A.6 Solution to Exercise 3.1

Consider the map

$$\mathcal{T}_s[\hat{\rho}(0)] = \hat{\rho}(s) = e^{-4s}\hat{\rho}(0) + \frac{\left(1 - e^{-4s}\right)}{2}\hat{\mathbb{1}}. \tag{A.20}$$

If we set $s = 0$, we trivially find

$$\mathcal{T}_0[\hat{\rho}(0)] = \hat{\rho}(0), \tag{A.21}$$

thus $\mathcal{T}_0 = \mathrm{id}$. Then, we apply $\mathcal{T}_t$ to the expression in Eq. (A.20):

$$\begin{aligned}
\mathcal{T}_t[\hat{\rho}(s)] &= e^{-4t}\hat{\rho}(s) + \frac{\left(1 - e^{-4t}\right)}{2}\hat{\mathbb{1}}, \\
&= e^{-4t}\left(e^{-4s}\hat{\rho}(0) + \frac{\left(1 - e^{-4s}\right)}{2}\hat{\mathbb{1}}\right) + \frac{\left(1 - e^{-4t}\right)}{2}\hat{\mathbb{1}}, \\
&= e^{-4(t+s)}\hat{\rho}(0) + \frac{e^{-4t}\left(1 - e^{-4s}\right)}{2}\hat{\mathbb{1}} + \frac{\left(1 - e^{-4t}\right)}{2}\hat{\mathbb{1}}, \\
&= e^{-4(t+s)}\hat{\rho}(0) + \frac{\left(1 - e^{-4(t+s)}\right)}{2}\hat{\mathbb{1}},
\end{aligned} \tag{A.22}$$

which covers $\mathcal{T}_t\mathcal{T}_s = \mathcal{T}_{t+s}$. Finally, $\lim_{h\to 0}\mathcal{T}_{t+h}[\hat{\rho}(0)] = \mathcal{T}_t[\hat{\rho}(0)]$ can be obtained by applying the limit to Eq. (A.22).

A.7 Solution to Exercise 3.2

For $\epsilon = i\epsilon_0$ with $\epsilon_0 \in \mathbb{R}$, we have that Eq. (A.27) becomes

$$\mathrm{d}\left|\psi_t\right\rangle = \left[-\tfrac{i}{\hbar}\hat{H}\mathrm{d}t + \sum_k\left(-\epsilon_0\hat{L}_k\mathrm{d}W_{k,t} - \tfrac{1}{2}\epsilon_0^2\hat{L}_k^\dagger\hat{L}_k\mathrm{d}t\right)\right]\left|\psi_t\right\rangle, \tag{A.23}$$

where, notably, in the last term one has ϵ_0^2 in place of $\epsilon^2 = -\epsilon_0^2$. This is due to the constraint on the conservation of the norm for $|\psi_t\rangle$, which reflects in the preservation of the trace of $\hat{\rho}_t$. Similarly, the equation for $\langle\psi_t|$ reads

$$\mathrm{d}\left\langle\psi_t\right| = \left\langle\psi_t\right|\left[\tfrac{i}{\hbar}\hat{H}\mathrm{d}t + \sum_k\left(-\epsilon_0\hat{L}_k^\dagger\mathrm{d}W_{k,t} - \tfrac{1}{2}\epsilon_0^2\hat{L}_k^\dagger\hat{L}_k\mathrm{d}t\right)\right]. \tag{A.24}$$

By following the calculations already performed, we find the following master equation

$$\mathrm{d}\hat{\rho}_t = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}_t\right]\mathrm{d}t + \epsilon_0^2\sum_k\left(\hat{L}_k\hat{\rho}_t\hat{L}_k^\dagger - \tfrac{1}{2}\left\{\hat{L}_k^\dagger\hat{L}_k,\hat{\rho}_t\right\}\right)\mathrm{d}t, \tag{A.25}$$

which has the same for of Eq. (3.79) with ϵ substituted by ϵ_0.

A.8 Solution to Exercise 3.3

Let us consider Eq. (3.79) with $\hat{L}_k^\dagger = \hat{L}_k$. In such a case, the master equation can be rewritten as

$$
\mathrm{d}\hat{\rho}_t = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}_t\right]\mathrm{d}t - \frac{\epsilon^2}{2}\sum_k\left[\hat{L}_k,\left[\hat{L}_k,\hat{\rho}_t\right]\right]\mathrm{d}t, \tag{A.26}
$$

which can be obtained from the unravelling

$$
\mathrm{d}\left|\psi_t\right\rangle = \left[-\tfrac{i}{\hbar}\hat{H}\mathrm{d}t + \sum_k\left(i\epsilon\hat{L}_k\mathrm{d}W_{k,t} - \tfrac{1}{2}\epsilon^2\hat{L}_k^2\mathrm{d}t\right)\right]\left|\psi_t\right\rangle. \tag{A.27}
$$

The relation between the two holds also for the specific case of $\hat{L}_k = \hat{A}_k - \langle\psi_t|\hat{A}_k|\psi_t\rangle$, for which the unravelling becomes non-linear in $|\psi_t\rangle$:

$$
\mathrm{d}\left|\psi_t\right\rangle = \left\{-\tfrac{i}{\hbar}\hat{H}\mathrm{d}t + \sum_k\left[i\epsilon\left(\hat{A}_k - \langle\psi_t|\hat{A}_k|\psi_t\rangle\right)\mathrm{d}W_{k,t} - \tfrac{1}{2}\epsilon^2\left(\hat{A}_k - \langle\psi_t|\hat{A}_k|\psi_t\rangle\right)^2\mathrm{d}t\right]\right\}\left|\psi_t\right\rangle. \tag{A.28}
$$

However, in such a case, in the commutators appearing in Eq. (A.26) have the following structure:

$$
\left[\hat{A}_k - \langle\psi_t|\hat{A}_k|\psi_t\rangle, \hat{X}\right] = \left[\hat{A}_k, \hat{X}\right], \tag{A.29}
$$

which is valid for any arbitrary operator $\hat{X}$. It follows that the master equation becomes

$$
\mathrm{d}\hat{\rho}_t = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}_t\right]\mathrm{d}t - \frac{\epsilon^2}{2}\sum_k\left[\hat{A}_k,\left[\hat{A}_k,\hat{\rho}_t\right]\right]\mathrm{d}t, \tag{A.30}
$$

which notably is linear in the state $\hat{\rho}_t$.