which resolves the exercise.

## A.5 Solution to Exercise 2.1

We notice that

$$\hat{\sigma}_i \hat{\sigma}_j \hat{\sigma}_i = \begin{cases} \hat{\sigma}_j & i = j, \\ -\hat{\sigma}_j & i \neq j. \end{cases} \tag{A.18}$$

Then, we have

$$\begin{aligned} \hat{\rho} + \hat{\sigma}_x \hat{\rho} \hat{\sigma}_x + \hat{\sigma}_y \hat{\rho} \hat{\sigma}_y + \hat{\sigma}_z \hat{\rho} \hat{\sigma}_z &= \hat{\rho} + \tfrac{1}{2}(\hat{\sigma}_x \hat{\mathbb{1}} \hat{\sigma}_x + \hat{\sigma}_x \mathbf{r} \cdot \hat{\boldsymbol{\sigma}} \hat{\sigma}_x) + \tfrac{1}{2}(\hat{\sigma}_y \hat{\mathbb{1}} \hat{\sigma}_y + \hat{\sigma}_y \mathbf{r} \cdot \hat{\boldsymbol{\sigma}} \hat{\sigma}_y) + \tfrac{1}{2}(\hat{\sigma}_z \hat{\mathbb{1}} \hat{\sigma}_z + \hat{\sigma}_z \mathbf{r} \cdot \hat{\boldsymbol{\sigma}} \hat{\sigma}_z), \\ &= \tfrac{1}{2}(\hat{\mathbb{1}} + r_x \hat{\sigma}_x + r_y \hat{\sigma}_y + r_z \hat{\sigma}_z) + \tfrac{1}{2}(\hat{\mathbb{1}} + r_x \hat{\sigma}_x - r_y \hat{\sigma}_y - r_z \hat{\sigma}_z) \\ &+ \tfrac{1}{2}(\hat{\mathbb{1}} - r_x \hat{\sigma}_x + r_y \hat{\sigma}_y - r_z \hat{\sigma}_z) + \tfrac{1}{2}(\hat{\mathbb{1}} - r_x \hat{\sigma}_x - r_y \hat{\sigma}_y + r_z \hat{\sigma}_z), \\ &= 2\hat{\mathbb{1}}, \end{aligned} \tag{A.19}$$

from which we obtain the expression in Eq. (2.38).