Statistical Analysis of Networks

Lecture 3 – Basic concepts

NETWORK ANALYSIS: TYPE OF RELATIONS

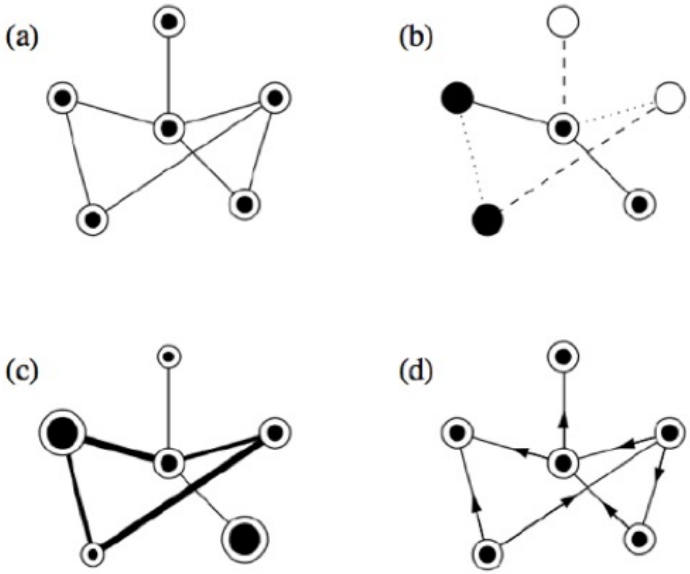
Relations – measurement level:

- binary (dichotomous): relation is present or absent
- weighted (valued): a (numerical) value attached to the edge indicating its '*strenght*' (intensity, frequency)
- signed: positive (+) and negative (-) signs attached to the edge

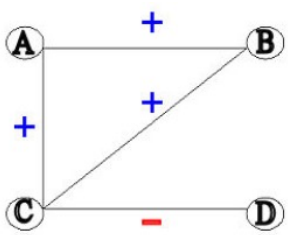
Measurement level can be combined with the type of relations: different types of network

- additional data can be available on nodes (node' s attributes) and/or eventually on edges/arcs (edges' attributes)

TYPE OF NETWORKS/GRAPHS:



(e)



(a) unweighted, undirected

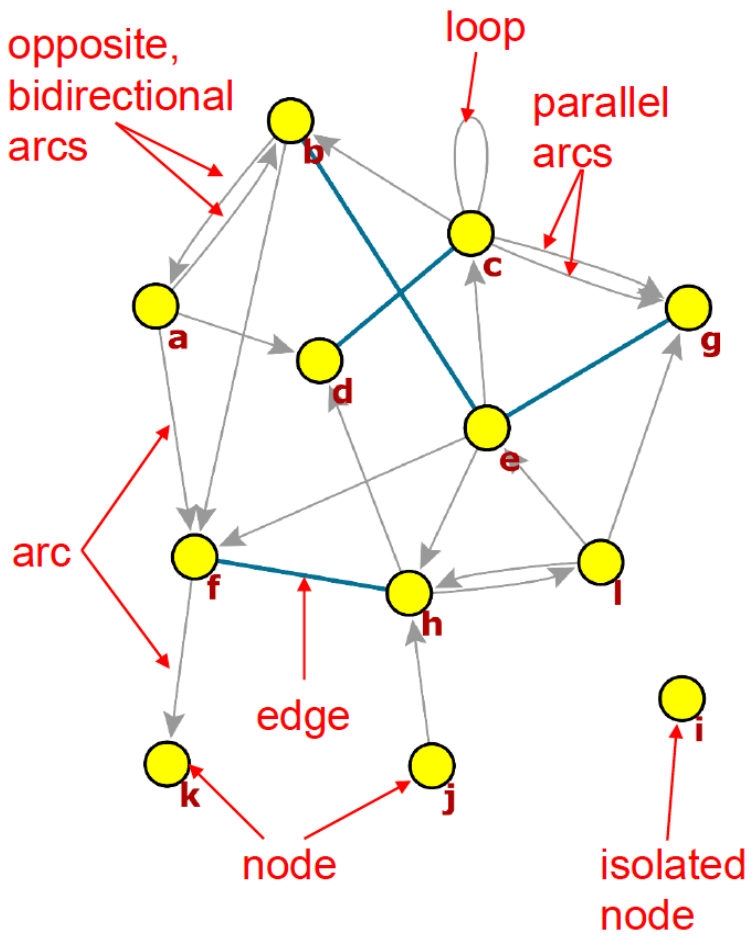
(b) discrete vertex and edge types, undirected

(c) varying vertex and edge weights, undirected

(d) directed

(e) signed

EXAMPLE: MIXED GRAPH

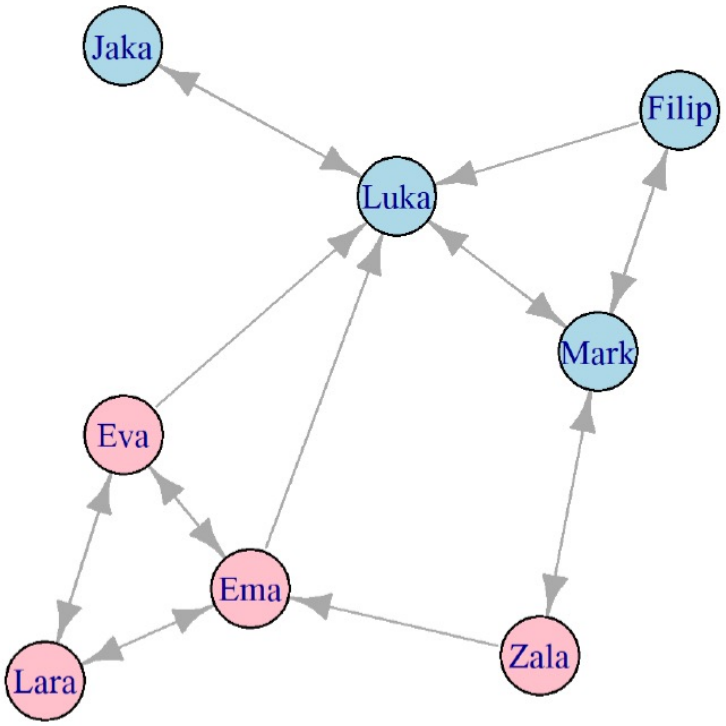


- Unit, actor, node, vertex – *a, b, …, k*
- Tie, link:
  - arc = directed link/edge, (*a, d*)
    - a – initial node
    - d – terminal node
  - edge = undirected link/edge, (*c: d*)

(i,i) relations (self-relations or loops) usually not considered/defined

EXAMPLE: FRIENDSHIP NOMINATION

E.g., answers to questions: who do you consider a friend?

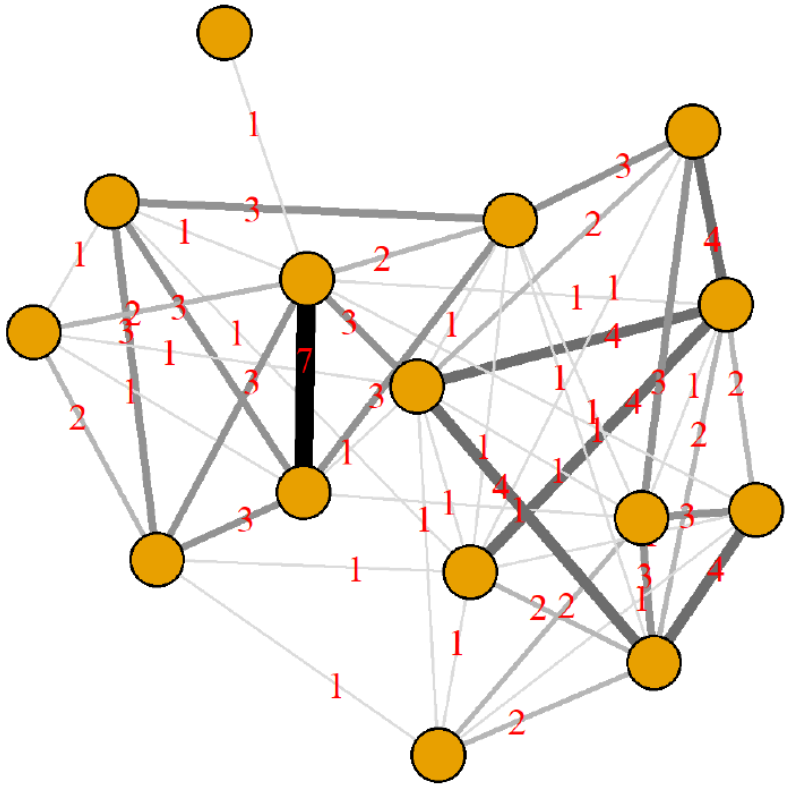


- one-mode
- binary
- directed
- small
- no-loops

directed graph:
also called DIGRAPH

EXAMPLE: INTERACTION NETWORK

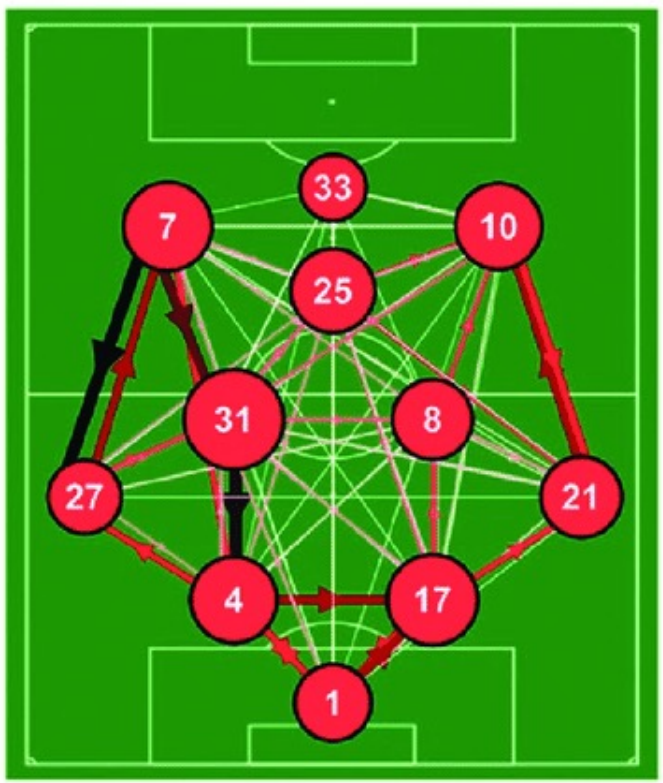
E.g., number of times observed kids (in kindergarden) playing together?



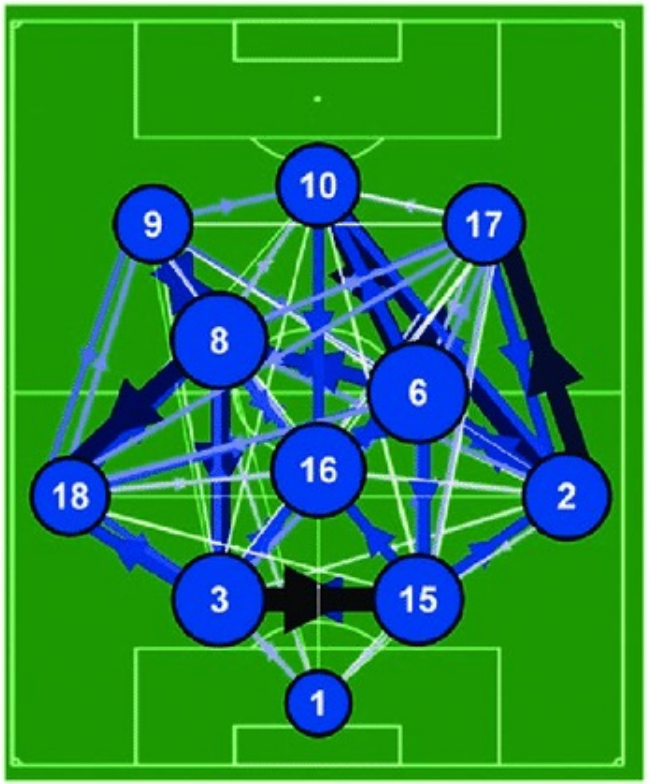
- one-mode
- valued/weighted
- undirected
- small
- no-loops

WEIGHTED GRAPH

EXAMPLE: FOOTBALL PASSING NETWORKS



7 - Ribery, 33 - Gómez, 10 - Robben,
31 - Schweinsteiger, 25 - Müller, 8 - Martínez,
27 - Alaba, 4 - Dante, 17 - Boateng, 21 - Lahm, 1 - Neuer



9 - Alexis, 10 - Messi, 17 - Pedro,
8 - Iniesta, 16 - Busquets, 6 - Xavi,
18 - Alba, 3 - Piqué, 15 -Bartra, 2 - Alves, 1 - Valdés

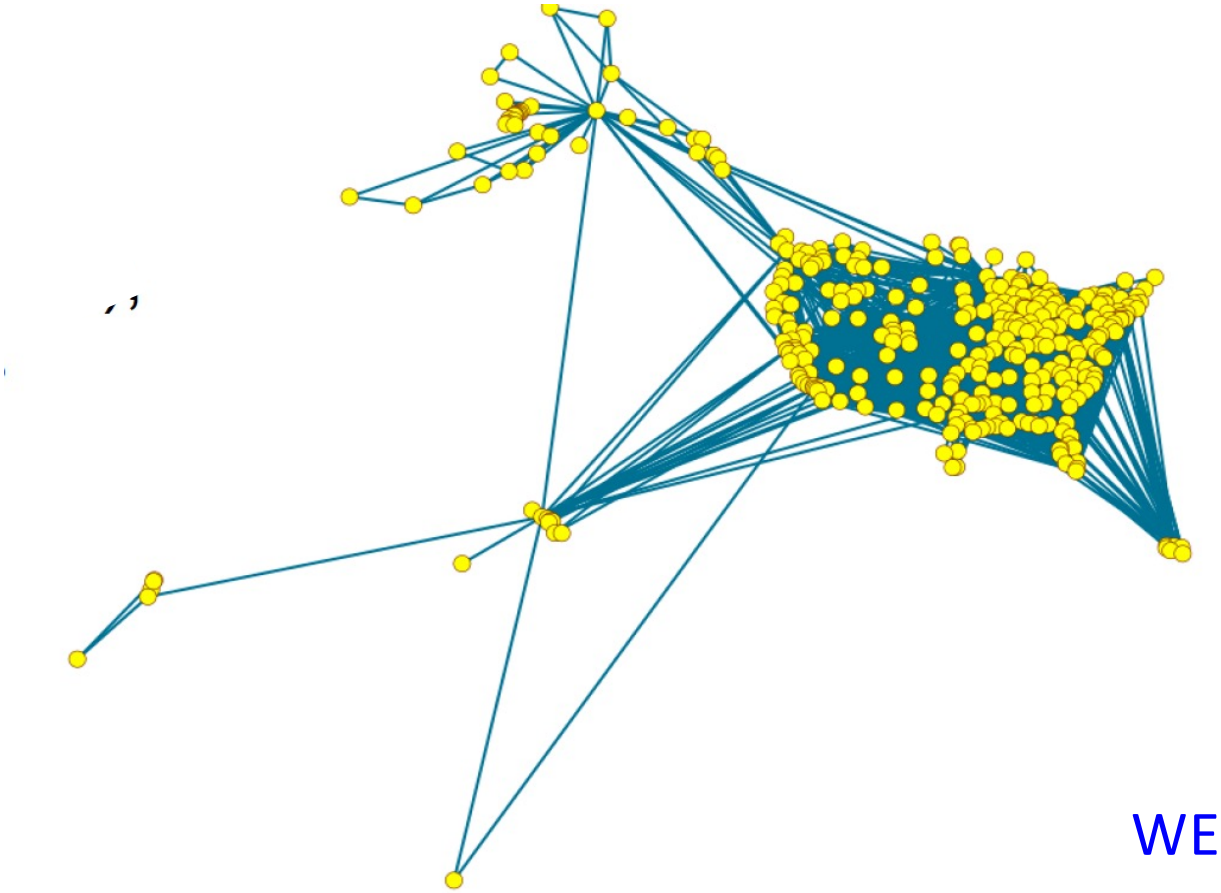
- one-mode
- valued
- small
- no-loops
- directed

WEIGHTED DIGRAPH

(https://grafos-da-bola.netlify.app/)

EXAMPLE: TRANSPORT NETWORK

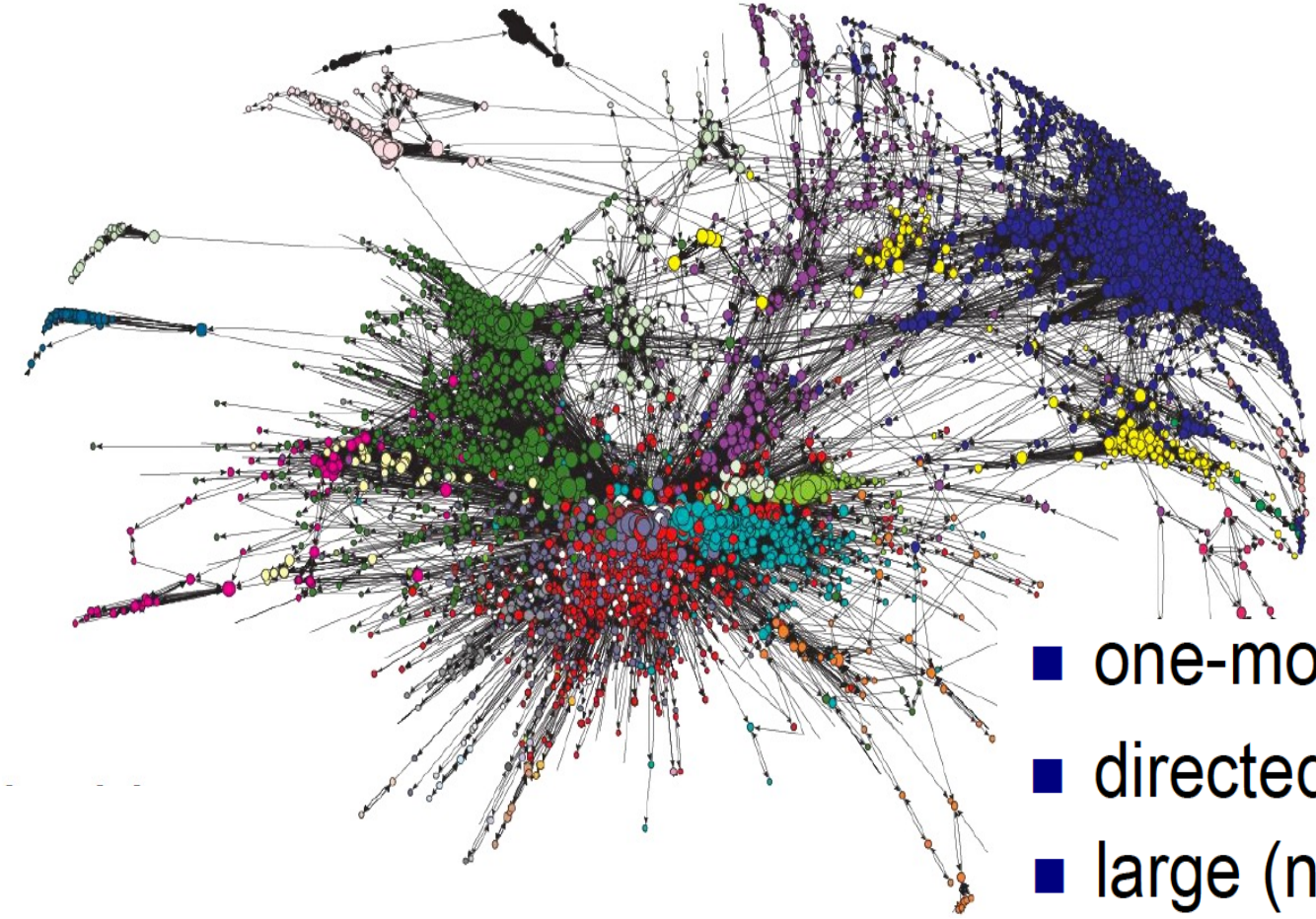
E.g., USA air roots



- one-mode
- valued (distance), but plotted as binary (values determines the layout)
- medium
- no-loops
- undirected

WEIGHTED (DISTANCE) GRAPH

EXAMPLE OF LARGE NETWORK: SLOVENIAN TWITTER NETWORK



- one-mode, binary
- directed (followers)
- large (n = **11211, m = 534895)**

EXAMPLE: CITATION NETWORK (DAG)

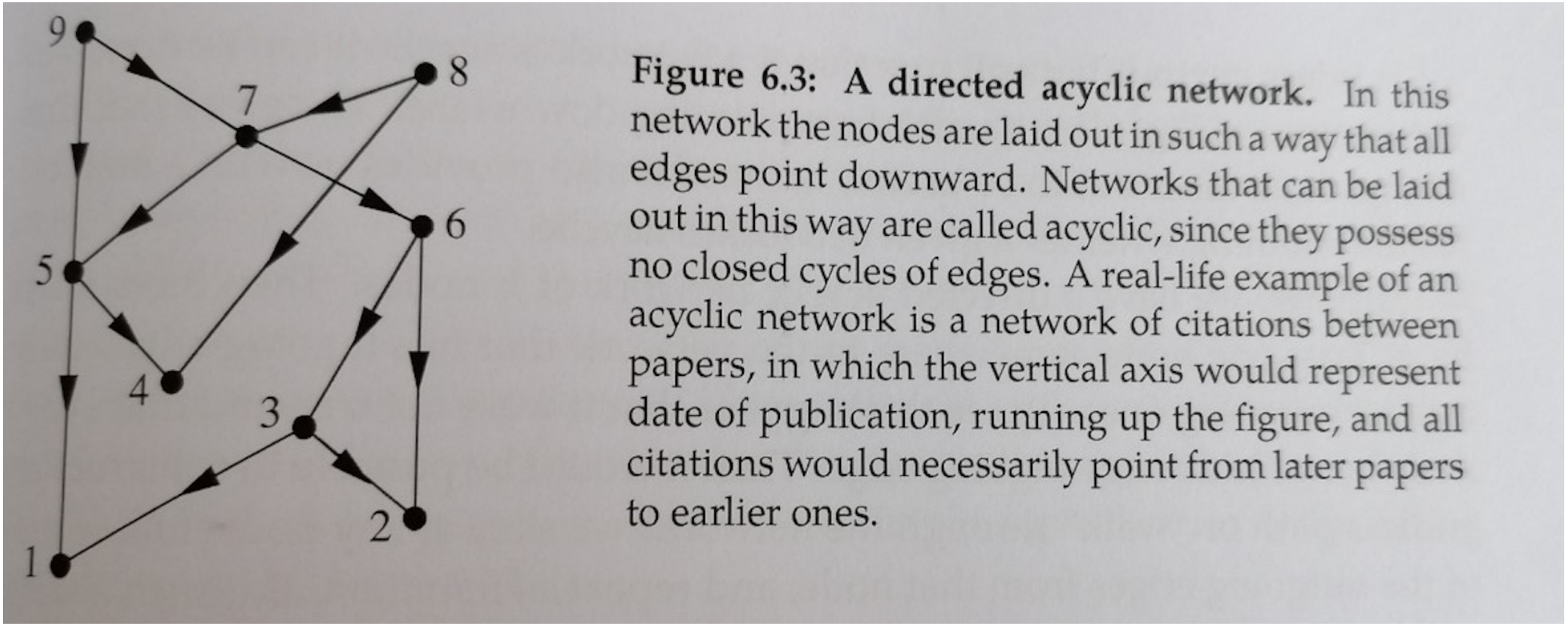


Figure 6.3: A directed acyclic network. In this network the nodes are laid out in such a way that all edges point downward. Networks that can be laid out in this way are called acyclic, since they possess no closed cycles of edges. A real-life example of an acyclic network is a network of citations between papers, in which the vertical axis would represent date of publication, running up the figure, and all citations would necessarily point from later papers to earlier ones.

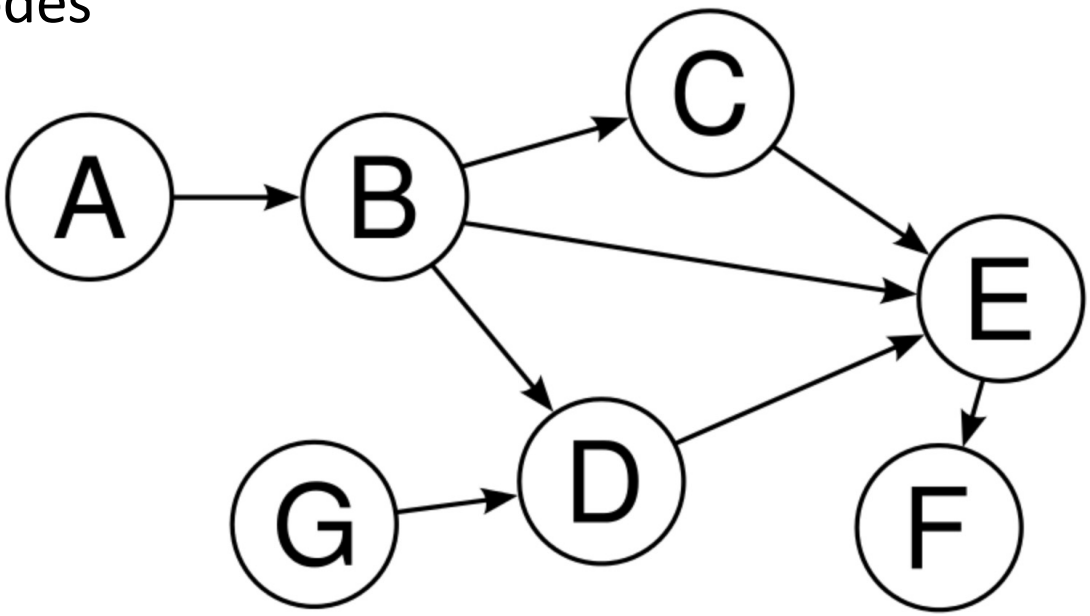
Newman M. (2018) *Networks*, Oxford, University Press, 2nd Edition

- one-mode
- binary
- directed (acyclic)

DIRECTED ACYCLIC GRAPH - DAG

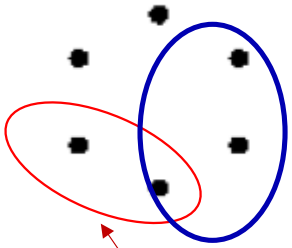
DAG: A directed network with no cycles (all distinct nodes following directed edges)

DAG is very useful in representing a clear dependency structure between the nodes



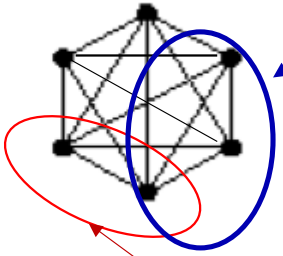
GRAPH: BASIC SUBNETWORKS

Empty graph



null dyad

Complete graph



Full dyad

Triad: a subset of three nodes (triplet) and the (possible) links between them

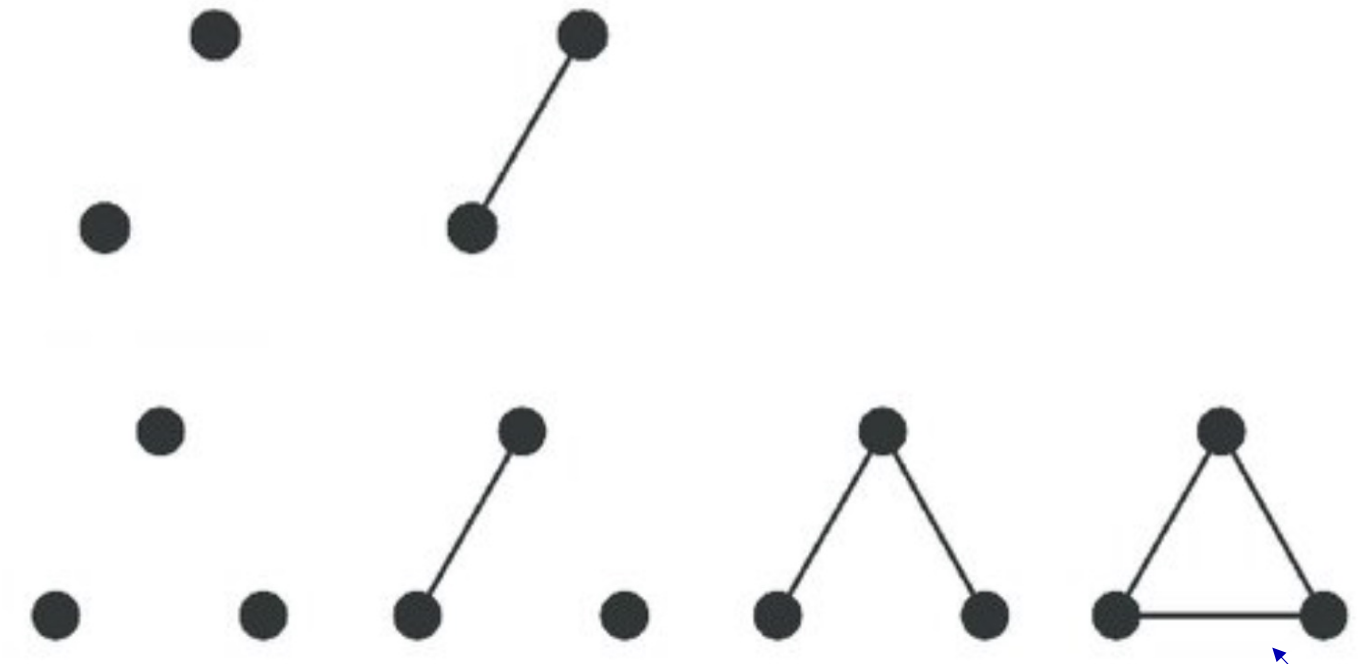
Dyad: a pair of nodes and the (possible) links between them

(the fondamental unit in networks)

n nodes: *n(n - 1)/ 2* dyads
(n = 6, 15 dyads)

n nodes: *n(n - 1)(n - 2)/ 6* triads
(n = 6, 20 triads)

DYADIC AND TRIADIC CONFIGURATION – UNDIRECTED GRAPH



Triangle (complete subgraph of 3 nodes)

The presence of ‘small’ network patterns reflects the underlyng structural processes in the network

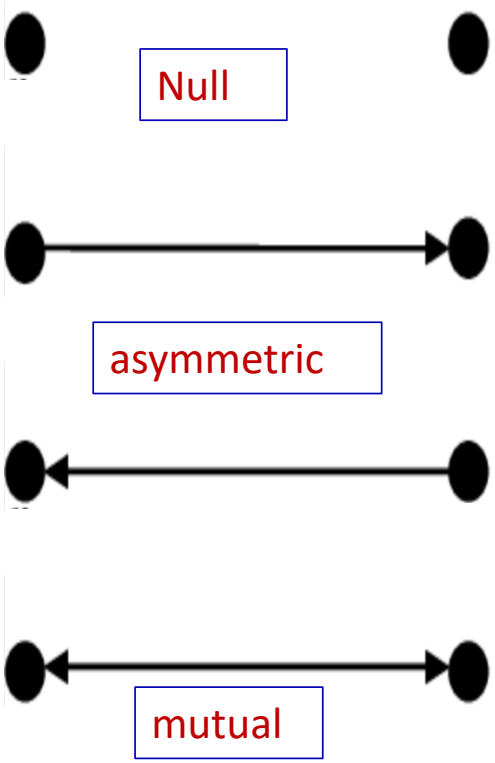
TRIANGLE – DIRECTED GRAPH

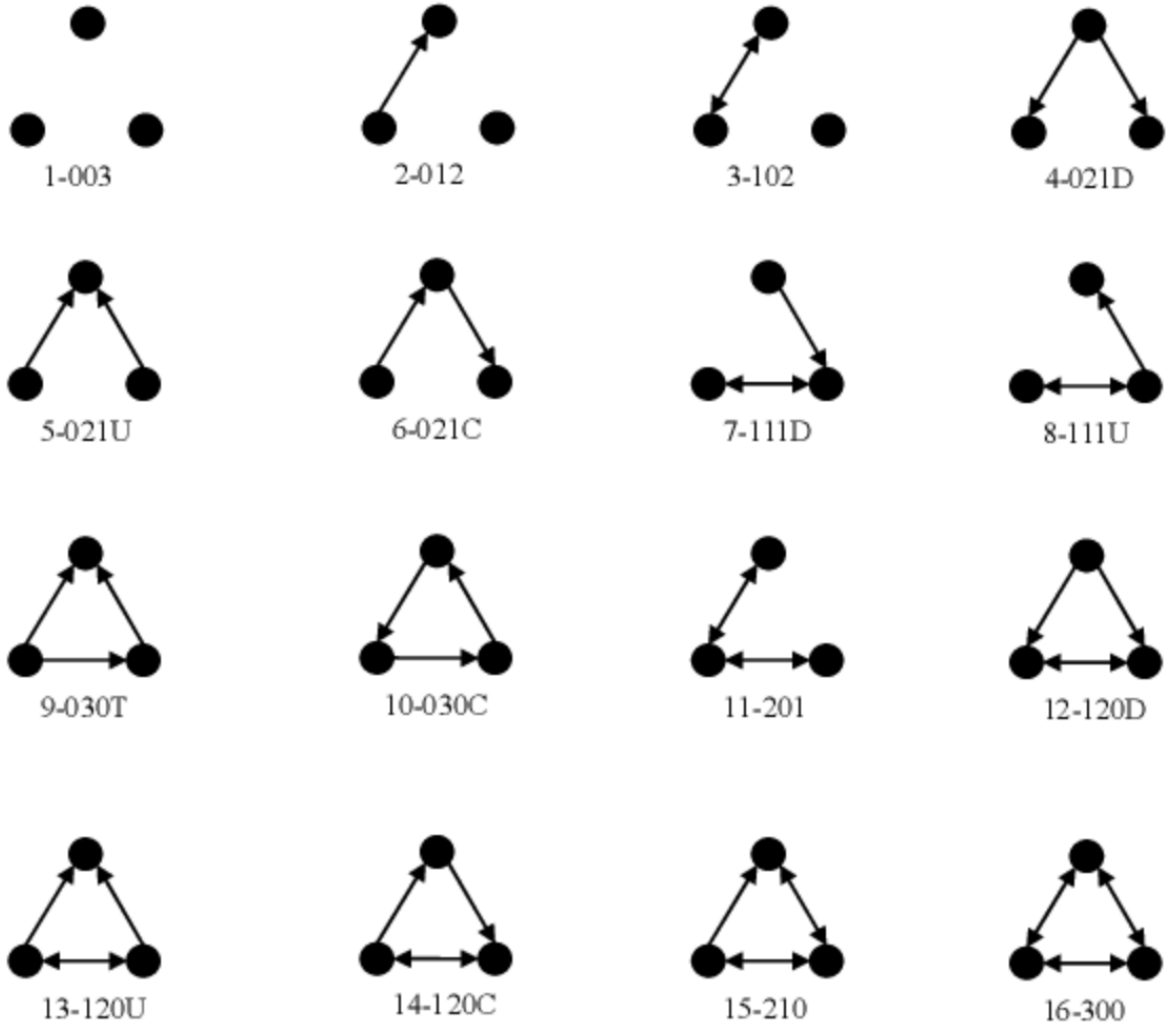
Cyclic triad: arcs follow the same direction

Transitive triad: not all arcs follow the same direction

The presence of ‘small’ network patterns reflects the underlyng structural processes in the network

DYADIC AND TRIADIC CONFIGURATION – DIRECTED GRAPH





16 TRIADIC CONFIGURATION – DIRECTED GRAPH

M-A-N labeling convention:
Mutual
Asymmetric
Null
the **first** character gives the number of mutual dyads,

the **second** character gives the number of asymmetric dyads,

the **third** character gives the number of null dyads

the **letter** (if present) stands for **D**own, **U**p, **T**ransitive, **C**ycle