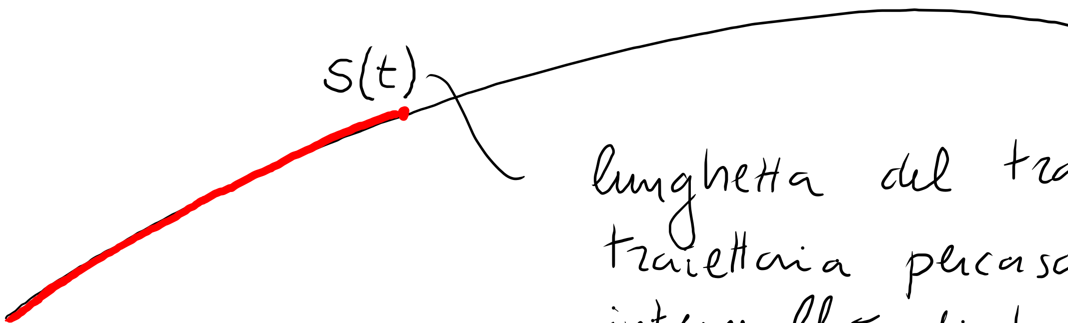
CINEMATICA

Meccanica
- cinematica ✓
- statica
- dinamica

punto materiale: punto geometrico dotato di massa

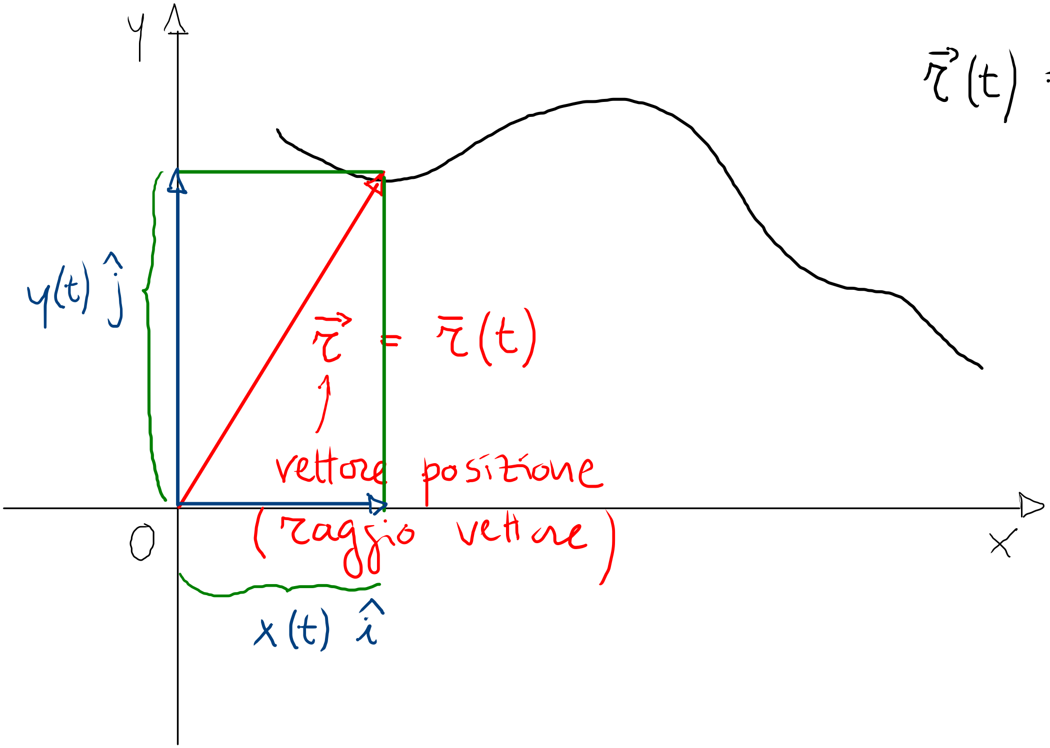
moto punto materiale

traiettoria:



lunghezza del tratto di traiettoria percorso nell'intervallo di tempo t

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

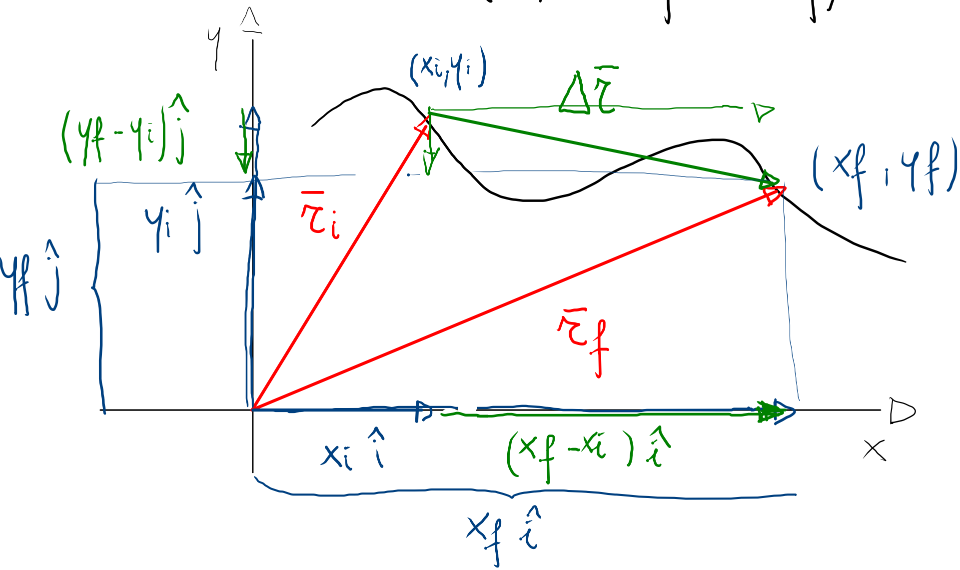


SPOSTAMENTO

$\bar{r}_i = \bar{r}(t_i) \qquad \bar{r}_f = \bar{r}(t_f)$

$t_i \quad t_f$

$\Delta t = t_f - t_i$



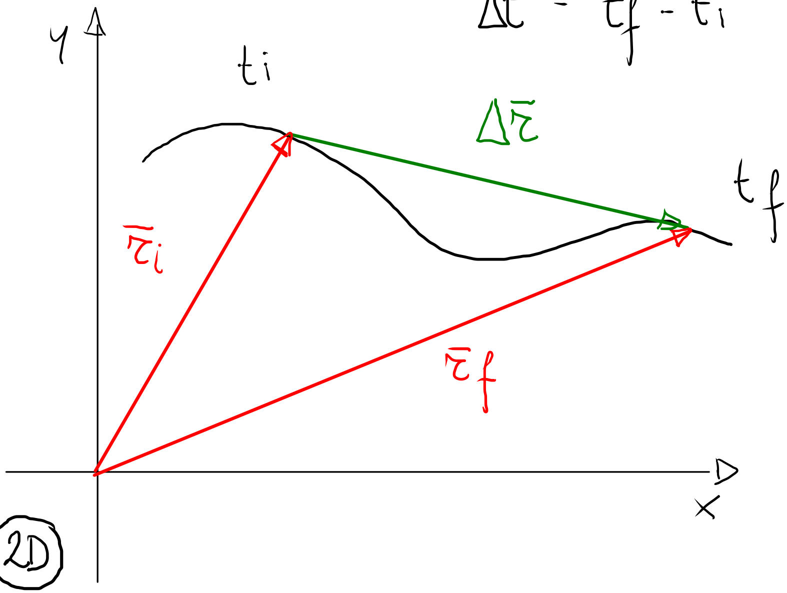
$$\boxed{\Delta\vec{r} = \bar{r}_f - \bar{r}_i}$$

$$\Delta\bar{r} = \bar{r}_f - \bar{r}_i$$
$$= (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$$

$|\Delta\bar{r}|$ non rappresenta la lunghezza della traiettoria percorsa

VELOCITÀ MEDIA

$$\Delta t = t_f - t_i$$



2D

$$\vec{v}_m = \frac{\Delta \bar{r}}{\Delta t} = \frac{\bar{r}_f - \bar{r}_i}{t_f - t_i}$$

$$|\vec{v}_m| = \frac{|\Delta \bar{r}|}{\Delta t}$$

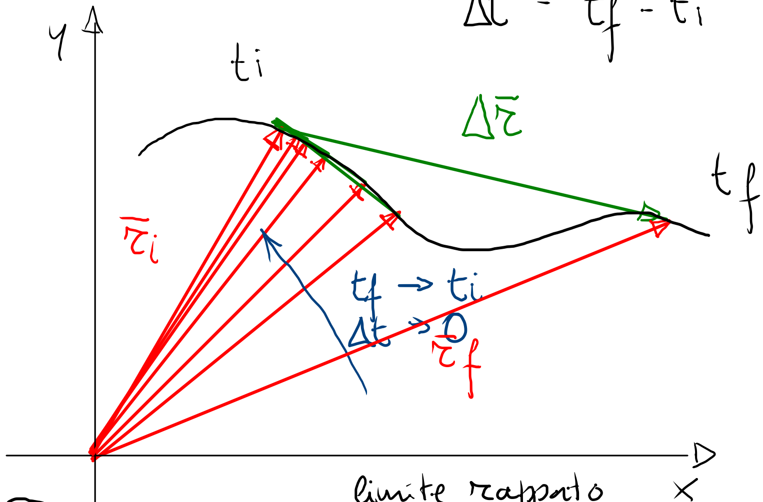
direzione e verso sono di $\Delta \bar{r}$

1D

$$v_{x\,m} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

VELOCITÀ ISTANTANEA

$\Delta t = t_f - t_i \qquad \Delta t \to 0$



2D

limite rapporto incrementale

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \bar{r}}{\Delta t} = \frac{d\bar{r}}{dt} = \frac{d\bar{r}}{dt}$$

derivata

rapporto tra infinitesimi

1D

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{d}$$

SI: $\frac{m}{s}$ cgs: $\frac{cm}{s}$

$$1 \frac{Km}{h} = \frac{1000\ m}{3600\ s} = \frac{1}{3{,}6} \frac{m}{s}$$

$$1 \frac{m}{s} = 3{,}6 \frac{Km}{h}$$

$$|\vec{v}| = \left| \frac{d\bar{r}}{dt} \right|$$

direzione tangente

verso del moto

ACCELERAZIONE

media

$$\bar{a}_m = \frac{\bar{v}_f - \bar{v}_i}{t_f - t_i} = \frac{\Delta \bar{v}}{\Delta t}$$

$t_i , t_f \qquad \Delta t = t_f - t_i$

istantanea

$$\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta \bar{v}}{\Delta t} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\frac{d\bar{r}}{dt}\right) = \frac{d^2 \bar{r}}{dt^2}$$

SI: $\frac{m}{s^2}$ $\qquad$ cgs: $\frac{cm}{s^2}$ $\qquad$ $g = 9,8 \ \frac{m}{s^2}$

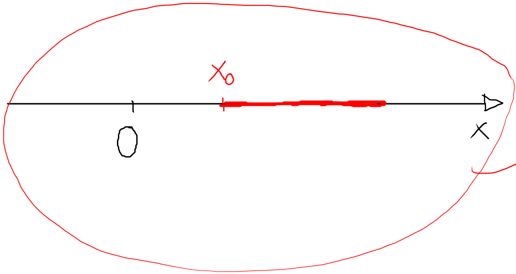
1D

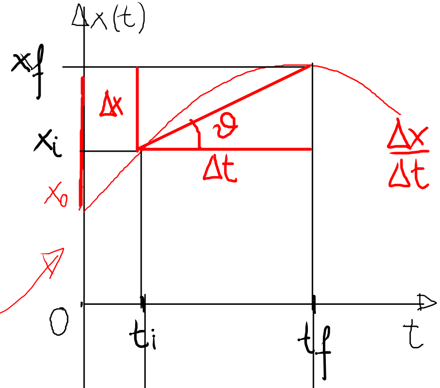
$$a_{xm} = \frac{\Delta v_x}{\Delta t}$$

$$a_x = \frac{d^2 x}{dt^2}$$

(2) DIAGRAMMA DEL MOTO

moto 1D $\quad x = x(t)$ (3)



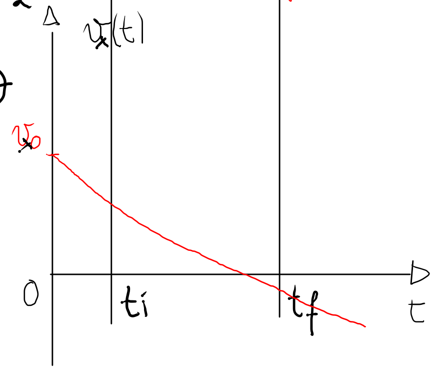


$$\frac{\Delta x}{\Delta t} = \mathrm{tg}\,\theta$$

t	x
0	x_0
t_1	x_1
t_2	x_2
...	...

tabella oraria (1)

(3) $x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$ esempio di legge oraria



$$v_{mx} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \mathrm{tg}\,\theta$$

pendenza media nel Δt considerato

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

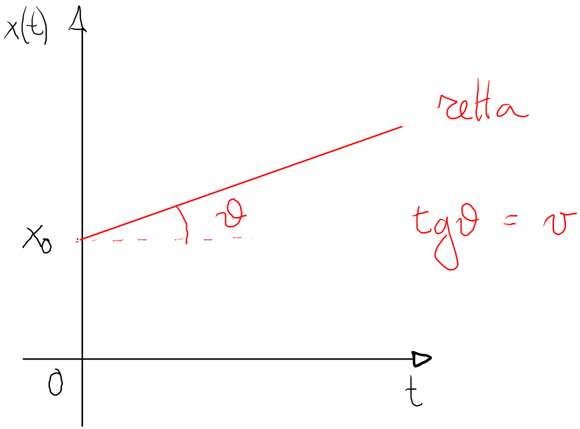
pendenza in un punto

$$a_{mx} = \frac{\Delta v_x}{\Delta t}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

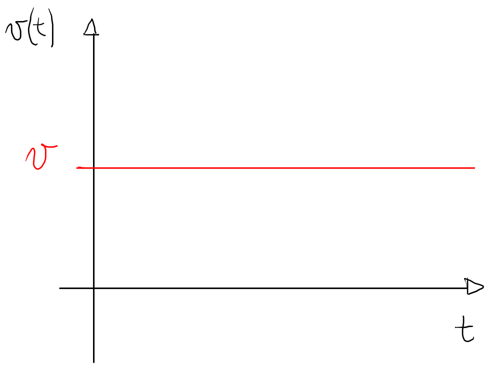
MOTO RETTILINEO UNIFORME (MRU)

1D $\qquad \vec{v}$ è costante nel tempo



$$\boxed{x(t) = x_0 + v\,t}$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}\cdot t$$

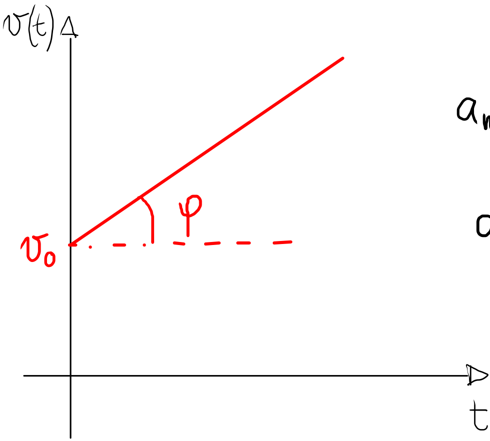


nel MRU, la velocità media su qualsiasi intervallo coincide con la velocità istantanea (costante):

$$\vec{v}_m = \vec{v}$$

MOTO RETTILINEO UNIFORMEMENTE ACCELERATO (MRUA)

1D $\qquad \vec{a}$ è costante $\qquad \vec{a} \neq 0$



$$a_{mx} = \frac{\Delta v_x}{\Delta t}$$

$$a = tg\,\varphi$$

$$\boxed{v(t) = v_0 + at} \qquad \mathrm{I}$$

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$

$$x(t) = \ ?$$

$$\boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2} \qquad \mathrm{II}$$

$$x(t=0) = x_0 \quad ✓$$

$$v(t) = \frac{dx}{dt} = v_0 + \frac{1}{\cancel{2}}a\cancel{2}t \stackrel{!}{=} v_0 + at \quad ✓$$

I $\quad v(t) = v_0 + at$

II $\quad x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

per t_f $\quad \begin{cases} v(t_f) = v_f = v_0 + a t_f \\ x(t_f) = x_f = x_0 + v_0 t_f + \frac{1}{2} a t_f^2 \end{cases}$

$$\begin{cases} t_f = \dfrac{v_f - v_0}{a} \\ x_f = x_0 + v_0 \left(\dfrac{v_f - v_0}{a} \right) + \dfrac{1}{2} a \left(\dfrac{v_f - v_0}{a} \right)^2 \end{cases}$$

$$= x_0 + \frac{v_0 v_f - v_0^2}{a} + \frac{1}{2} \cancel{a} \frac{v_f^2 - 2 v_f v_0 + v_0^2}{a^{\cancel{2}}}$$

$$= x_0 + \frac{\cancel{2 v_0 v_f} - 2 v_0^2 + v_f^2 \cancel{- 2 v_f v_0} + v_0^2}{2a}$$

$$x_f = x_0 + \frac{v_f^2 - v_0^2}{2a}$$

$$x_f - x_0 = \frac{v_f^2 - v_0^2}{2a}$$

$$2a(x_f - x_0) = v_f^2 - v_0^2$$

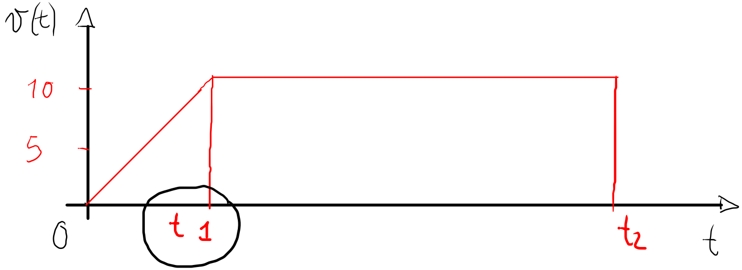
$$\boxed{v_f^2 = v_0^2 + 2a(x_f - x_0)}$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)} \quad \text{III}$$

problema: "il velocista", dal Ragozzino, "Elementi di Fisica", Es. 2.27 a pag 56

- 200 m in 20,3 s

$x_1 = 20\ m$ MRUA a cost $\rightarrow$ v 39 Km/h $= 39\ \dfrac{1000\ m}{3600\ s} = 10{,}83\ \dfrac{m}{s}$



$v(t_1) = v(t_2) = v_1 = v_2 = v_f$

a) velocità media $v_m = \dfrac{200\ m}{20{,}3\ s} = 9{,}85\ \dfrac{m}{s}$

b) $a = ?$

$a = \dfrac{\Delta v}{\Delta t}$

$v^2 = v_0^2 + 2a\underbrace{(x - x_0)}_{20m}$ $\qquad v_0 = 0$ $\qquad x_0 = 0$

$v_1^2 = 2a\,x_1$

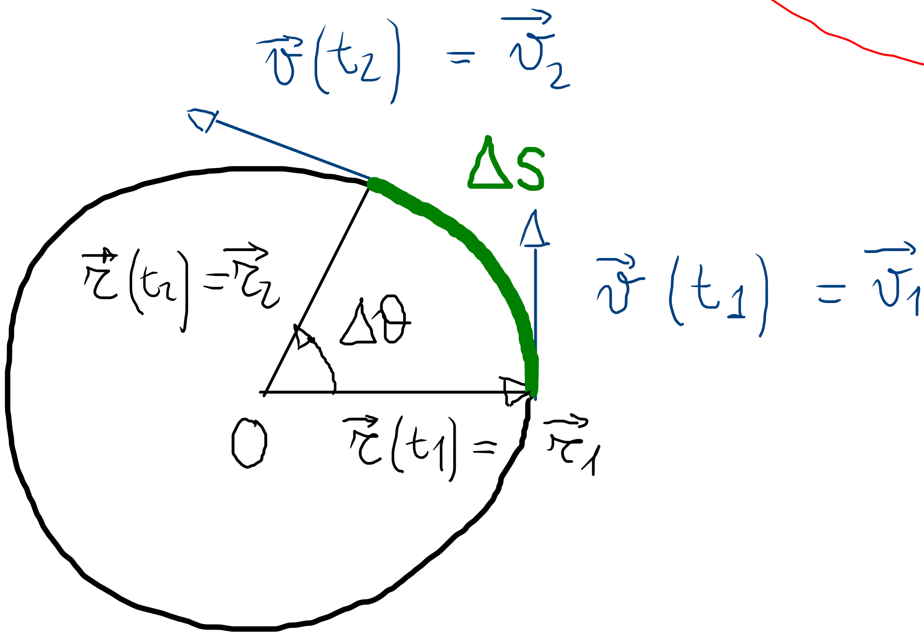
$a = \dfrac{v_1^2}{2x_1} = \dfrac{v_f^2}{2x_1} = \dfrac{(10{,}83)^2\ \frac{m^{\cancel{2}}}{s^2}}{2 \cdot 20\ \cancel{m}} = 2{,}93\ \dfrac{m}{s^2}$

c) $t_1 = ?$ $\qquad t_1 = \dfrac{\Delta v}{a} = \dfrac{\cancel{v_1}}{\frac{v_1^{\cancel{2}}}{2x_1}} = \dfrac{2x_1}{v_1} = \dfrac{2 \cdot 20\ \cancel{m}}{10{,}83\ \frac{\cancel{m}}{s}} = 3{,}69\ s$

MOTO CIRCOLARE UNIFORME

$|\vec{r}_1| = |\vec{r}_2| = R$

$|\vec{v}_1| = |\vec{v}_2| = v$ nota: $\vec{v}_1 \neq \vec{v}_2$!



$$\Delta t = t_2 - t_1$$

$$v = \frac{\Delta s}{\Delta t}$$

• velocità angolare: $\omega_m = \dfrac{\Delta\vartheta}{\Delta t}$ media

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\vartheta}{\Delta t} = \frac{d\vartheta}{dt} \quad \text{istantanea}$$

$$\Delta\vartheta = \frac{\Delta s}{R}$$

$$\omega = \frac{\Delta\vartheta}{\Delta t} = \frac{1}{\Delta t} \cdot \frac{\Delta s}{R} = \frac{1}{R}\frac{\Delta s}{\Delta t} = \frac{1}{R} v = \frac{v}{R}$$

$\boxed{\omega = \dfrac{v}{R}}$ o $\boxed{v = R\omega}$ $\Rightarrow$ ω è costante !

- accelerazione angolare: $\alpha_m = \frac{\Delta\omega}{\Delta t}$ media

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\vartheta}{dt^2}$$ istantanea

nel moto circolare uniforme ω cost.

$$\Delta\omega = 0$$

$$\alpha_m = \alpha = 0$$

Non c'è accelerazione angolare ma c'è ....

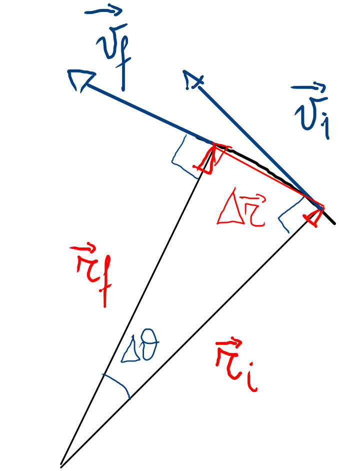
- accelerazione $\vec{a}_m = \frac{\Delta\vec{v}}{\Delta t}$

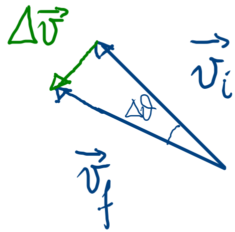
$\Delta\vec{v} \neq 0$

$\vec{v}_1 \neq \vec{v}_2$ $\Rightarrow$ c'è acc.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

ACCELERAZIONE NEL MOTO CIRCOLARE UNIFORME





$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

$$|\vec{r}_i| = |\vec{r}_f| = R \qquad |\vec{v}_i| = |\vec{v}_f| = v$$

Sono triangoli simili: $\dfrac{|\Delta\vec{r}|}{R} = \dfrac{|\Delta\vec{v}|}{v}$

$$\vec{a}_m = \frac{\Delta\vec{v}}{\Delta t}$$

$$|\vec{a}_m| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v}{\Delta t}\,\frac{|\Delta\vec{r}|}{R} = \frac{v}{R}\,\frac{|\Delta\vec{r}|}{\Delta t} \qquad (*)$$

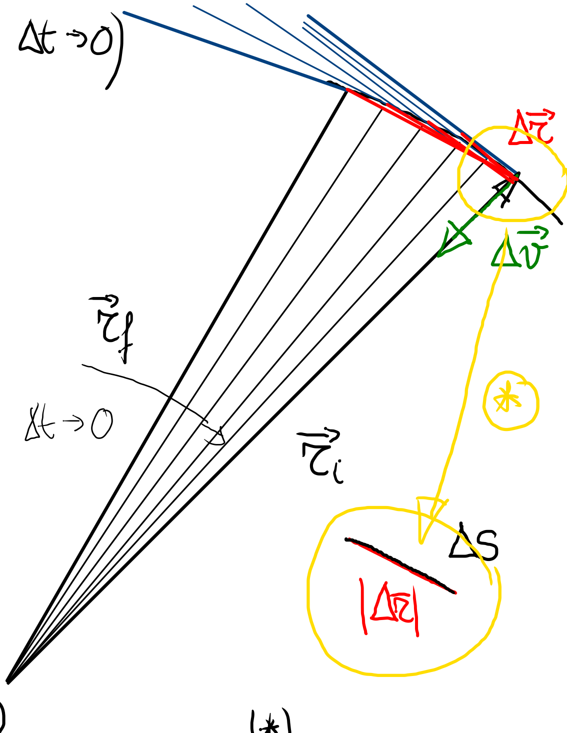
$\vec{a}_m \to \vec{a}$
$(\lim \Delta t \to 0)$

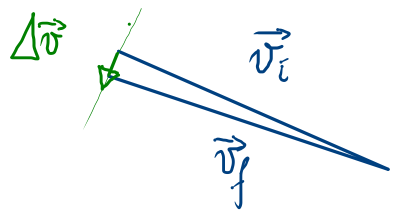
$\vec{a}$ ha direzione radiale
punta verso O
} é CENTRIPETA

$\Delta t \to 0$

$\vec{v}_f \to \vec{v}_i$

$\vec{r}_f - \vec{r}_i$





$\Delta\vec{v}$ risulta ortogonale a $\vec{v}$

$$|\vec{a}| = \lim_{\Delta t \to 0} |\vec{a}_m| \overset{(*)}{=} \lim_{\Delta t \to 0} \frac{v}{R}\frac{|\Delta\vec{r}|}{\Delta t} = \frac{v}{R}\lim_{\Delta t \to 0}\frac{|\Delta\vec{r}|}{\Delta t} \overset{*}{=} \frac{v}{R}\underbrace{\lim_{\Delta t \to 0}\frac{\Delta S}{\Delta t}}_{v}$$

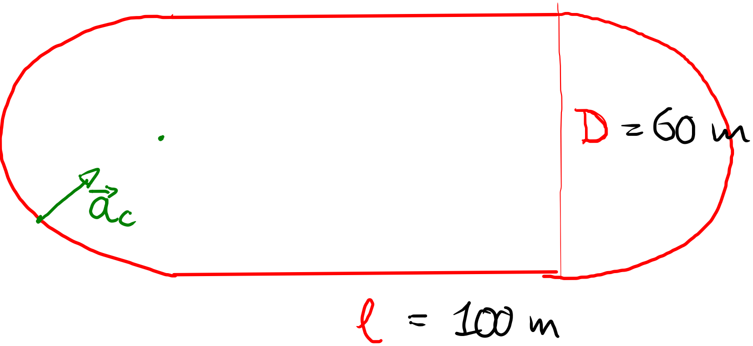
$$\boxed{|\vec{a}| = \frac{v^2}{R}}$$

$$\boxed{v = \omega R}$$

$$\boxed{|\vec{a}| = \frac{\omega^2 R^2}{R} = \omega^2 R}$$

La pattinatrice

(esercizio 2,39 a pag. 57 del Ragozzino, Elementi di Fisica)



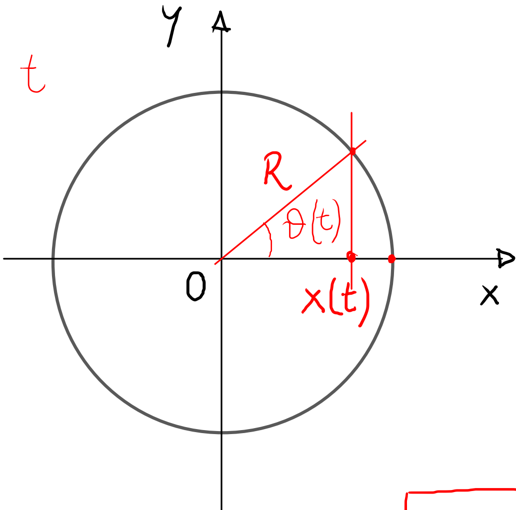
$$v = 45\ \frac{Km}{h} = 45\ \frac{\cancel{1000}^{1}\ m}{\cancel{3600}_{3,6}\ s} = 12{,}5\ \frac{m}{s}$$

a) $$T = \frac{2 \cdot \ell + \pi D}{v} = \frac{2 \cdot 100\ m + \pi \cdot 60\ m}{12{,}5\ \frac{m}{s}} = 31{,}1\ s$$

b) $$|\vec{a}_c| = \frac{v^2}{R} = \frac{2v^2}{D} = \frac{2\left(12{,}5\ \frac{m}{s}\right)^2}{60\ m} = 5{,}21\ \frac{m}{s^2}$$

MOTO ARMONICO (1D)

$t = 0 \Rightarrow \vartheta = 0$



$$\vartheta(t) = \omega \cdot t$$

$$\left(\omega = \frac{\Delta\vartheta}{\Delta t} = \frac{\vartheta}{t} \right)$$

$$x(t) = R\cos\left(\vartheta(t)\right)$$

$$\left[\, y(t) = R\,\mathrm{sen}\left(\vartheta(t)\right) \right]$$

$$x(t) = R\cos\left(\vartheta(t)\right) = R\cos(\omega t)$$

(esempio di moto armonico)

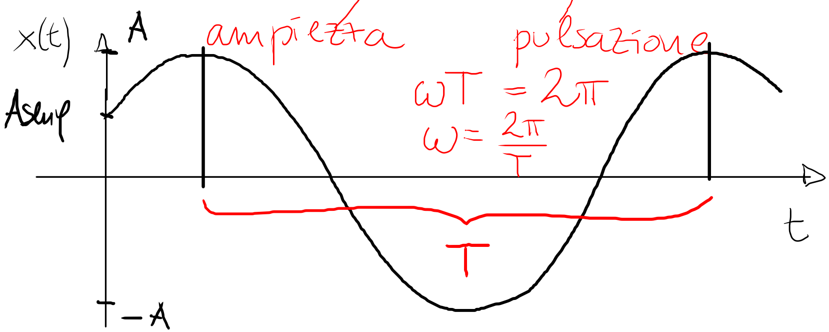
In generale:

$$\boxed{x(t) = A\,\mathrm{sen}(\omega t + \varphi)}$$

ampiezza

pulsazione

fase
(condizioni iniziali)



$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$

Frequenza $\nu = \frac{1}{T}$ si misura in $s^{-1} = Hz$

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

$$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$$

$$\boxed{x(t) = A\,\mathrm{sen}(\omega t + \varphi)}$$

$$v(t) = \frac{dx(t)}{dt}$$

$$\boxed{v(t) = A\omega \cos(\omega t + \varphi)}$$

$$a(t) = \frac{dv(t)}{dt}$$

$$\boxed{a(t) = -A\omega^2\,\mathrm{sen}(\omega t + \varphi)}$$

Esempio: $\varphi = \frac{\pi}{2}$

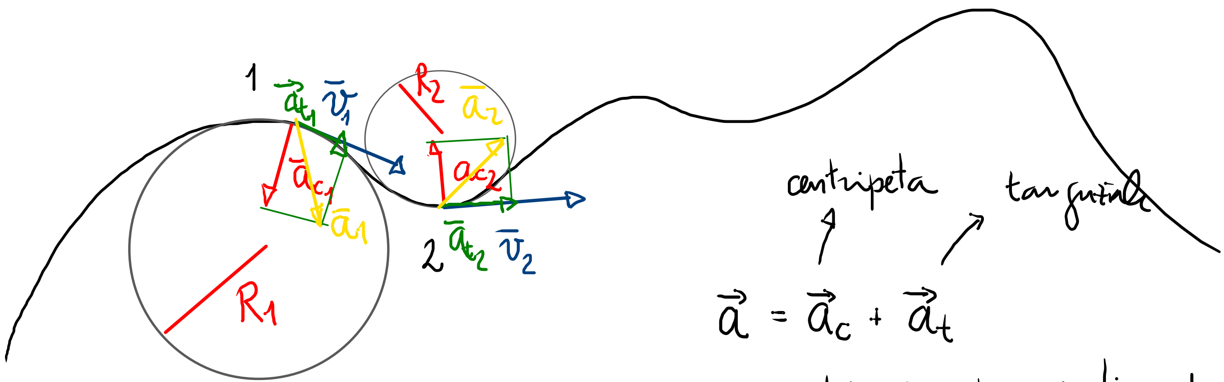
$$x(t) = A\,\mathrm{sen}\left(\omega t + \frac{\pi}{2}\right)$$

$$= A\left(\cancel{\mathrm{sen}(\omega t) \cdot \cos\frac{\pi}{2}} + \cos(\omega t)\,\mathrm{sen}\frac{\pi}{2}\right)$$

$$= A\cos(\omega t)$$

$$\boxed{a(t) = -\omega^2 x(t)}$$

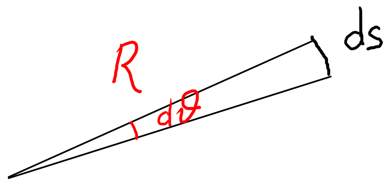
MOTO GENERICO (2D)



centripeta tangenziale

$$\vec{a} = \vec{a}_c + \vec{a}_t$$

due componenti $\perp$



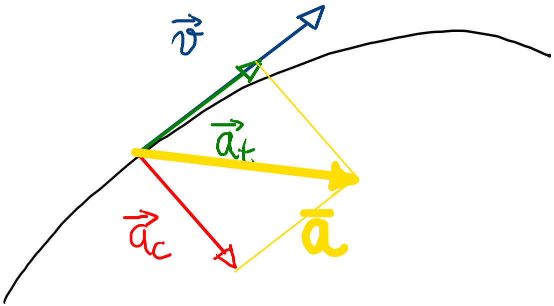
$$|\bar{a}_c| = \frac{v^2}{R}$$

$$|\vec{a}_t| = \frac{d|\vec{v}|}{dt} = \frac{d}{dt}\frac{ds}{dt} = \frac{d}{dt}\frac{R\,d\theta}{dt} = R\frac{d\omega}{dt} = R\alpha$$

accel. angolare

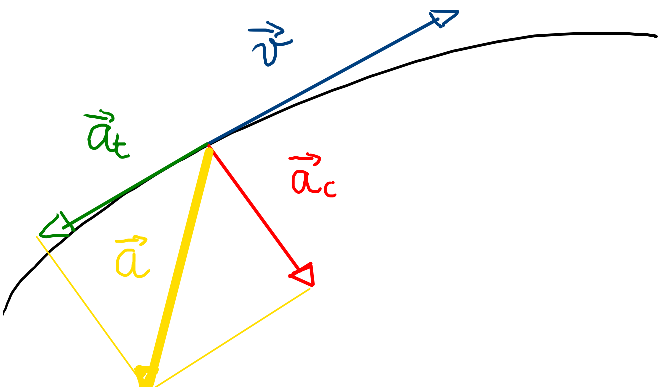
AUTOMOBILE IN CURVA

premendo sull'acceleratore



$$\vec{a} = \vec{a}_c + \vec{a}_t$$

premendo sul freno



$$\vec{a} = \vec{a}_c + \vec{a}_t$$