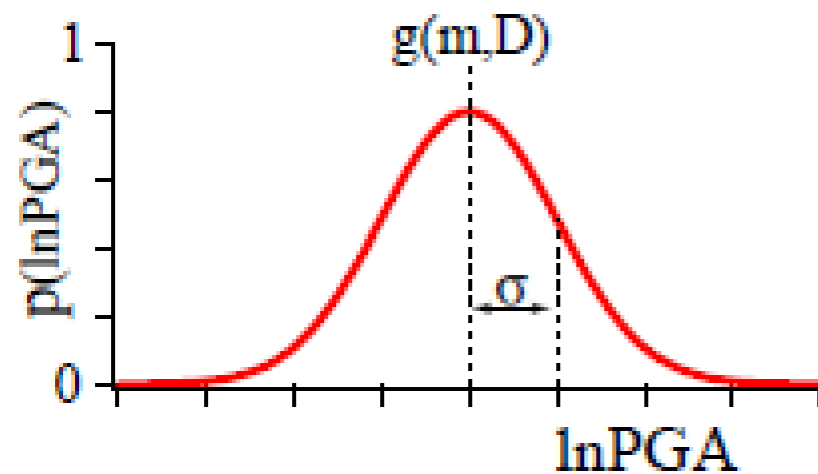
$$p_n(\ln\text{PGA}) = \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{(\ln\text{PGA} - g(m_n, D_n))^2}{2\sigma_n^2}}$$



Basic of Probabilistic Seismic Hazard Assessment (2)

Probability of exceeding a certain lnPGA

$$P_n(> \ln\text{PGA}) = \frac{1}{\sigma_n \sqrt{2\pi}} \int_{\ln\text{PGA}}^{\infty} e^{-(\ln\text{PGA} - g(m_n, D_n))^2 / 2\sigma_n^2} d\ln\text{PGA} =$$



Basic of Probabilistic Seismic Hazard Assessment (2)

Multiplying for the annual rate r_n one get **annual rate R_n at which a certain $\ln\text{PGA}$ will be exceeded** for that specific M and Location scenario at the considered site

$$R_n (>\ln\text{PGA}) = r_n P_n (>\ln\text{PGA})$$

Summing over the N scenarios (all considered Magnitudes and locations, and rates) one get the

Total annual rate of exceeding a certain $\ln\text{PGA}$

$$R_{\text{tot}} (>\ln\text{PGA}) = \sum_{n=1}^N R_n (>\ln\text{PGA}) = \sum_{n=1}^N r_n P_n (>\ln\text{PGA})$$

Basic of Probabilistic Seismic Hazard Assessment (2)

Considering the Poissonian distribution one can compute the **Probability of exceeding each ground motion level in T years** using the total annual rate

$$P_{\text{pois}}(> \ln \text{PGA}, T) = 1 - e^{-R_{\text{tot}} T}$$

If $P_{\text{pois}}=10\%$ in 50 years

$T= 50$ years

$$R_{\text{tot}}=(-\ln(1-0.1))/T=0.00210721$$

From which one get a **return period of 475 years**

Basic of Probabilistic Seismic Hazard Assessment (2)

Example Two scenarios only

R1=M=6 every 22 years

$$\ln(\text{PGA}) = 0.53(M-6) - 0.39\ln(D^2+31) + 0.25$$

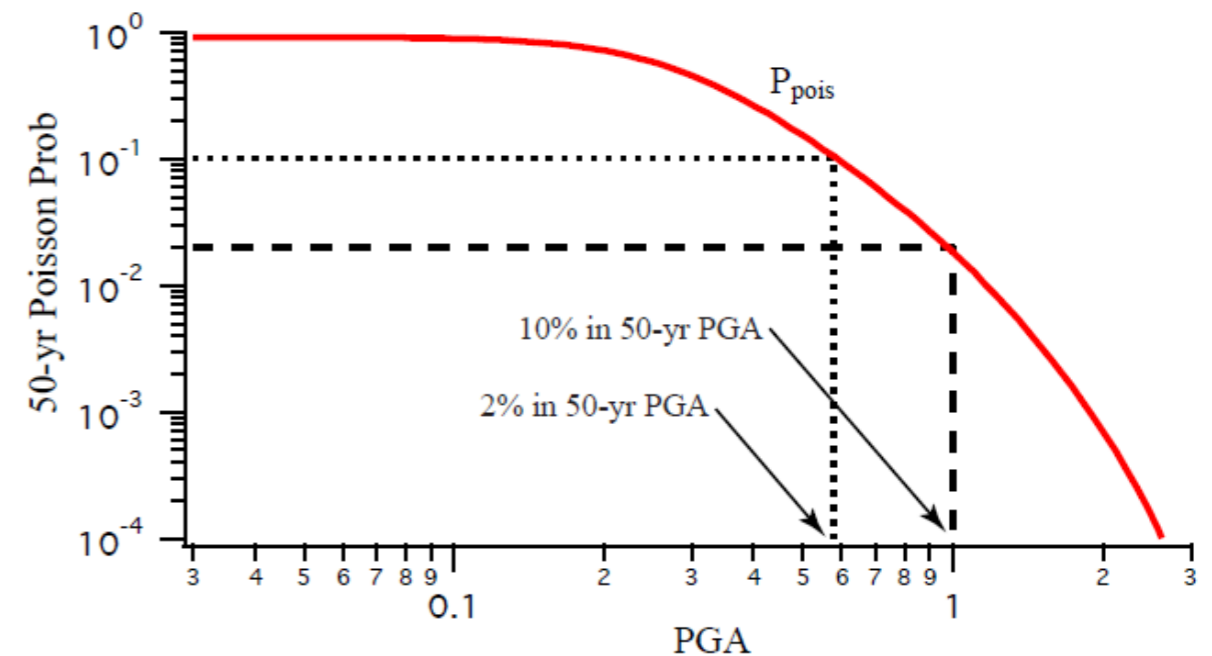
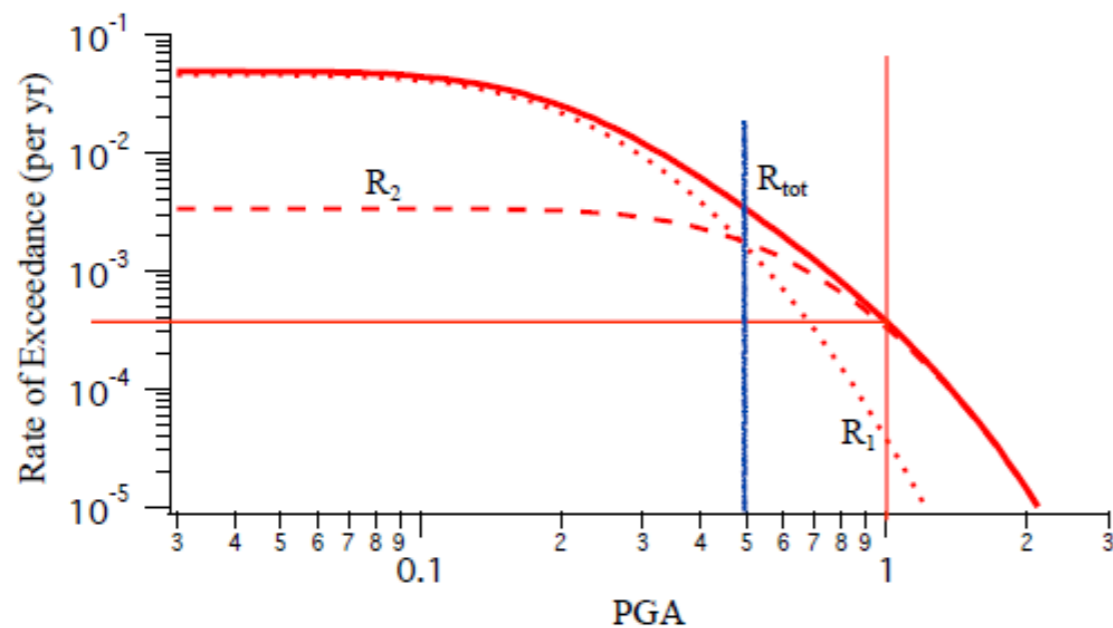
➔ 0.19 g

R2=M= 7.8 every 300 years

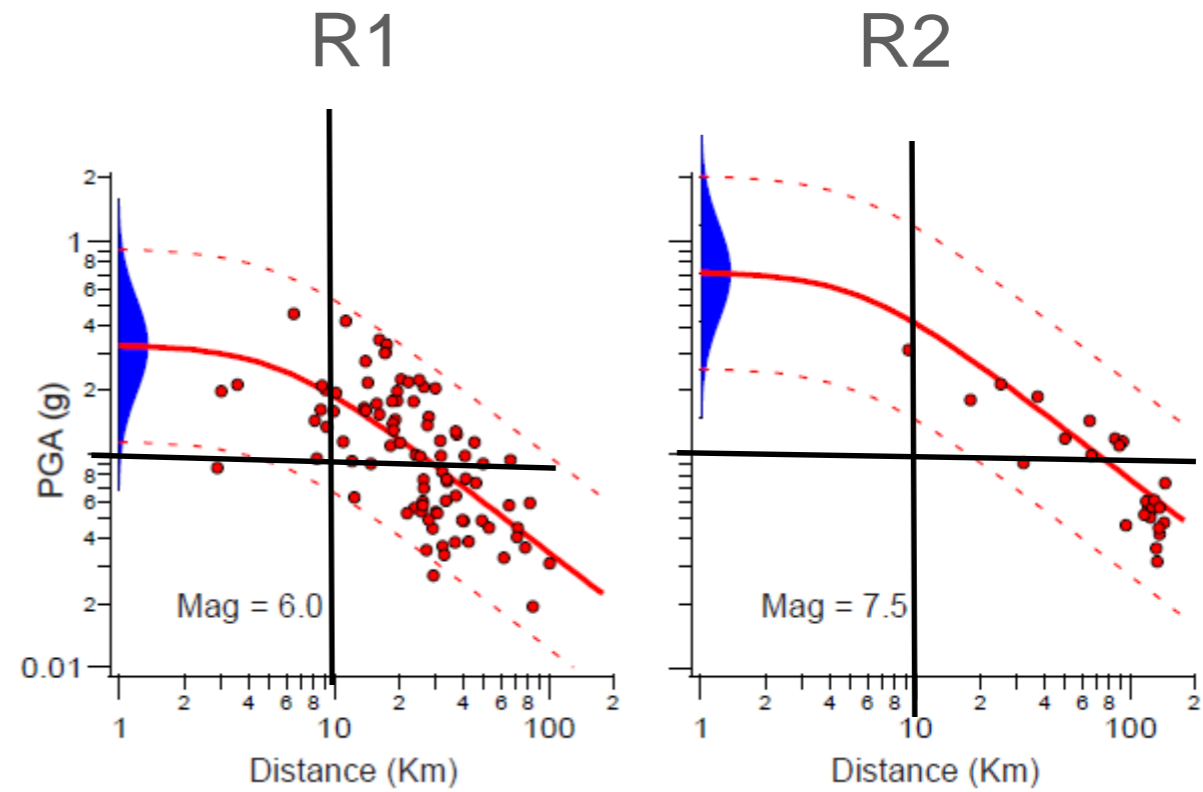
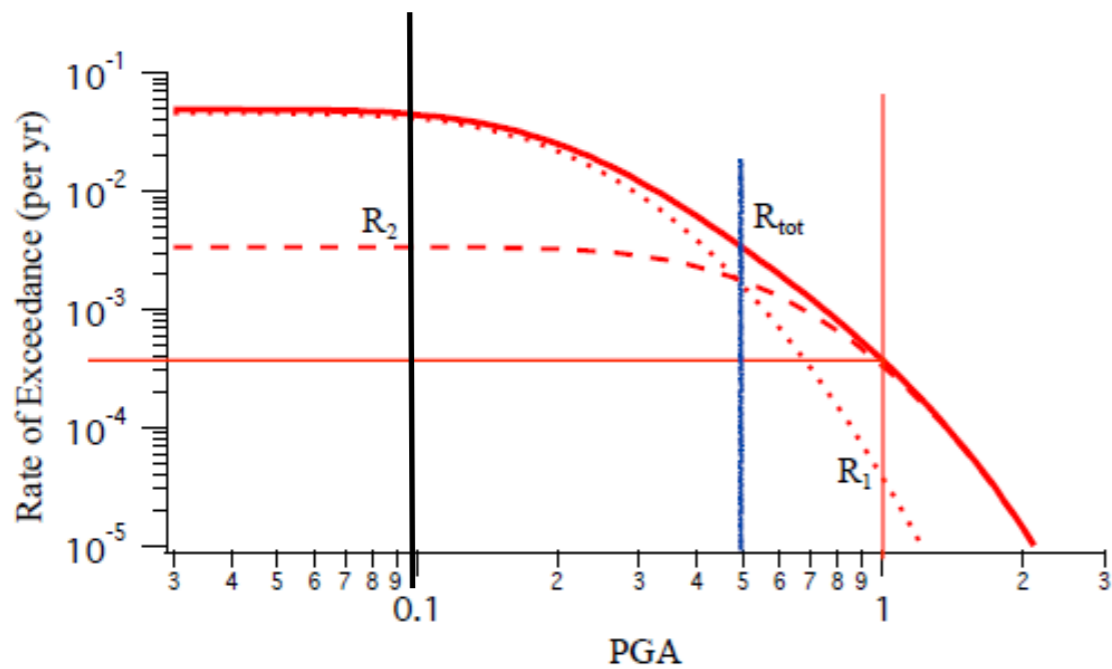
➔ 0.5 g

Both at 10 km from the target site

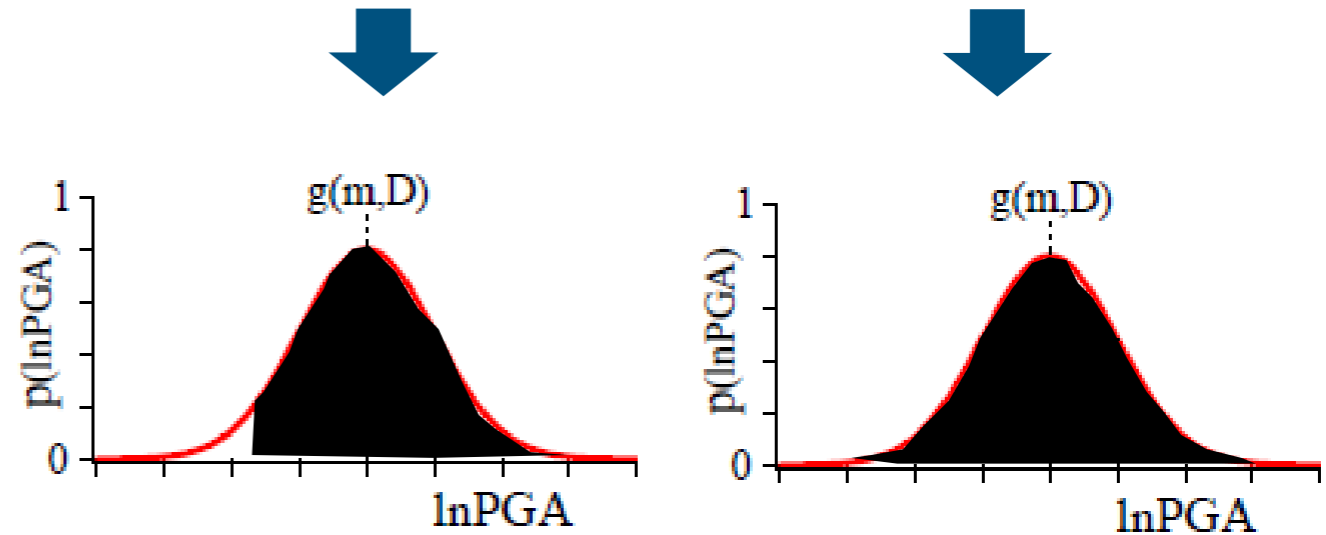
$$\sigma (\ln\text{pga})= 0.52$$



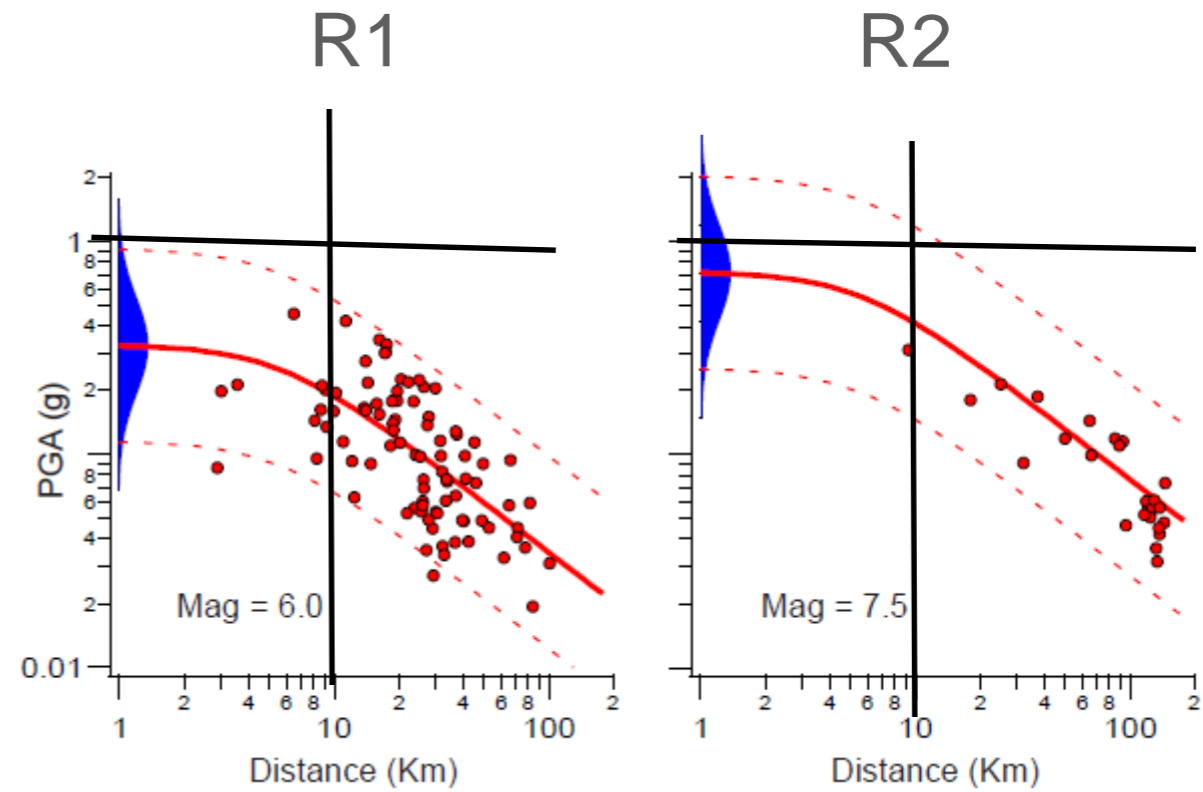
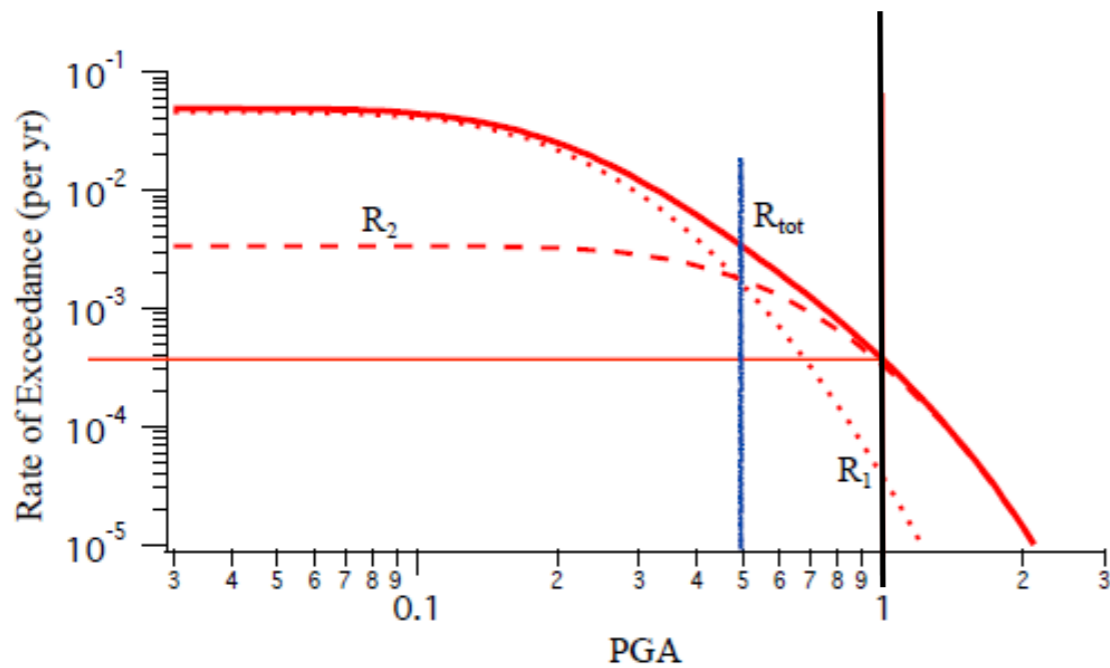
Basic of Probabilistic Seismic Hazard Assessment (2)



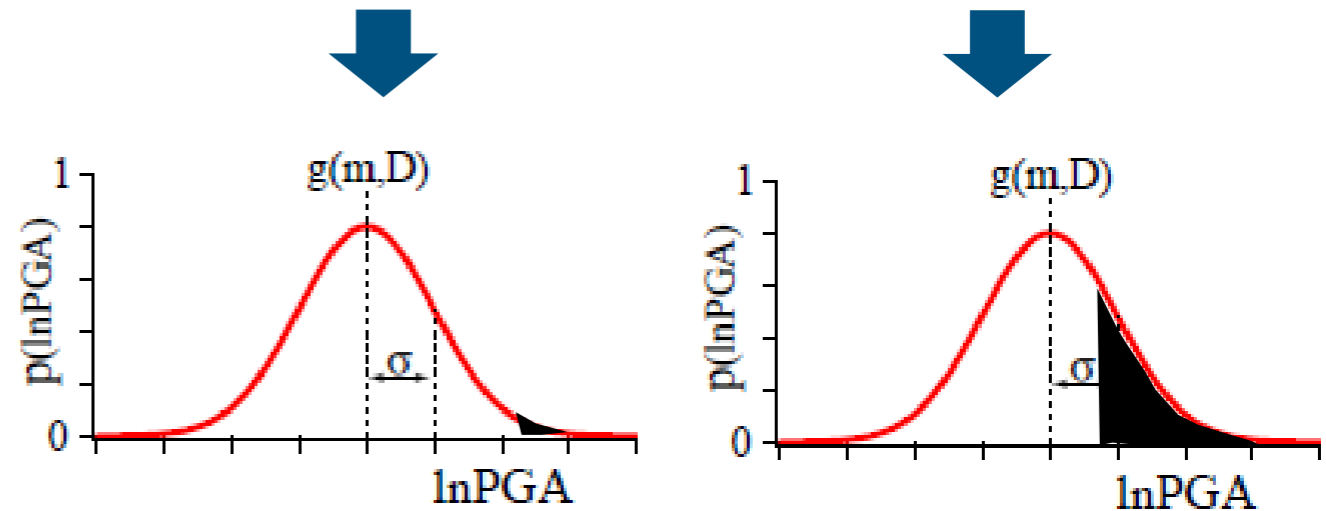
For small PGA (e.g. 0.1 g) although the **probability of exceedence** is larger for the M6, the **annual rate of exceedence** of R1 is larger than that of R2 because the **annual rate** of R1, $r_1=1/22$ is much larger than the **annual rate** of R2, $r_2=1/300$!



Basic of Probabilistic Seismic Hazard Assessment (2)

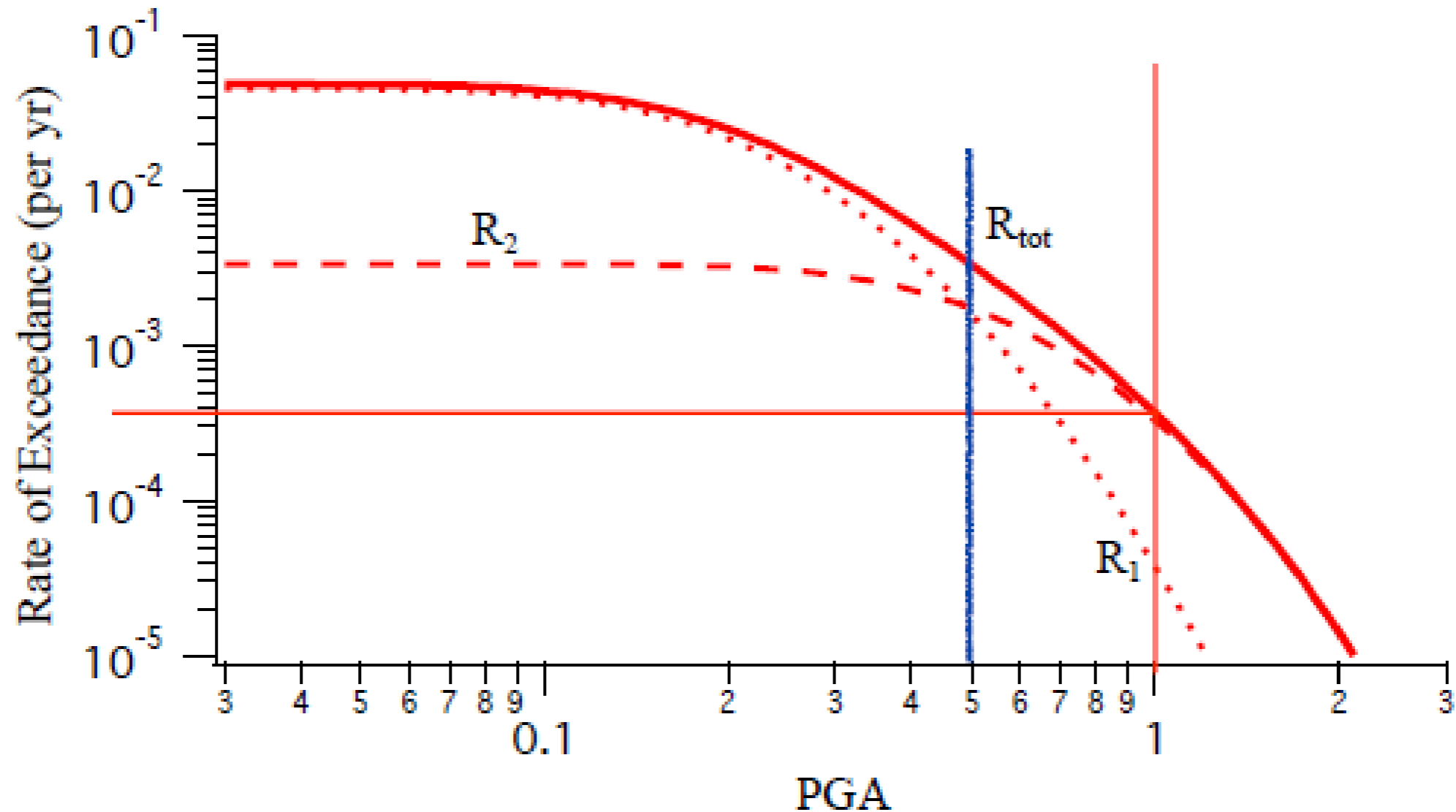


For large PGA (e.g. 1 g) the **probability of exceedance** is larger for the M7.8, although the **annual rate** of R1 is larger than that of R2, because **probability of exceedance** of R1 is very small



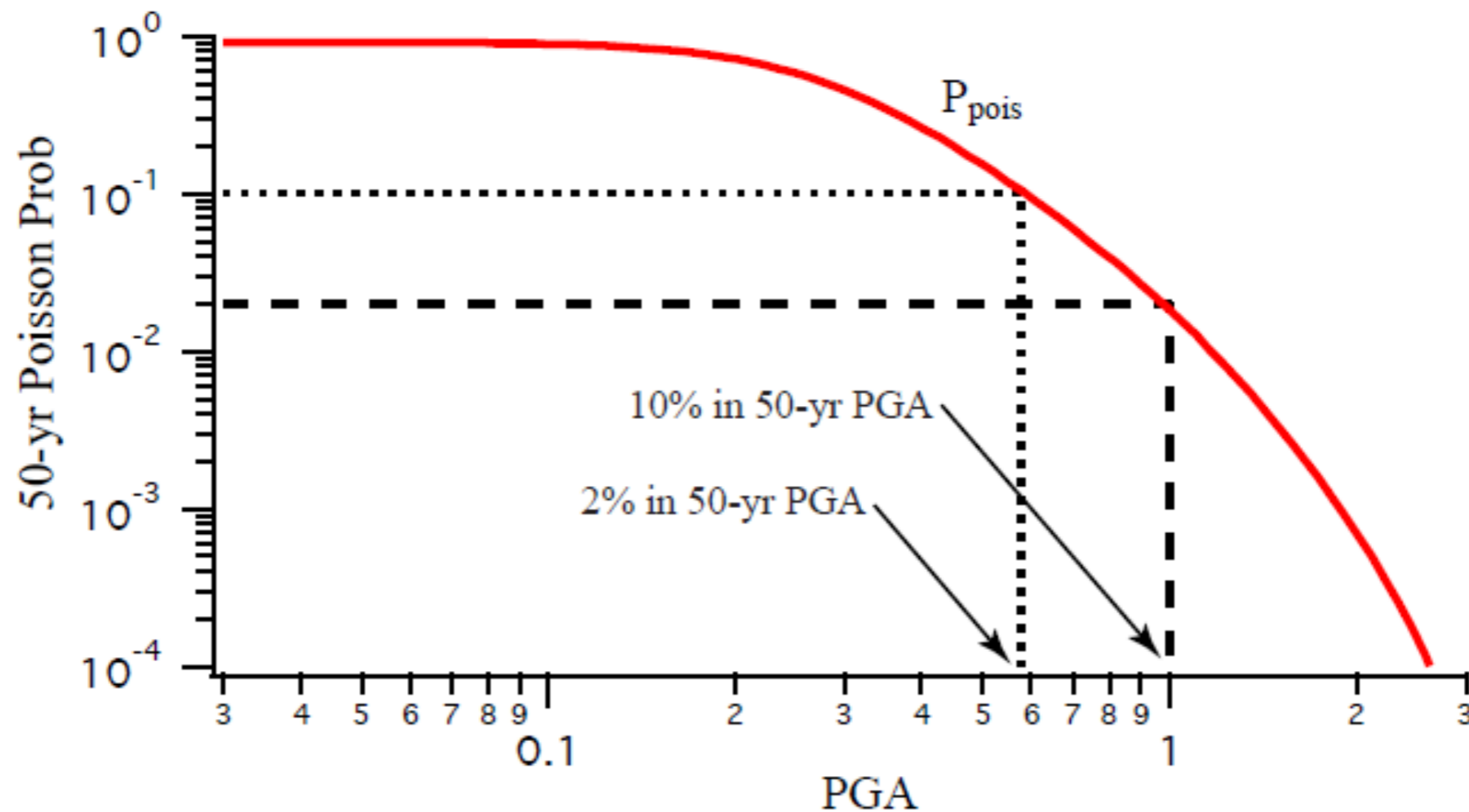
Basic of Probabilistic Seismic Hazard Assessment (2)

R_{tot} = sum of the two scenarios is dominated at low PGA by the small but frequent events and for high pga by the strong but rare events



Basic of Probabilistic Seismic Hazard Assessment (2)

$$P_{\text{pois}}(> \ln\text{PGA}, T) = 1 - e^{-R_{\text{tot}}T}$$



Extending this analysis for several sites we obtain the seismic hazard maps

Italian building code (NTC08/18)

- Seismic classification

<https://rischi.protezionecivile.gov.it/it/sismico/attivita/classificazione-sismica>

- Seismic hazard

<http://esse1.mi.ingv.it>

- NTC08 Seismic code (§ 2.*; 3.2; 7.*)

<https://www.gazzettaufficiale.it/eli/id/2008/02/04/08A00368/sg>

- NTC18 Seismic code (§ 2.*; 3.2; 7.*)

<https://www.gazzettaufficiale.it/eli/gu/2018/02/20/42/so/8/sg/pdf>

<https://www.gazzettaufficiale.it/eli/id/2019/02/11/19A00855/sg>

Italian code NTC18 - Seismic Action

L'azione sismica è caratterizzata da 3 componenti traslazionali, due orizzontali contrassegnate da X ed Y ed una verticale contrassegnata da Z, da considerare tra di loro indipendenti. Le componenti possono essere descritte, in funzione del tipo di analisi adottata, mediante una delle seguenti rappresentazioni:

- accelerazione massima in superficie;
- accelerazione massima e relativo spettro di risposta in superficie;
- **storia temporale del moto del terreno.**

Le due componenti ortogonali indipendenti che descrivono il moto orizzontale sono caratterizzate dallo stesso spettro di risposta o dalle due componenti accelerometriche orizzontali del moto sismico.

Italian code NTC18 - Elastic spectra

Lo spettro di risposta elastico in accelerazione è espresso da una forma spettrale (spettro normalizzato) riferita ad uno smorzamento convenzionale del 5%, moltiplicata per il valore della accelerazione orizzontale massima a_g su sito di riferimento rigido orizzontale.

Sia la forma spettrale che il valore di a_g variano al variare della probabilità di superamento nel periodo di riferimento P_{VR} (vedi § 2.4 e § 3.2.1).

Gli spettri così definiti possono essere utilizzati per strutture con periodo fondamentale minore o uguale a 4,0 s. Per strutture con periodi fondamentali superiori lo spettro deve essere definito da apposite analisi oppure l'azione sismica deve essere descritta mediante **storie temporali del moto del terreno**.

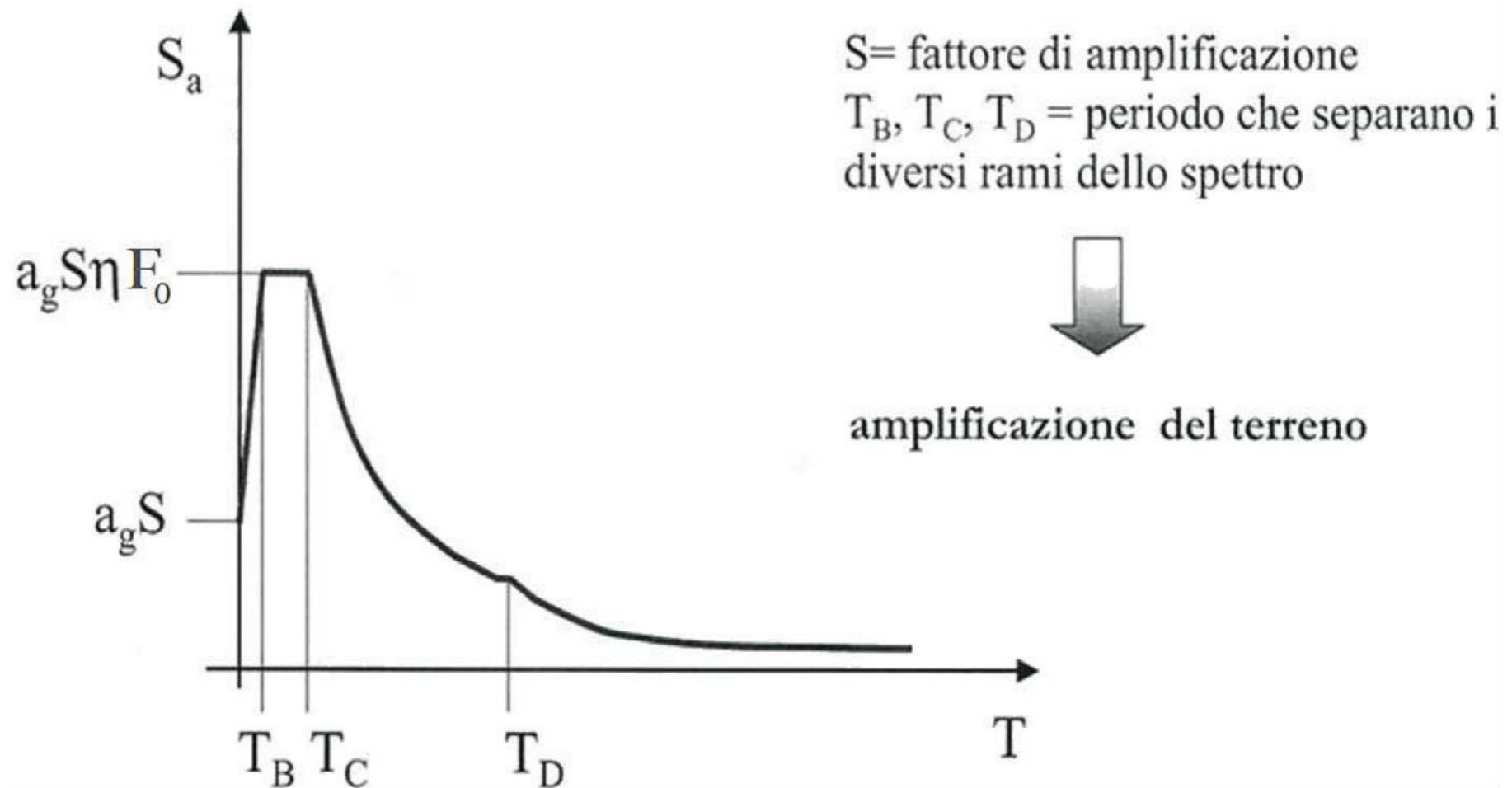
Italian code NTC18 - Elastic spectra

Lo spettro di risposta (componente orizzontale) è definito a partire dai valori dei seguenti parametri, validi per sito di riferimento su suolo rigido:

- a_g accelerazione orizzontale massima al sito
- F_0 è il fattore che quantifica l'amplificazione spettrale massima, su sito di riferimento rigido orizzontale, ed ha valore minimo pari a 2,2
- T_C^* (valore di riferimento per la determinazione del) periodo di inizio del tratto a velocità costante dello spettro in accelerazione orizzontale.
Viene quindi definito: $T_C = C_C T_C^*$
dove C_C dipende dalla categoria del sottosuolo

I valori di tali parametri sono forniti dalla NTC18, per tutti i siti considerati, in forma tabellare. Per la pericolosità in particolare (a_g): <http://esse1.mi.ingv.it>

Italian code NTC18 - Elastic spectra



$$S_e(T) = a_g \cdot S \cdot \eta \cdot F_0 \cdot \left[\frac{T}{T_B} + \frac{1}{\eta \cdot F_0} \cdot \left(1 - \frac{T}{T_B} \right) \right] \quad S_e(T) = a_g \cdot S \cdot \eta \cdot F_0 \cdot \left(\frac{T_C}{T} \right)$$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot F_0$$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot F_0 \cdot \left(\frac{T_C \cdot T_D}{T^2} \right)$$

- T_B è il periodo corrispondente all'inizio del tratto dello spettro ad accelerazione costante, $T_B = T_C / 3$; T_D è il periodo corrispondente all'inizio del tratto a spostamento costante dello spettro, espresso in secondi mediante la relazione: $T_D = 4.0 \cdot a_g / g + 1.6$
- η è il fattore che altera lo spettro elastico per coefficienti di smorzamento viscosi convenzionali ξ diversi dal 5%, ($\eta = [10 / (5 + \xi)]^{0.5} \geq 0,55$), e valutato sulla base di materiali, tipologia strutturale e terreno di fondazione

Italian code NTC18 - from hazard to “design”

- Per ciascun nodo del reticolo di riferimento e per ciascuno dei periodi di ritorno T_R considerati dalla pericolosità sismica, i tre parametri si ricavano riferendosi ai valori corrispondenti al 50–esimo percentile ed attribuendo a F_0 e T_C^* i valori ottenuti imponendo che...
- le forme spettrali in accelerazione, velocità e spostamento previste dalle NTC scartino al minimo dalle corrispondenti forme spettrali previste dalla pericolosità sismica (la condizione di minimo è imposta operando ai minimi quadrati, su spettri di risposta normalizzati ad uno, per ciascun sito e ciascun periodo di ritorno).

Site effects and NTC18 - Elastic spectra & soil

S è il coefficiente che tiene conto della categoria di sottosuolo e delle condizioni topografiche mediante la relazione: $S = S_S \cdot S_T$

S_S è il coefficiente di amplificazione stratigrafica

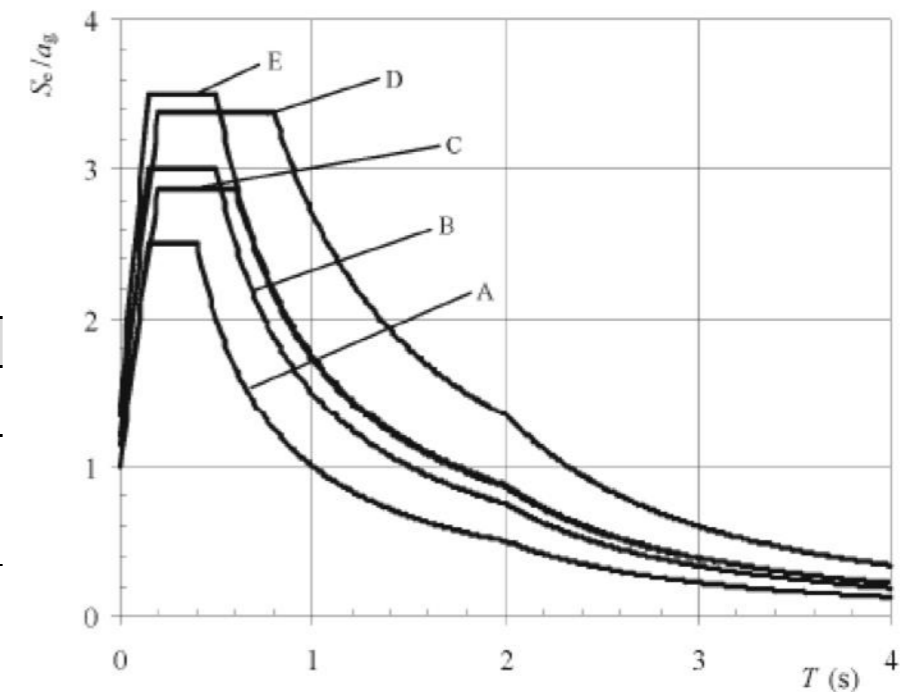
S_T è il coefficiente di amplificazione topografica

Tab. 3.2.V – Valori massimi del coefficiente di amplificazione topografica S_T

Categoria topografica	Ubicazione dell'opera o dell'intervento	S_T
T1	-	1,0
T2	In corrispondenza della sommità del pendio	1,2
T3	In corrispondenza della cresta di un rilievo con pendenza media minore o uguale a 30°	1,2
T4	In corrispondenza della cresta di un rilievo con pendenza media maggiore di 30°	1,4

Tab. 3.2.IV – Espressioni di S_S e di C_C

Categoria sottosuolo	S_S	C_C
A	1,00	1,00
B	$1,00 \leq 1,40 - 0,40 \cdot F_0 \cdot \frac{a_g}{g} \leq 1,20$	$1,10 \cdot (T_C^*)^{-0,20}$
C	$1,00 \leq 1,70 - 0,60 \cdot F_0 \cdot \frac{a_g}{g} \leq 1,50$	$1,05 \cdot (T_C^*)^{-0,33}$
D	$0,90 \leq 2,40 - 1,50 \cdot F_0 \cdot \frac{a_g}{g} \leq 1,80$	$1,25 \cdot (T_C^*)^{-0,50}$
E	$1,00 \leq 2,00 - 1,10 \cdot F_0 \cdot \frac{a_g}{g} \leq 1,60$	$1,15 \cdot (T_C^*)^{-0,40}$



Site effects and NTC18 - Soil classification

3.2.2 CATEGORIE DI SOTTOSUOLO E CONDIZIONI TOPOGRAFICHE

Categorie di sottosuolo

Ai fini della definizione dell'azione sismica di progetto, l'effetto della risposta sismica locale si valuta mediante specifiche analisi, da eseguire con le modalità indicate nel § 7.11.3. In alternativa, qualora le condizioni stratigrafiche e le proprietà dei terreni siano chiaramente riconducibili alle categorie definite nella Tab. 3.2.II, si può fare riferimento a un approccio semplificato che si basa sulla classificazione del sottosuolo in funzione dei valori della velocità di propagazione delle onde di taglio, V_S . I valori dei parametri meccanici necessari per le analisi di risposta sismica locale o delle velocità V_S per l'approccio semplificato costituiscono parte integrante della caratterizzazione geotecnica dei terreni compresi nel volume significativo, di cui al § 6.2.2.

Tab. 3.2.II – *Categorie di sottosuolo che permettono l'utilizzo dell'approccio semplificato.*

Categoria	Caratteristiche della superficie topografica
A	<i>Ammassi rocciosi affioranti o terreni molto rigidi caratterizzati da valori di velocità delle onde di taglio superiori a 800 m/s, eventualmente comprendenti in superficie terreni di caratteristiche meccaniche più scadenti con spessore massimo pari a 3 m.</i>
B	<i>Rocce tenere e depositi di terreni a grana grossa molto addensati o terreni a grana fina molto consistenti, caratterizzati da un miglioramento delle proprietà meccaniche con la profondità e da valori di velocità equivalente compresi tra 360 m/s e 800 m/s.</i>
C	<i>Depositi di terreni a grana grossa mediamente addensati o terreni a grana fina mediamente consistenti con profondità del substrato superiori a 30 m, caratterizzati da un miglioramento delle proprietà meccaniche con la profondità e da valori di velocità equivalente compresi tra 180 m/s e 360 m/s.</i>
D	<i>Depositi di terreni a grana grossa scarsamente addensati o di terreni a grana fina scarsamente consistenti, con profondità del substrato superiori a 30 m, caratterizzati da un miglioramento delle proprietà meccaniche con la profondità e da valori di velocità equivalente compresi tra 100 e 180 m/s.</i>
E	<i>Terreni con caratteristiche e valori di velocità equivalente riconducibili a quelle definite per le categorie C o D, con profondità del substrato non superiore a 30 m.</i>

Site effects and NTC18 - $V_{S,eq}$

La classificazione del sottosuolo si effettua in base alle condizioni stratigrafiche ed ai valori della velocità equivalente di propagazione delle onde di taglio, $V_{S,eq}$ (in m/s), definita dall'espressione:

$$V_{S,eq} = \frac{H}{\sum_{i=1,N} \frac{h_i}{V_{S,i}}} \quad [\text{m / s}]$$

con h_i spessore dell' i -esimo strato; $V_{S,i}$ velocità delle onde di taglio nell' i -esimo strato; N numero di strati; H profondità del substrato, definito come quella formazione costituita da roccia o terreno molto rigido, caratterizzata da V_S non inferiore a 800 m/s.

Per depositi con profondità H del substrato superiore a 30 m, la velocità equivalente delle onde di taglio $V_{S,eq}$ è definita dal parametro $V_{S,30}$, ottenuto ponendo $H=30$ m nella precedente espressione e considerando le proprietà degli strati di terreno fino a tale profondità.

V_{S30}

Nelle definizioni precedenti V_{S30} è la velocità media di propagazione dei primi 30 m di profondità delle onde di taglio e viene calcolata con la seguente espressione:

$$V_{S30} = \frac{30}{\sum_{i=1,N} \frac{h_i}{V_i}} \quad [\text{m / s}]$$

dove h_i e V_i indicano lo spessore (in m) e la velocità delle onde di taglio (per deformazioni di taglio $\gamma < 10^{-6}$) dello strato i -esimo, per un totale di N strati presenti nei 30 m superiori.

Site effects and NTC18 - Topography

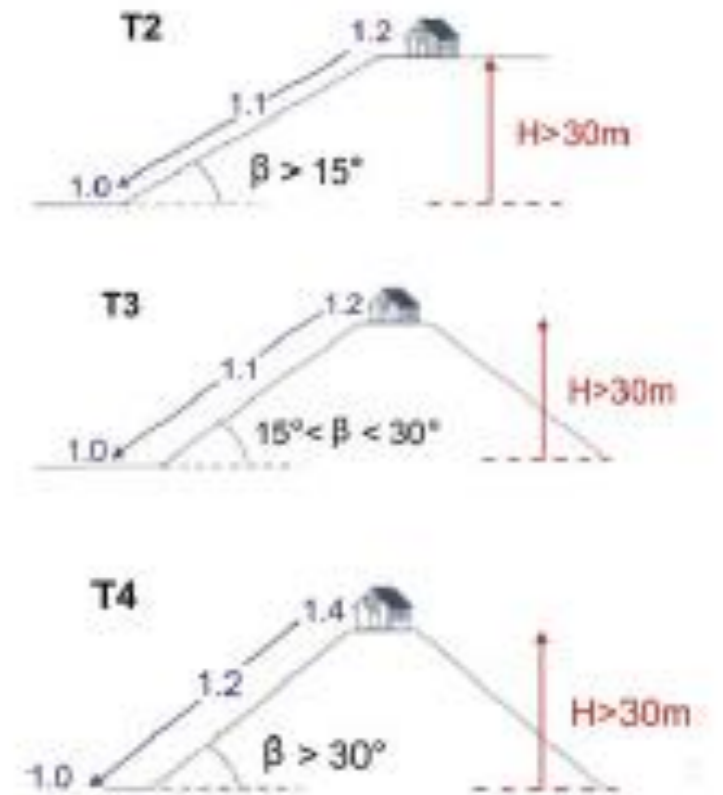
Per condizioni topografiche complesse è necessario predisporre specifiche analisi di risposta sismica locale. Per configurazioni superficiali semplici si può adottare la seguente classificazione (Tab. 3.2.III):

T1 Superficie pianeggiante, pendii e rilievi isolati con inclinazione media $i \leq 15^\circ$

T2 Pendii con inclinazione media $i > 15^\circ$

T3 Rilievi con larghezza in cresta molto minore che alla base e inclinazione media $15^\circ \leq i \leq 30^\circ$

T4 Rilievi con larghezza in cresta molto minore che alla base e inclinazione media $i > 30^\circ$



Le su esposte categorie topografiche si riferiscono a configurazioni geometriche prevalentemente bidimensionali, creste o dorsali allungate, e devono essere considerate nella definizione dell'azione sismica se di altezza maggiore di 30 m.

Site effects and NTC18 - Topography

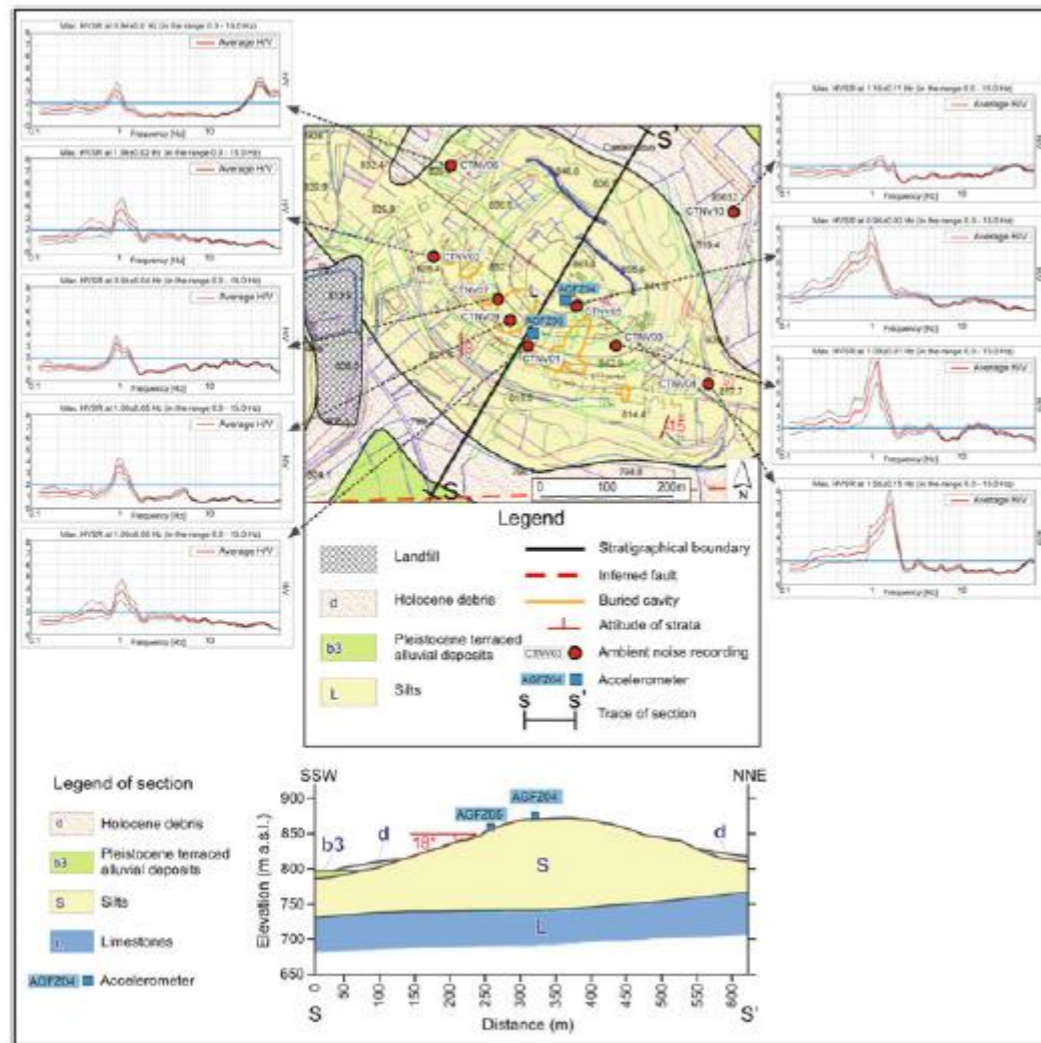


Fig. 2 Geological map and section for Castelnuovo (modified from Gallipoli et al. 2011)

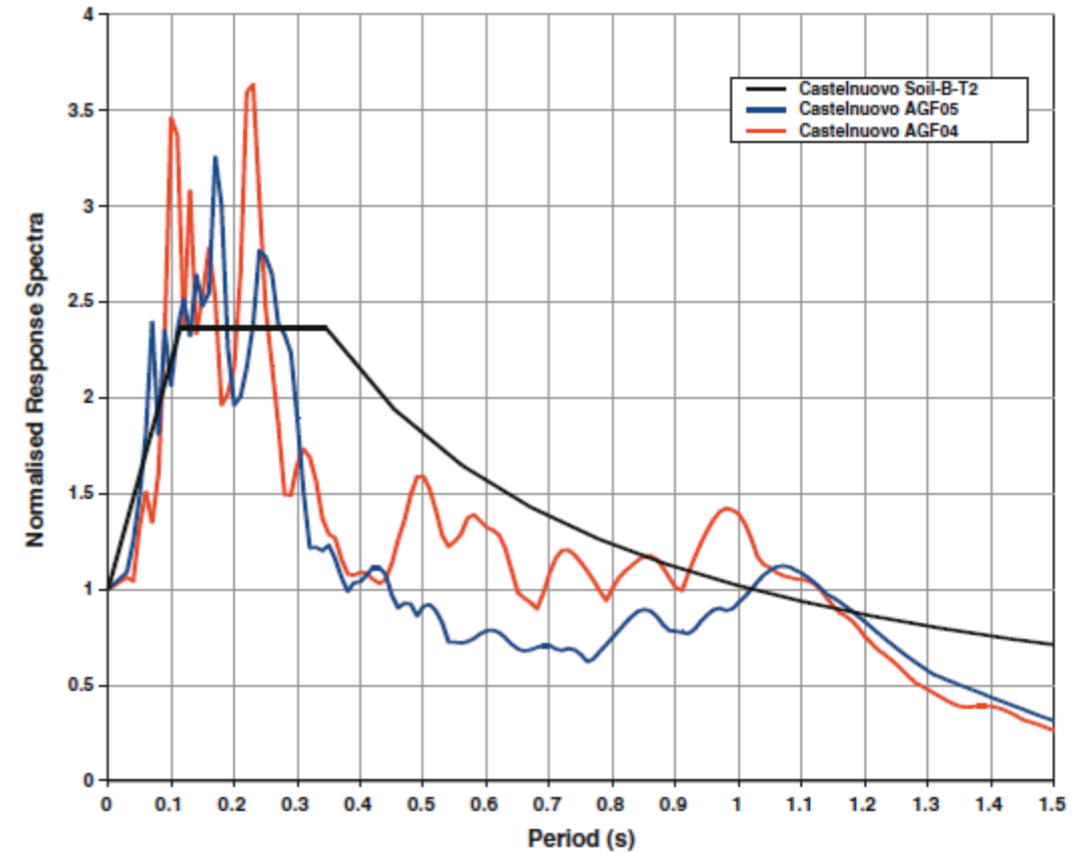


Fig. 5 Normalised Response spectra of the M 5.1 event of April 9, 2009 recorded at two sites in Castelnuovo compared with code provision

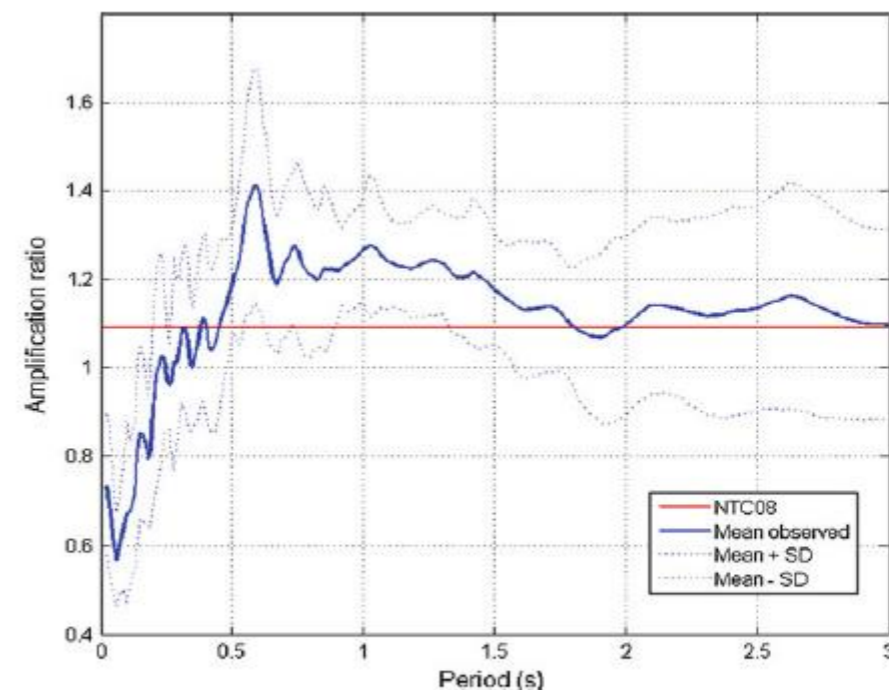


Fig. 6 Comparison between code provisions ratio (red) and observed amplification ratio (blue) in Castelnuovo

Site effects and NTC18 - Topography

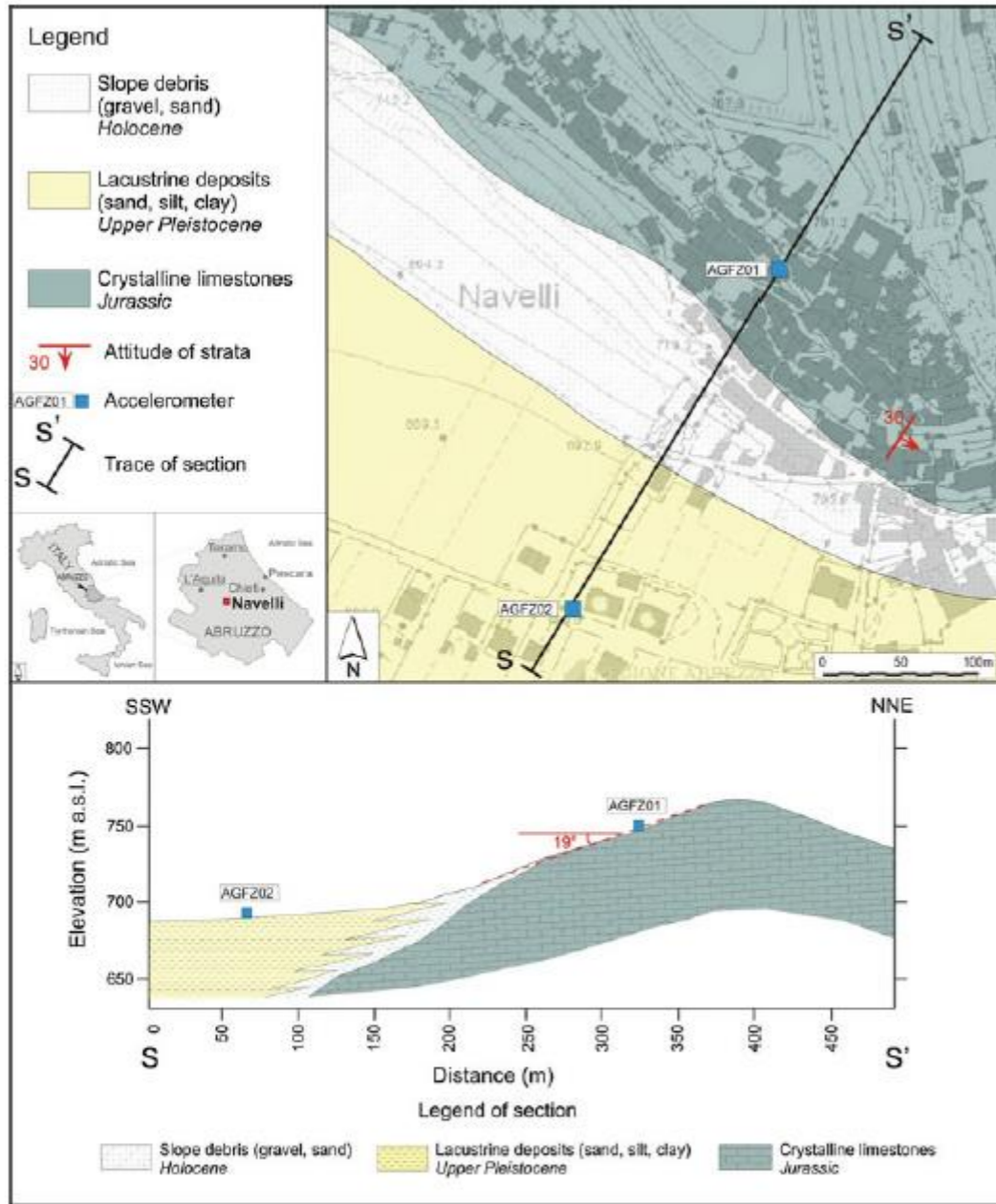


Fig. 7 Geological map and section for Navelli

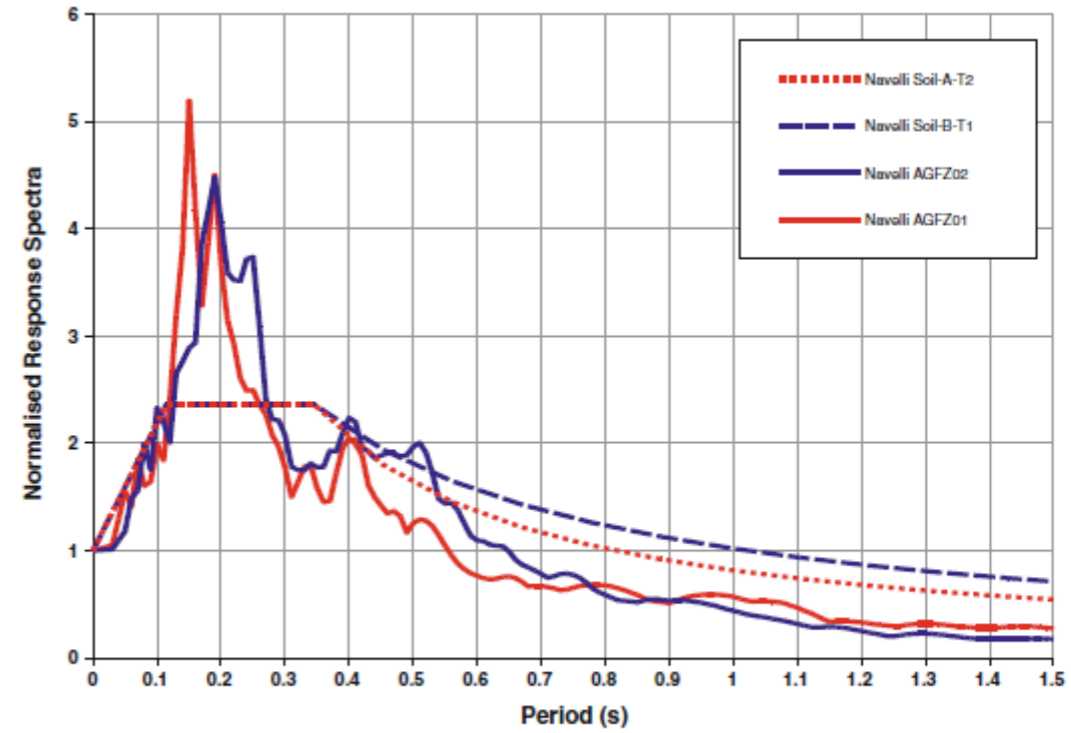


Fig. 10 Normalised Response spectra of the M 5.1 event of April 9, 2009 recorded at two sites in Navelli compared with code provision

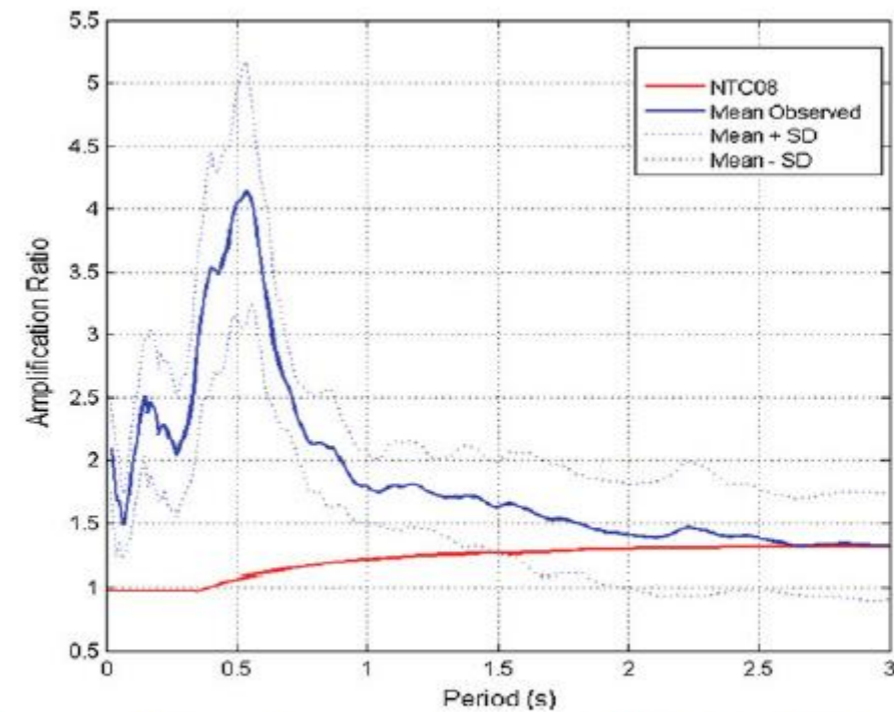


Fig. 11 Comparison between code provisions (red) and observed amplification ratio (blue) in Navelli

NTC18 - Time histories (3.2)

Gli stati limite, ultimi e di esercizio, possono essere verificati mediante l'uso di storie temporali del moto del terreno artificiali o naturali...

L'uso di storie temporali del moto del terreno artificiali non è ammesso nelle analisi dinamiche di opere e sistemi geotecnici.

L'uso di storie temporali del moto del terreno generate mediante simulazione del meccanismo di sorgente e della propagazione è ammesso a condizione che siano adeguatamente giustificate le ipotesi relative alle caratteristiche sismogenetiche della sorgente e del mezzo di propagazione e che, negli intervalli di periodo sopraindicati, l'ordinata spettrale media non presenti uno scarto in difetto superiore al 20% rispetto alla corrispondente componente dello spettro elastico.

L'uso di storie temporali del moto del terreno naturali o registrate è ammesso a condizione che la loro scelta sia rappresentativa della sismicità del sito e sia adeguatamente giustificata in base alle caratteristiche sismogenetiche della sorgente, alle condizioni del sito di registrazione, alla magnitudo, alla distanza dalla sorgente e alla massima accelerazione orizzontale attesa al sito.

NTC18 - Space variability (3.2.4.1)

Nei punti di contatto con il terreno di opere con sviluppo planimetrico significativo, il moto sismico può avere caratteristiche differenti, a causa del carattere asincrono del fenomeno di propagazione, delle disomogeneità e delle discontinuità eventualmente presenti, e della diversa risposta locale del terreno.

Degli effetti sopra indicati deve tenersi conto quando essi possono essere significativi e in ogni caso quando le condizioni di sottosuolo siano così variabili lungo lo sviluppo dell'opera da richiedere l'uso di accelerogrammi o di spettri di risposta diversi.

NTC18 - Local response (7.11.3)

Il moto generato da un terremoto in un sito dipende dalle particolari condizioni locali, cioè dalle caratteristiche topografiche e stratigrafiche del sottosuolo e dalle proprietà fisiche e meccaniche dei terreni e degli ammassi rocciosi di cui è costituito. Alla scala della singola opera o del singolo sistema geotecnico, l'analisi della risposta sismica locale consente quindi di definire le modifiche che il segnale sismico di ingresso subisce, a causa dei suddetti fattori locali.

...

Nelle analisi di risposta sismica locale, l'azione sismica di ingresso è descritta in termini di storia temporale dell'accelerazione (accelerogrammi) su di un sito di riferimento rigido ed affiorante con superficie topografica orizzontale.

L'applicazione del metodo richiede la valutazione dell'accelerazione critica, che deve essere valutata con i valori caratteristici dei parametri di resistenza, e dell'azione sismica di progetto, che deve essere rappresentata mediante storie temporali delle accelerazioni. Gli accelerogrammi impiegati nelle analisi, in numero non inferiore a 7, devono essere rappresentativi della sismicità del sito e la loro scelta deve essere adeguatamente giustificata (vedi § 3.2.3.6). Non è ammesso l'impiego di accelerogrammi artificiali.

SURFACE TOPOGRAPHY EFFECTS

(convexity) sensitivity to:
a) type of wavefield
b) angle of incidence
c) shape and sharpness

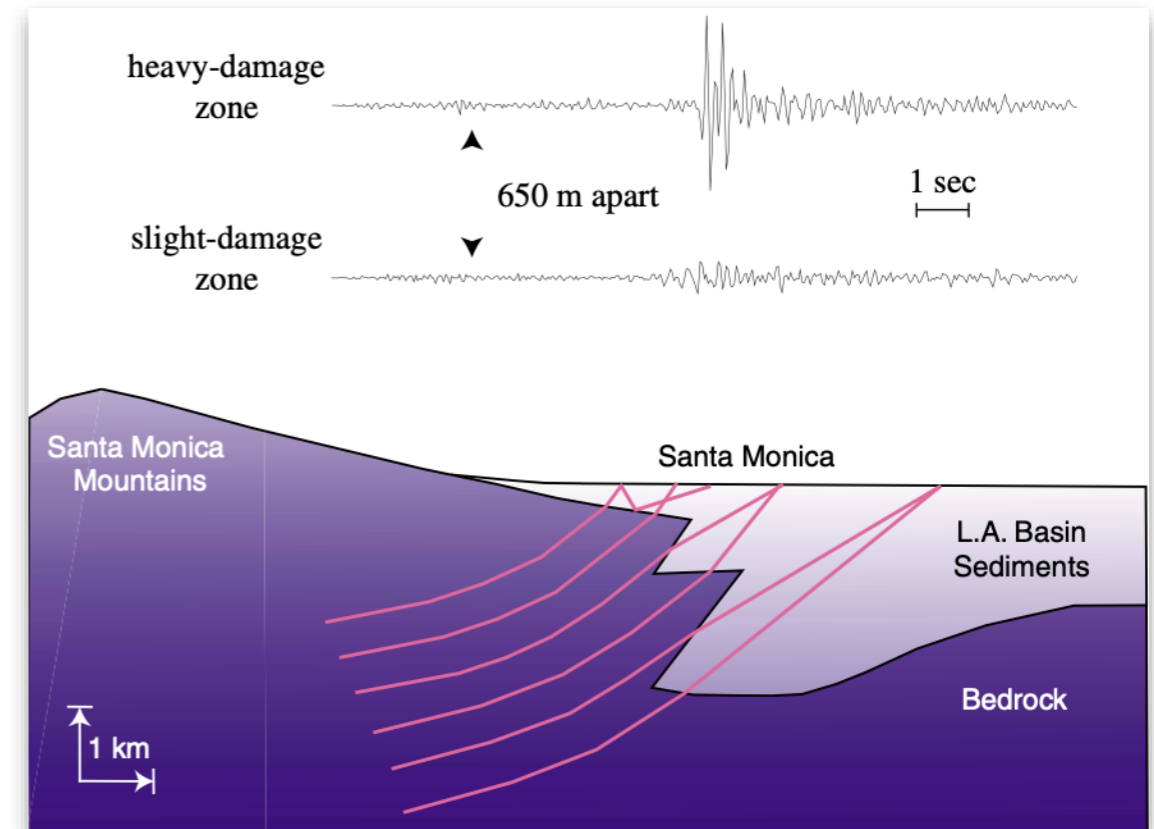
GROUNDSHAKING SITE EFFECTS

SOFT SURFACE LAYERING

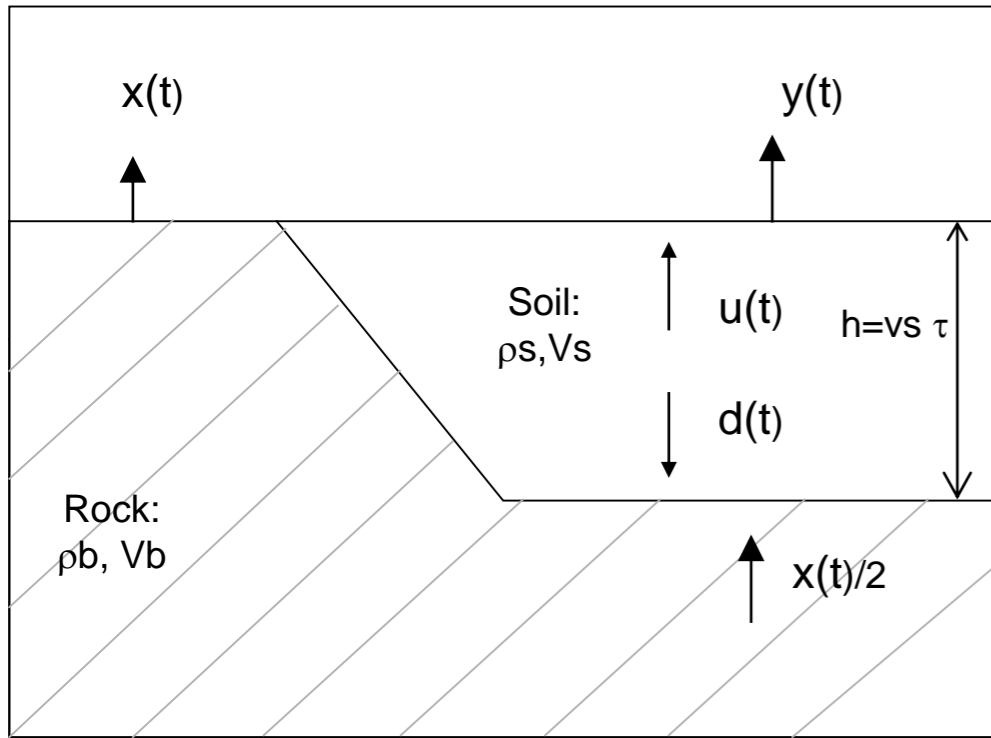
a) 1-D: trapping of waves for impedance contrast; vertical resonances

$$f_n = [(2n+1)\beta]/4H; \quad A \approx (\rho_2 v_2)/(\rho_1 v_1)$$

b) 2-D, 3-D: complex energy focusing; diffraction effects; basin edge waves



Effetti di sito 1D



$$|H(f)| = \left(\frac{(1+r)^2}{1 + 2r \cos(4\pi f \tau) + r^2} \right)^{1/2}$$

$$r = \frac{\rho_b V_b - \rho_s V_s}{\rho_b V_b + \rho_s V_s} = \frac{c-1}{1+c}$$

$$\tau = h/V_s$$

$$f_o = 400 / (4 * 100) = 1 \text{ Hz}$$

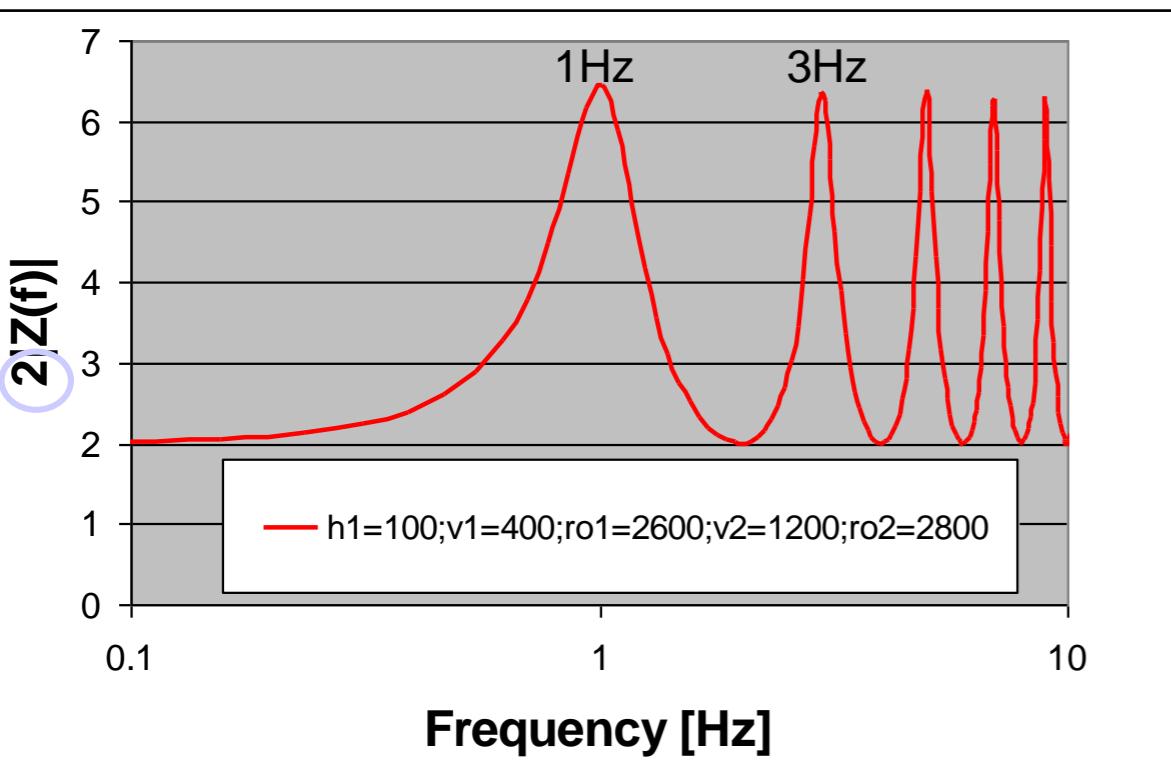
$$C = \left(\frac{2800 * 1200}{2600 * 400} \right) \sim 3.23$$

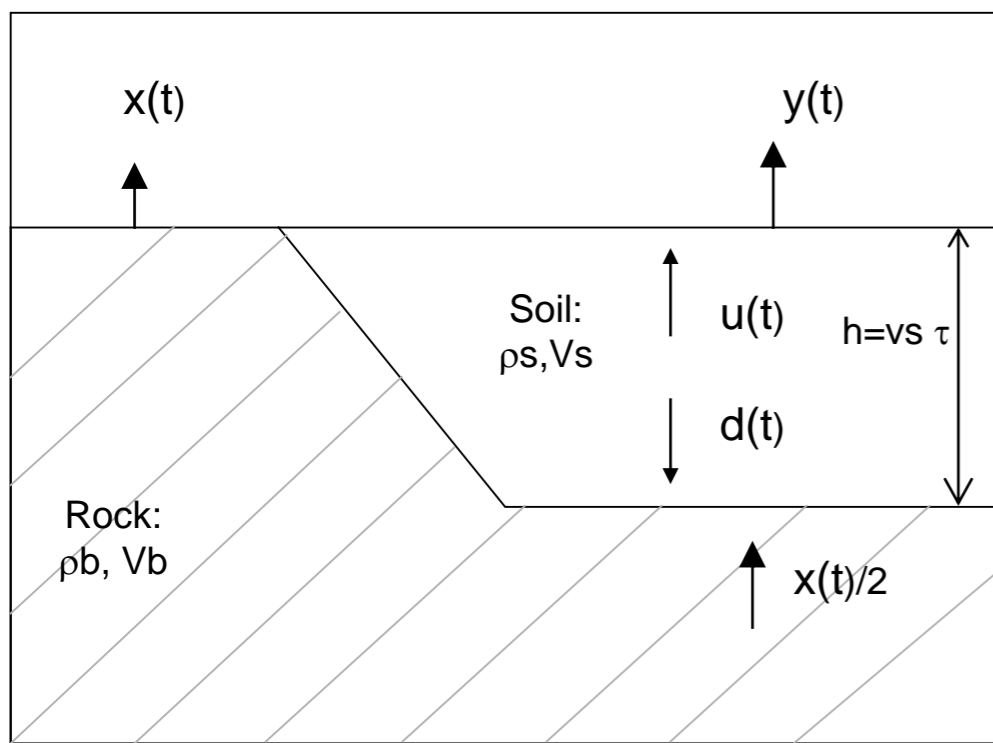
$|H(f)|$ Massimo per $f=f_o$
Tale che $\cos(4\pi f \tau) = -1 \rightarrow f_o = \frac{1}{4\tau} = \frac{V_s}{4h}$

Esistono diversi massimi a frequenze:

$$f_n = \frac{V_s}{4h} (2n + 1) \quad \text{with } n = 0, 1, 2, 3, \dots$$

($n=0$ modo fondamentale
 $n>0$ modi superiori)





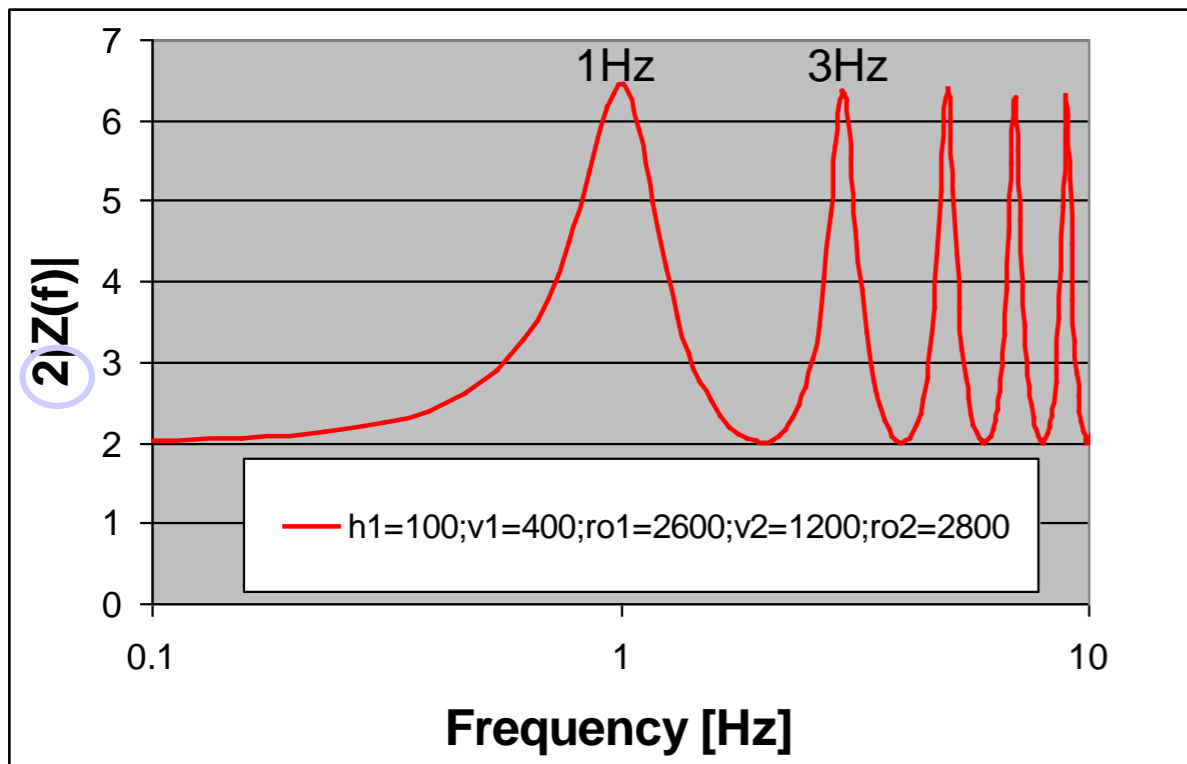
$$|H(f)| = \left(\frac{(1+r)^2}{1+2r \cos(4\pi ft) + r^2} \right)^{1/2}$$

Per $f=f_n$ $|H(f)|$ diventa

$$f_0 = 400 / (4 \cdot 100) = 1 \text{ Hz}$$

$$C = \left(\frac{2800 \cdot 1200}{2600 \cdot 400} \right) \sim 3.23$$

$$|H(f_n)| = \left(\frac{(1+r)^2}{1-2r+r^2} \right)^{1/2} = \frac{1+r}{1-r} = C$$



Il contrasto di impedenza determina l'ampiezza del picco (modello elastic)

1D site effects

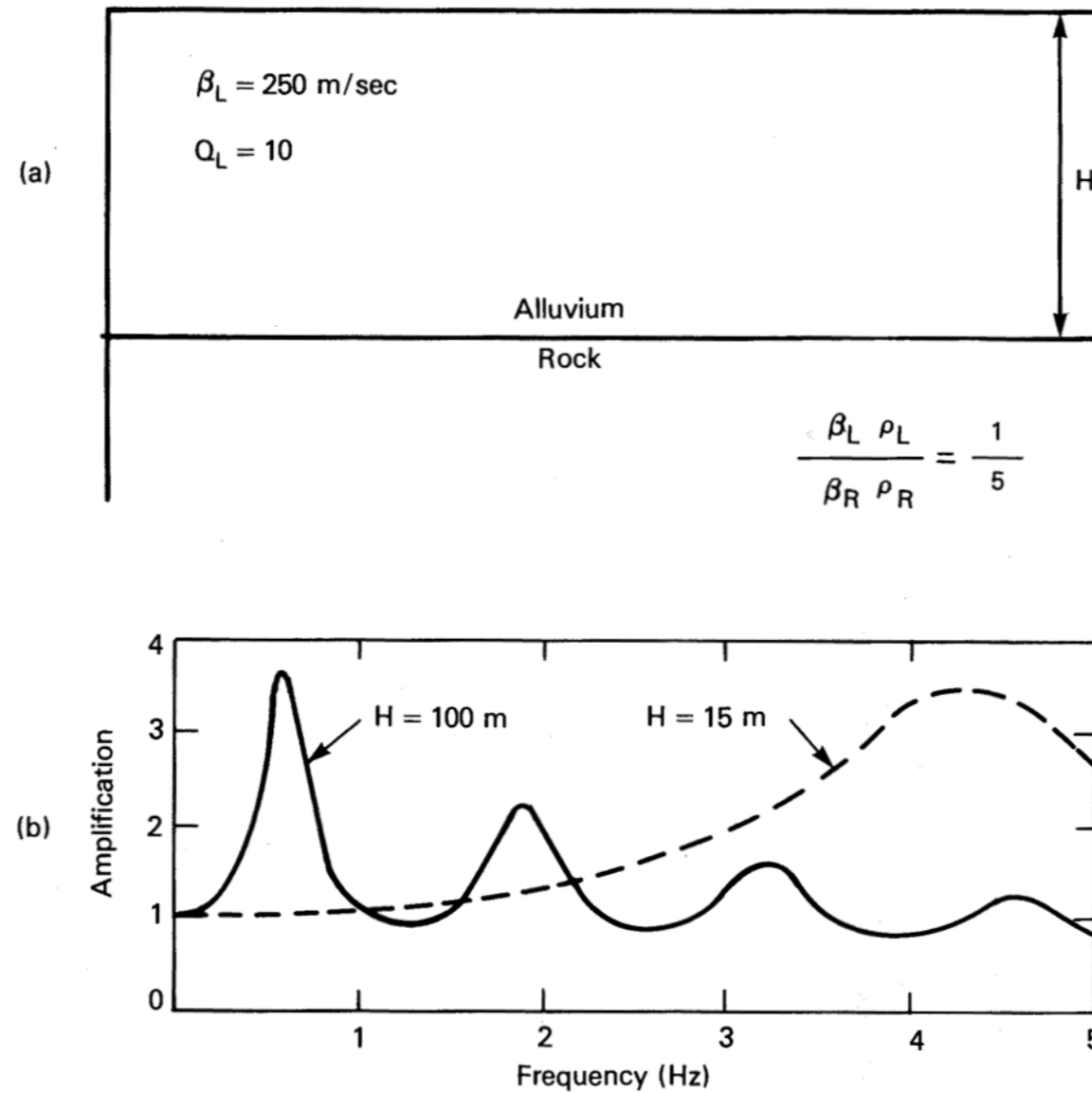


FIGURE 8.2 Model of site amplification. (a) Cross-section of alluvial layer of thickness H overlying rock. Impedance of rock is five times impedance of alluvium. (b) Amplification factors in the frequency domain for two thicknesses of alluvium (after Murphy and O'Brien 1978).

$$|U_L(\omega)| = 2.0 \left[\cos^2(k_L H) + \left(\frac{\rho_L v_L}{\rho_H v_H} \right) \sin^2(k_L H) \right]^{-1/2}$$

Site effect estimation

1) Direct methods:

Earthquake based:

With reference site: Standard Spectral Ratio (SSR), Generalised Inversion Technique (GIT),

Without a reference site: Horizontal-to-Vertical Spectral Ratio (H/V)

Seismic noise based:

With reference site: Standard Spectral Ratio (SSR), Spectra analysis,

Without a reference site: Horizontal-to-Vertical Spectral Ratio (H/V)

2) Indirect methods: active (SASW, MASW) and passive (seismic noise) array analysis. Numerical simulations

WEAK (AND STRONG) MOTION

- a) S/B spectral ratio (Borcherdt, 1970)
- b) generalized inversion scheme (Andrews, 1986)
- c) coda waves analysis (Margheriti et al., 1994)
- d) parametrized source and path inversion (Boatwright et al., 1991)
- e) H/V spectral ratio (receiver function) (Lermo et al., 1993)

$$R_{ij}(\omega) = E_i(\omega) \cdot P_{ij}(\omega) \cdot S_j(\omega)$$

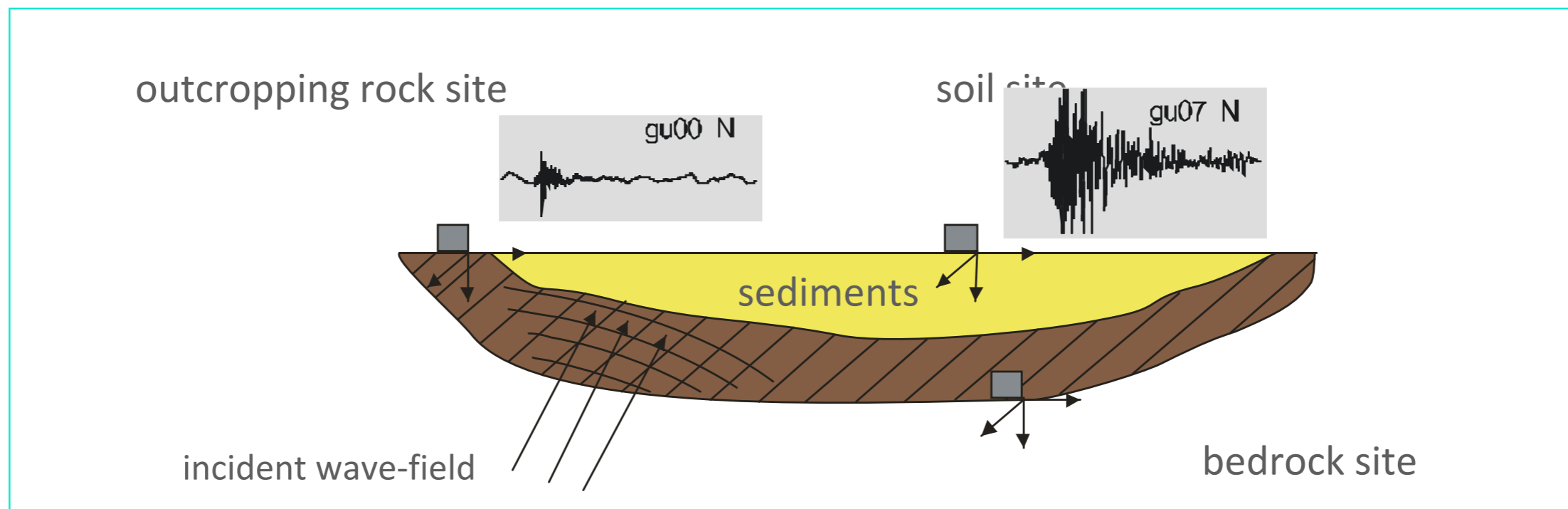
EMPIRICAL TECHNIQUES FOR SITE EFFECT ESTIMATION

MICROTREMORS

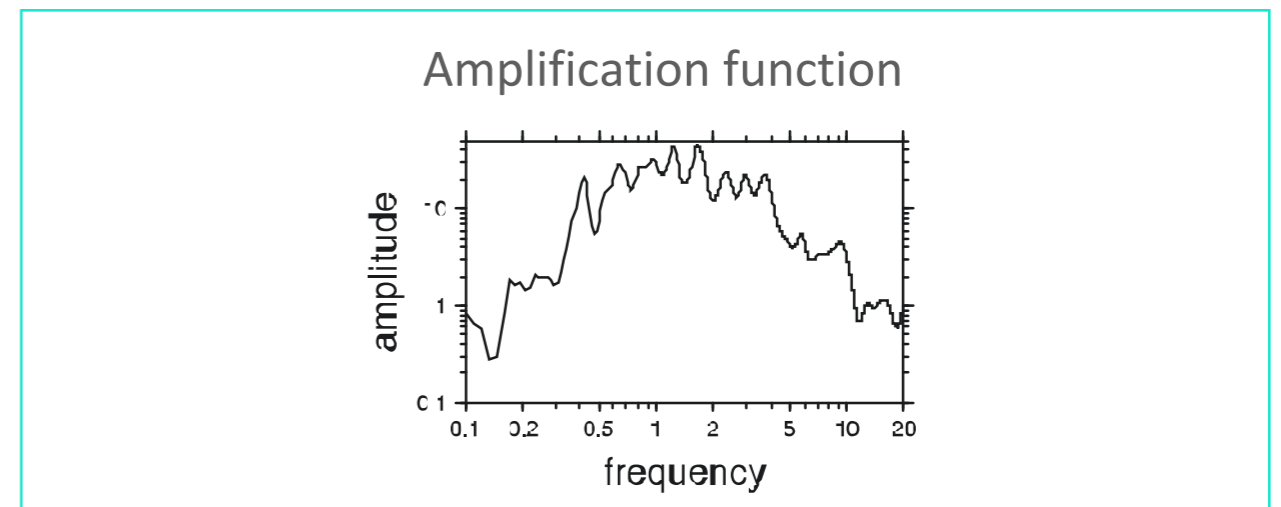
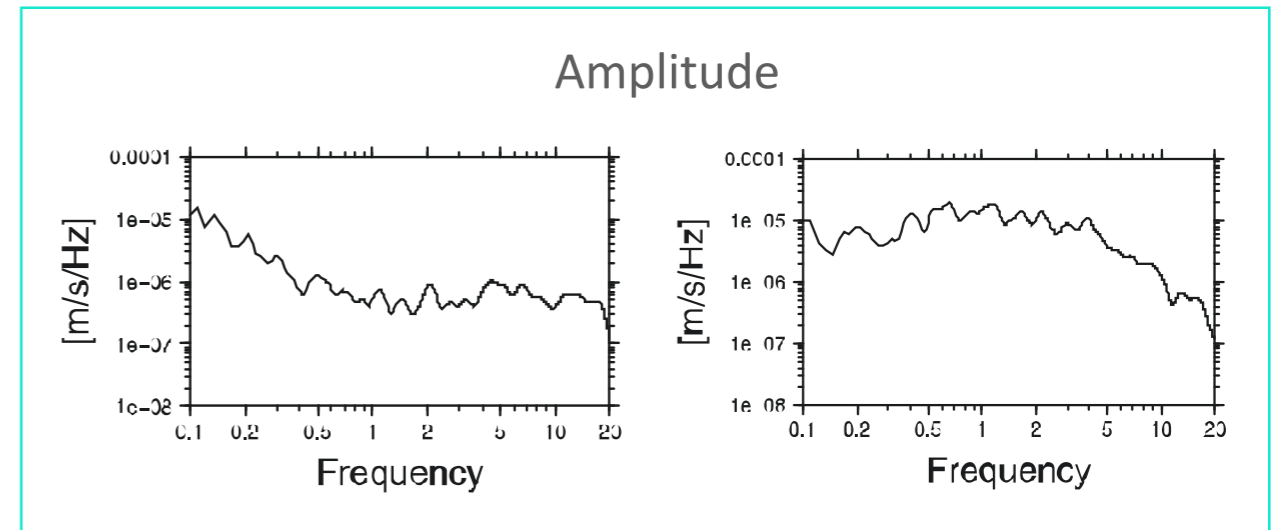
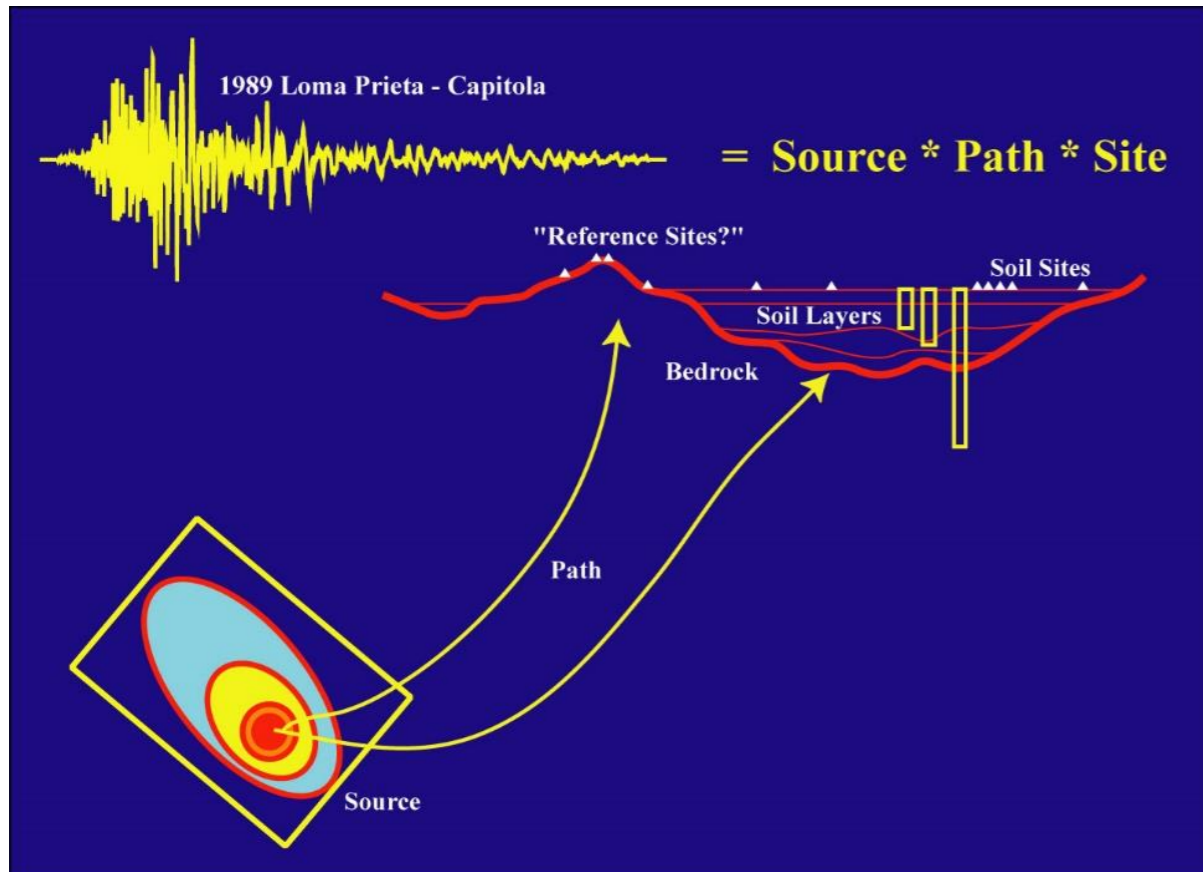
- a) peak frequencies examination
- b) S/B spectral ratio
- c) H/V spectral ratio (Nagoshi, 1971; Nakamura, 1989)
- d) array analysis (Malagnini et al., 1993)

Earthquake based Reference Site methods

- 1) Standard spectral ratio: spectral ratio between the same ground motion components of 2 close stations
- 2) Generalized inversion techniques: a spectral inversion is performed in order to correct for the path effects if the reference station is faraway from the actual one.



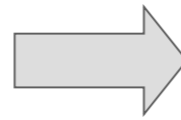
Fourier Amplitude Spectra A(f)



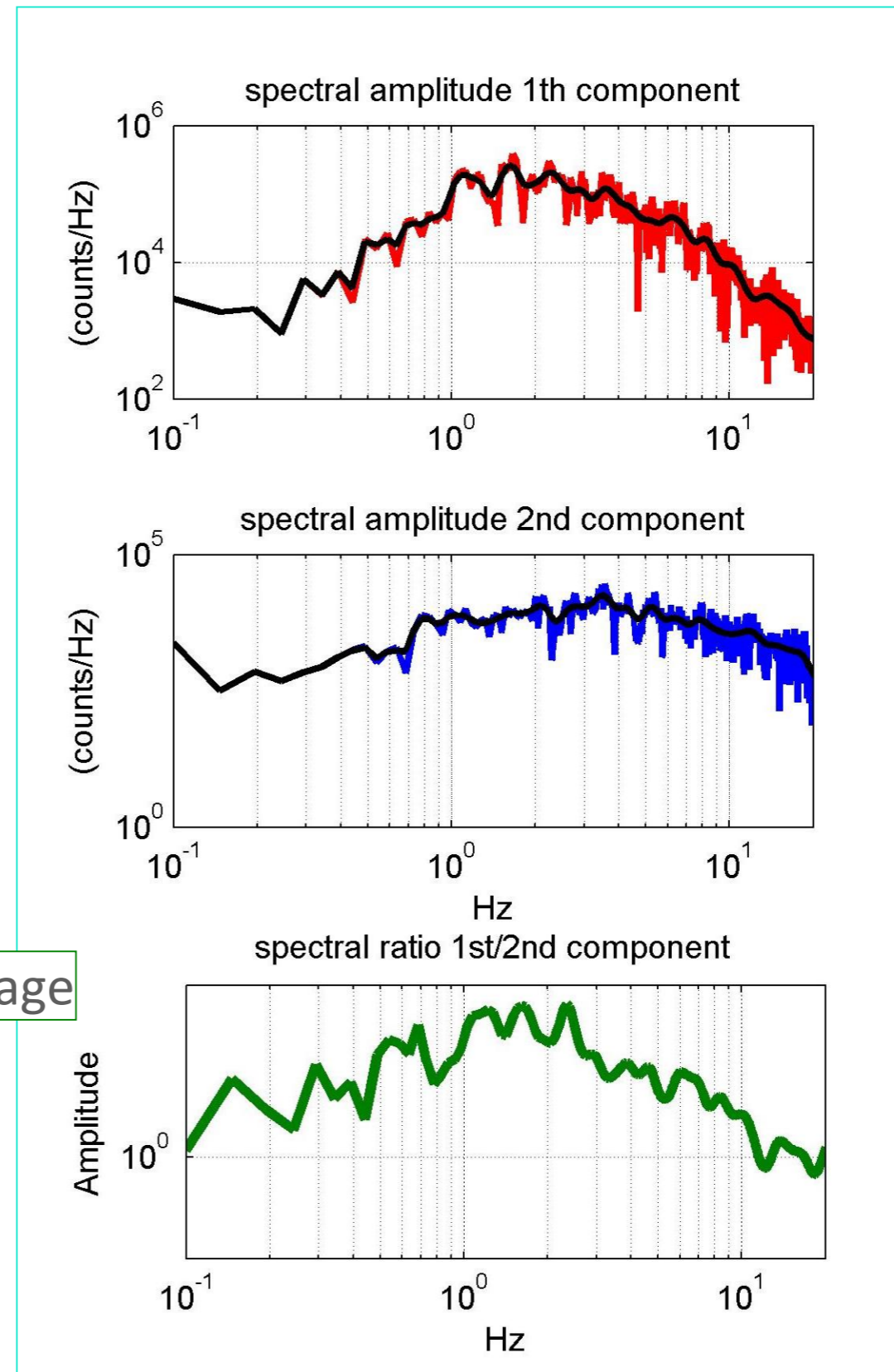
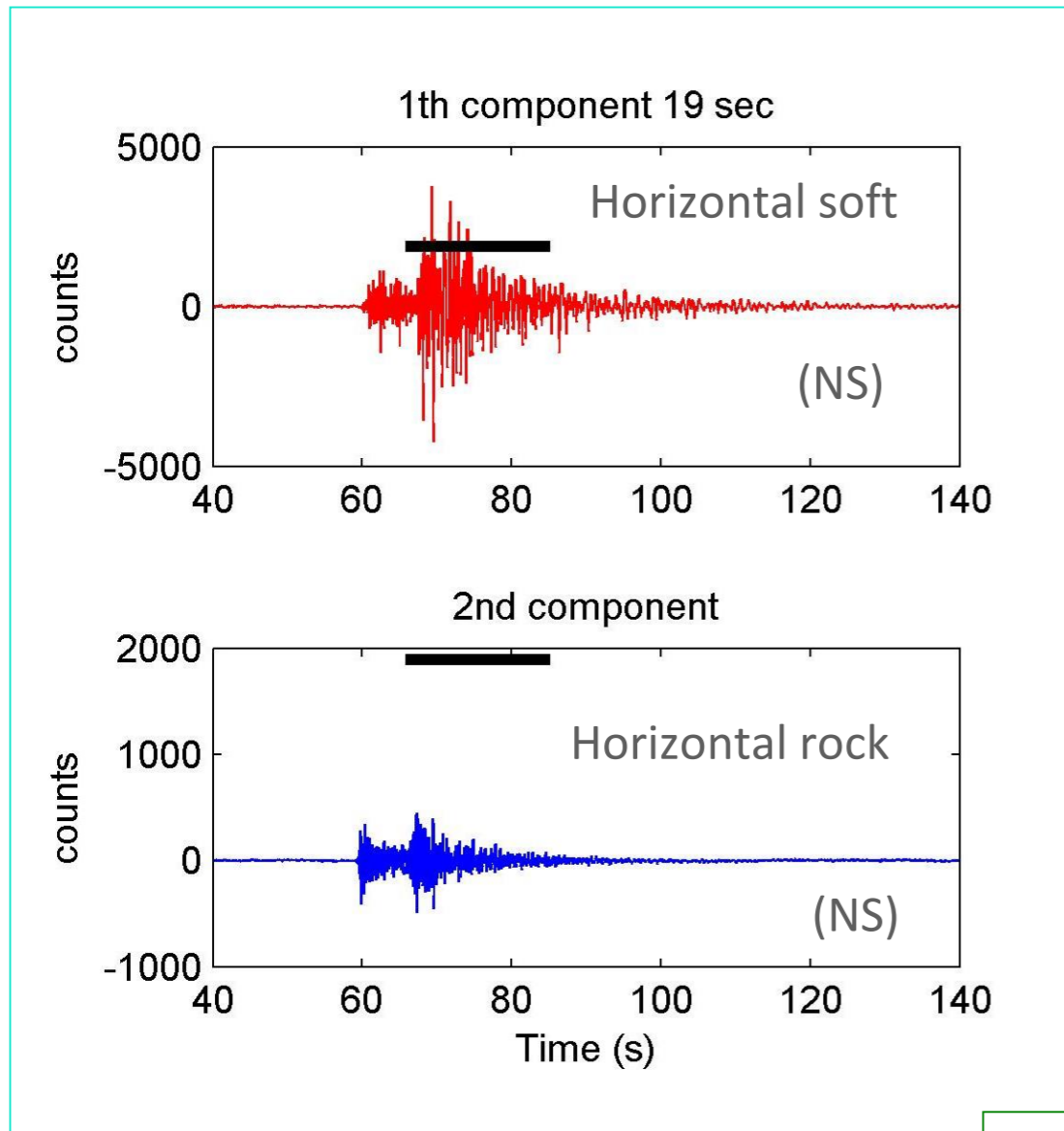
$$\frac{A_{soil}(f)}{A_{rock}(f)} = \frac{\cancel{\text{Source}}_{soil} \cancel{\text{Path}}_{soil} \cancel{\text{Site}}_{soil}}{\cancel{\text{Source}}_{rock} \cancel{\text{Path}}_{rock} \text{Site}_{rock}} = \frac{\cancel{\text{Path}}_{soil} \cancel{\text{Site}}_{soil}}{\cancel{\text{Path}}_{rock}} = \text{Site}_{soil}$$

=1
(reference)

Window selection in time domain

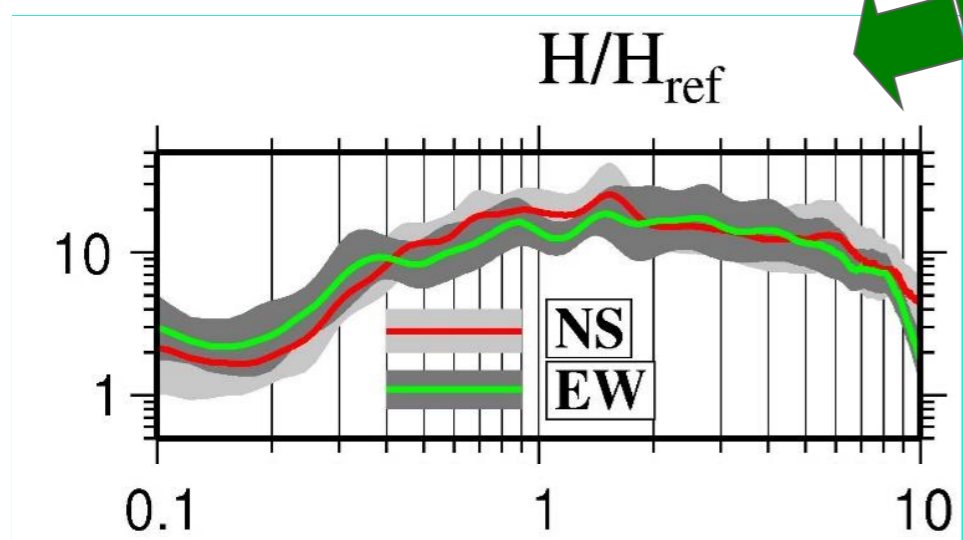


Fourier amplitude and smoothing



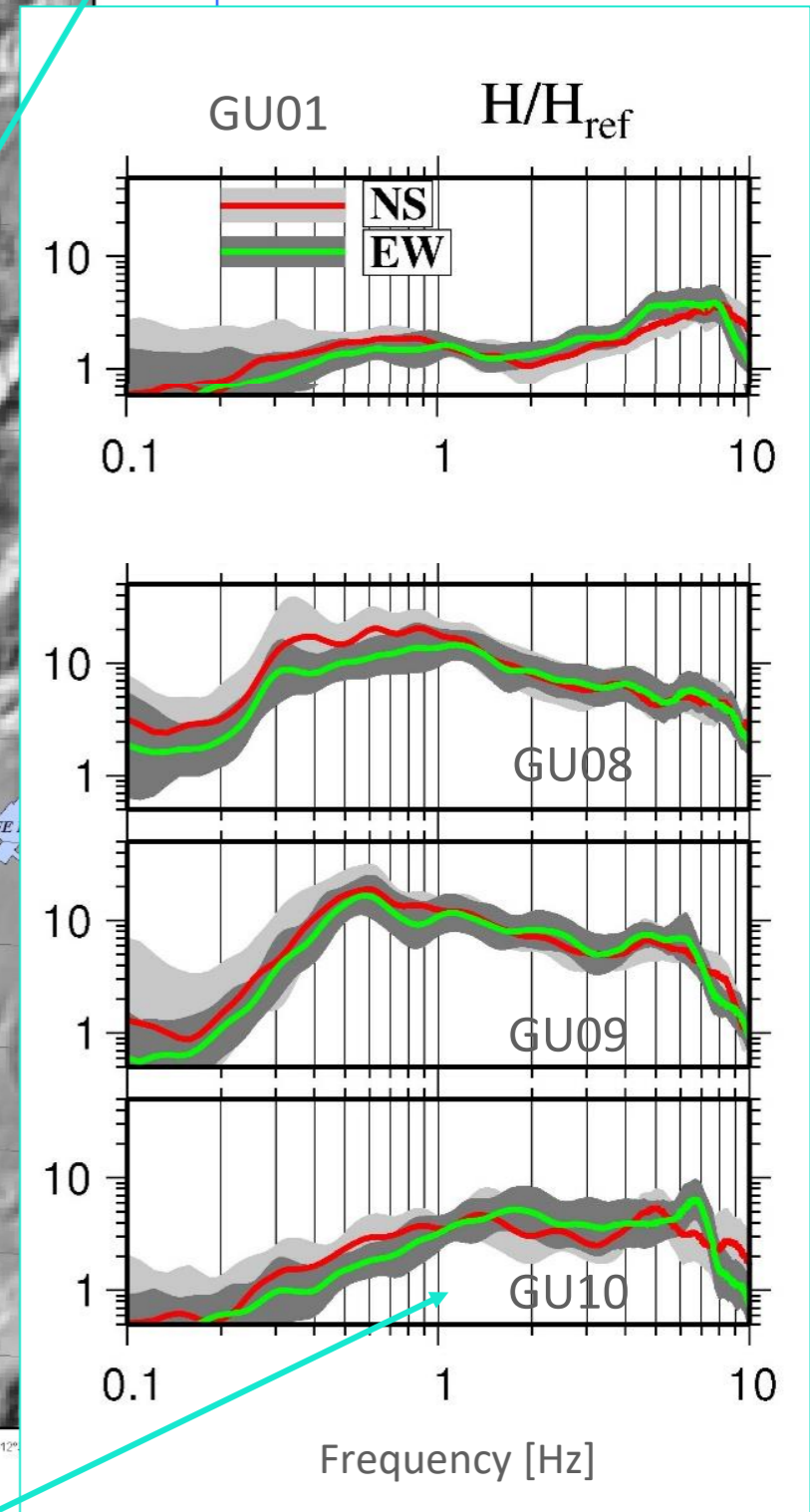
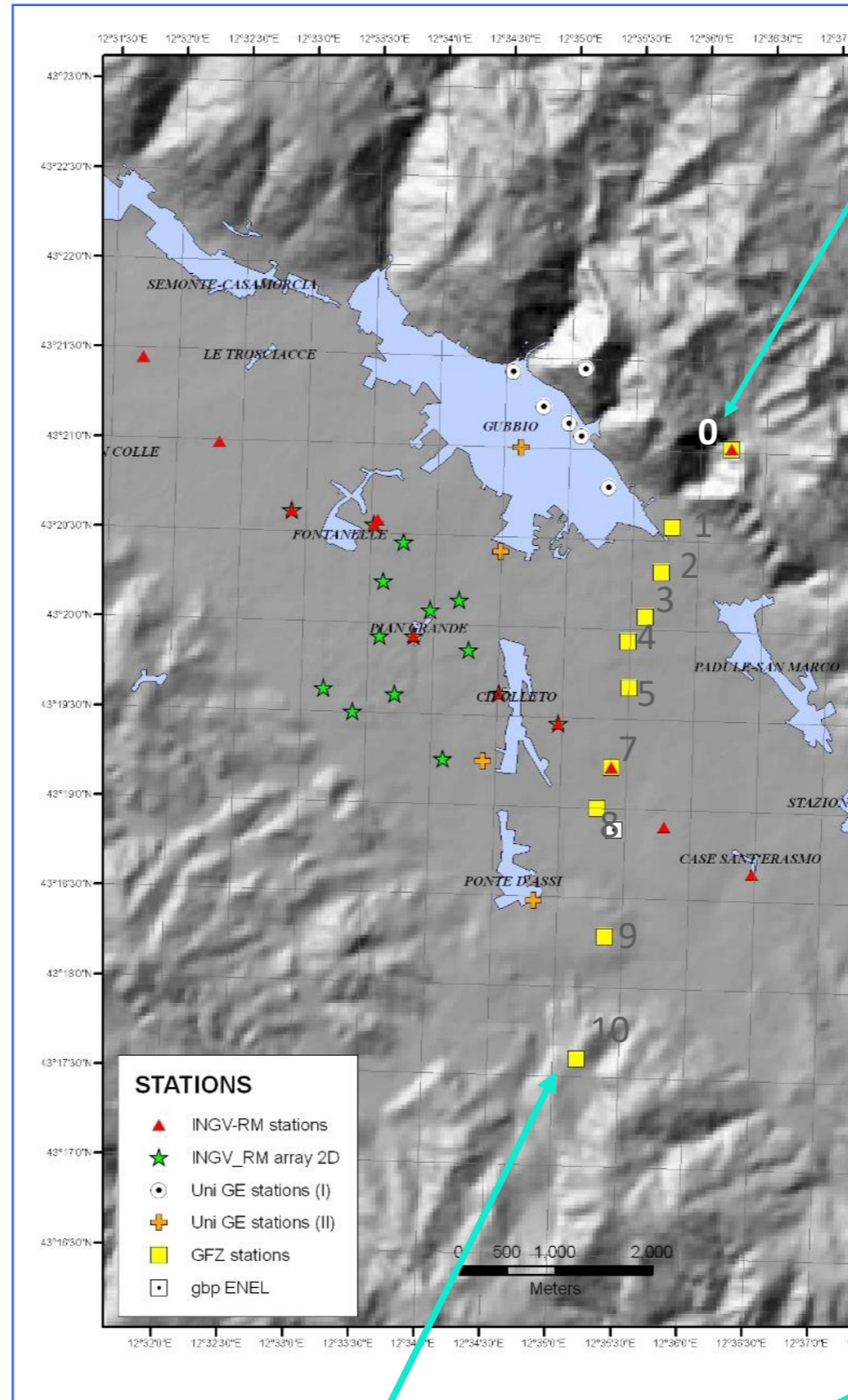
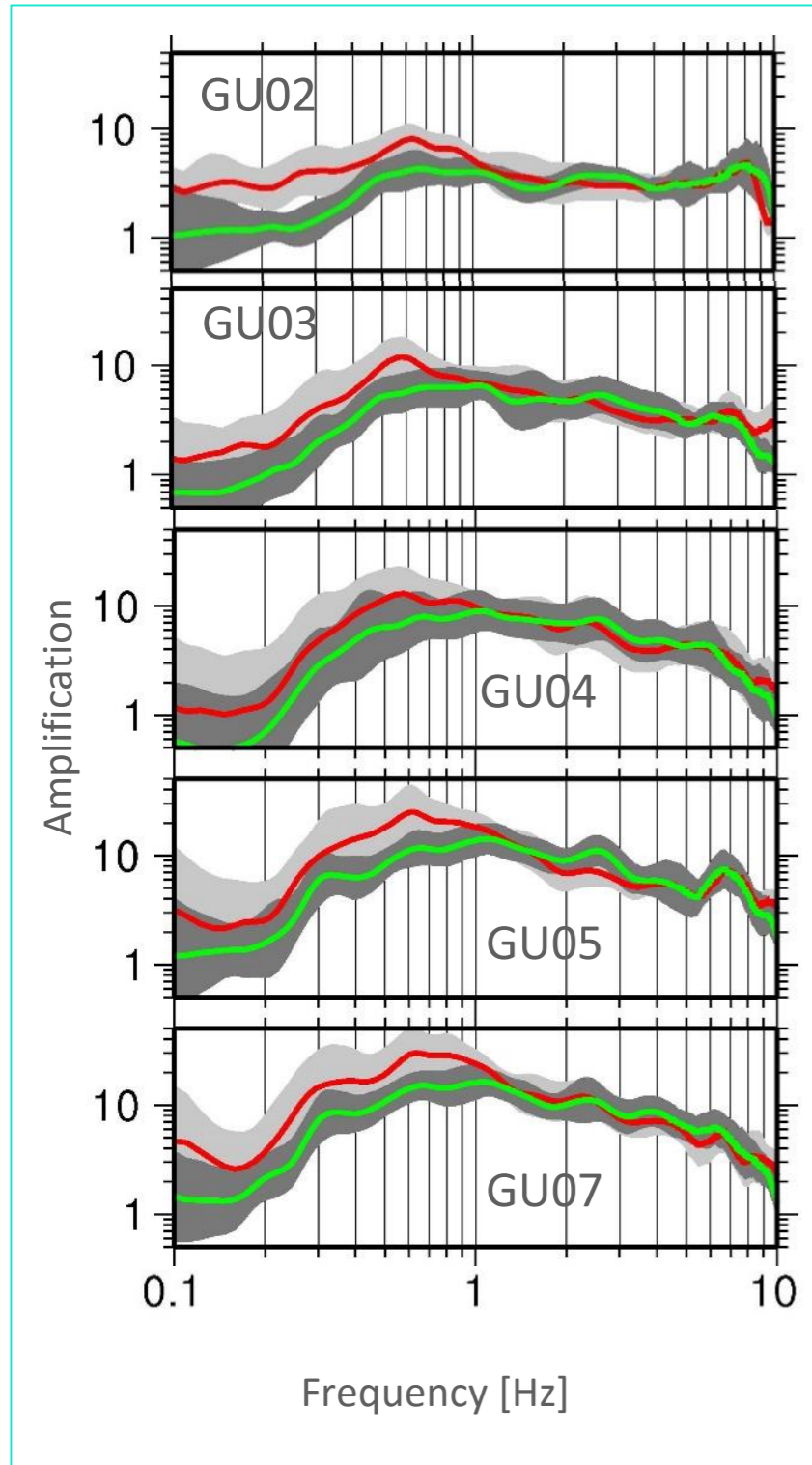
$$\begin{array}{c}
 H(f) \\
 \hline
 H(f) \\
 = \\
 |SSR(f)|
 \end{array}$$

Average



Standard spectral ratios: the example of Gubbio basin (Italy)

“Good” reference site



another rock station but

Important issues in SRE

- Near surface effects: impedance contrast, velocity
 - geological maps, V_{S30}
 - Basin effects
- Basin-edge induced waves
 - Subsurface focusing

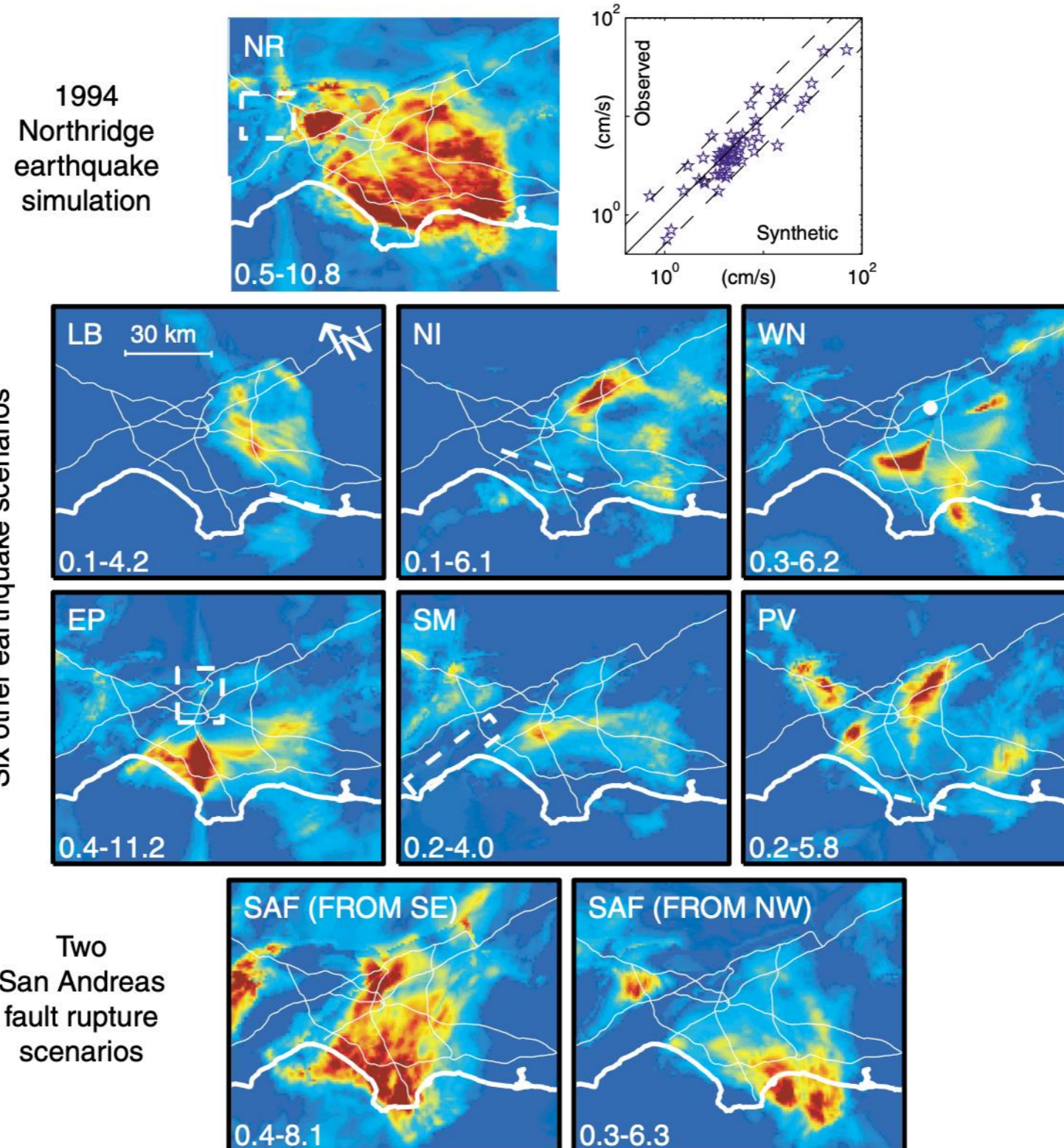
In SHA the site effect should be defined as the average behavior, relative to other sites, given all potentially damaging earthquakes.

This produces an **intrinsic variability** with respect to different earthquake locations, that cannot exceed the difference between sites

Amplification patterns...

....may vary greatly among the earthquake scenarios, considering different source locations (and rupture ...)

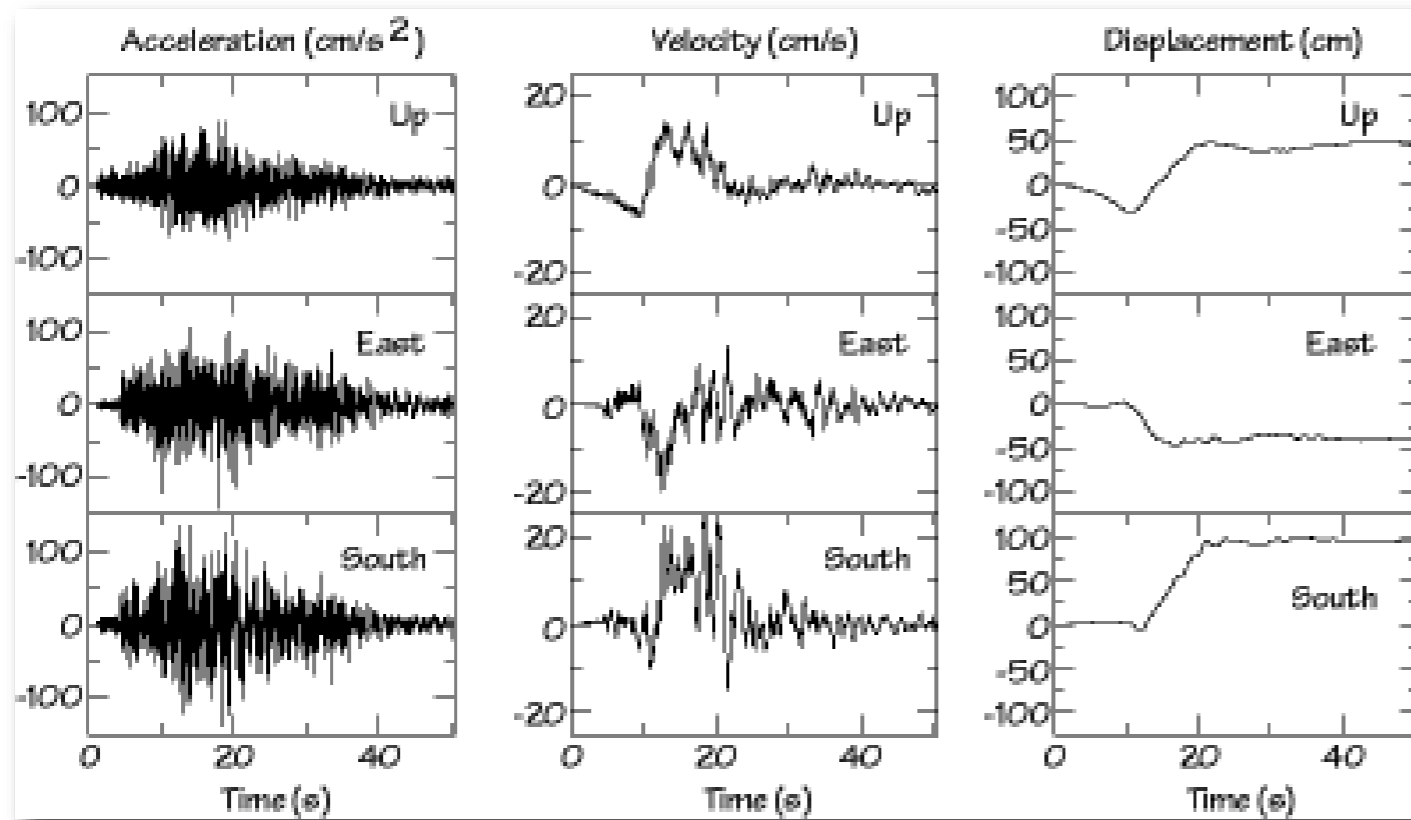
Peak Velocity Amplification from the 3D Simulations of Olsen (2000)



SCEC
Phase 3
Report

Six other earthquake scenarios

Seismic Source effects



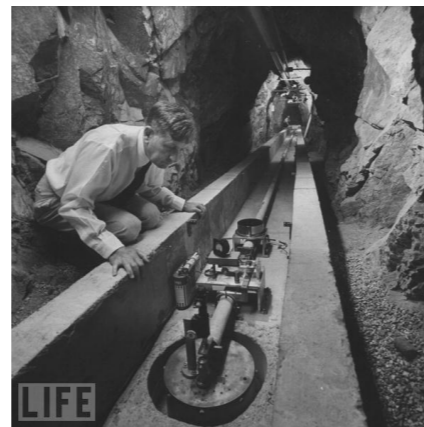
Michoacan, 1985

Fling & Directivity aka

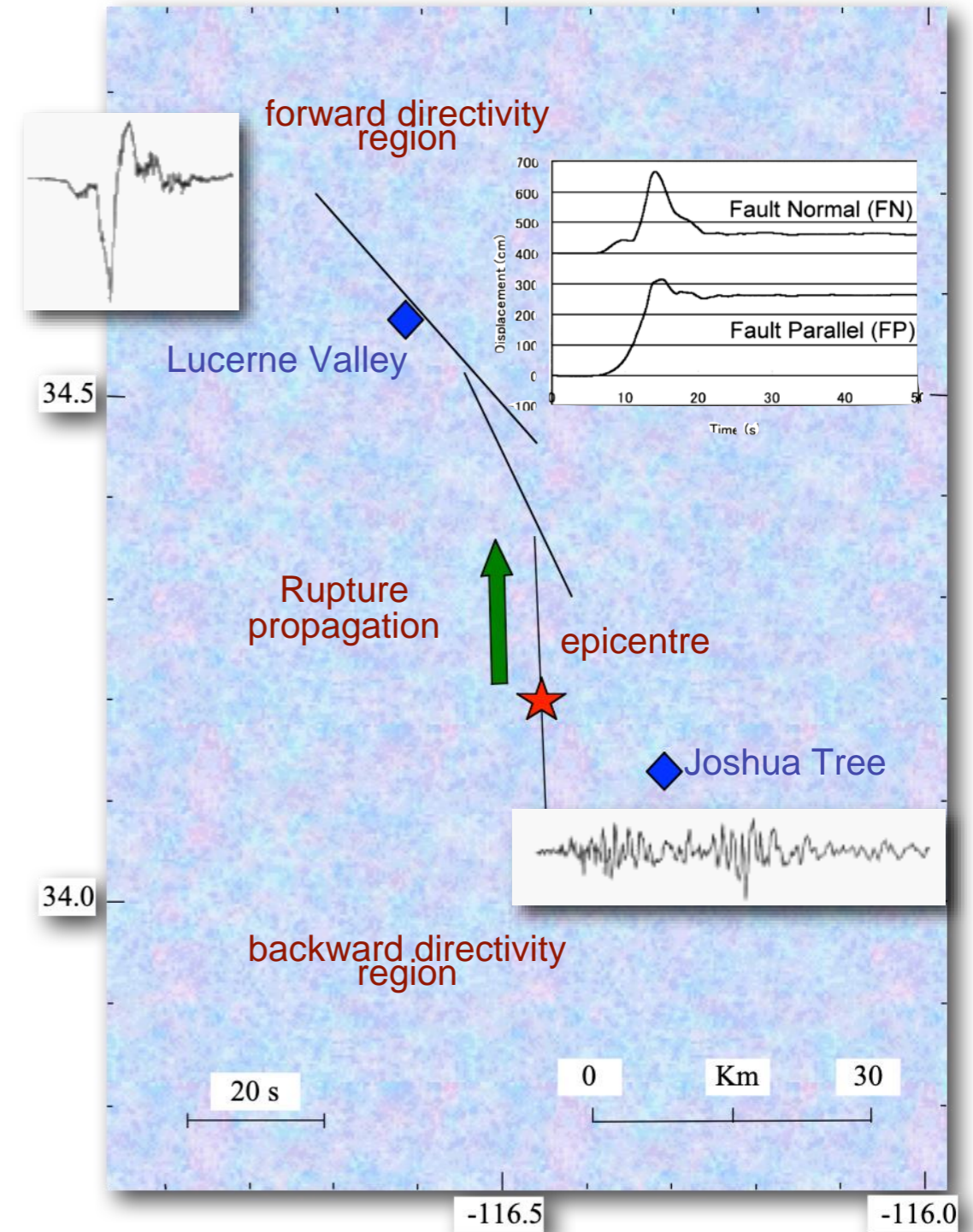
Near-field & Near-source



Sir Georges Stokes



Hugo Benioff



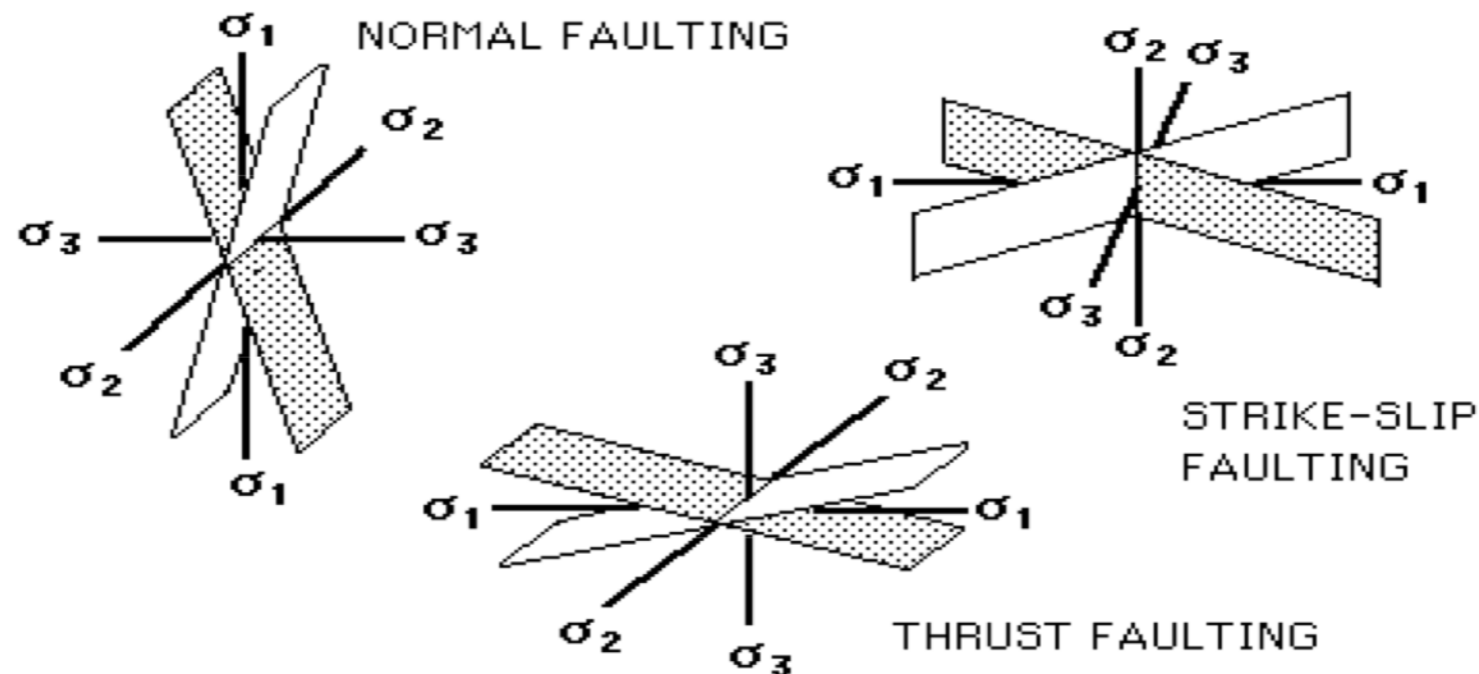
Landers, 1992

Seismic source: Friction & Stresses

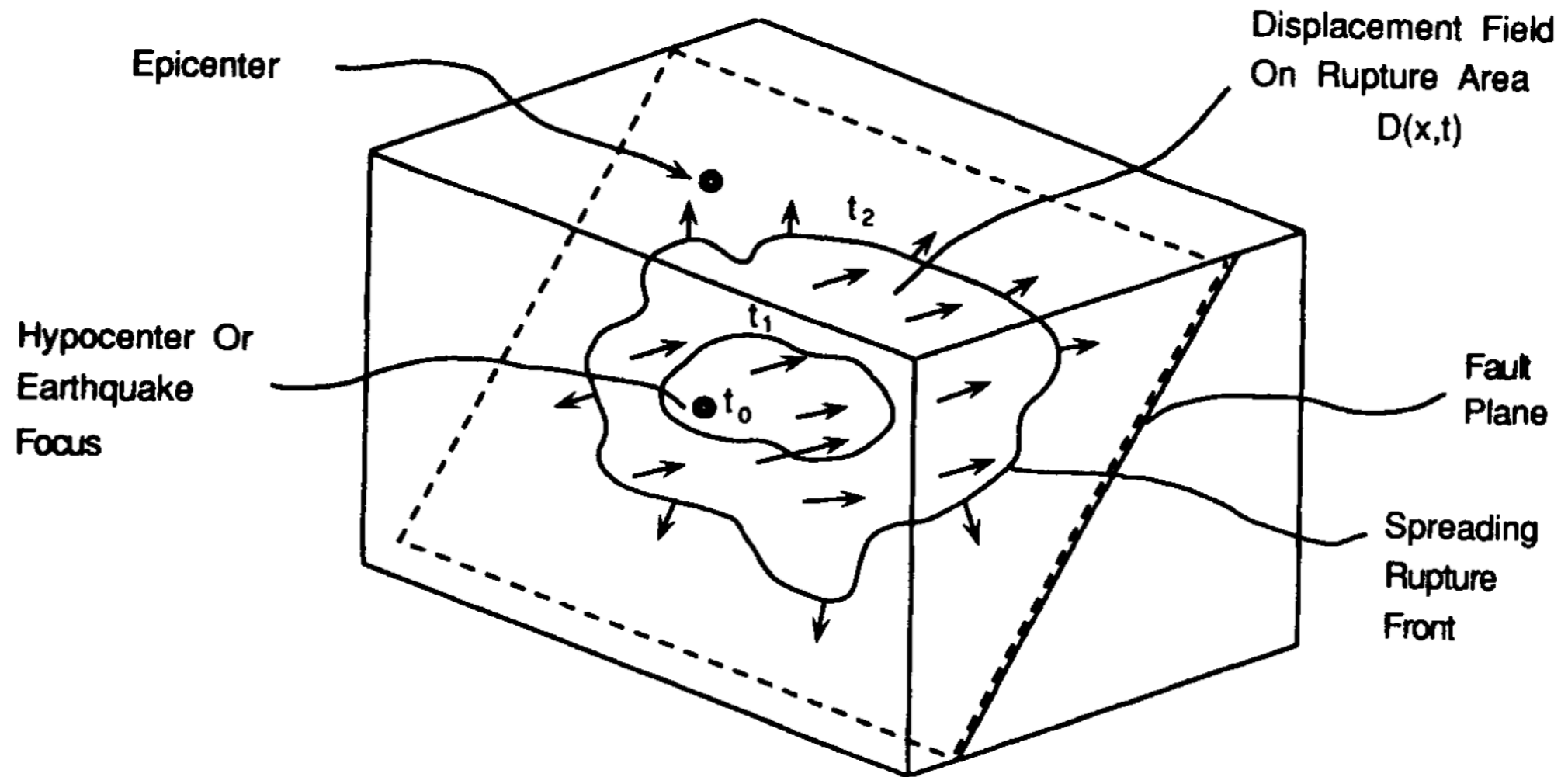
A number of factors can control friction: temperature, slip rate and slip history. Many materials become weaker with repeated slip (slip weakening).

They may exhibit an inverse dependence of friction on slip velocity (velocity weakening). Stick slip behaviour is observed only at temperatures below 300°C.

Anderson's theory of faulting: he recognized that principal stress orientations could vary among geological provinces within the upper crust of the earth. He deduced the connection between three common fault types: normal, strike-slip, and thrust and the three principal stress systems arising as a consequence of the assumption that one principal stress must be normal to the earth's surface.



Rupture process



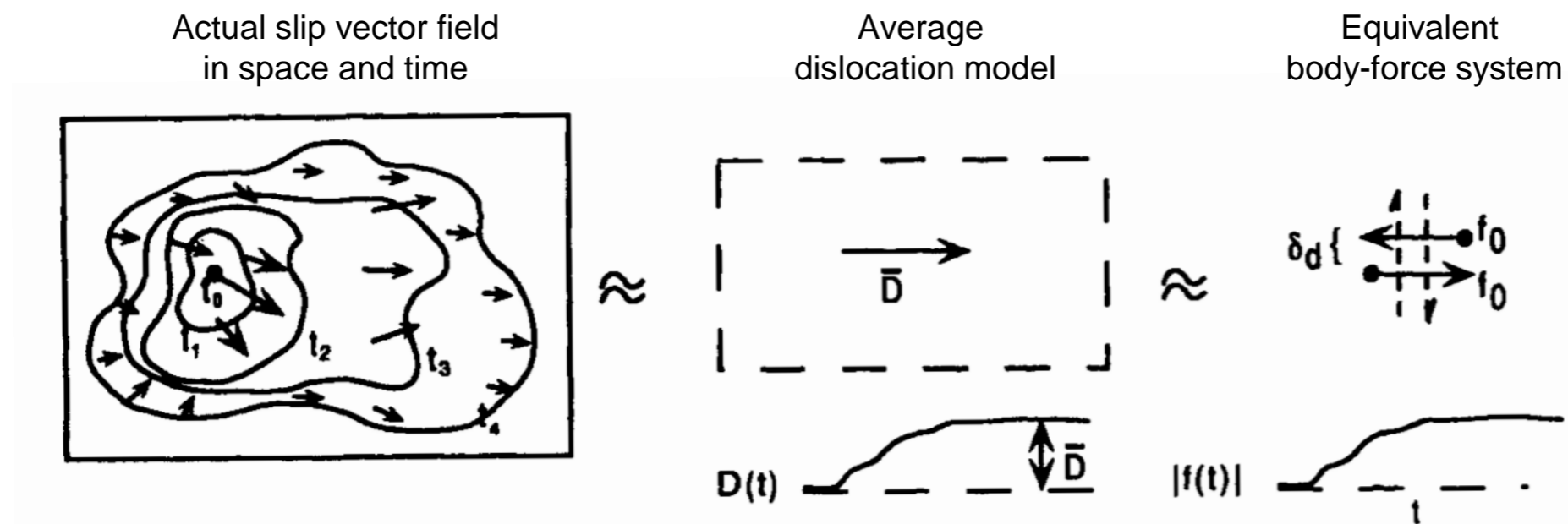
Schematic diagram of rupture on a fault plane. Slipping points radiate outgoing P- and S-waves. In general, rupture wavefront is not regular and slip vector, as well as slipping time, is different for the points on the fault.

Fault slip involves three main stages:

- 1) initiation of fault sliding
- 2) rupture front expansion
- 3) termination of rupture process.

Equivalent Forces

The observable seismic radiation is through energy release as the fault surface moves: formation and propagation of a crack. This complex dynamical problem can be studied by kinematical equivalent approaches.



The scope is to develop a representation of the displacement generated in an elastic body in terms of the quantities that originated it: body forces and applied tractions and displacements over the surface of the body.

The actual slip process will be described by superposition of equivalent body forces acting in space (over a fault) and time (rise time).

Fundamental papers

- Maruyama T. (1963). On the force equivalents of dynamical elastic dislocations with reference to the earthquake mechanism. *Bulletin of the Earthquake Research Institute* 41: 467–486.
- Burridge R. and Knopoff L. (1964). Body force equivalents for seismic dislocations. *Bulletin of the Seismological Society of America* 54: 1875–1878.

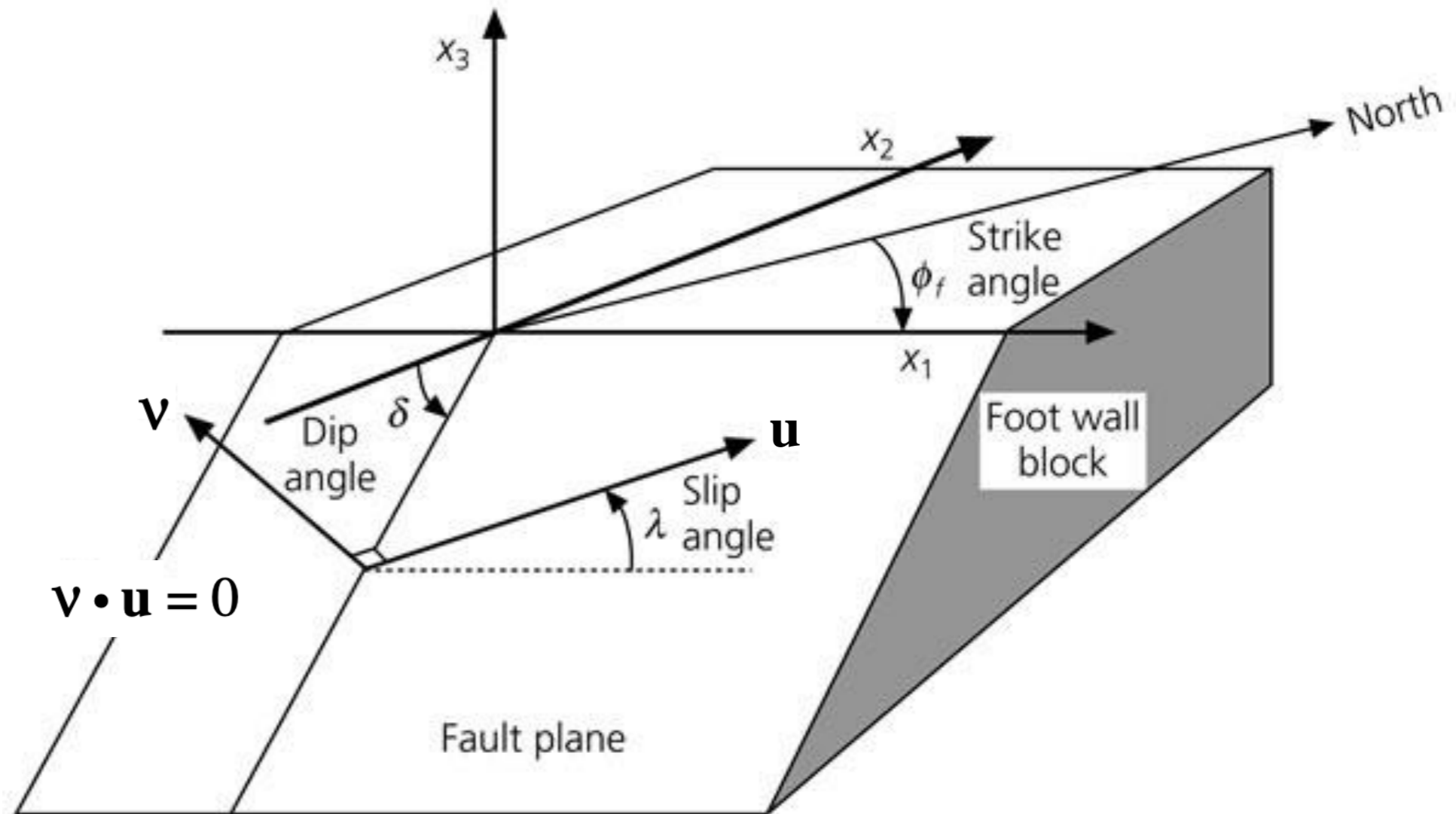
“An explicit expression is derived for the body force to be applied in the absence of a dislocation, which produces radiation identical to that of the dislocation. This equivalent force depends only upon the source and the elastic properties of the medium in the immediate vicinity of the source and not upon the proximity of any reflecting surfaces. The theory is developed for dislocations in an anisotropic inhomogeneous medium; in the examples isotropy is assumed. For displacement dislocation faults, the double couple is an exact equivalent body force.”



Leon Knopoff

Fault plane and slip vector

Figure 4.2-2: Fault geometry used in earthquake studies.



Final (point) source representation

And if the source can be considered a point-source (for distances greater than fault dimensions), the contributions from different surface elements can be considered in phase.

Thus for an effective point source, one can define the moment tensor, to be convolved with the (spatially derived) Green's function of the medium:

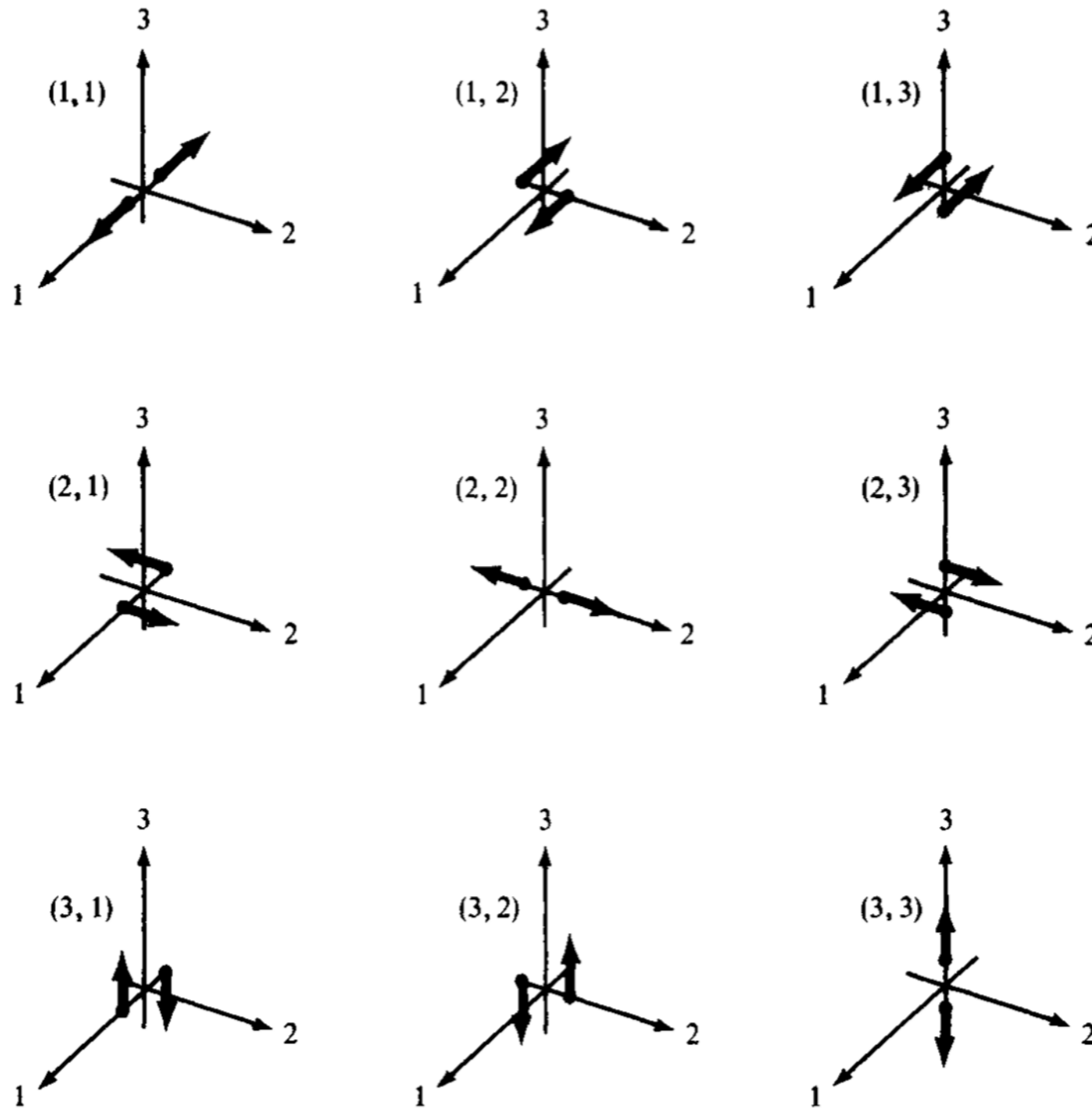
$$u_n(\mathbf{x}, t) = M_{pq} * G_{np,q}$$

For a shear dislocation, the equivalent point force is a **double-couple**, since internal faulting implies that the total force $\mathbf{f}^{[u]}$ and its total moment are null. The seismic moment has a **null trace** and **one of the eigenvalues is 0**.

$$M_{pq}(\text{doublecouple}) = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix} \quad \text{with } M_0 = \mu A[\bar{u}]$$

M_0 is called **seismic moment**, a scalar quantity related to the area of the fault and to the slip, averaged over the fault plane.

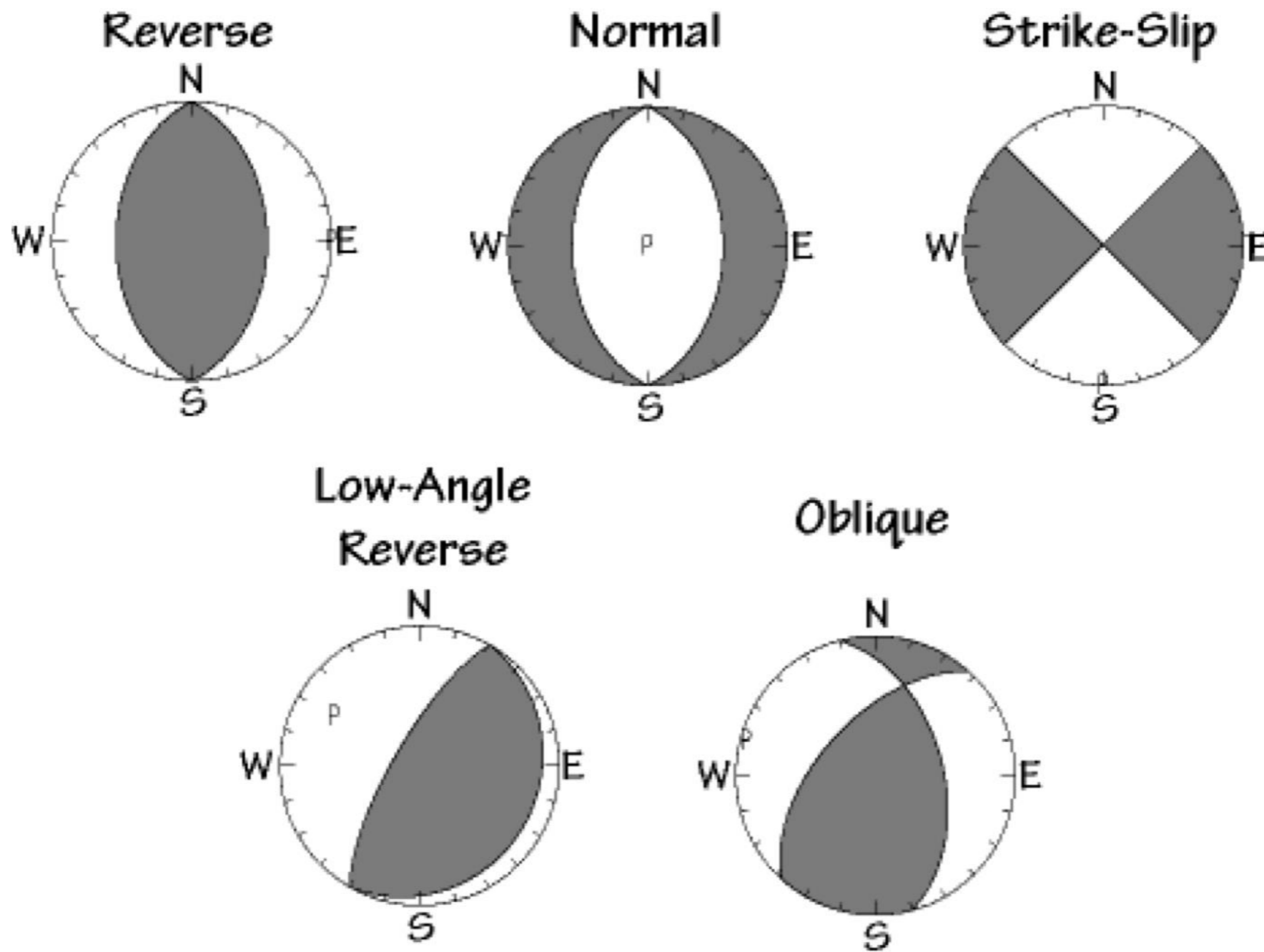
Moment tensor components



Point sources can be described by the seismic moment tensor M_{pq} , whose elements have clear physical meaning of **forces acting on particular planes.**

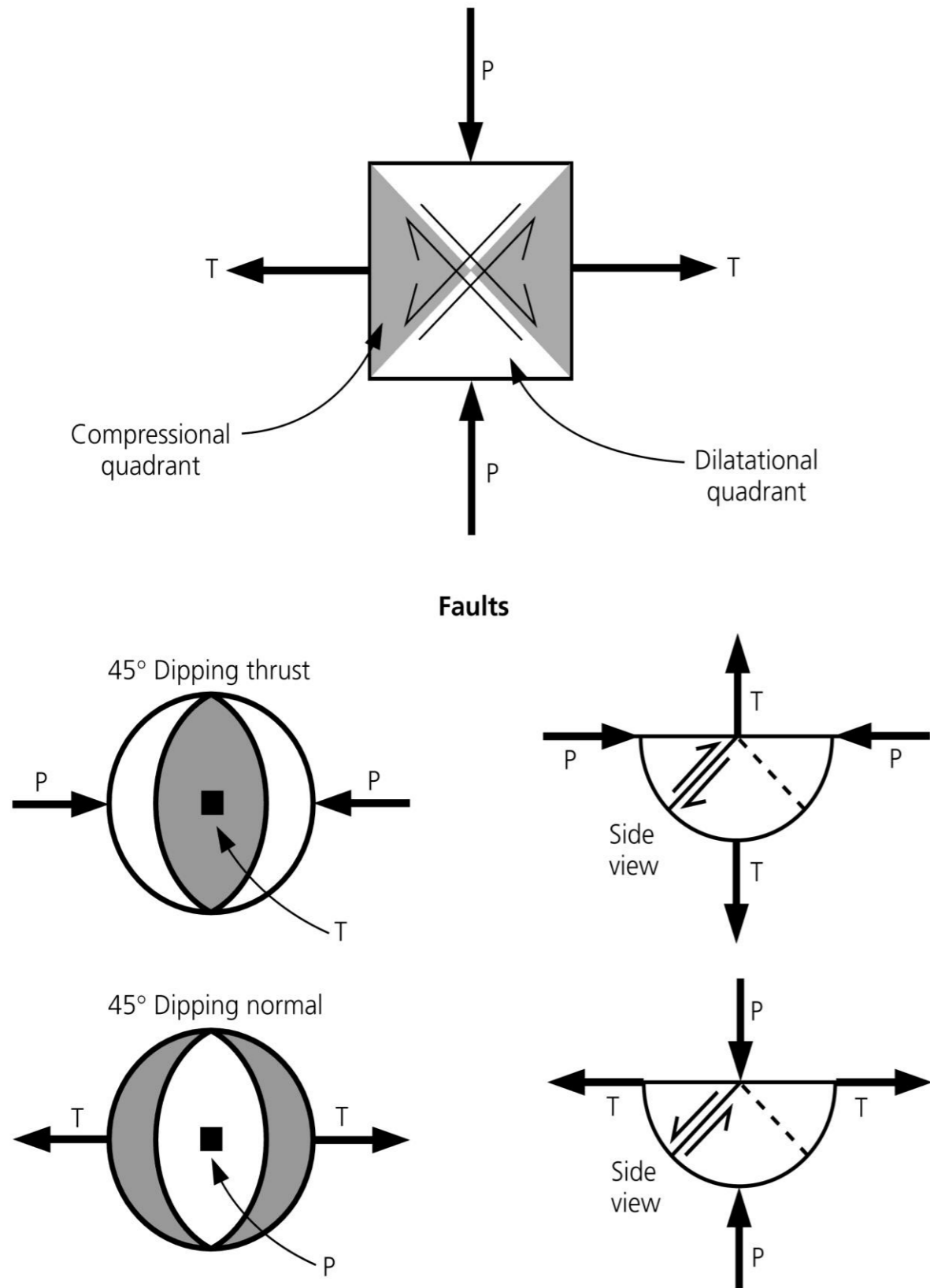
The nine possible couples that are required to obtain equivalent forces for a generally oriented displacement discontinuity in anisotropic media.

The Principal Focal Mechanisms

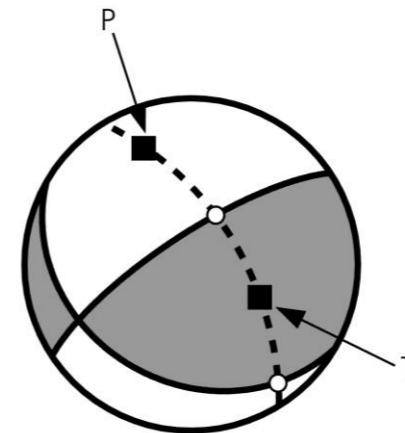


FM & stress axes

Figure 4.2-16: Relation between fault planes and stress axes.



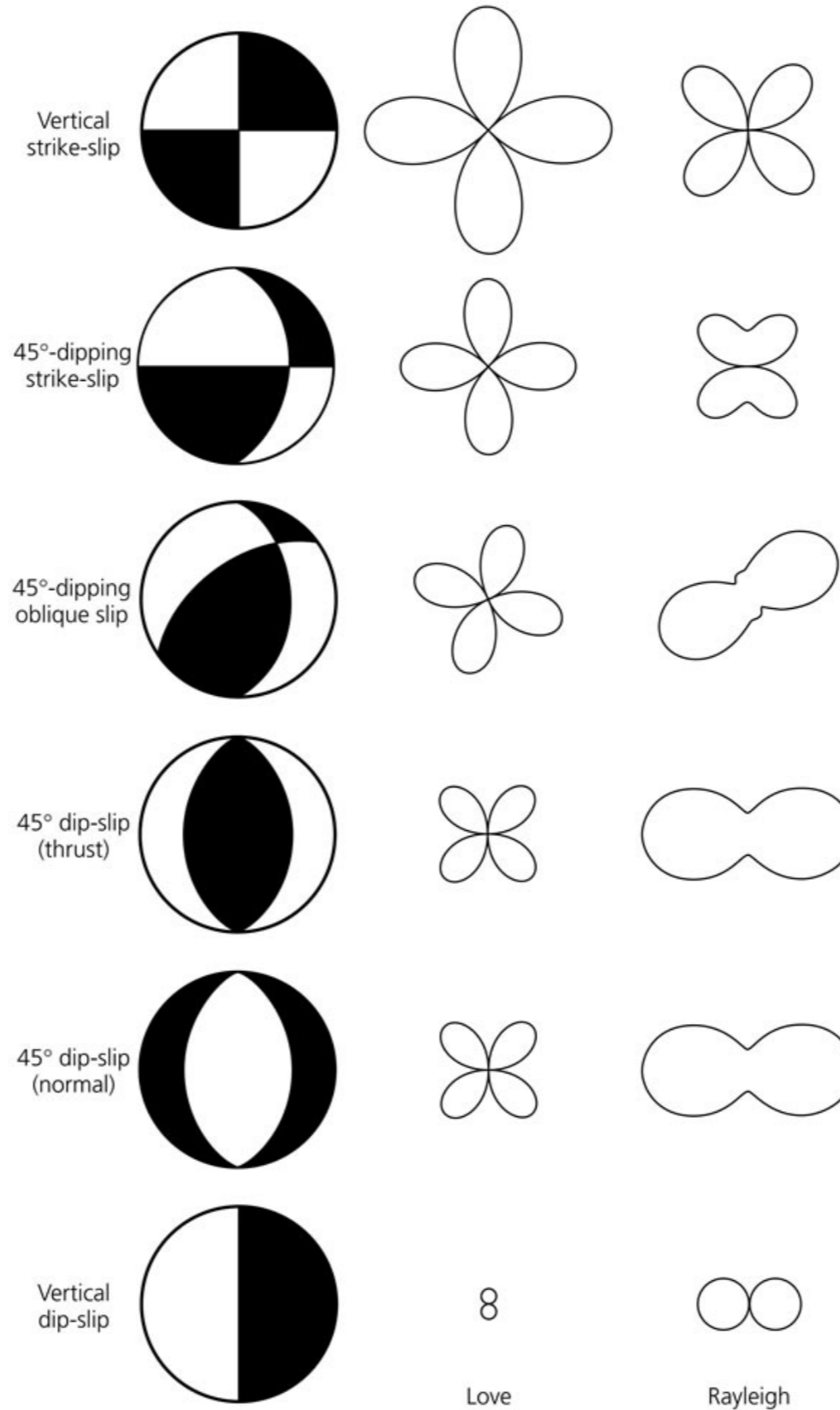
To obtain P and T axes:



On the meridian connecting the poles, the points half-way between the nodal planes are the **P** and **T** axes

Radiation pattern & surface waves

Figure 4.3-12: Surface wave amplitude radiation patterns for several focal mechanisms.



Haskell dislocation model

Haskell N. A. (1964). Total energy spectral density of elastic wave radiation from propagating faults, Bull. Seism. Soc. Am. **54**, 1811-1841

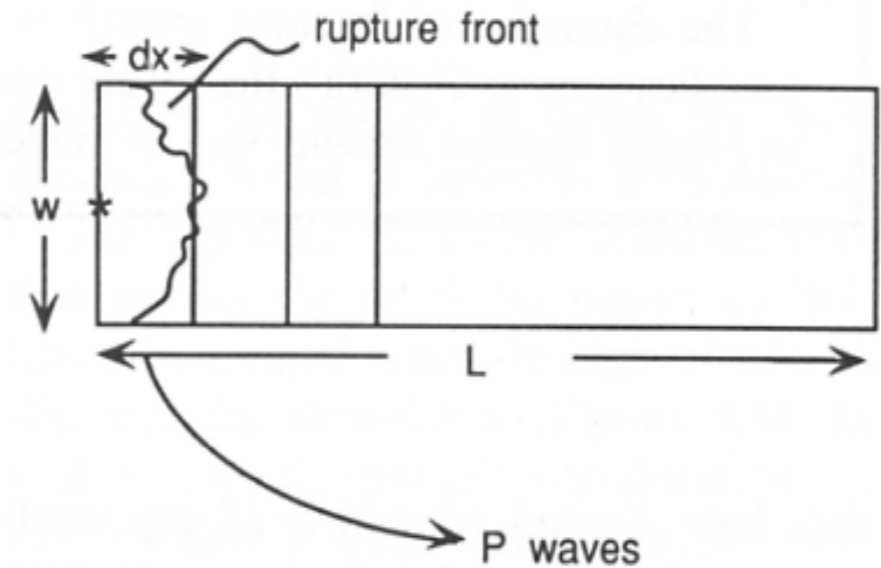
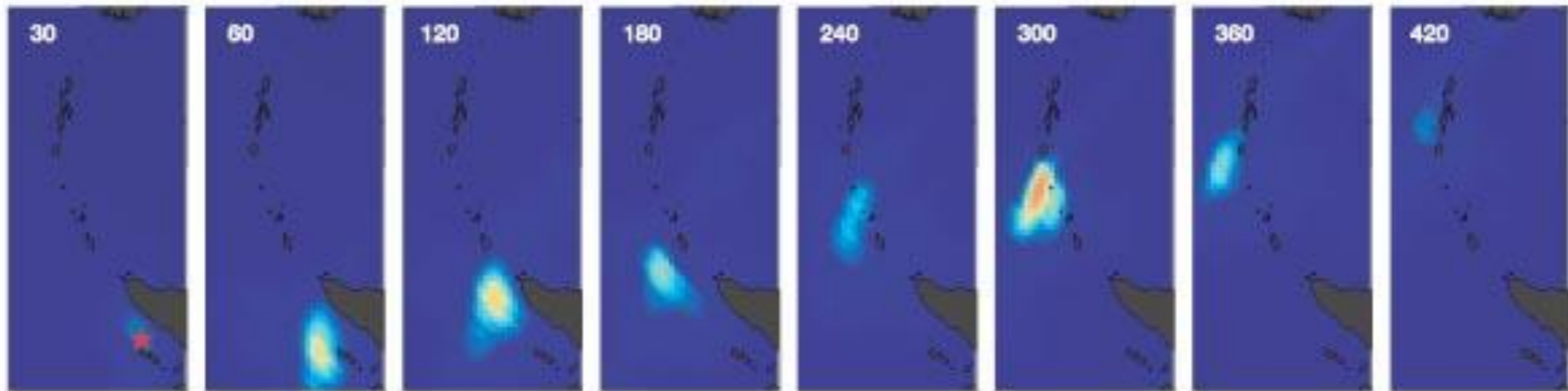


FIGURE 9.5 Geometry of a one-dimensional fault of width w and length L . The individual segments of the fault are of length dx , and the moment of a segment is $m dx$. The fault ruptures with velocity v_r .

Sumatra earthquake, Dec 26, 2004



Ishii et al., Nature 2005 doi:10.1038/nature03675

Haskell source model: far field

resulting in the convolution of two boxcars: the first with duration equal to the rise time and the second with duration equal to the **rupture time** (L/v_r)

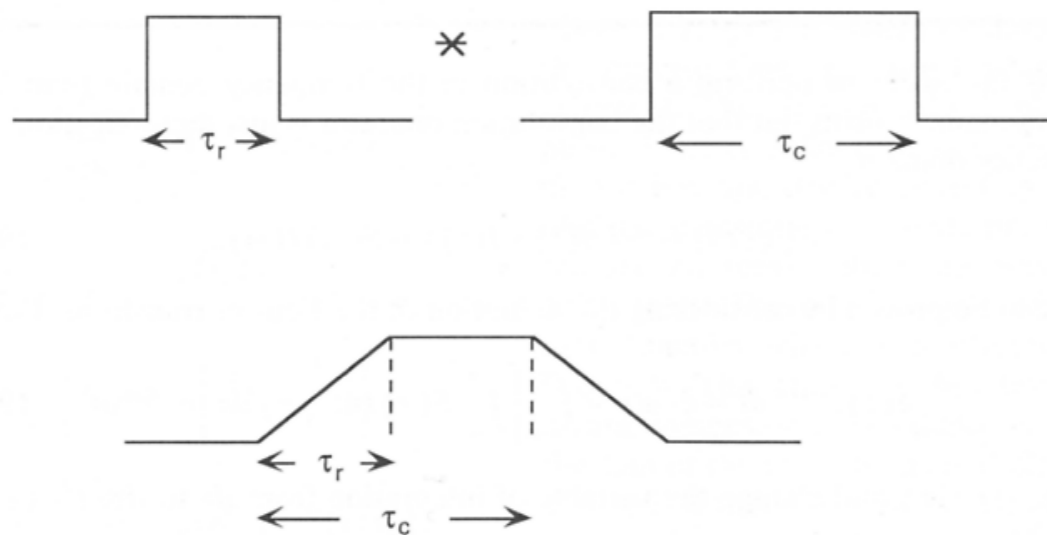


FIGURE 9.6 The convolution of two boxcars, one of length τ_r and the other of length τ_c ($\tau_c > \tau_r$). The result is a trapezoid with a rise time of τ_r , a top of length $\tau_c - \tau_r$, and a fall of width τ_r .

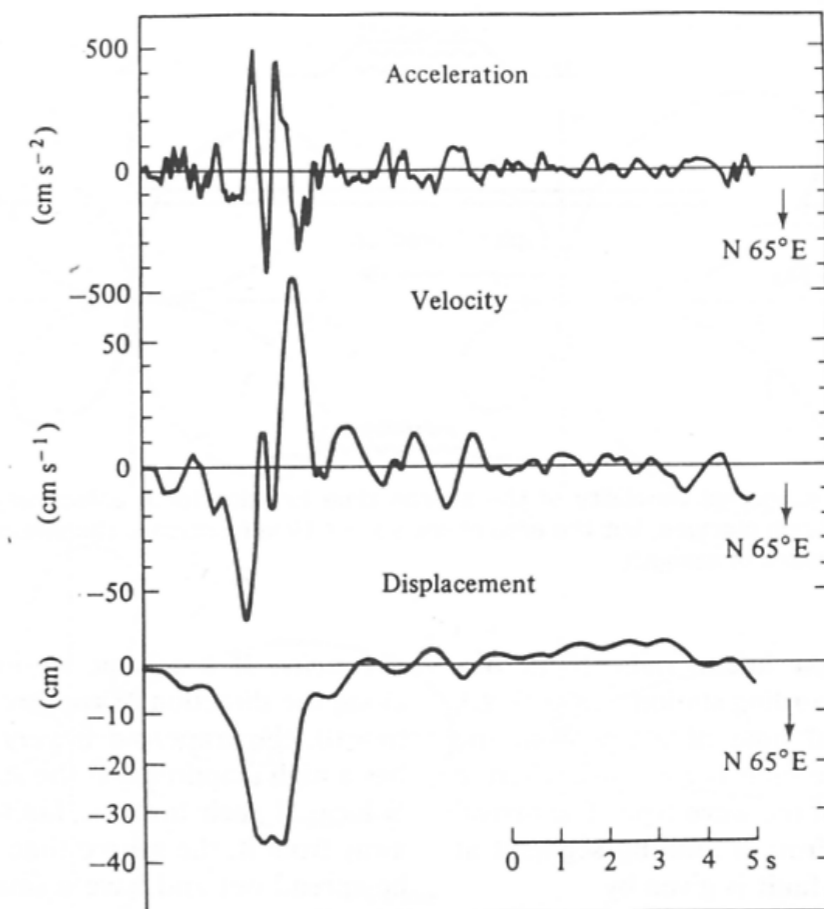
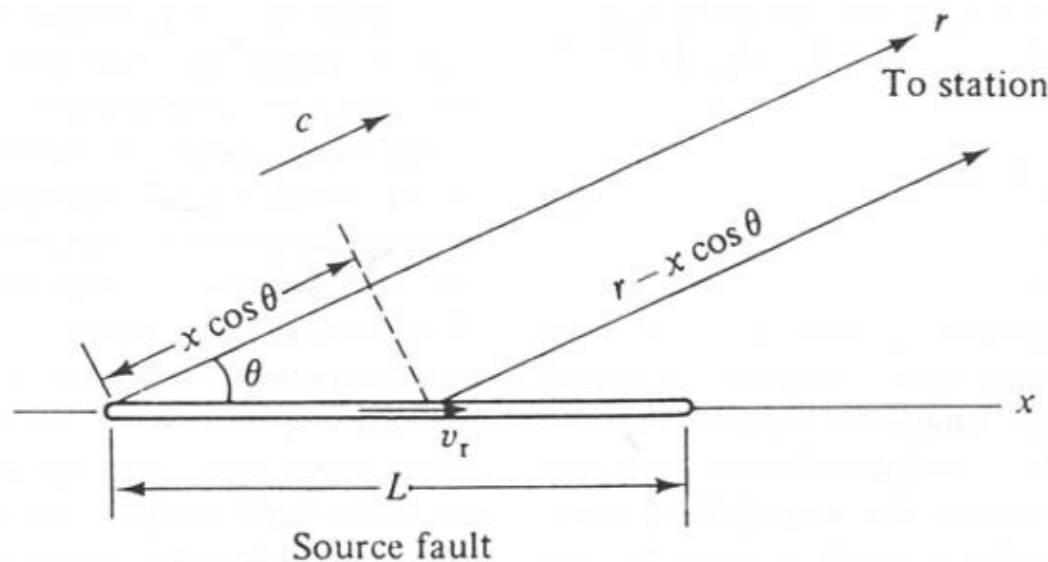


FIGURE 9.7 A recording of the ground motion near the epicenter of an earthquake at Parkfield, California. The station is located on a node for P waves and a maximum for SH . The displacement pulse is the SH wave. Note the trapezoidal shape. (From Aki, *J. Geophys. Res.* 73, 5359–5375, 1968; © copyright by the American Geophysical Union.)

Haskell source model: directivity

The body waves generated from a breaking segment will arrive at a receiver before than those that are radiated by a segment that ruptures later.

If the path to the station is not perpendicular, the waves generated by different segments will have different path lengths, and then unequal travel times.



$$T_r = \left[\frac{L}{v_r} + \left(\frac{r - L \cos \theta}{c} \right) \right] - \frac{r}{c} =$$

$$= \frac{L}{v_r} - \left(\frac{L \cos \theta}{c} \right) = \frac{L}{v_r} \left(1 - \frac{v_r}{c} \cos \theta \right)$$

FIGURE 9.8 Geometry of a rupturing fault and the path to a remote recording station.
(From Kasahara, 1981.)

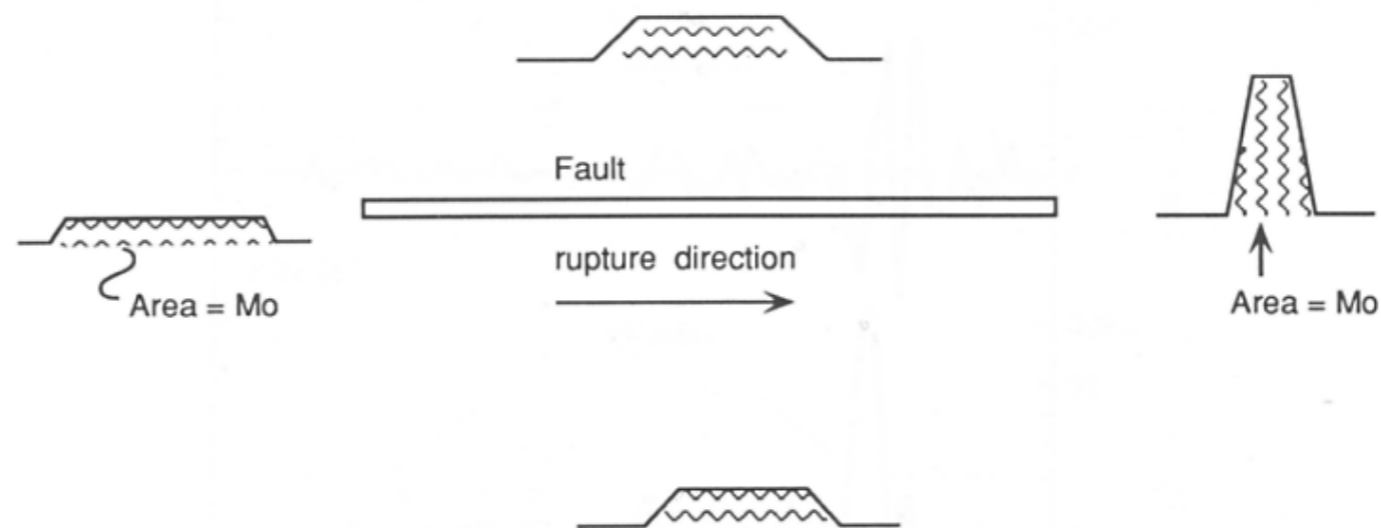


FIGURE 9.9 Azimuthal variability of the source time function for a unilaterally rupturing fault. The duration changes, but the area of the source time function is the seismic moment and is independent of azimuth.

Directivity example

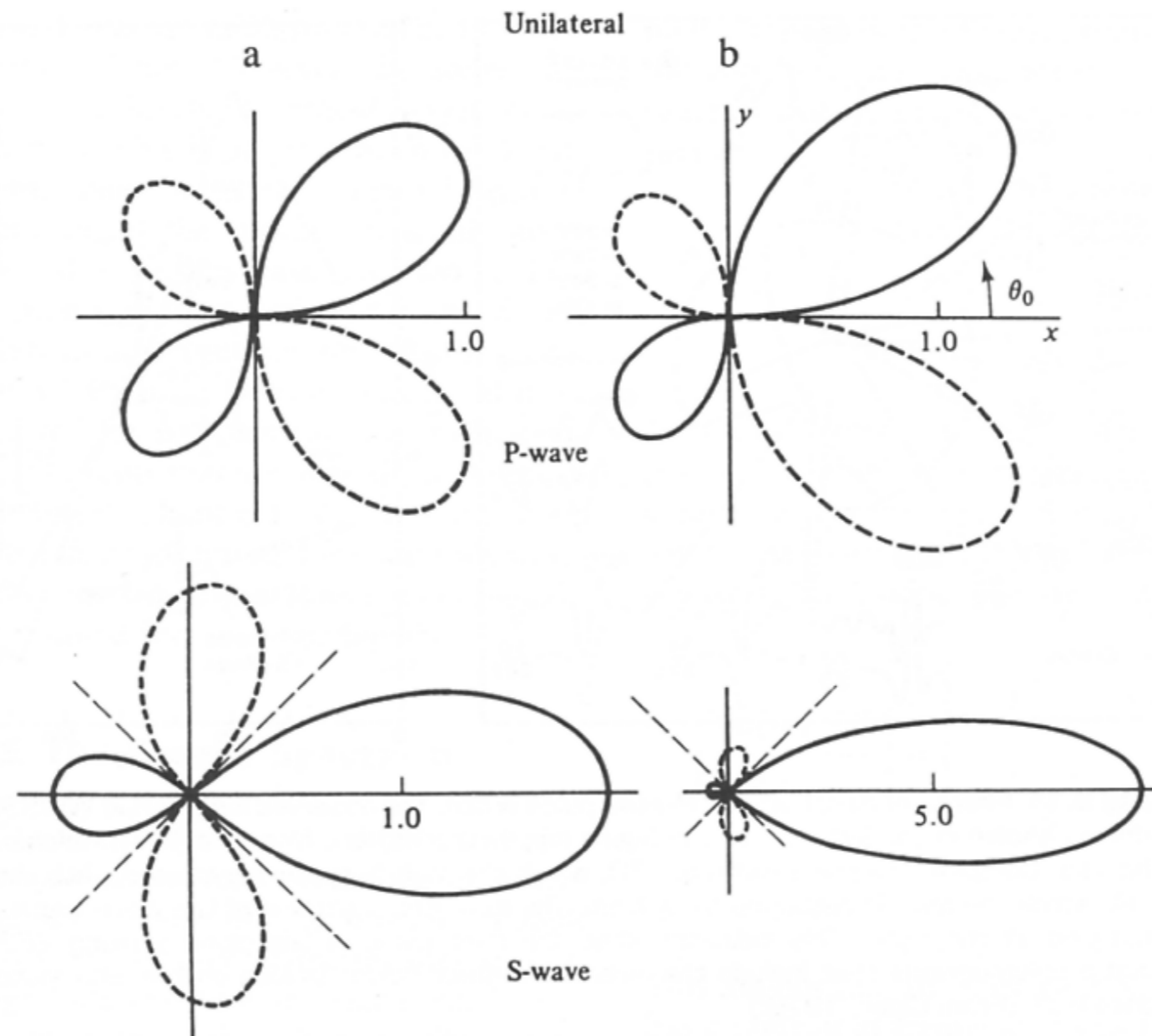


FIGURE 9.10 The variability of *P*- and *SH*-wave amplitude for a propagating fault (from left to right). For the column on the left $v_r/v_s = 0.5$, while for the column on the right $v_r/v_s = 0.9$. Note that the effects are amplified as rupture velocity approaches the propagation velocity. (From Kasahara, 1981.)

Source spectrum (amplitude)

The displacement pulse, corrected for the geometrical spreading and the radiation pattern can be written as:

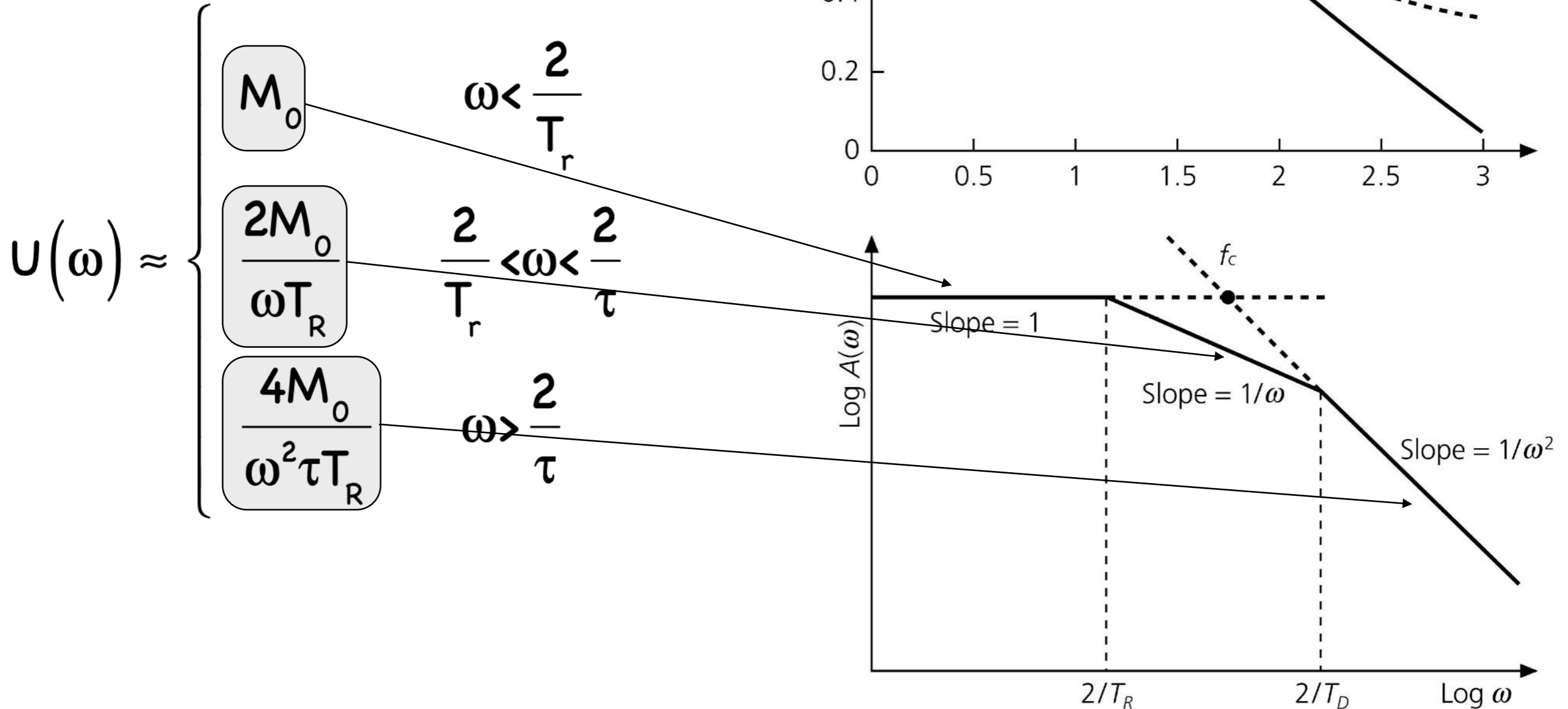
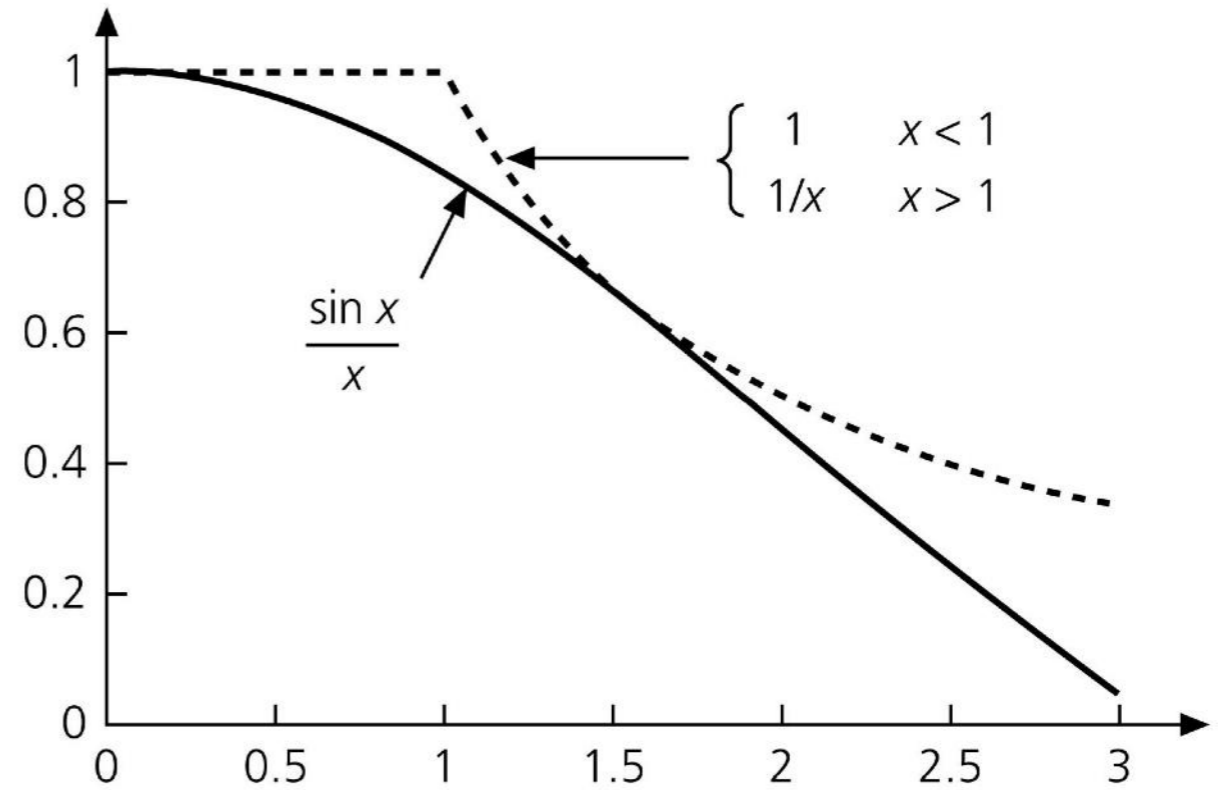
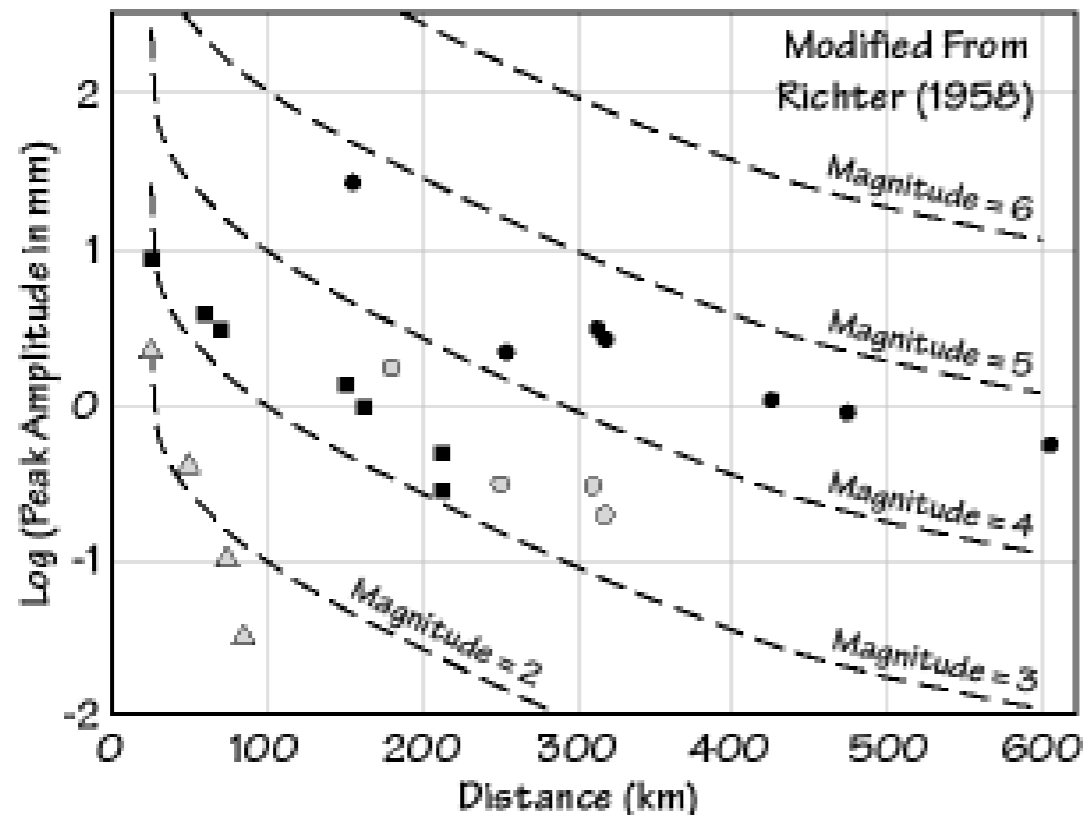


Figure 4.6-4: Approximation of the $(\sin x)/x$ function, and derivation of corner frequencies.



Magnitude Scales - Richter

The concept of magnitude was introduced by Richter (1935) to provide an objective instrumental measure of the size of earthquakes. Contrary to seismic intensity, I , which is based on the assessment and classification of shaking damage and human perceptions of shaking, the magnitude M uses instrumental measurements of earth ground motion adjusted for epicentral distance and source depth.



The original Richter scale was based on the observation that the amplitude of seismic waves systematically decreases with epicentral distance.

Data from local earthquakes in California



The relative size of events is calculated by comparison to a reference event, with $M_L=0$, such that A_0 was $1 \mu\text{m}$ at an epicentral distance, Δ , of 100 km with a Wood-Anderson instrument:

$$M_L = \log(A/A_0) = \log A - 2.48 + 2.76\Delta.$$

Magnitude Scales - Richter

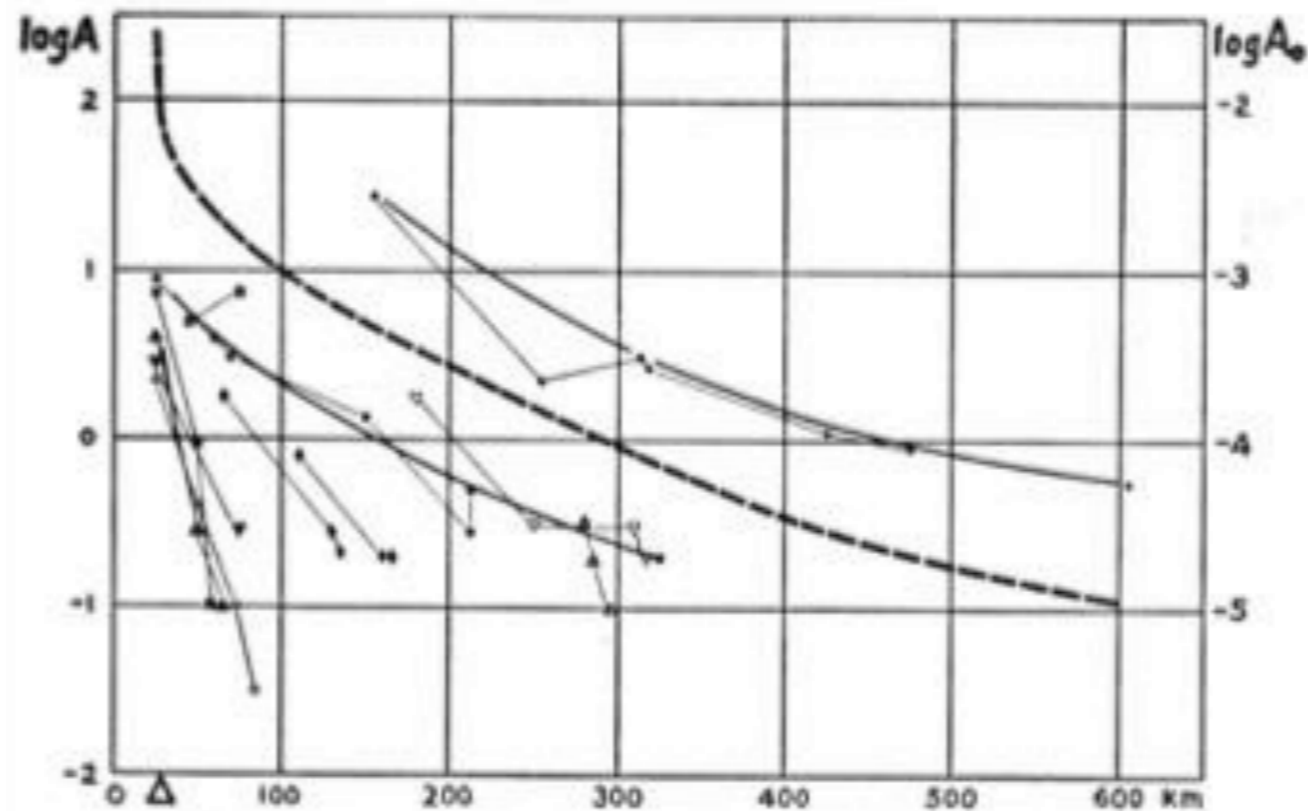
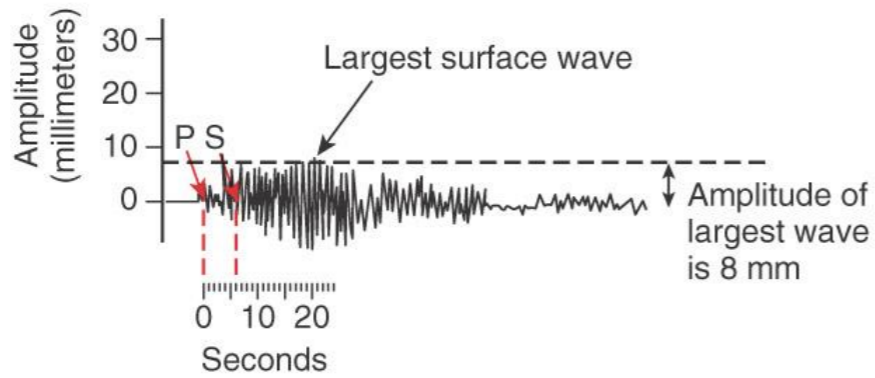


FIGURE 22-2 Origin of the magnitude scale. Data for Southern California earthquakes of January, 1932. [Redrafted from the original notes.]

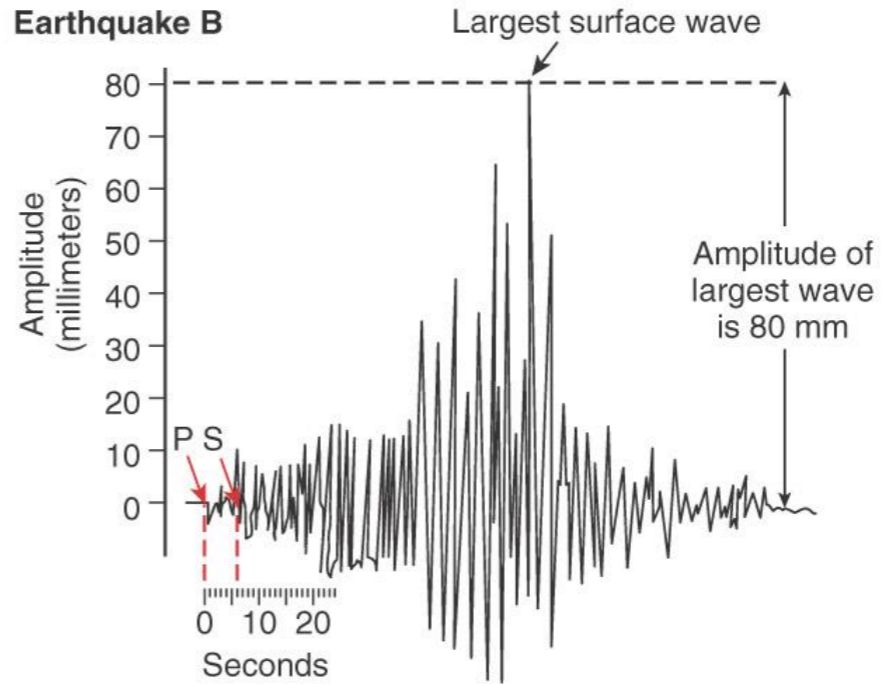
“I found a paper by Professor K. Wadati of Japan in which he compared large earthquakes by plotting the maximum ground motion against distance to the epicenter. I tried a similar procedure for our stations, but the range between the largest and smallest magnitudes seemed unmanageably large. Dr. Beno Gutenberg then made the natural suggestion to plot the amplitudes logarithmically. I was lucky because **logarithmic plots are a device of the devil**. I saw that I could now rank the earthquakes one above the other. Also, quite unexpectedly the attenuation curves were roughly parallel on the plot. By moving them vertically, a representative mean curve could be formed, and individual events were then characterized by individual logarithmic differences from the standard curve. This set of logarithmic differences thus became the numbers on a new instrumental scale. Very perceptively, Mr. Wood insisted that this new quantity should be given a distinctive name to contrast it with the intensity scale. My amateur interest in astronomy brought out the term "magnitude," which is used for the brightness of a star.”

Earthquake A

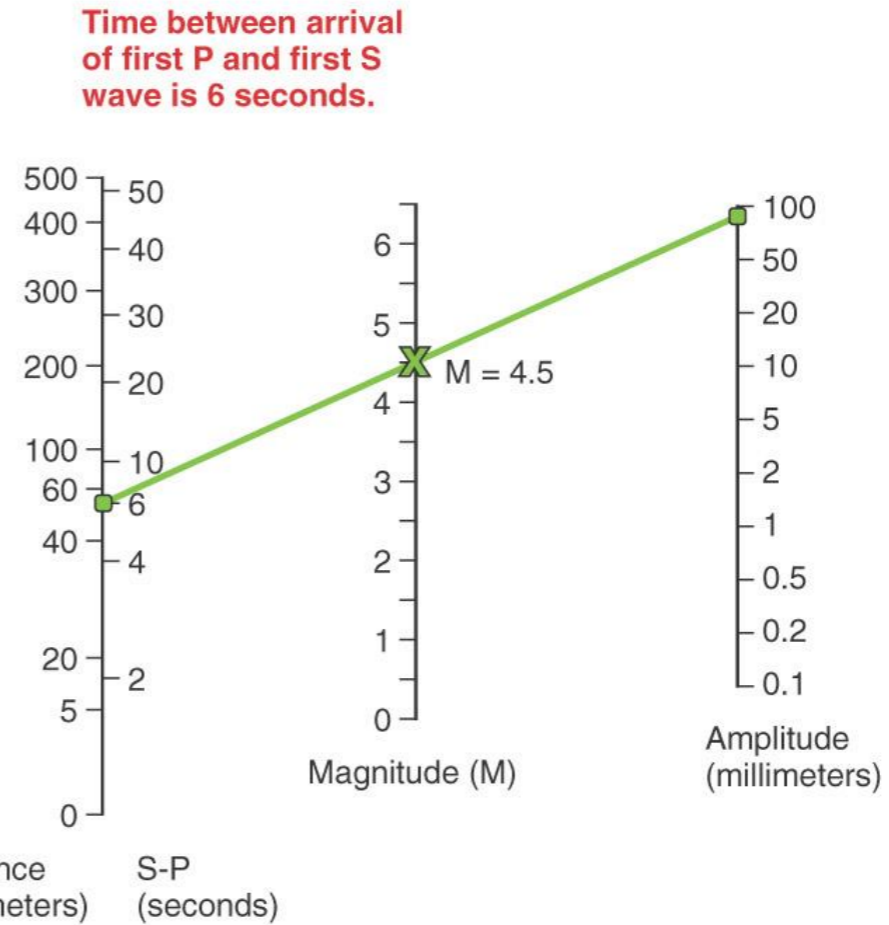
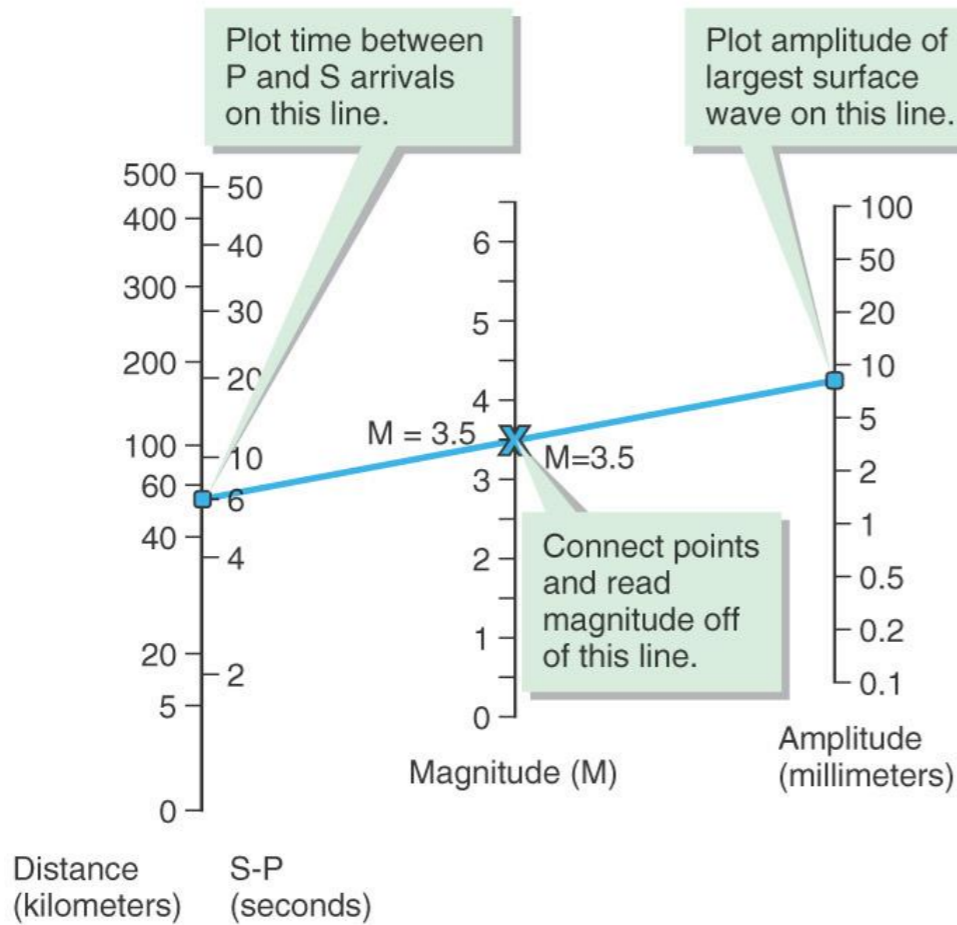


Time between arrival of first P and first S wave is 6 seconds.

Earthquake B

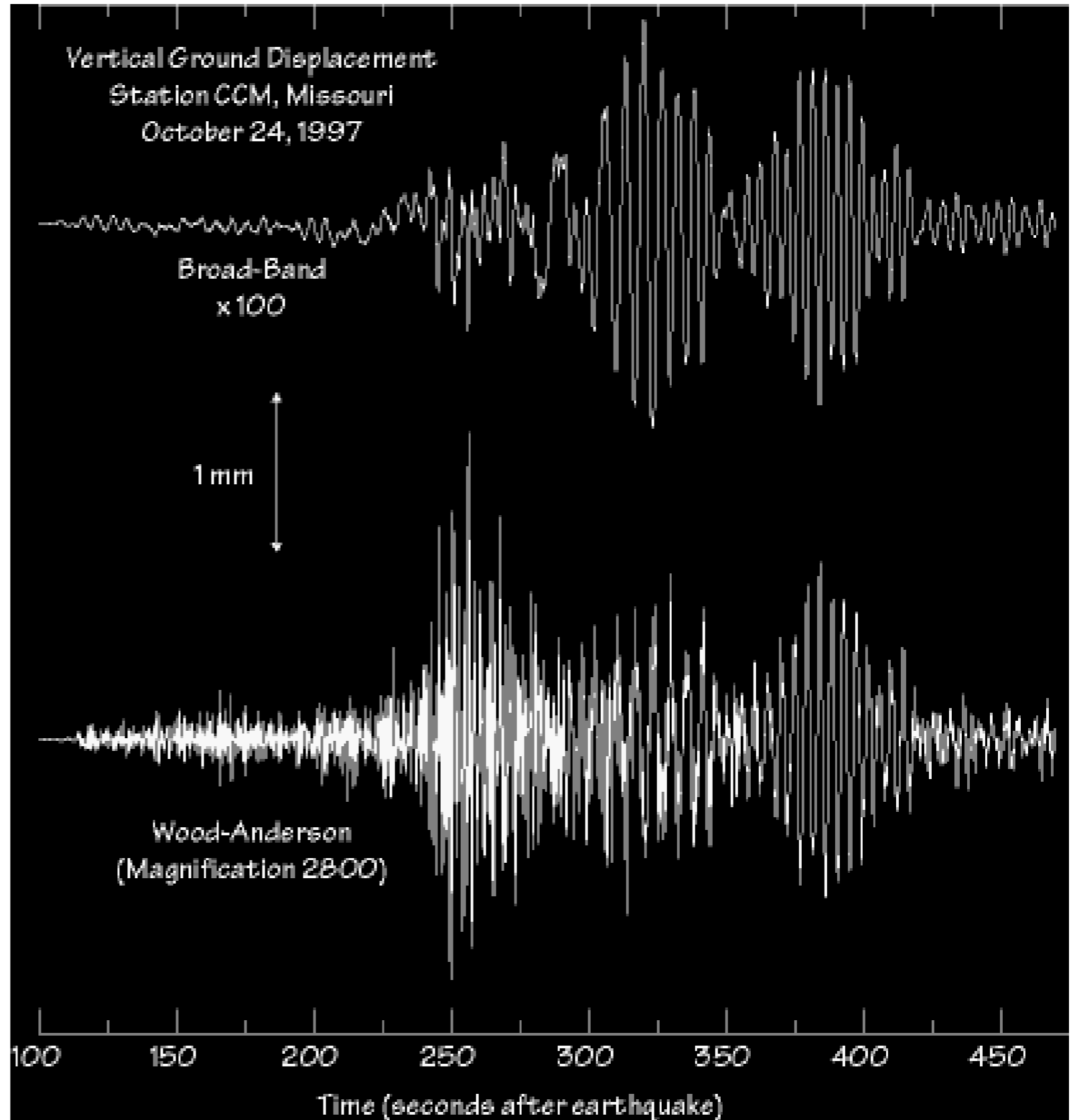


Time between arrival of first P and first S wave is 6 seconds.



Wood-Anderson Seismometer

Richter also tied his formula to a specific seismic instrument.



Magnitude Scales

The original M_L is suitable for the classification of local shocks in Southern California only since it used data from the standardized short-period Wood-Anderson seismometer network. The magnitude concept has then been extended so as to be applicable also to ground motion measurements from medium- and long-period seismographic recordings of both surface waves (M_s) and different types of body waves (m_b) in the teleseismic distance range.

The general form of all magnitude scales based on measurements of ground displacement amplitudes A and periods T is:

$$M = \log\left(\frac{A}{T}\right) + f(\Delta, h) + C_r + C_s$$

M seismic magnitude

A amplitude

T period

f correction for distance and depth

C_s correction for site

C_r correction for source region

M_L **Local magnitude**

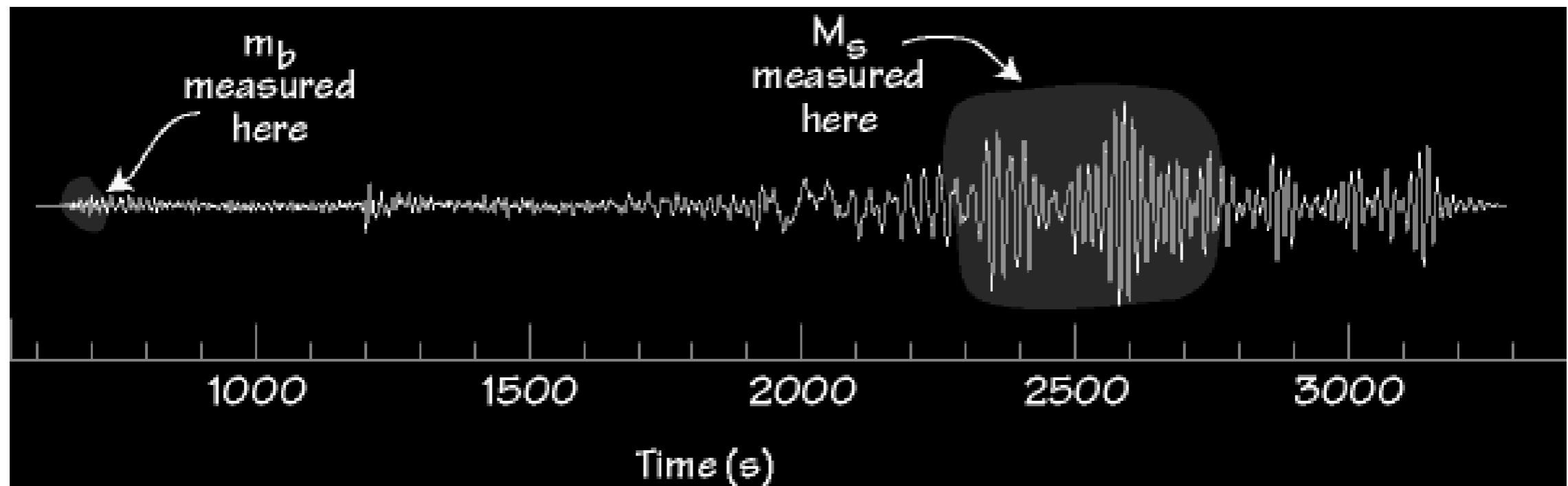
m_b **body-wave magnitude (1s)**

M_s **surface wave magnitude (20s)**

Teleseismic M_s and m_b

The two most common modern magnitude scales are:

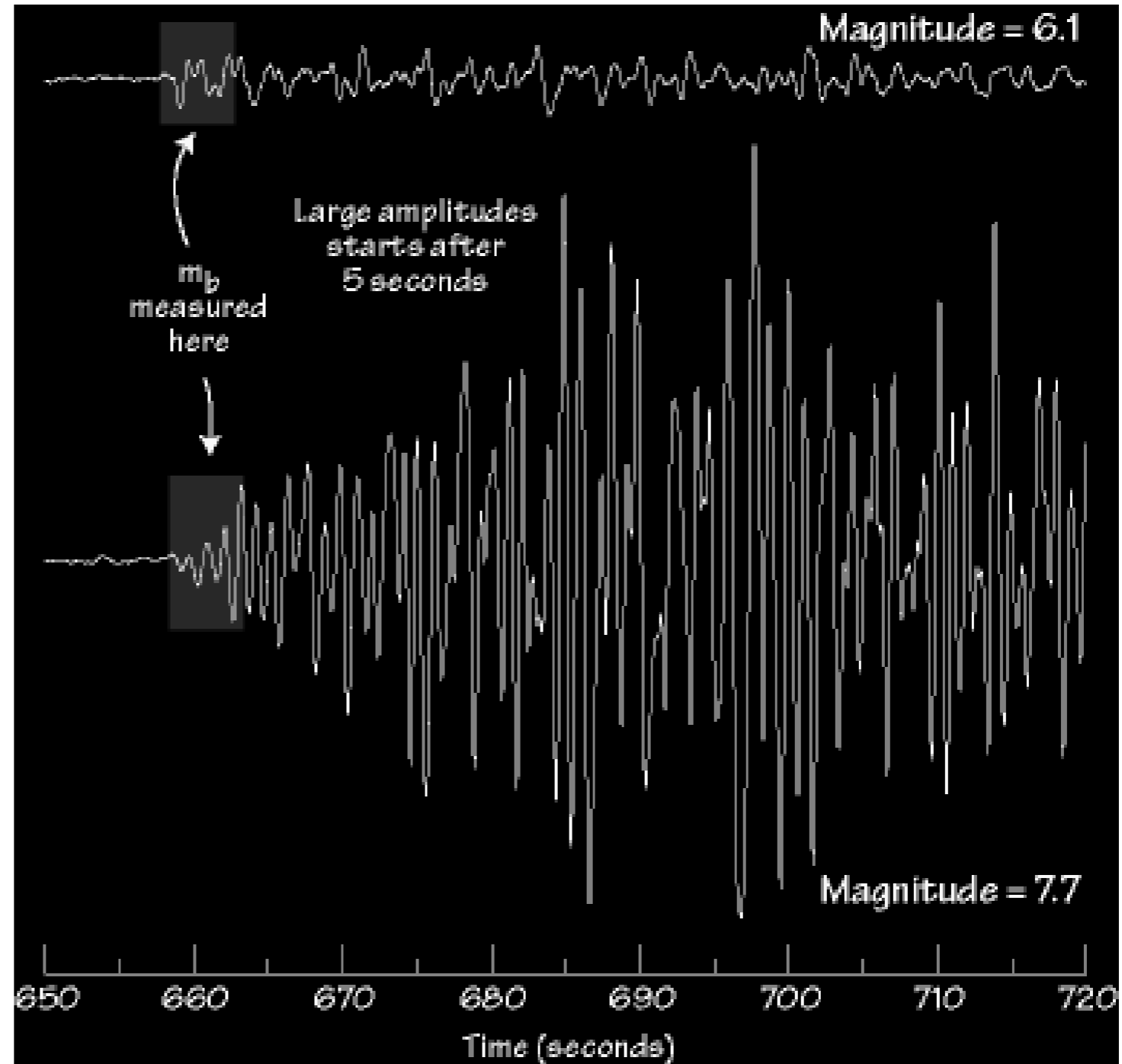
- M_s , Surface-wave magnitude (Rayleigh Wave, 20s)
- m_b , Body-wave magnitude (P-wave)



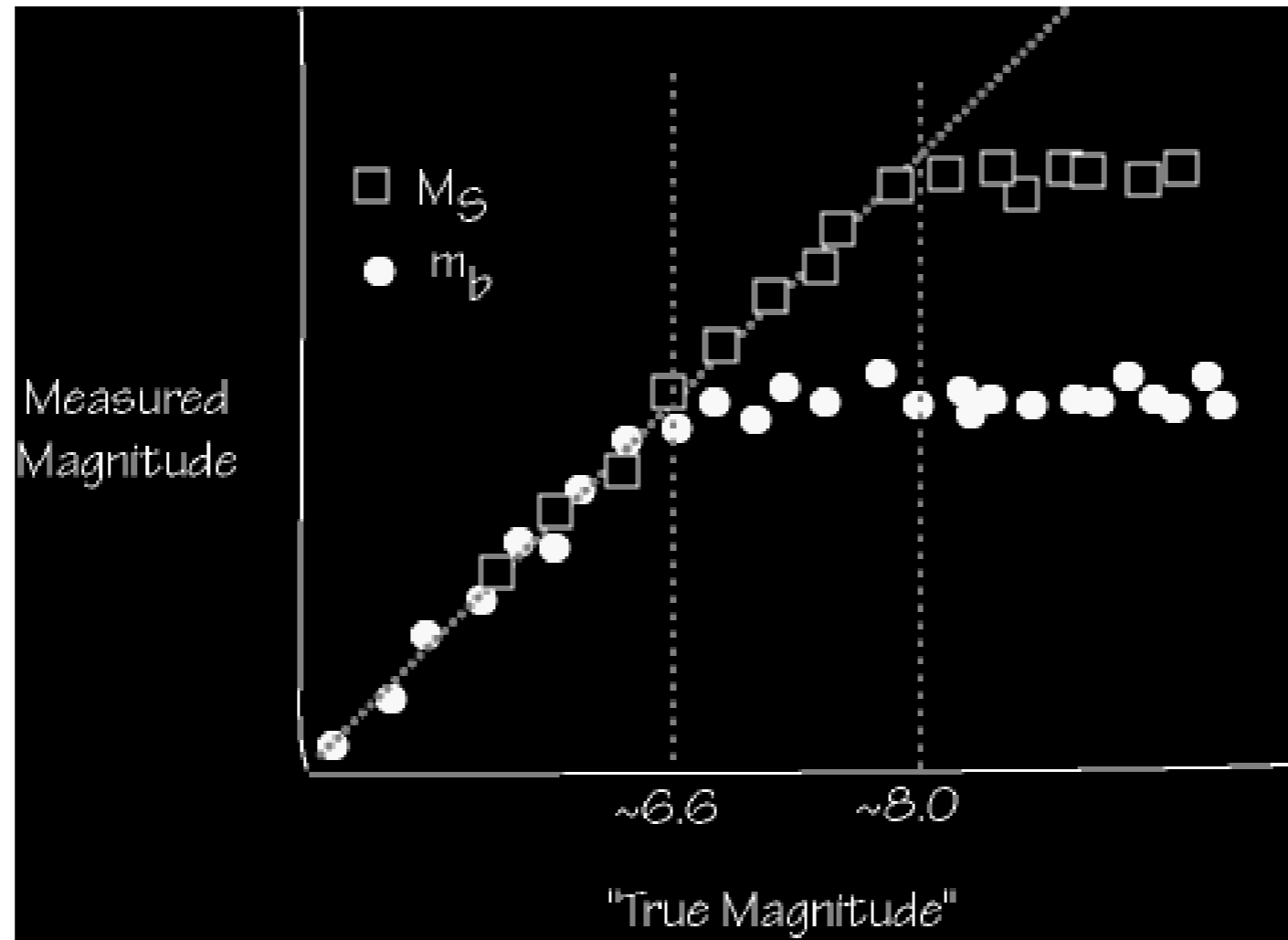
Example: m_b “Saturation”

m_b seldom gives values above 6.7 - it “saturates”.

m_b must be measured in the first 5 seconds - that’s the rule.

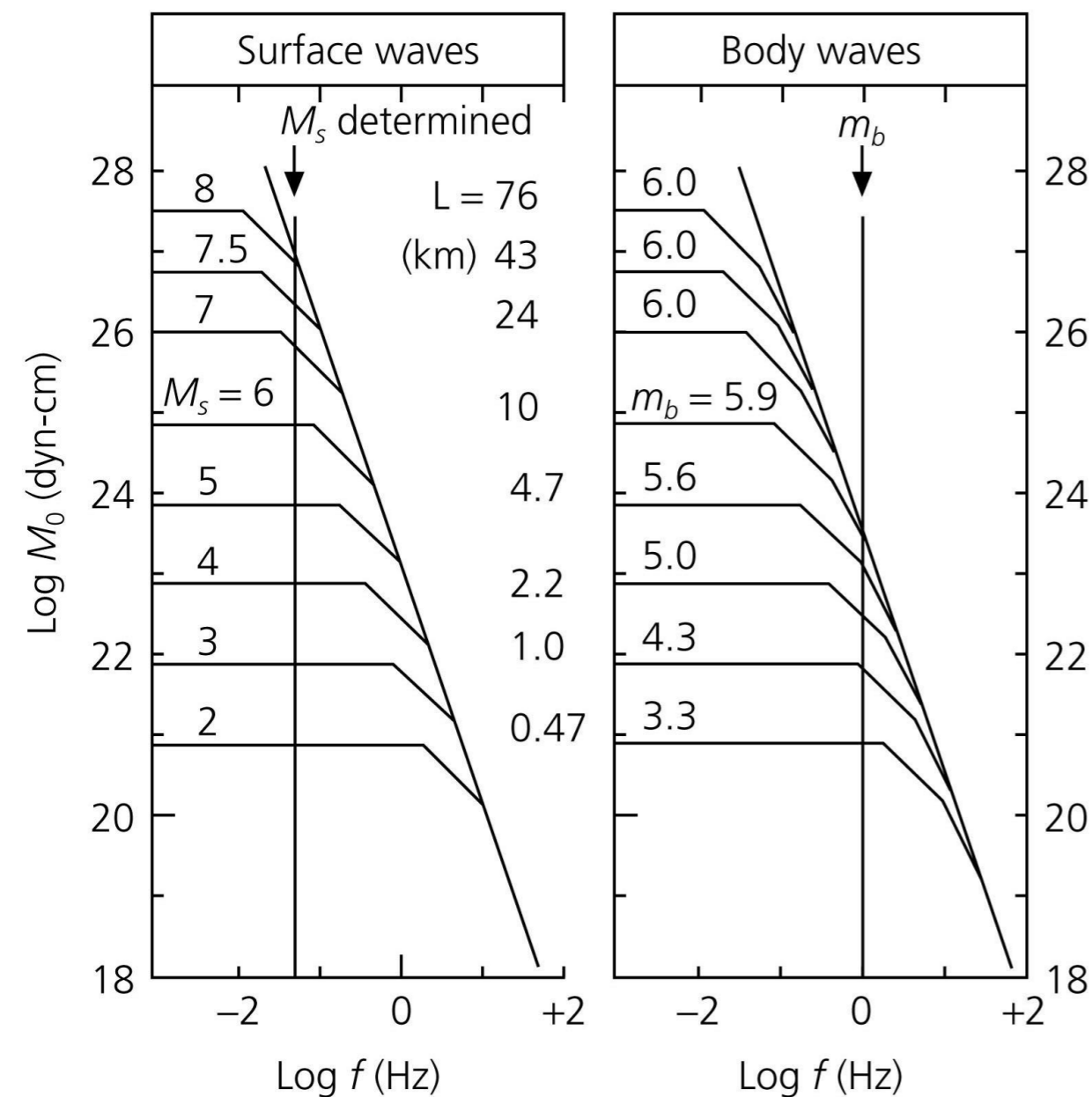


Saturation



Magnitude saturation

Nature limits the maximum size of tectonic earthquakes which is controlled by the maximum size of a brittle fracture in the lithosphere. A simple seismic shear source with linear rupture propagation has a typical "source spectrum".



M_s is not linearly scaled with M_0 for $M_s > 6$ due to the beginning of the so-called saturation effect for spectral amplitudes with frequencies $f > f_c$. This saturation occurs already much earlier for m_b which are determined from amplitude measurements around 1 Hz.

Moment magnitude

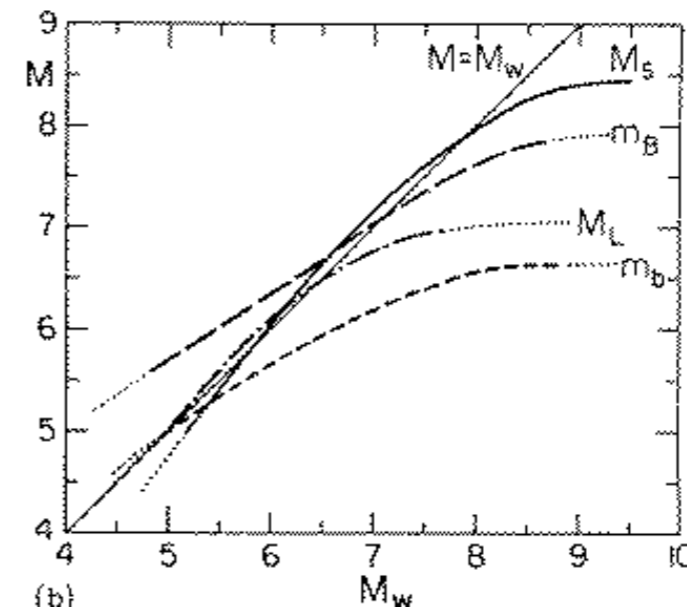
Empirical studies (Gutenberg & Richter, 1956; Kanamori & Anderson, 1975) lead to a formula for the released seismic energy (in Joule), and for moment, with magnitude:
 $\log E = 4.8 + 1.5M_s$ $\log M_0 = 9.1 + 1.5M_s$
 resulting in

$$u(x, t) = A \cos\left(\frac{2\pi t}{T}\right) \Rightarrow v(x, t) \propto \frac{A}{T} u$$

$$\Rightarrow e \propto v^2 \propto \left(\frac{A}{T}\right)^2 \Rightarrow \log E = C + 2 \log\left(\frac{A}{T}\right)$$

$$M_w = 2/3 \log M_0 - 6.07$$

when the Moment is measured in N·m (otherwise the intercept becomes 10.73); it is related to the final static displacement after an earthquake and consequently to the tectonic effects of an earthquake.



Earthquake	Body wave magnitude m_b	Surface wave magnitude M_s	Fault area (km ²) length × width	Average dislocation (m)	Moment (dyn-cm) M_0	Moment magnitude M_w
Truckee, 1966	5.4	5.9	10 × 10	0.3	8.3×10^{24}	5.8
San Fernando, 1971	6.2	6.6	20 × 14	1.4	1.2×10^{26}	6.7
Loma Prieta, 1989	6.2	7.1	40 × 15	1.7	3.0×10^{26}	6.9
San Francisco, 1906		8.2	320 × 15	4	6.0×10^{27}	7.8
Alaska, 1964	6.2	8.4	500 × 300	7	5.2×10^{29}	9.1
Chile, 1960		8.3	800 × 200	21	2.4×10^{30}	9.5

Seismic moment (1)

Remember . . . the displacement equation for the P and S wave radiation patterns:

$$u_r = \frac{1}{4\pi\alpha^3 r} \dot{M}(t - r/\alpha) \sin(2\theta)\cos(\varphi) \quad \text{e.g. P waves}$$

Amplitude term Source time function Describes the pattern

Considering the **seismic moment rate function** or **source time function**

$$\dot{M}(t - r/v)$$

which is the time derivative of the **seismic moment function**

$$M(t) = \mu D(t) S(t)$$

where μ is rigidity, and $D(t)$ and $S(t)$ are the slip and fault area histories, respectively.

(Lay & Wallace, 1995; Stein & Wyssession, 2003)

Seismic moment (2)

This leads to the best measure of an earthquake's size and energy,

$M(t) = \mu D_{av} S$ the **seismic moment**, where D_{av} is the average slip or dislocation and S is the fault area.

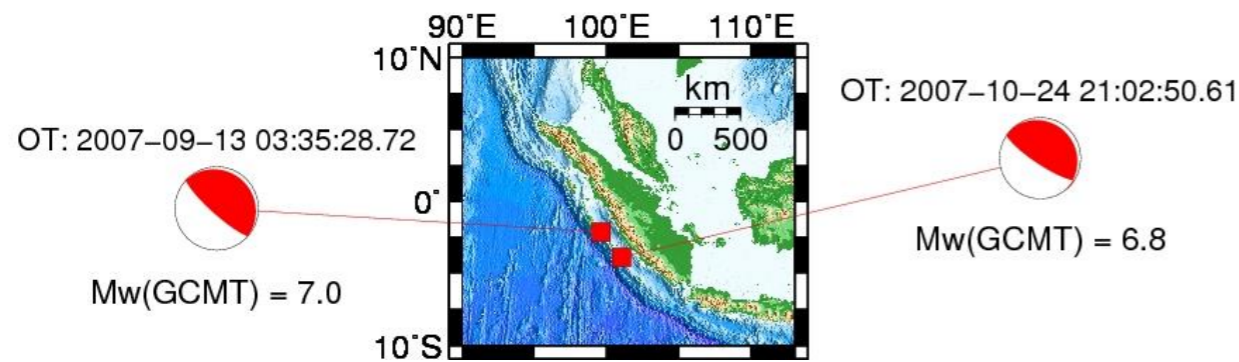
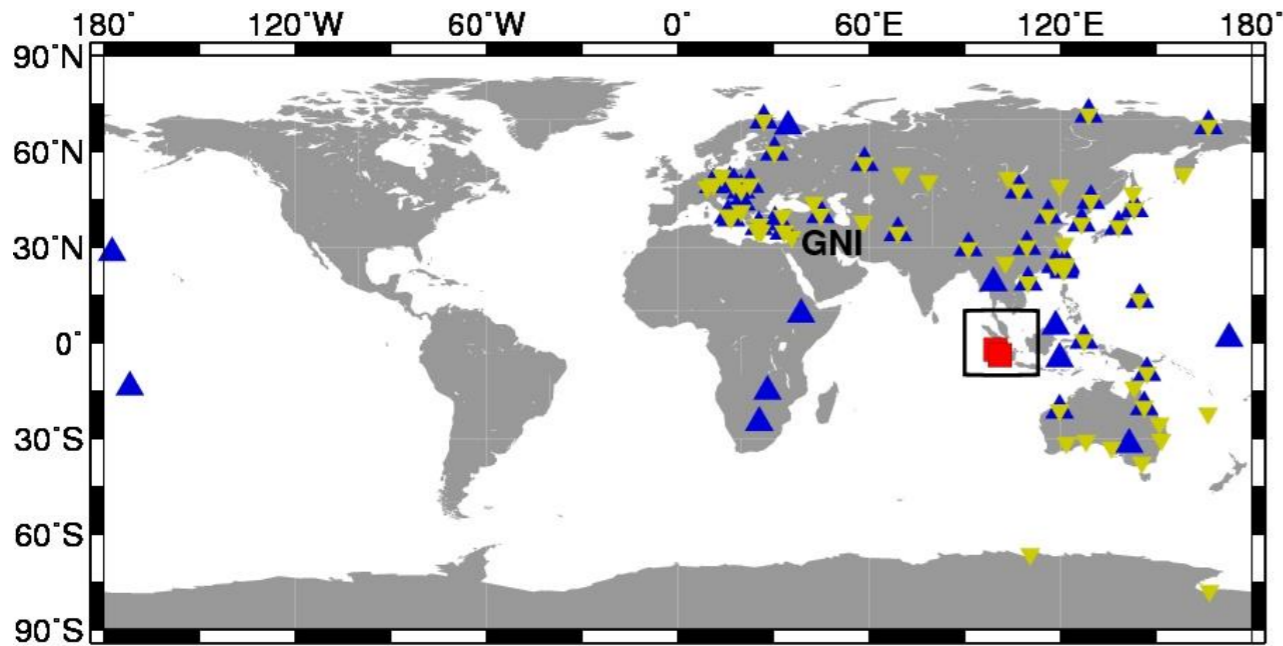
which in turn gives the **moment magnitude** M_w

$$M_w = \frac{\log M_o}{1.5} - 10.73 \quad \text{where } M_o \text{ is in dyn-cm.}$$

and which we will discuss again with respect to other magnitude scales.

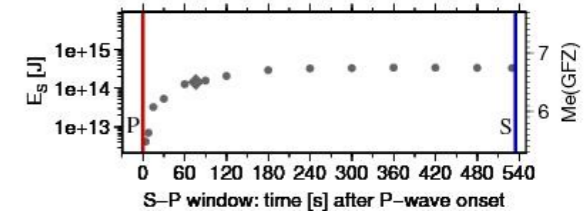
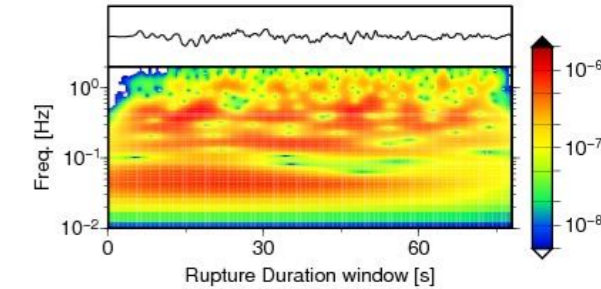
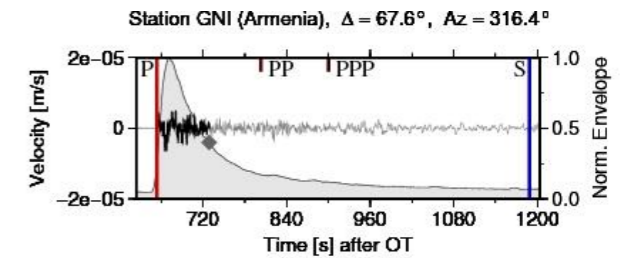
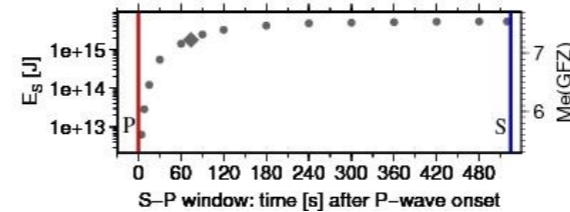
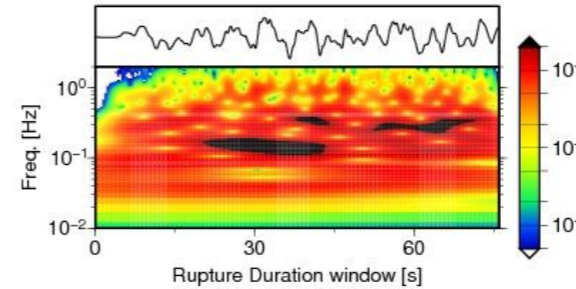
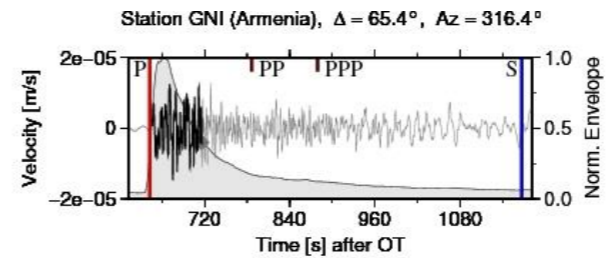
(Lay & Wallace, 1995; Stein & Wyssession)

Importance of comparing Mw and Me



Mw(GCMT) = 7.0
Me(GFZ) = 7.1

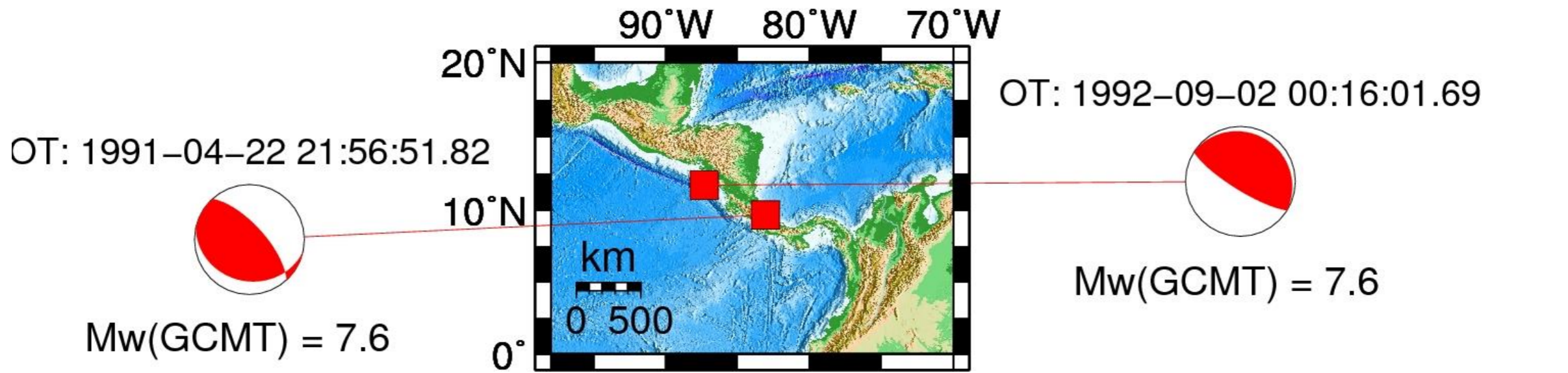
Mw(GCMT) = 6.8
Me(GFZ) = 6.4



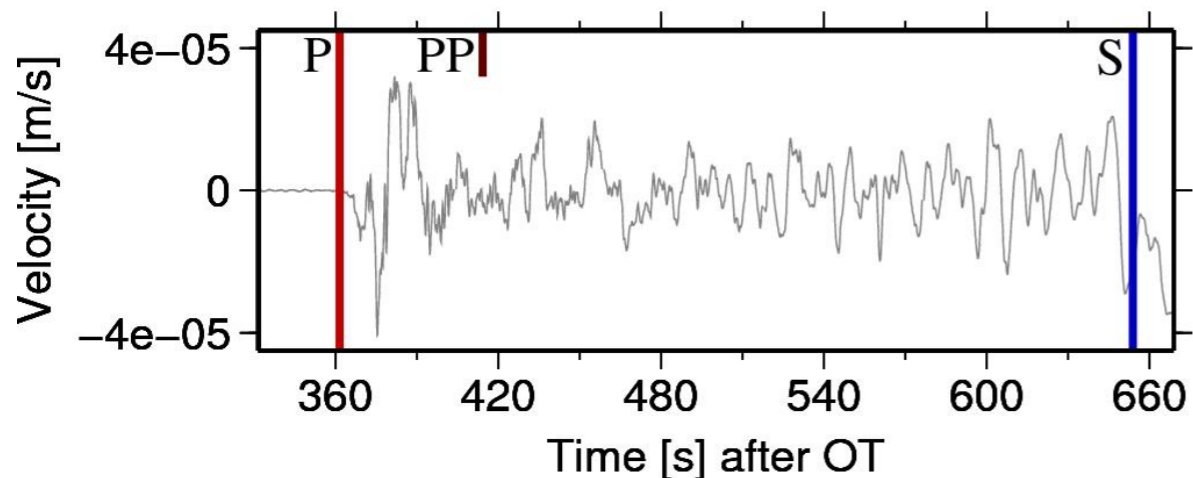
However, the high frequency content observed in the seismograms is significantly different and cannot be explained by Mw only.

The locations differ by about 250 km and the moment magnitudes Mw and the fault plane solutions are very similar.

Importance of comparing Mw and Me

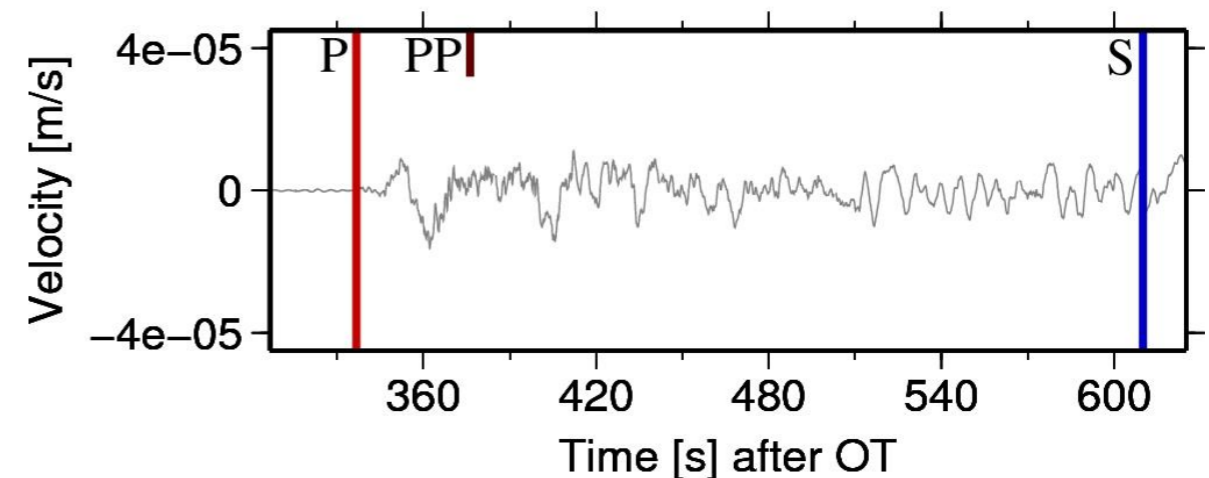


Station CCM (USA), $\Delta = 29.2^\circ$, Az = 346.7°



Mw(GCMT) = 7.6, Me(GFZ) = 7.19

Station CCM (USA), $\Delta = 26.4^\circ$, Az = 353.2°



Mw(GCMT) = 7.6, Me(GFZ) = 6.75

The locations differ by about 500 km and the moment magnitudes Mw are nearly identical, therefore the differences in the high frequency content observed in the seismograms can be attributed to different source characteristics.

Strong motion seismology

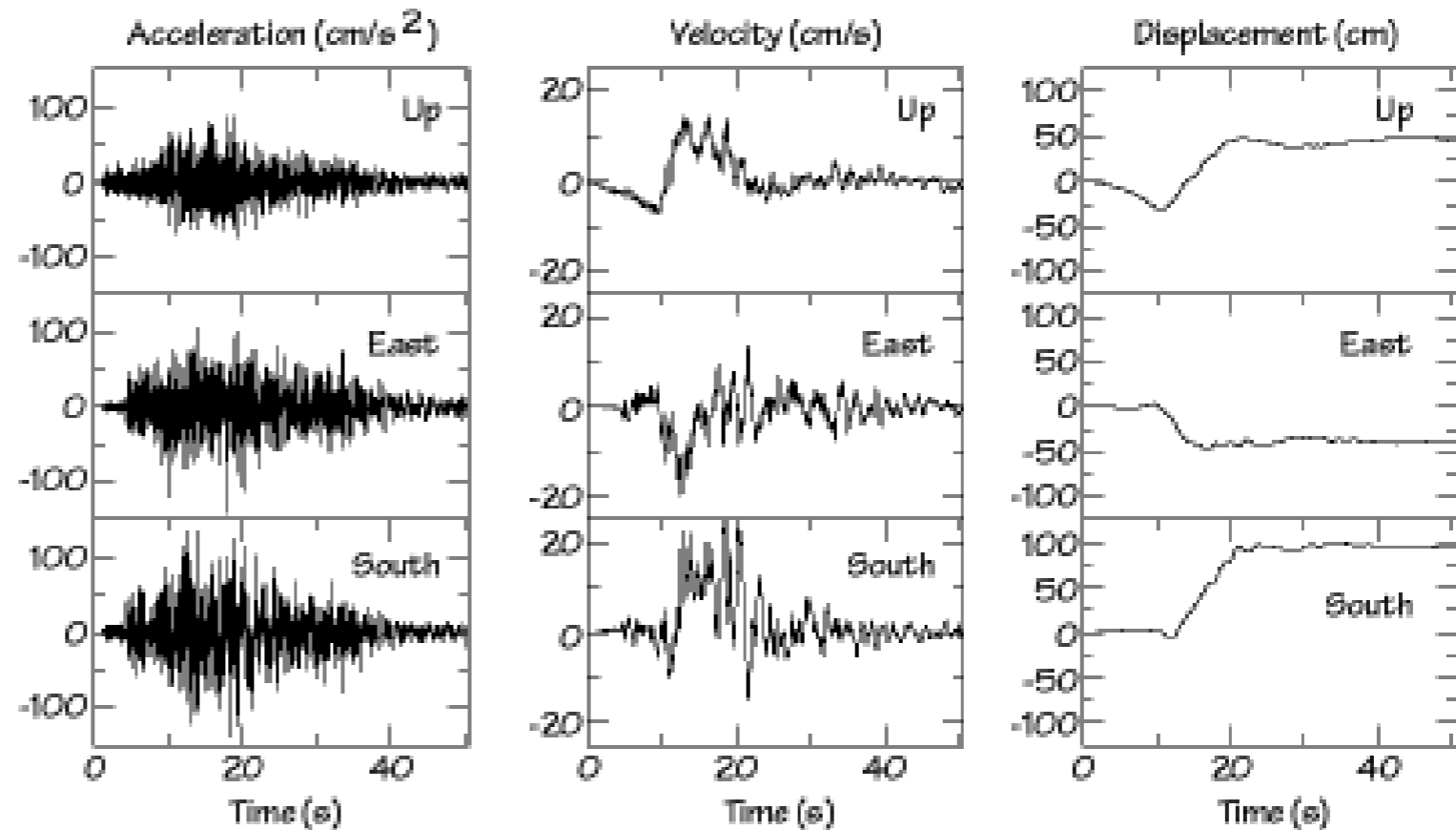
- Strong ground motion is an event in which an earthquake cause the ground to shake at least strongly enough for people to feel the motion or to damage or destroy man-made structures.
- The goal of strong motion seismology is to be able to understand and predict seismic motions sufficiently well that the predictions can be used for engineering applications
- The field of strong-motion seismology could initially be identified with a type of instrument, designed to remain on-scale and record the ground motion with fidelity under the conditions of the strongest ground motions experienced in earthquakes.

Strong motion seismology

- Early instruments were typically designed so that ground motions up to the acceleration of gravity (1g) would be on-scale.
- The lower limit of ground motion considered by the early strong motion seismology studies was roughly defined by the thickness of the light beam read until the edge of a recorded film. The minimum acceleration resolved is somewhat less than 0.01g, that approximately coincided with minimum ground motions that humans are able to feel.
- Since much smaller ground motions can be recorded on modern instruments, the distinction between strong-motion seismology and traditional seismology is blurred.

Example of Recordings

Ground acceleration, velocity and displacement, recorded at a strong-motion seismometer that was located directly above the part of a fault that ruptured during the 1985 Mw = 8.1, Michaoacan, Mexico earthquake.

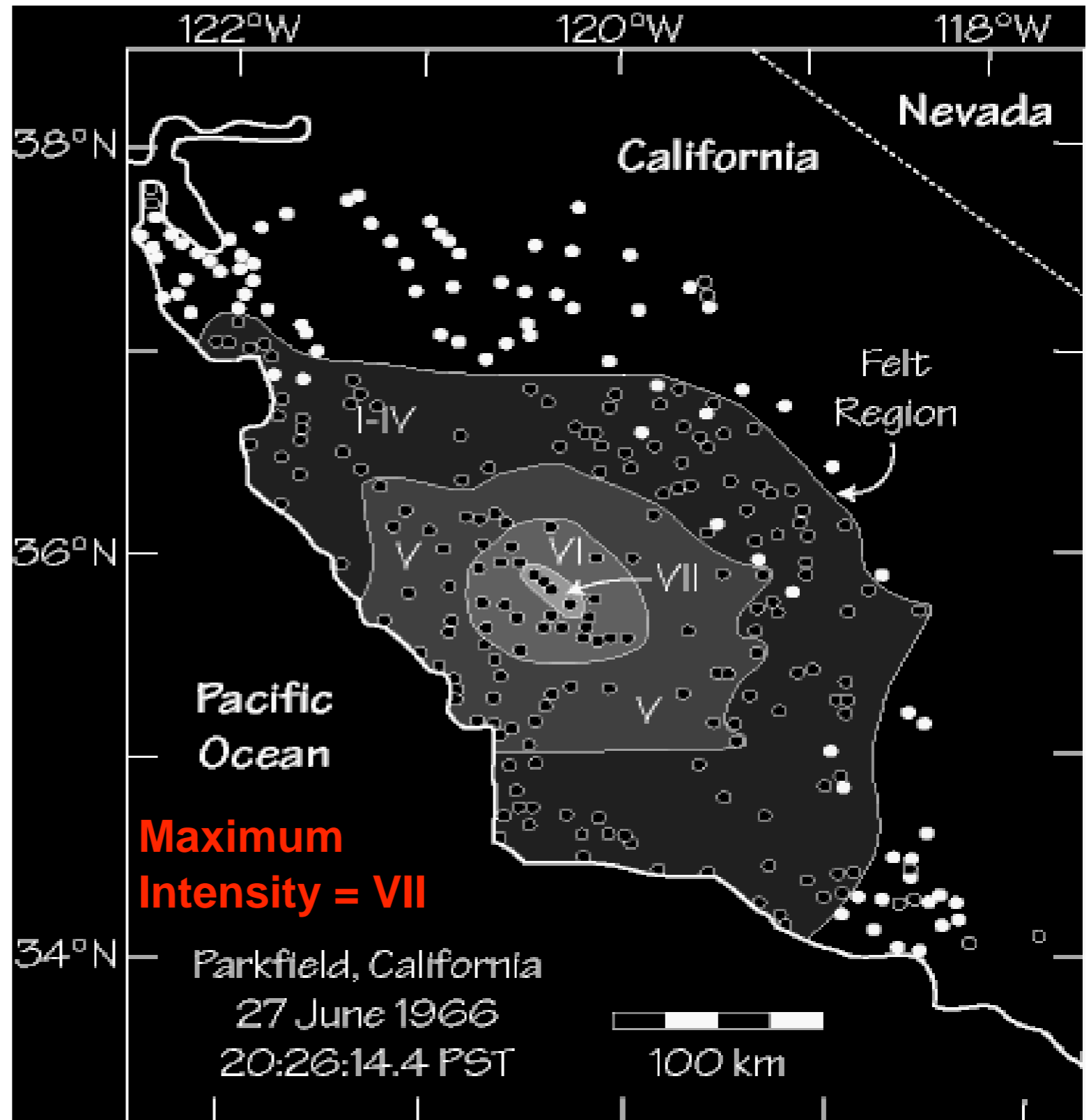


The left panel is a plot of the three components of acceleration: strong, high-frequency shaking lasted almost a minute and the peak acceleration was about 150 cm/s² (or about 0.15g). The middle panel shows the velocity of ground movement: the peak velocity for this site during that earthquake was about 20-25 cm/sec.

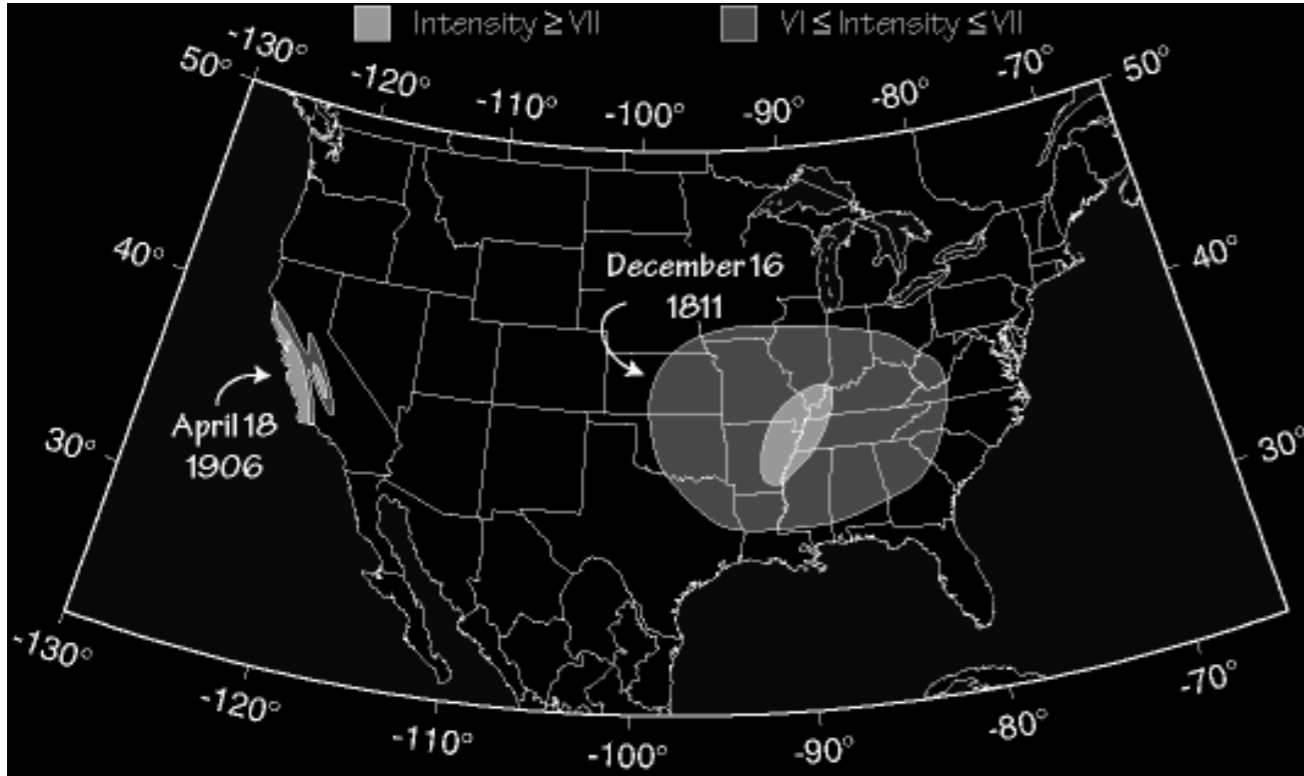
Integrating the velocity, we can compute the displacement, which is shown in the right-most panel: the permanent offsets near the seismometer were up, west, and south, for a total distance of about 125 centimeters.

Maximum Intensity

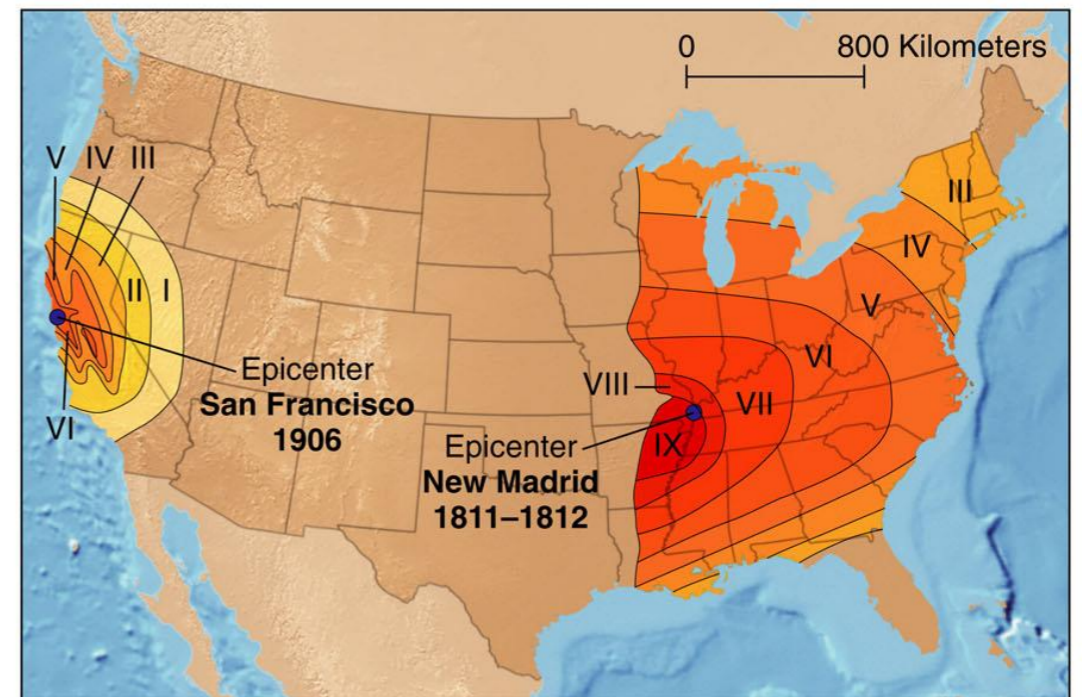
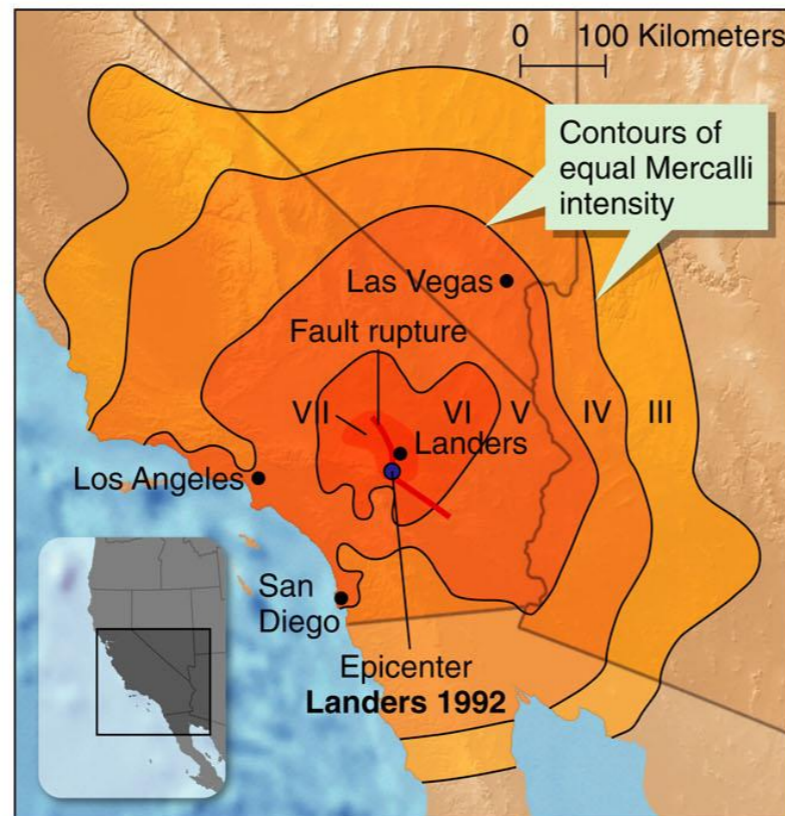
Maximum Intensity is used to estimate the size of historical earthquakes, but suffers from dependence on depth, population, construction practices, site effects, regional geology, etc.



1906 SF and 1811-12 New Madrid



These earthquakes were roughly the same size, but the intensity patterns in the east are broader than in the west (wait for Q...)



Mercalli Intensity and Richter Magnitude

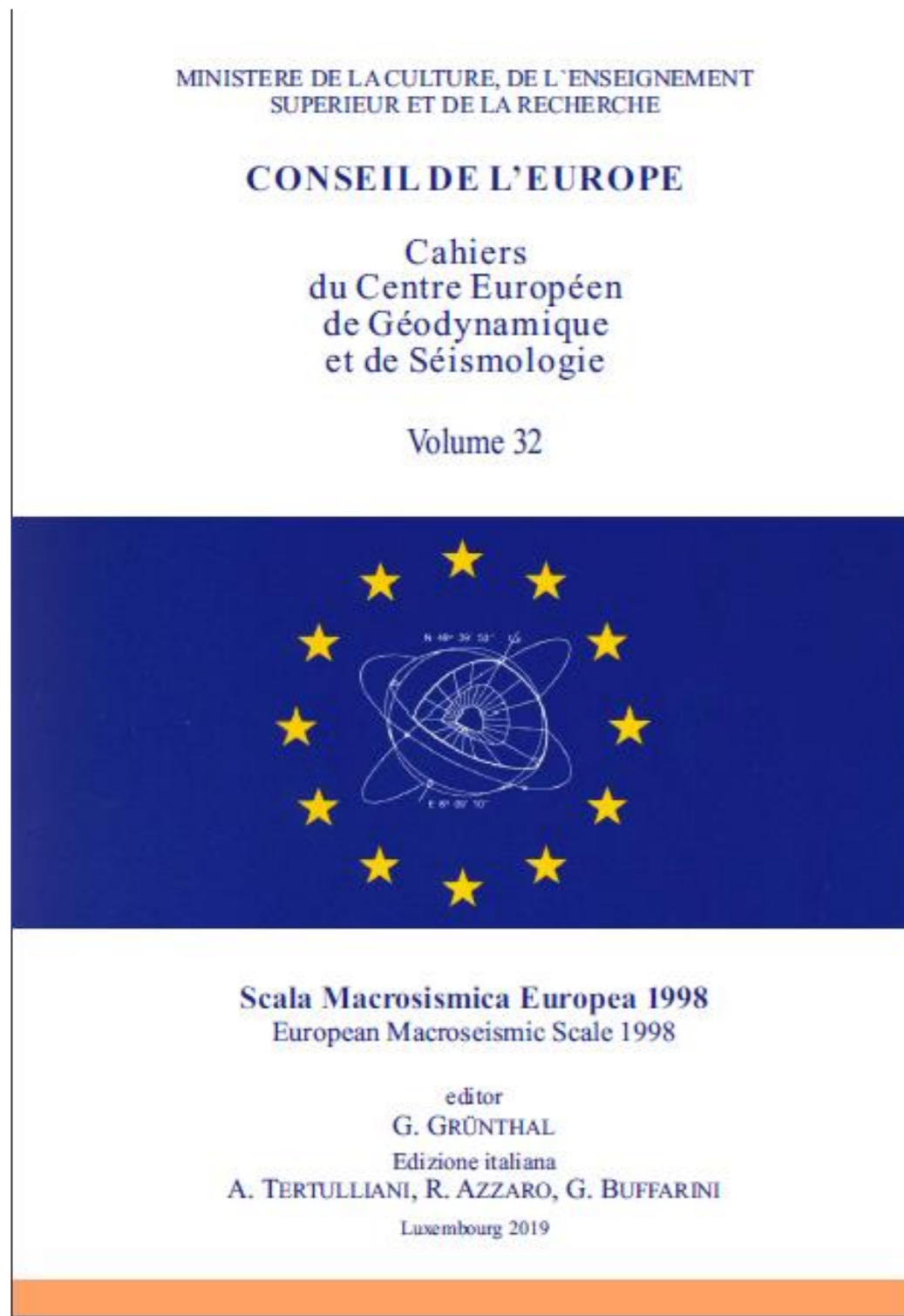
Magnitude	Intensity	Description
1.0-3.0 Micro	I	I. Not felt except by a very few under especially favorable conditions.
3.0 - 3.9 Minor	II - III	II. Felt only by a few persons at rest, especially on upper floors of buildings. III. Felt quite noticeably by persons indoors, especially on upper floors of buildings. Many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibrations similar to the passing of a truck. Duration estimated.
4.0 - 4.9 Light	IV - V	IV. Felt indoors by many, outdoors by few during the day. At night, some awakened. Dishes, windows, doors disturbed; walls make cracking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably. V. Felt by nearly everyone; many awakened. Some dishes, windows broken. Unstable objects overturned. Pendulum clocks may stop.
5.0 - 5.9 Moderate	VI - VII	VI. Felt by all, many frightened. Some heavy furniture moved; a few instances of fallen plaster. Damage slight. VII. Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable damage in poorly built or badly designed structures; some chimneys broken.
6.0 - 6.9 Strong	VII - IX	VIII. Damage slight in specially designed structures; considerable damage in ordinary substantial buildings with partial collapse. Damage great in poorly built structures. Fall of chimneys, factory stacks, columns, monuments, walls. Heavy furniture overturned. IX. Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb. Damage great in substantial buildings, with partial collapse. Buildings shifted off foundations.
7.0 and higher Major great	VIII or higher	X. Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations. Rails bent. XI. Few, if any (masonry) structures remain standing. Bridges destroyed. Rails bent greatly. XII. Damage total. Lines of sight and level are distorted. Objects thrown into the air.

Intensity scales

MM	RF	JMA	MCS	MSK
I	I		II	I
II			III	II
III				III
IV	IV	I	IV	IV
V	V		II	V
VI	VI	III	VI	V
VII	VII	IV	VII	VI
VIII		VIII	V	VIII
IX	IX	VI		IX
X			X	IX
XI			XI	X
XII	X	VII	XII	X
XI				XI
XII				XII

MM – Modified Mercalli; RF – Rossi-Forel; JMA – Japanese Meteorological Agency;
MCS – Mercalli-Cancani-Sieberg; MSK – Medvedev-Sponheuer-Karnik

Intensity scales

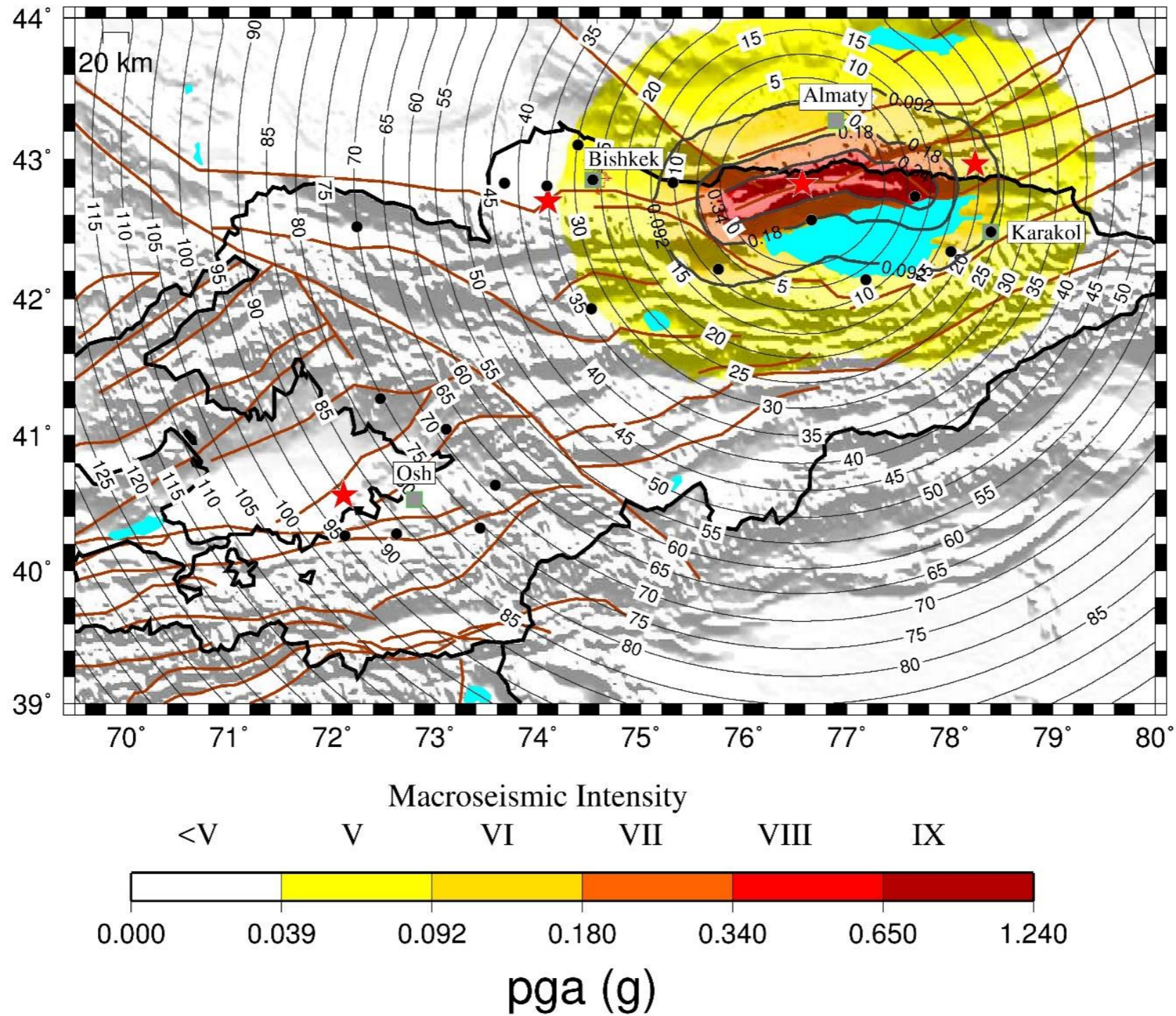


8 Tabella sintetica della EMS-98

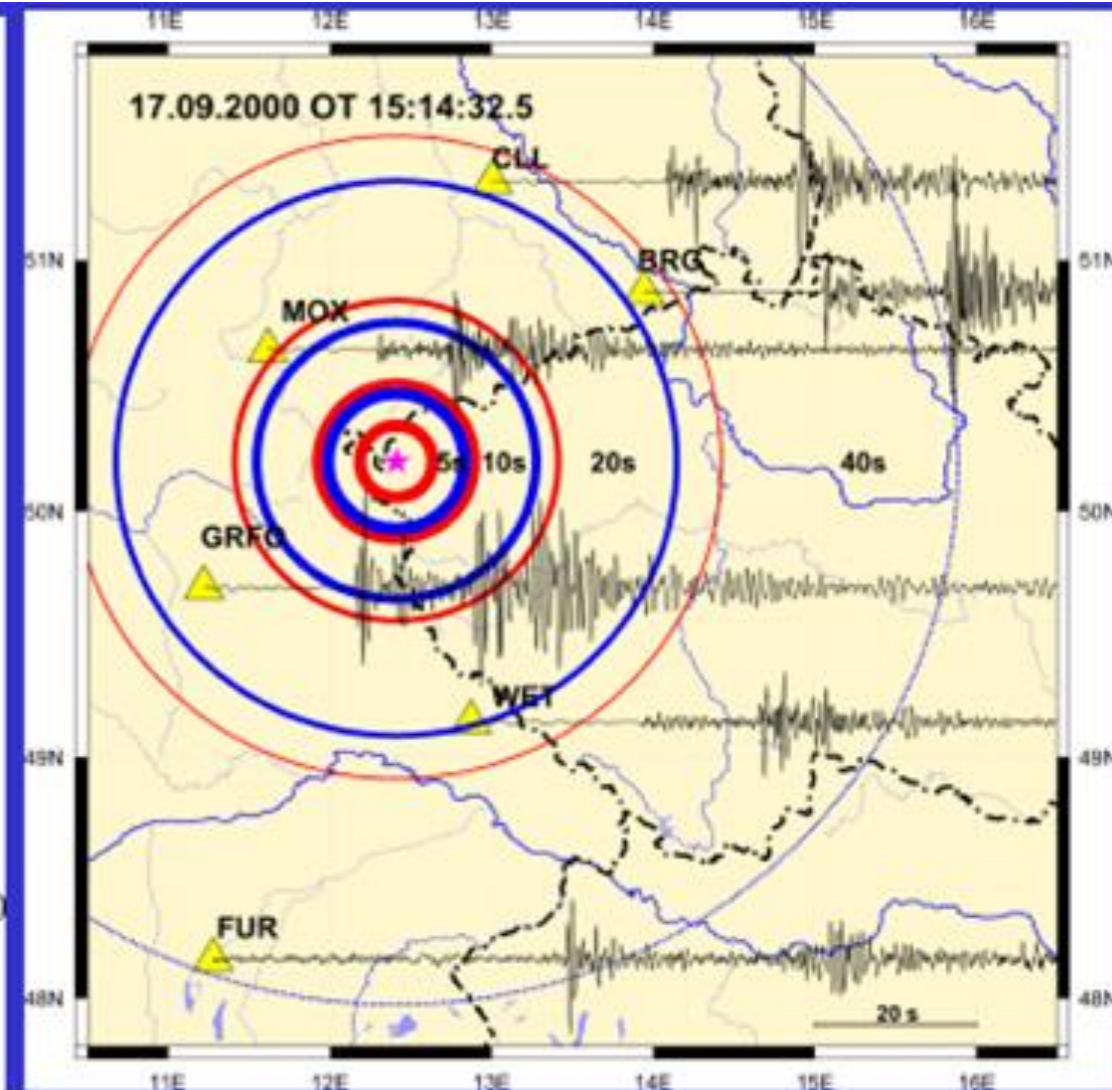
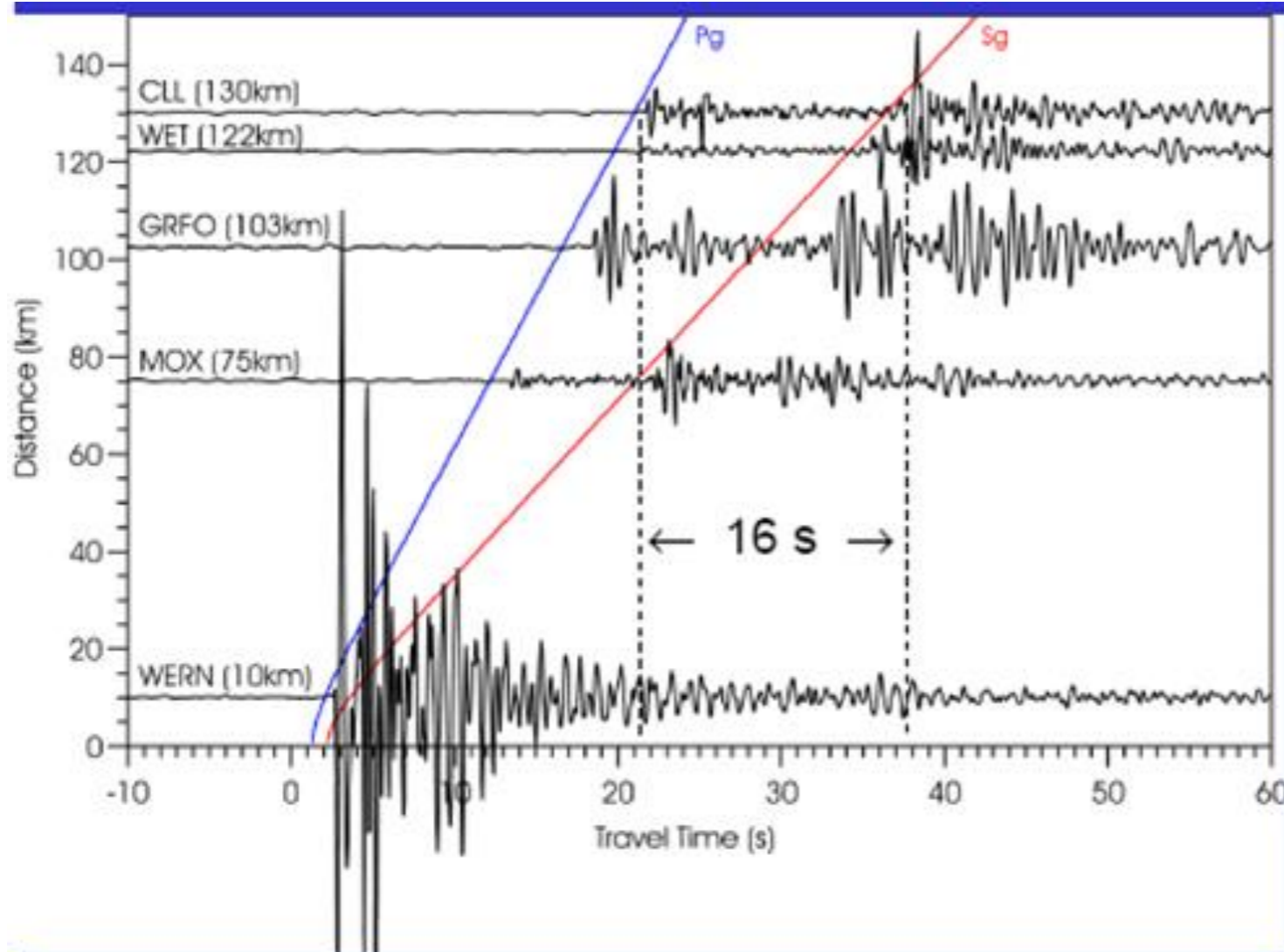
La forma sintetica della Scala Macrosismica Europea, estratta dalla parte centrale, è fornita per dare una panoramica molto semplificata e generalizzata della Scala EM. Essa può, per esempio, essere usata per scopi divulgativi. *Questa forma sintetica ma non è adatta per le assegnazioni d'intensità.*

Intensità EMS	Definizione	Descrizione degli effetti tipici osservati (riassunto)
I	Non avvertito	Non avvertito.
II	Appena avvertito	Avvertito solo da poche persone in stato di riposo al chiuso.
III	Debole	Avvertito da alcune persone in casa. Persone a riposo avvertono una oscillazione o un leggero tremore.
IV	Ampiamente osservato	Avvertito all'interno da molta gente, da pochissimi all'esterno. Alcune persone si svegliano. Finestre, porte e piatti sbattono.
V	Forte	Avvertito all'interno dalla maggior parte delle persone, all'esterno da pochi. Molte persone che dormivano si svegliano. Alcuni si spaventano. Gli edifici tremano nel loro complesso. Oggetti appesi oscillano notevolmente. Piccoli oggetti vengono spostati. Porte e finestre si spalancano o si chiudono.
VI	Danni lievi	Molte persone si spaventano e corrono all'aperto. Alcuni oggetti cadono. Molti edifici subiscono leggeri danni non strutturali come sottilissime fessure capillari e caduta di piccoli pezzi di intonaco.
VII	Danni diffusi	La maggior parte delle persone si spaventano e corrono fuori. I mobili si spostano e gli oggetti cadono dalle mensole in grande numero. Molti edifici ben costruiti subiscono danni moderati: piccole crepe nei muri, caduta di intonaco, caduta di parti di camini; gli edifici più vecchi possono mostrare grandi crepe nei muri e cedimento dei tramezzi.
VIII	Danni gravi	Molte persone hanno difficoltà a stare in piedi. Molti edifici presentano grandi fenditure nei muri. Alcuni edifici ben costruiti mostrano cedimenti gravi dei muri, mentre strutture deboli e più vecchie possono crollare.
IX	Distruttivo	Panico generale. Molte costruzioni deboli crollano. Anche edifici ben costruiti mostrano danni molto gravi: gravi lesioni dei muri e parziali cedimenti strutturali.
X	Molto distruttivo	Molti edifici ben costruiti crollano.
XI	Devastante	La maggior parte degli edifici ben costruiti crollano; anche alcuni con un buon livello di progettazione antisismica vengono distrutti.
XII	Completamente devastante	Quasi tutti gli edifici vengono distrutti.

Offline application to Kyrgyzstan: Lead time for Repetition of the M 7.8 1911 Kemin Earthquake

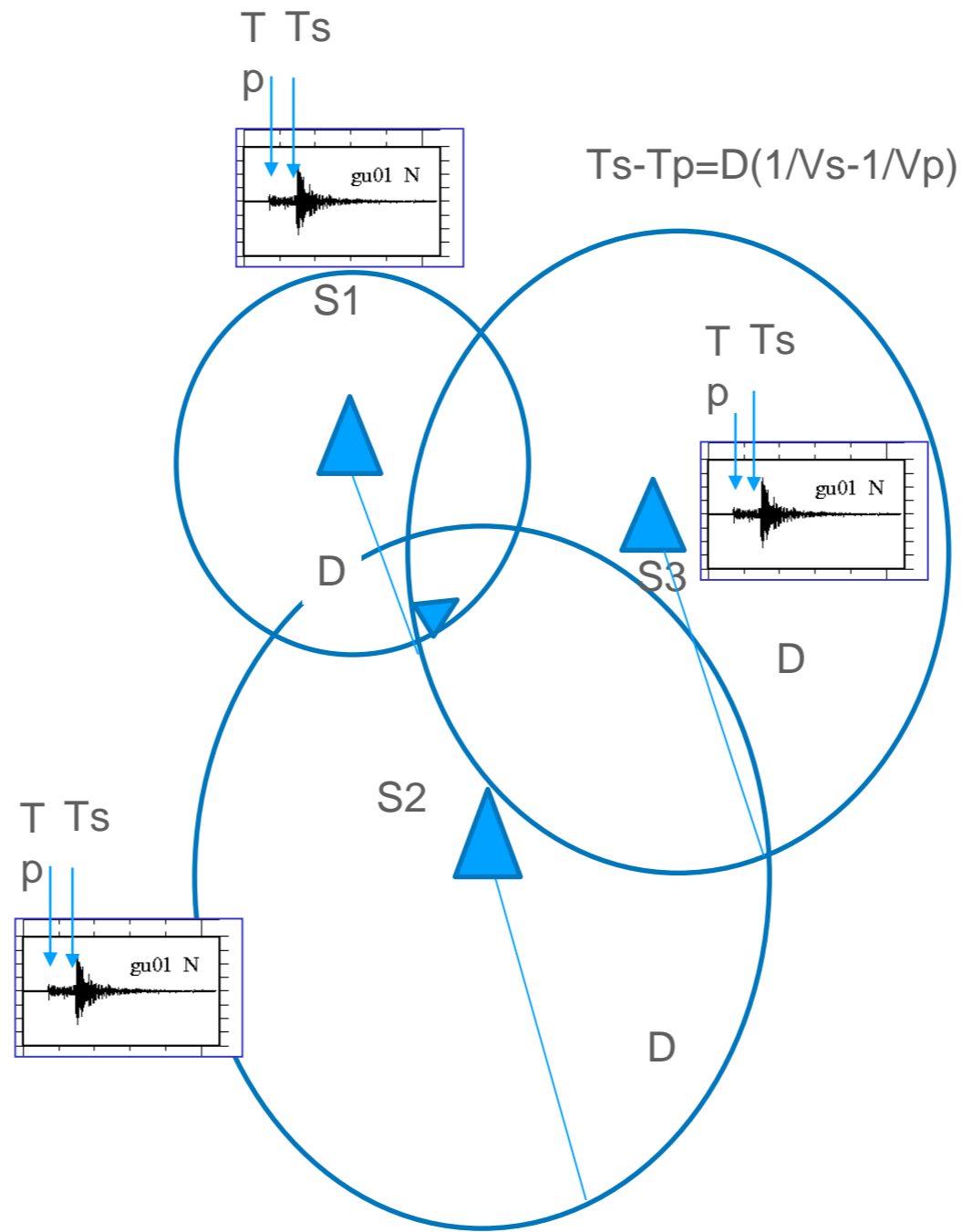


from
Parolai et al., 2017, Frontiers

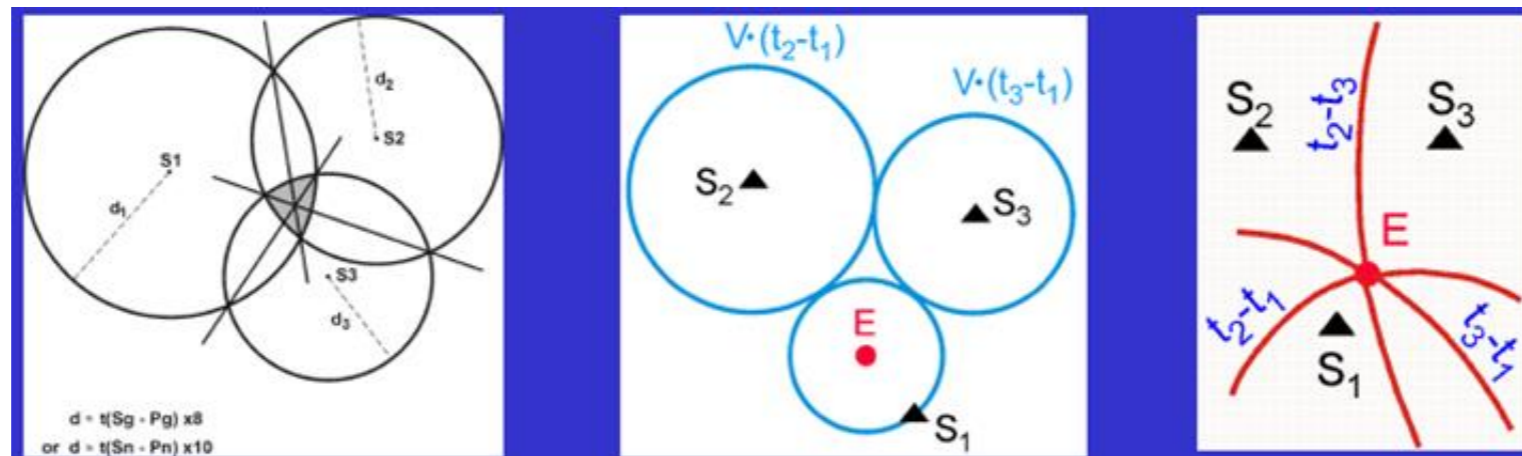


$$T_p = D/V_p$$

$$T_s = D/V_s$$



Confronto tra diversi metodi per la localizzazione dei terremoti



Tutti i metodi illustrati non considerano la forma della terra, e si basano su distanze planari
cioè possono essere applicati solo su piccola scala, a livello locale.

Allo stesso modo, tutti i metodi si basano sul modello calcolo dei tempi di percorso
mostrato nella diapositiva precedente

PlanetPhysics/Geigers Method

< PlanetPhysics

Geiger's method ^[1] is an iterative procedure using Gauss-Newton optimization to determine the location of an earthquake, or seismic event. Originally his method was developed to obtain the origin time and Epicentre but it is easily extended to include the Focal Depth for Hypocentre determination.

Given a set of M arrival times t_i find the origin time t_0 and the hypocentre in cartesian coordinates (x_0, y_0, z_0) which minimize

the objective function

$$F(\mathbf{X}) = \sum_{i=1}^M r_i^2.$$

Here, r_i is the difference between observed and calculated arrival times

$$r_i = t_i - t_0 - T_i,$$

and the unknown parameter vector is

$$\mathbf{X} = (t_0, x_0, y_0, z_0)^T$$

In matrix form (1) becomes

$$F(\mathbf{X}) = \mathbf{r}^T \mathbf{r}$$

The Gauss-Newton procedure requires an initial guess of the sought parameters, denoted here as

$$\mathbf{X}^* = (t_0^*, x_0^*, y_0^*, z_0^*)^T,$$

which are then used to calculate the adjustment vector

$$\delta \mathbf{X} = (\delta t_0, \delta x_0, \delta y_0, \delta z_0)^T$$

in

$$(1) \mathbf{A}^T \mathbf{A} \delta \mathbf{X} = -\mathbf{A}^T \mathbf{r}.$$

The Jacobian matrix \mathbf{A} is defined as

$$\mathbf{A} = \begin{pmatrix} \partial r_1 / \partial t_0 & \partial r_1 / \partial x_0 & \partial r_1 / \partial y_0 & \partial r_1 / \partial z_0 \\ \partial r_2 / \partial t_0 & \partial r_2 / \partial x_0 & \partial r_2 / \partial y_0 & \partial r_2 / \partial z_0 \\ \vdots & \vdots & \vdots & \vdots \\ \partial r_M / \partial t_0 & \partial r_M / \partial x_0 & \partial r_M / \partial y_0 & \partial r_M / \partial z_0 \end{pmatrix}.$$

The partial derivatives are evaluated at the initial guess, or trial vector, \mathbf{X}^* . Equation (45) can be rewritten as

$$(2) \mathbf{G} \delta \mathbf{X} = \mathbf{g}.$$

Using (46) and an initial guess \mathbf{X}^* an adjustment vector can be calculated. The initial guess can then be updated $\mathbf{X}^* + \delta \mathbf{X}$ and used as the initial guess in the next run of the algorithm. In this manner the sought parameters \mathbf{X} can be determined to some tolerance.

De aggregazione

$$\lambda(IM > x) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{min}) \sum_{j=1}^{n_M} \sum_{k=1}^{n_R} P(IM > x | m_j, r_k) P(M_i = m_j) P(R_i = r_k) \quad (2.25)$$

where the range of possible M_i and R_i have been discretized into n_M and n_R intervals, respectively, using the discretization technique discussed earlier.

One of the primary advantages of PSHA—that it accounts for all possible earthquake sources in an area when computing seismic hazard—is also a disadvantage. Once the PSHA computations are complete, a natural question to ask is “which earthquake scenario is most likely to cause $PGA > x$?” Because we have aggregated all scenarios together in the PSHA calculations, the answer is not immediately obvious. We saw in the example calculations above, however, that some of the intermediate calculation results indicated the relative contribution of different earthquake sources and magnitudes to the rate of exceedance of a given ground motion intensity. Here we will formalize those calculations, through a process known as deaggregation¹ (Bazzurro & Cornell, 1999; McGuire, 1995).

Let us start with magnitude deaggregation. In this case, we are interested in the probability that an earthquake’s magnitude is equal to m , given that a ground motion of $IM > x$ has occurred. Intuitively, this is equal to the rate of earthquakes with $IM > x$ and $M = m$, divided by the rate of all earthquakes with $IM > x$

$$P(M = m | IM > x) = \frac{\lambda(IM > x, M = m)}{\lambda(IM > x)} \quad (3.1)$$

$$\lambda(IM > x, M = m) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{\min}) \sum_{k=1}^{n_{R_i}} P(IM > x | m, r_k) P(M_i = m) P(R_i = r_k) \quad (3.2)$$

Per Magnitudo e Distanza

$$P(M = m, R = r | IM > x) = \frac{\lambda(IM > x, M = m, R = r)}{\lambda(IM > x)} \quad (3.8)$$

where the numerator of equation 3.8 is computed using the basic PSHA equation but not summing over either M or R

$$\lambda(IM > x, M = m, R = r) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{\min}) P(IM > x | m_j, r_k) P(M_i = m) P(R_i = r) \quad (3.9)$$