Cormen Problem 12-1. Equal keys pose a problem for the implementation of binary search trees.

**a.** What is the asymptotic performance of TREE-INSERT when used to insert *n* items with identical keys into an initially empty binary search tree?

**Cormen Problem 12-1.** We propose to improve TREE-INSERT by testing before line 5 to determine whether z.key = x.key and by testing before line 11 to determine whether z.key = y.key.

```
TREE-INSERT (T, z)

1  y = \text{NIL}

2  x = T.root

3  while x \neq \text{NIL}

4  y = x

5  if z.key < x.key

6  x = x.left

7  else x = x.right

8  z.p = y

9  if y == \text{NIL}

10  T.root = z

11  elseif z.key < y.key
```

y.left = z

else y.right = z

12

13

If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting *n* items with identical keys into an initially empty binary search tree. (The strategies are described for line 5, in which we compare the keys of *z* and *x*. Substitute *y* for *x* to arrive at the strategies for line 11.)

**b.** Keep a boolean flag *x.b* at node *x*, and set *x* to either *x.left* or *x.right* based on the value of *x.b*, which alternates between FALSE and TRUE each time we visit *x* while inserting a node with the same key as *x*.

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```
TREE-INSERT (T, z)
     y = NIL
    x = T.root
    while x \neq NIL

\begin{array}{ccc}
4 & y = x \\
\hline
5 & \text{if } z.key < x.key
\end{array}

      x = x.left
          else x = x.right
     z.p = y
     if y == NIL
10
           T.root = z
11
     elseif z.key < y.key
           y.left = z
12
```

else y.right = z

13

If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting *n* items with identical keys into an initially empty binary search tree. (The strategies are described for line 5, in which we compare the keys of *z* and *x*. Substitute *y* for *x* to arrive at the strategies for line 11.)

**c.** Keep a list of nodes with equal keys at *x*, and insert *z* into the list.

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```
TREE-INSERT (T, z)
   y = NIL
   x = T.root
3
   while x \neq NIL
x = x.left
  else x = x.right
   z.p = y
   if y == NIL
10
       T.root = z
11
   elseif z.key < y.key
       y.left = z
12
```

else y.right = z

13

If equality holds, we implement one of the following strategies. For each strategy, find the asymptotic performance of inserting *n* items with identical keys into an initially empty binary search tree. (The strategies are described for line 5, in which we compare the keys of *z* and *x*. Substitute *y* for *x* to arrive at the strategies for line 11.)

**d.** Randomly set x to either *x.left* or *x.right*. (Give the worst-case performance and informally derive the expected running time.)

A preorder traversal of a tree is given by the following procedure:

- Visit (print) the root node
- Traverse the left sub-tree in pre-order
- Traverse the right sub-tree in pre-order

A postorder traversal of a tree is given by the following procedure:

- Traverse the left subtree by calling the postorder function recursively.
- Traverse the right subtree by calling the postorder function recursively.
- Visit (print) the current node.

**EX.** Given a BST in pre-order as {13,5,3,2,11,7,19,23}, draw this BST and determine if this BST is the same as one described in post-order as {2,3,5,7,11,23,19,13}.