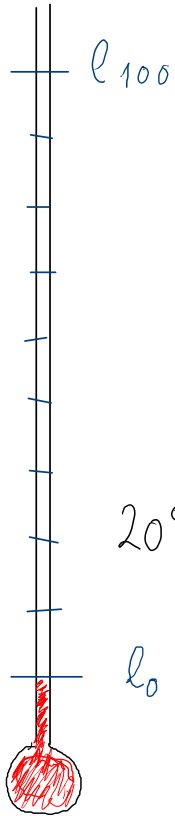


GRANDEZZE FISICHE



$$l = l_0 + a t$$

temperatura
in $^\circ C$

MISURA:
confronto con una opportuna
unità di misura

SI - MKS

grandezze fisiche fondamentali

lunghezza
 $1 \text{ m} = \frac{1}{299\,792\,458} \text{ s}$

massa kg
 $N_A = 6,022 \cdot 10^{23}$

tempo s
cesio 133

temperatura K

quantità di materia mol

intensità di corrente elettrica A

intensità luminosa cd

$$c = 299\,792\,458 \frac{\text{m}}{\text{s}}$$

$$\approx 3 \cdot 10^8 \text{ m/s}$$

$$\approx 0,3 \cdot 10^9 \text{ m/s}$$

$$\approx 0,3 \text{ m} / 10^{-9} \text{ s} = 30 \text{ cm/us}$$

grandette fisiche derivate

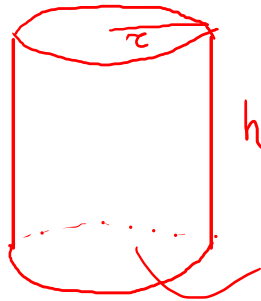
$$v \rightarrow \frac{m}{s}$$

$$a \rightarrow \frac{m}{s^2}$$

$$F \rightarrow \text{kg} \frac{m}{s^2} = N$$

$$E \rightarrow \text{kg} \frac{m^2}{s^2} = J$$

DIMENSIONI



$$r = 1 \text{ m}$$

$$h = 2 \text{ m}$$

$$A = \pi r^2 = \pi (1 \text{ m})^2 = \pi \text{ m}^2$$

$$V = A \cdot h = \pi r^2 \cdot h = \pi (1 \text{ m})^2 \cdot 2 \text{ m} = 2\pi \text{ m}^3$$

$$A + V =$$

$$K = \frac{1}{2} m v^2 = \dots \quad \frac{1}{2} 0,58 \text{ kg} \cdot \left(2,3 \frac{\text{m}}{\text{s}}\right)^2$$

$$\text{kg} \frac{\text{m}^2}{\text{s}^2} \rightarrow \text{J}$$

J

ANALISI DIMENSIONALE

$$[\tau] = [L]$$

$$[A] = [L^2]$$

$$[V] = [L^3]$$

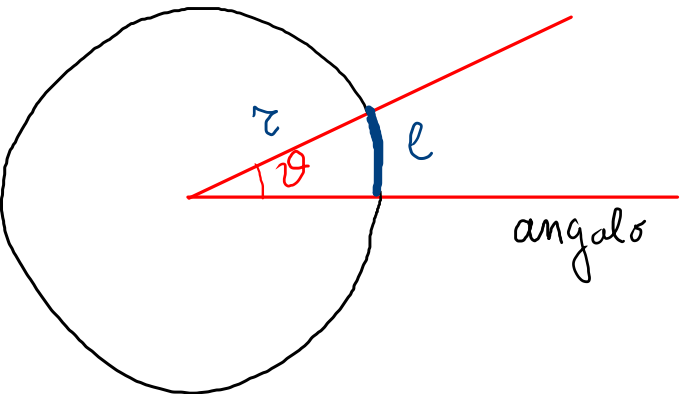
$$[K] = [L^2][M][T^{-2}]$$

$$[\text{grandezza fisica}] = [L^\alpha][M^\beta][T^\delta] \dots [U^\xi]$$

ANGOLI

$$\vartheta = \frac{l}{r}$$

$$[\vartheta] = [L^0] \text{ adimensionale!}$$



angolo giro



gradi

360°

radiani

$$\frac{2\pi r}{r} = 2\pi$$

piatto



180°

$$\frac{\pi r}{r} = \pi$$

retto



90°

$$\frac{\frac{1}{2}\pi r}{r} = \frac{\pi}{2}$$

45°

30°

60°

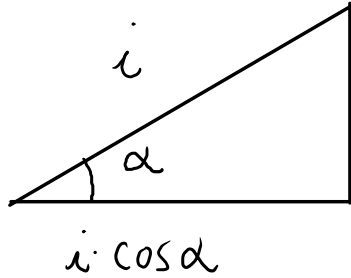
$\frac{\pi}{4}$

$\frac{\pi}{6}$

$\frac{\pi}{3}$

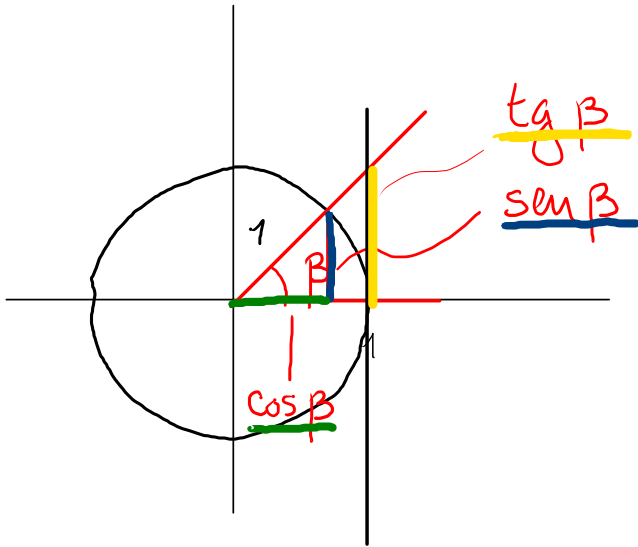
$$\alpha = 32^\circ = 32^\circ \cdot 1 = 32^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{32}{180} \pi = \frac{8}{45} \pi$$

TRIANGOLO



$$i \cdot \text{sen } \alpha$$

$$\text{tg } \alpha = \frac{\text{Sen } \alpha}{\text{cos } \alpha}$$



$$\text{sen } \beta \in [-1, 1]$$

$$\text{cos } \beta \in [-1, 1]$$

$$(\text{sen } \beta)^2 + (\text{cos } \beta)^2 = 1$$

$$\text{sen}^2 \beta + \text{cos}^2 \beta = 1$$

CIFRE SIGNIFICATIVE

$$(800 \pm 1) \text{ m}$$

$$(800 \pm 10) \text{ m}$$

$$(800 \pm 100) \text{ m}$$

Abramo Lincoln

$$h = 6 \text{ piedi e } 4 \text{ pollici}$$

$$\begin{array}{l} \downarrow \\ = 6 \text{ piedi} \left(\frac{12 \text{ pollici}}{1 \text{ piede}} \right) + 4 \text{ pollici} \end{array}$$

$$\begin{array}{l} \downarrow \\ = 72 \text{ pollici} + 4 \text{ pollici} = 76 \text{ pollici} \end{array}$$

$$h = 76 \text{ pollici} \left(\frac{2,54 \text{ cm}}{1 \text{ pollice}} \right) = 193,04 \text{ cm} \quad \times$$
$$\begin{array}{l} \downarrow \\ = 193 \text{ cm} \quad \checkmark \end{array}$$

I VETTORI

GRANDEZZE SCALARI: numero ed unità di misura
Es: massa, energia...

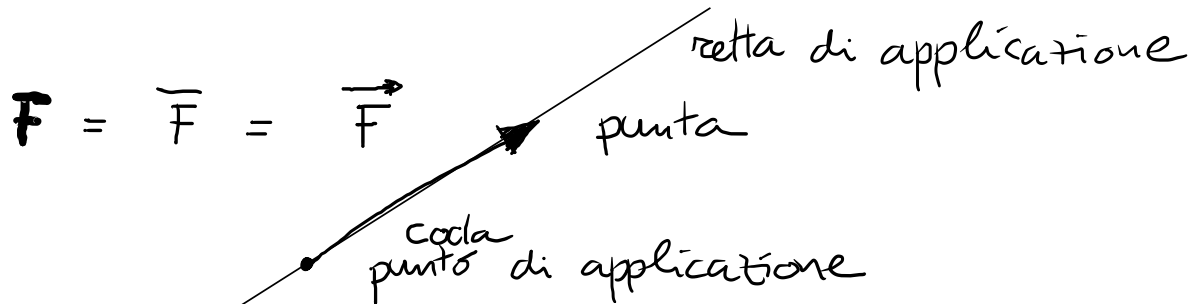
GRANDEZZE VETTORIALI: numero e unità di misura
determinano il **MODULO** (o l'intensità)
della grandezza fisica

ma servono anche: **DIREZIONE**

VERSO

Es: velocità, forza, etc...

$$\mathbf{F} = \overline{F} = \vec{F}$$



il modulo del vettore \vec{F} si indica $|\vec{F}|$ o sempl. F

Due vettori sono uguali quando hanno

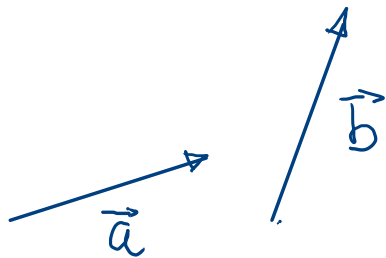
stesso modulo

stessa direzione

stesso verso

Dal punto di vista fisico talvolta sono importanti anche la retta di applicazione ed il punto di applicazione

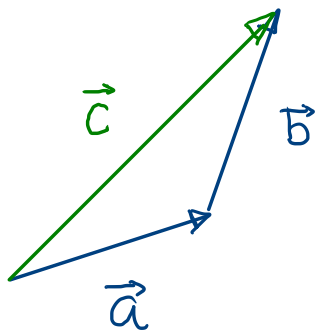
SOMMA DI VETTORI



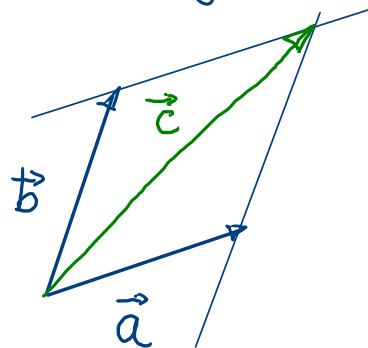
\vec{c} è la somma o composizione di \vec{a} e \vec{b}

$$\vec{a} + \vec{b} = \vec{c}$$

① Punta-coda



② Parallelogramma



COMMUTATIVA:

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} = \vec{c}$$

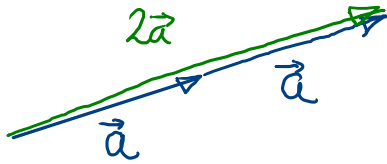
ASSOCIATIVA:

$$\vec{a} + (\vec{b} + \vec{e}) = (\vec{a} + \vec{b}) + \vec{e}$$

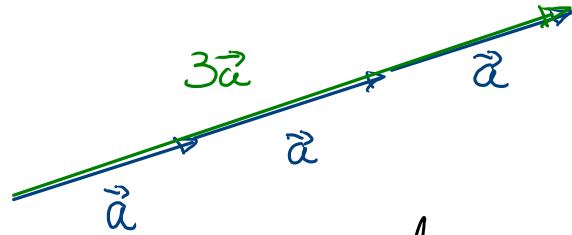
$$\forall \vec{a}, \vec{b}, \vec{e}$$

PRODOTTO DI UN VETTORE PER UNO SCALARE

$$\vec{a} + \vec{a} = 2\vec{a}$$



$$\vec{a} + \vec{a} + \vec{a} = 3\vec{a}$$



$$m\vec{a} \quad m \in \mathbb{R}$$

modulo
direzione
verso

$$|m\vec{a}| = |m| |\vec{a}|$$

la stessa di \vec{a}

lo stesso di \vec{a}

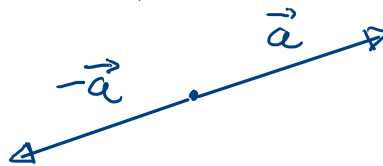
opposto

valore assoluto di m
modulo di \vec{a}

se $m > 0$

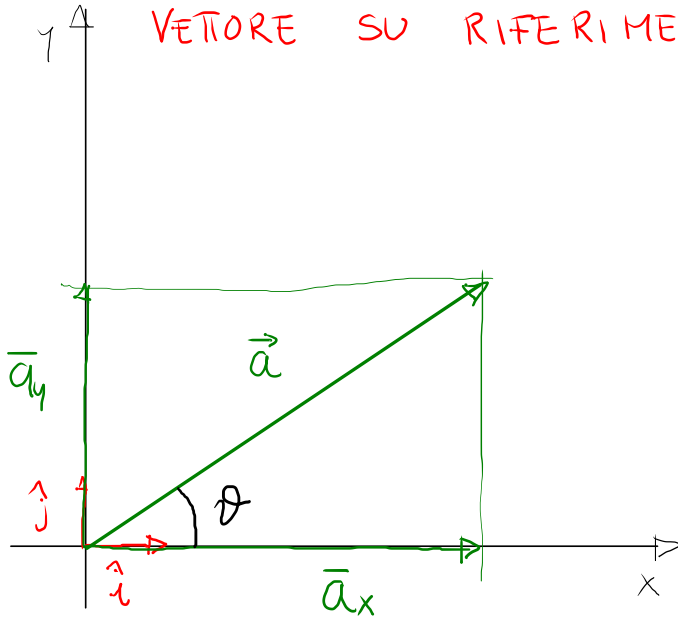
se $m < 0$

$$\text{In particolare} \quad -1 \cdot \vec{a} = -\vec{a}$$



VETORE SU RIFERIMENTO CARTESIANO

$$|\hat{i}| = |\hat{j}| = 1$$



dal vettore alle componenti

$$\begin{cases} a_x = |\vec{a}_x| = |\vec{a}| \cdot \cos \vartheta \\ a_y = |\vec{a}_y| = |\vec{a}| \cdot \sin \vartheta \end{cases}$$

$$\frac{a_y}{a_x} = \frac{|\vec{a}| \sin \vartheta}{|\vec{a}| \cos \vartheta} = \operatorname{tg} \vartheta$$

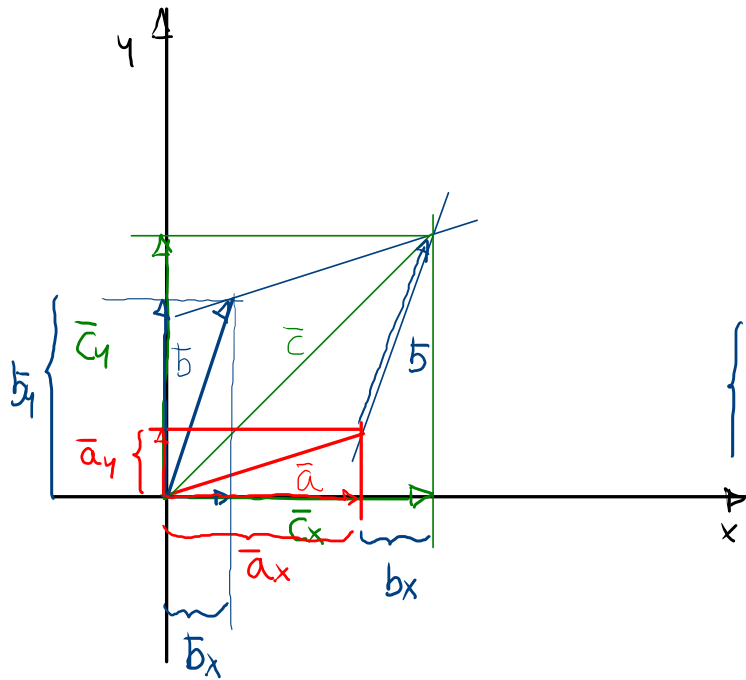
$$\begin{aligned} \vec{a} &= \vec{a}_x + \vec{a}_y \\ &= \underbrace{|\vec{a}_x|}_{\in \mathbb{R}^+} \cdot \hat{i} + \underbrace{|\vec{a}_y|}_{\in \mathbb{R}^+} \cdot \hat{j} \end{aligned}$$

$$\vec{a} = 6 \hat{i} + 4 \hat{j} = (6, 4)$$

↑ ↑
Componenti scalari

dalle componenti al vettore

$$\begin{cases} |\vec{a}| = \sqrt{|\vec{a}_x|^2 + |\vec{a}_y|^2} \\ \vartheta = \operatorname{arctg} \left(\frac{a_y}{a_x} \right) \end{cases}$$



$$\bar{c} = \bar{a} + \bar{b}$$

$$\bar{c} = \bar{c}_x + \bar{c}_y$$

$$= |\bar{c}_x| \hat{i} + |\bar{c}_y| \hat{j}$$

$$= c_x \hat{i} + c_y \hat{j}$$

$$\begin{cases} c_x = a_x + b_x \\ c_y = a_y + b_y \end{cases}$$

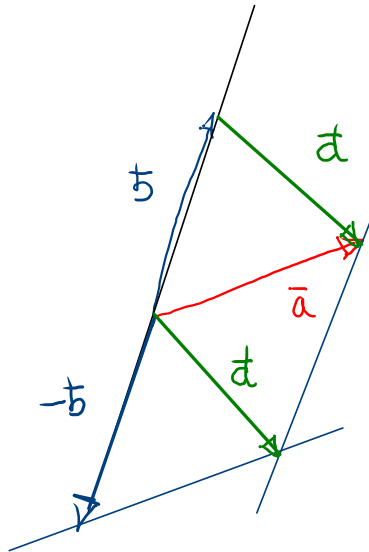
somma di vettori \bar{e} :

- commutativa $\bar{a} + \bar{b} = \bar{b} + \bar{a}$

- associativa $(\bar{a} + \bar{b}) + \bar{e} = \bar{a} + (\bar{b} + \bar{e}) = \bar{a} + \bar{b} + \bar{e}$

SOTTRAZIONE TRA VETTORI

$$\bar{a} - \bar{b} = \bar{d}$$

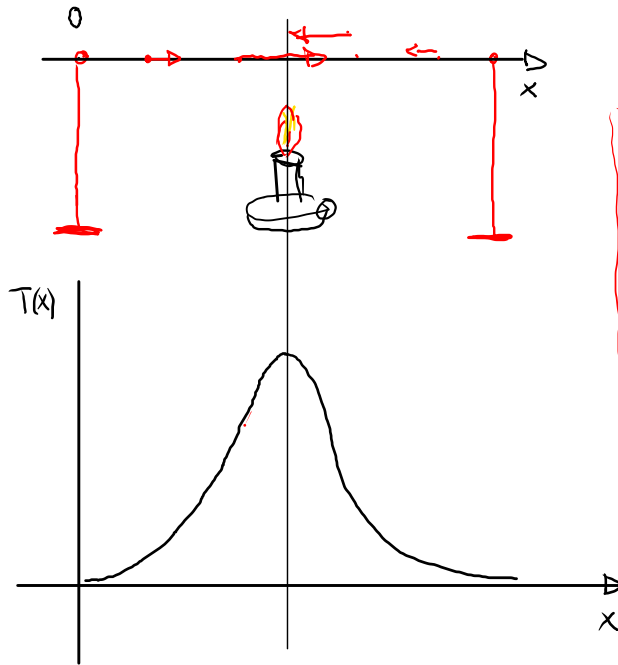


$$\begin{aligned}\bar{a} - \bar{b} &= \bar{a} + (-1)\bar{b} \\ &\stackrel{!}{=} \bar{a} + (-\bar{b})\end{aligned}$$

$$d_x = a_x - b_x$$

$$d_y = a_y - b_y$$

VEITORE GRADIENTE



$T(x)$ è una funzione scalare

$$\overline{\text{grad. } T(x)} \equiv \nabla T(x)$$

$$|\nabla T(x)| = \left| \frac{dT(x)}{dx} \right| \quad \text{modulo}$$

direzione: \hat{i}

$$\text{verso } +\hat{i} \quad \text{se } \frac{dT(x)}{dx} > 0$$

$$-\hat{i} \quad \text{se } \frac{dT(x)}{dx} < 0$$

$$T(x, y, z)$$

$$\begin{aligned} \text{grad } T(x, y, z) &= \nabla T(x, y, z) \\ &= \frac{\partial T(x, y, z)}{\partial x} \hat{i} + \frac{\partial T(x, y, z)}{\partial y} \hat{j} + \frac{\partial T(x, y, z)}{\partial z} \hat{k} \end{aligned}$$

Esempio: consideriamo la mappa di un territorio, e definiamo una funzione $h=h(x,y)$ (e non $h(x,y,z)$ come ho erroneamente detto a lezione, sorry) che definisce la quota sul livello del mare:

$h(x, y, z)$ quota sul livello del mare

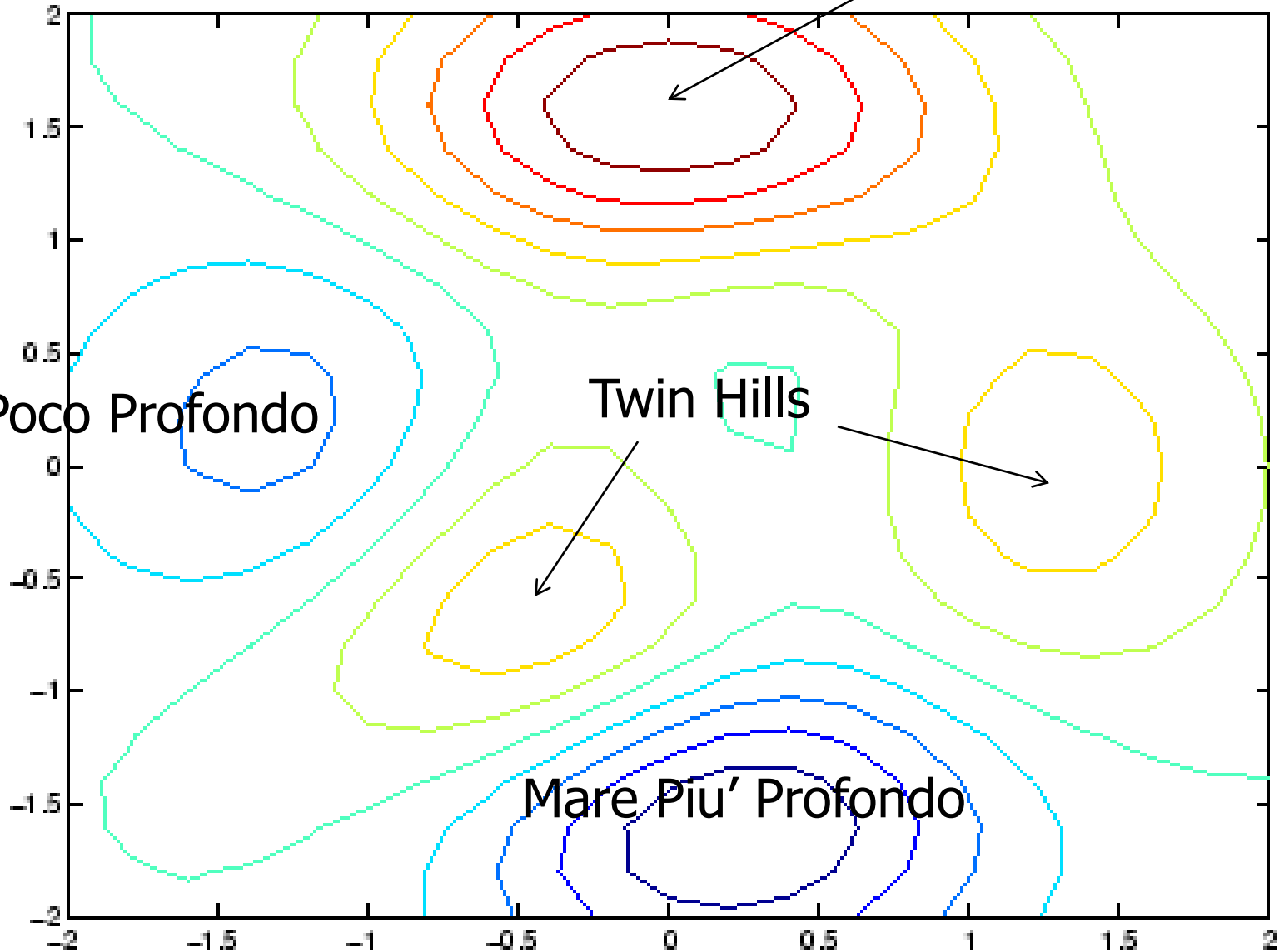
Mappa del territorio

Picco Alto

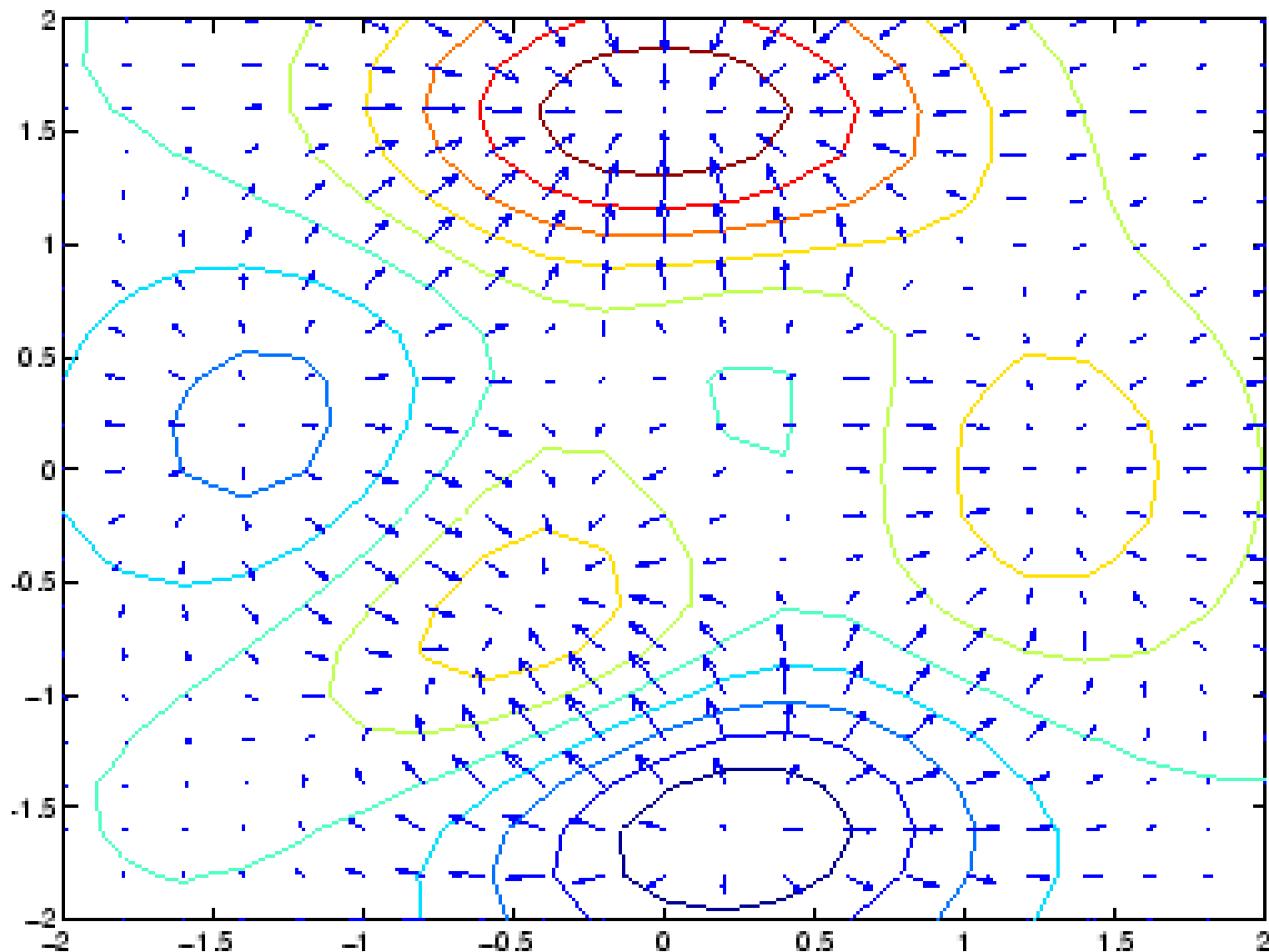
Mare Poco Profondo

Twin Hills

Mare Piu' Profondo

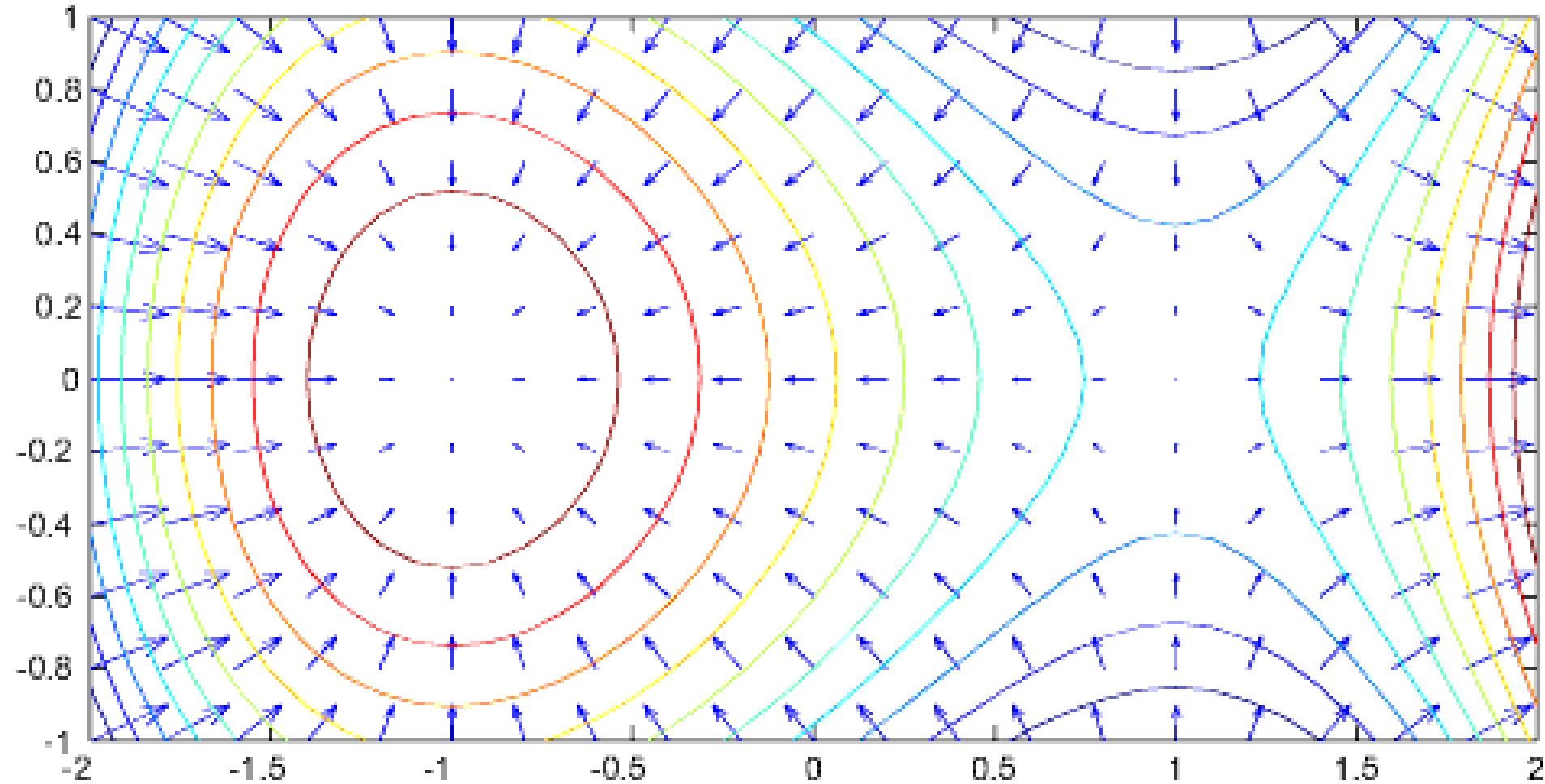


Mappa del territorio con Gradiente



Altro esempio: $f(x,y)=x^3-3x-2y^2$

Gradient vector field and level curves of $f(x,y)=x^3-3x-2y^2$



$$\mathbf{grad} f(x,y) = (3x^2-3)\mathbf{i} - 4y\mathbf{j}$$