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Proposed problem

A cylindrical (pin) fin, as shown in figure 1, is made with a uniform, isotropic material with a thermal conductivity $k = 40$ [W/(m K)]; it has a diameter d and a length L.

The fin is cooled only by convection, with a convective heat transfer coefficient $h = 400$ [W/(m^2 K)], and the surrounding fluid temperature $T_\infty = 25$ [°C]. The base of the fin is maintained at a temperature $T_b = 200$ [°C], while also the tip of the fin contribute, with the same heat transfer coefficient, to the overall heat flux.
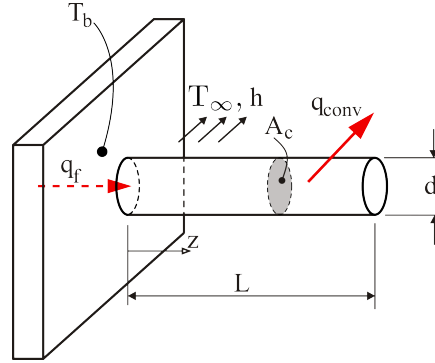

Figure 1: Cylindrical (pin) fin.

Assuming a *1D* temperature distribution, i.e. $T \approx T(z)$, compute, with the Finite Volume method (FV), the heat flux from the fin q_{num} [W], using a number N of FVs equal to $N = 10$, 20, 40, and 80.
Consider the two following cases:

1. $L = 30$ mm and $d = 3$ mm.

2. $L = 30$ mm and $d = 20$ mm.

Plot in a *log-log* graph the behavior of the error vs *N*, verifying its quadratic trend. The error is defined as the difference between the numerical value of the computed heat flux q_{num} and its analytical (exact) solution q_f given in [1, 2]:

$$q_f = M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL} \tag{1}$$

with

$$m = \sqrt{4h/kd}$$
$$M = \frac{\pi}{2} \sqrt{h\,k\,d^3}\,(T_b - T_\infty)$$

References

[1] F. P. Incropera, D. P. Dewitt, T. L. Bergman, A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Ed., Wiley, (2007).

[2] A. D. Kraus, A. Aziz, J. Welty, EXTENDED SURFACE HEAT TRANSFER, J. Wiley & Sons, (2001).