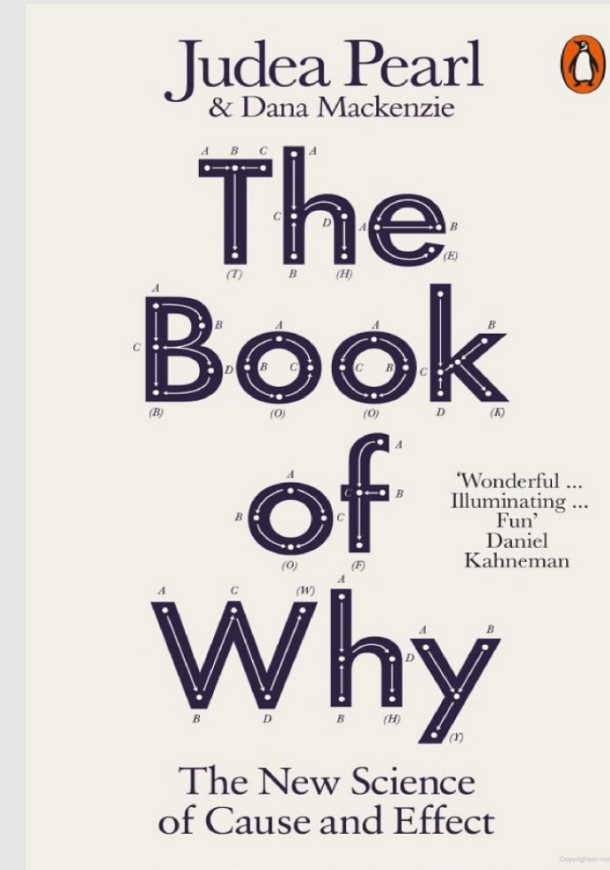
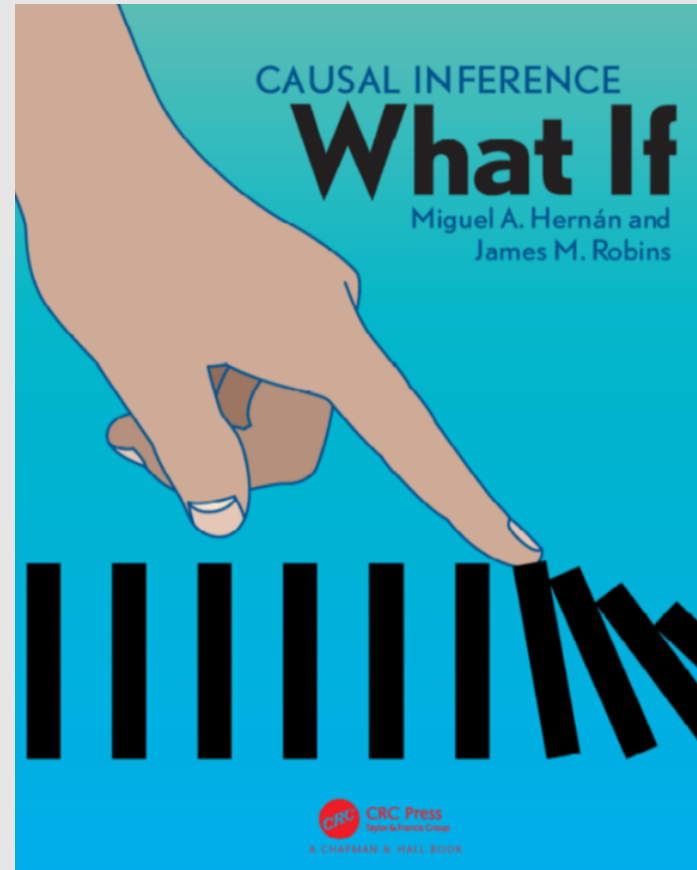
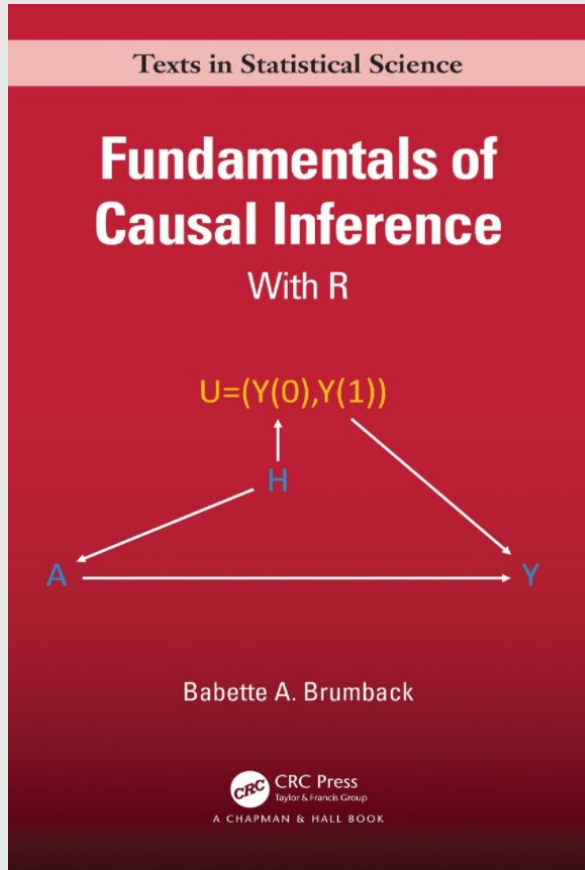


Causal Models in Epidemiological & Clinical Research (basic ideas...)



Explanatory/Causal Models (Observational Studies)

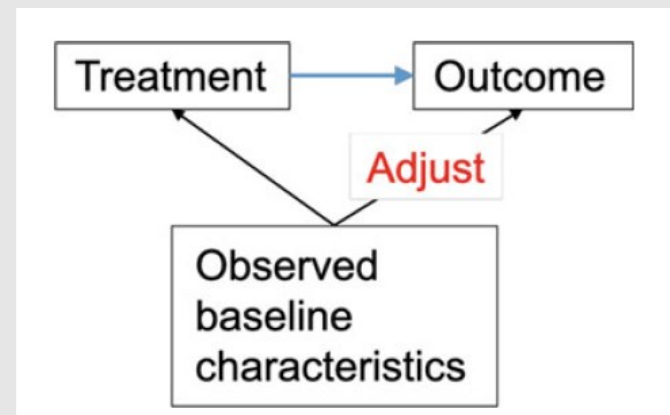
Confounding is one of the major concern in epidemiological analyses of observational studies, where we aim to estimate causal effects.

When treatments/exposures are compared, groups are often quite different **because of a lack** of randomization.

For example, subjects with specific characteristics **are more likely** to receive a certain treatment than other subjects (*confounding by indication*).

If these characteristics also affect the outcome, a direct comparison of treatments is **biased** and may merely reflect the lack of initial comparability.

Often, randomization is not possible, and **observational studies are the only possible design**. Dealing with confounding is an essential step in such analyses.



what is the treatment effect if baseline characteristics were *similar* between treatment groups?

In this context, **causal inference** methods must bridge a gap between goals and means.

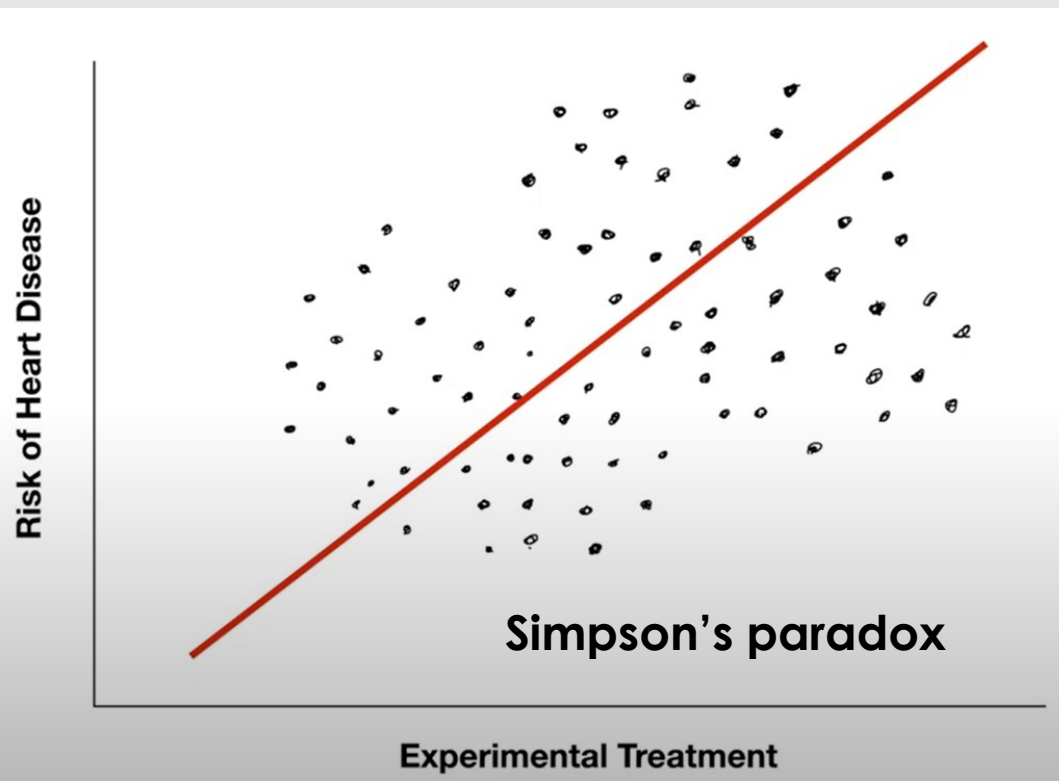
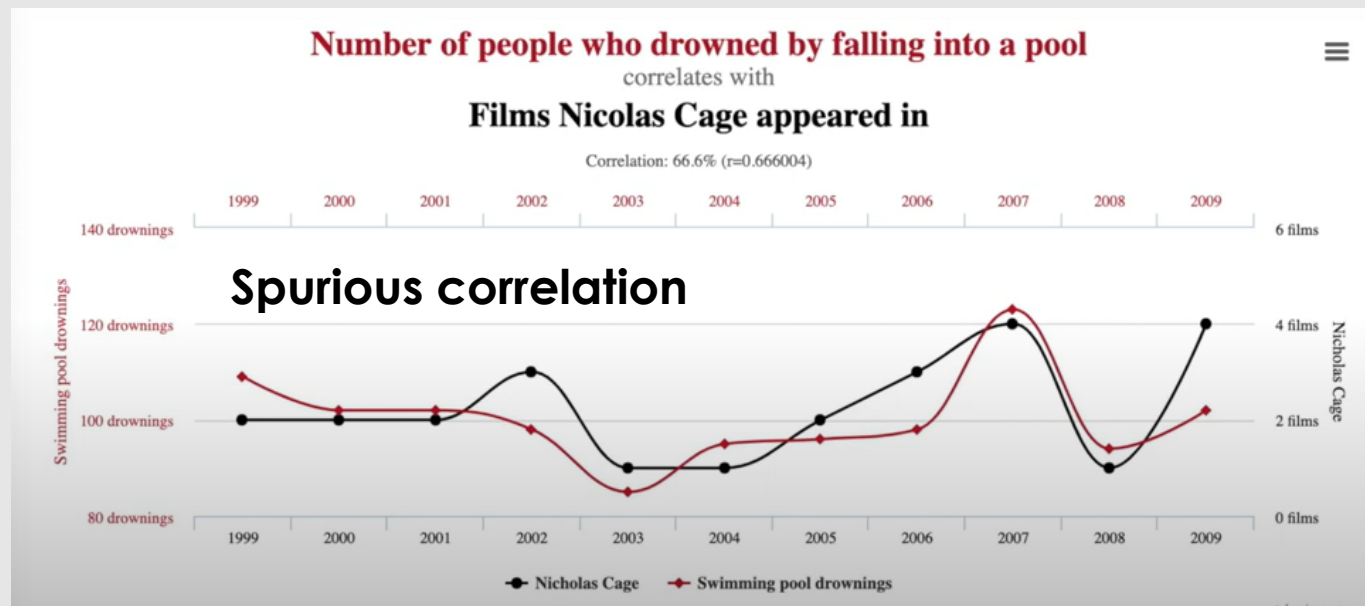
Researchers seek **causation**, but the data, on their own, only communicate **associations**.

Associations usually consist of a **mixture** of causal and non-causal (spurious) components.

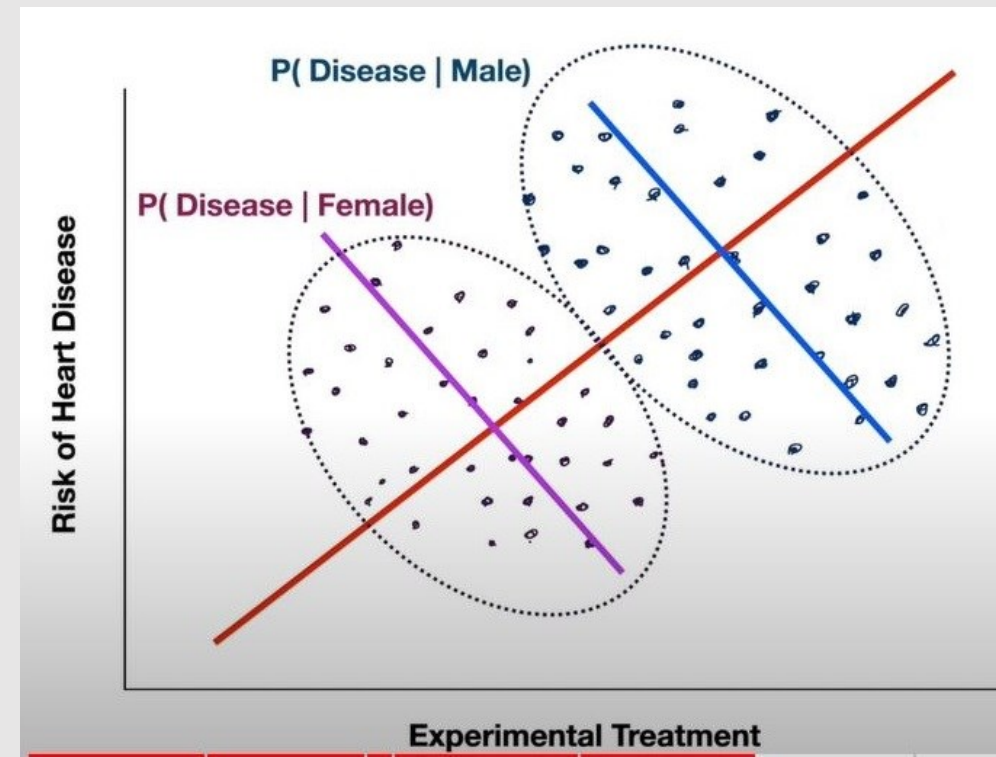
Therefore, a first step of **identification analysis** should determine whether, and under which conditions, it is possible to strip an observed association of all its spurious components.

Identification analysis requires causal **assumptions** about how the data were generated.

The sum of these causal assumptions is called a **causal model**, which must describe both *how the world works* (how observed and unobserved variables take their values) and *how the data were collected* (**which** variables and variable values are recorded/which **study design**).

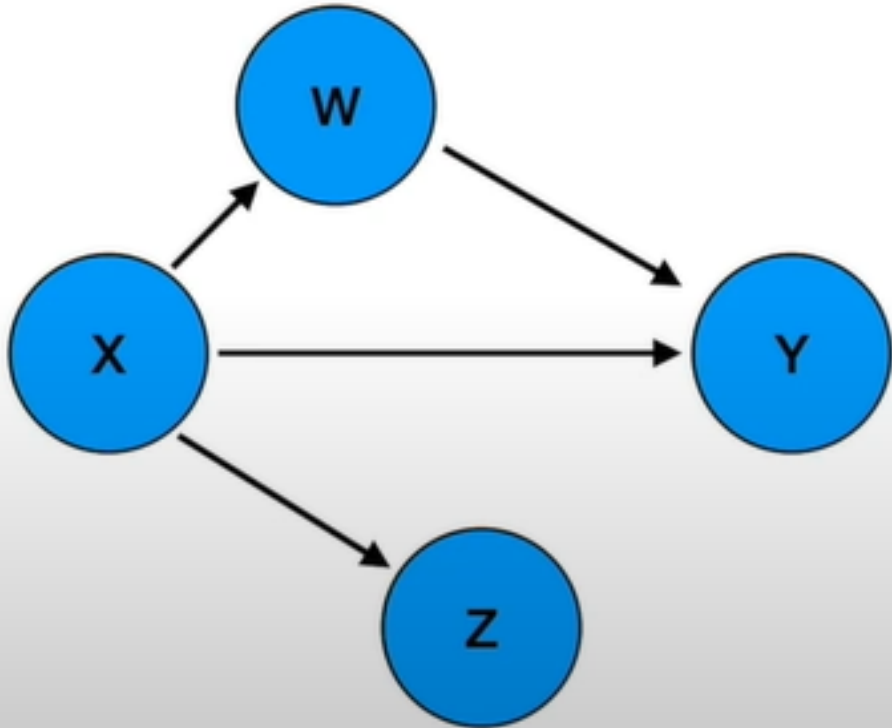


Treatment is **good**
for a man, **good**
for a woman but
bad for a person



Causality

X causes Y if *when all confounders are adjusted*, an intervention in X **results in a change** in Y, but intervention in Y **does not change** X



A causal model could be represented using a **DAG** (Directed **A**cyclic **G**raph)

Each arrow represents a causal influence

The graph is:

- **Directed** : each connection between two variables consists of an arrow
- **Acyclic** : no “reverse” cycles

A variable can't cause itself; however time varying processes can be depicted adding one *realization* of each variable per time unit

Knowledge of the data-generating mechanism has to be provided by **external theory** and **understanding**

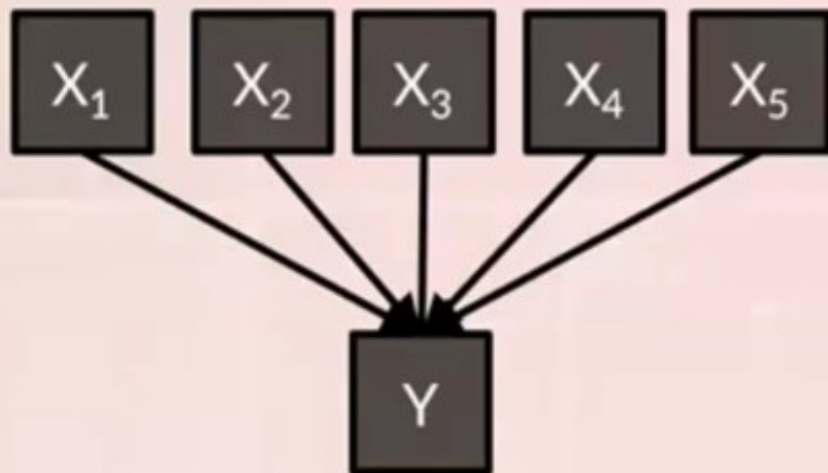


i.e. a **causal** model

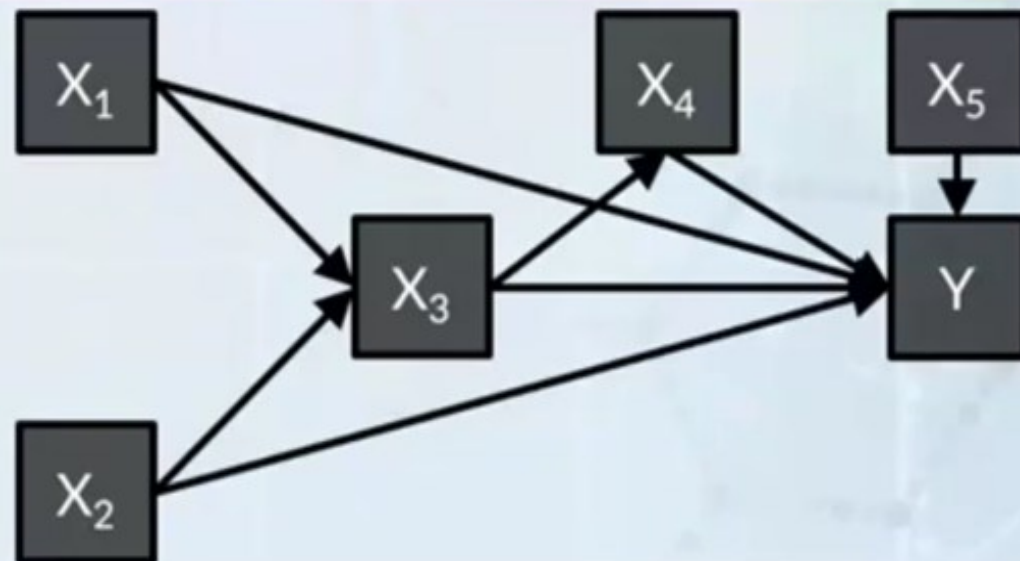
No software/algorithm can (currently) **understand** this

Prediction models can not be in general *causally interpreted* – however transparent they are ...

HOW THE ALGORITHM SEES IT



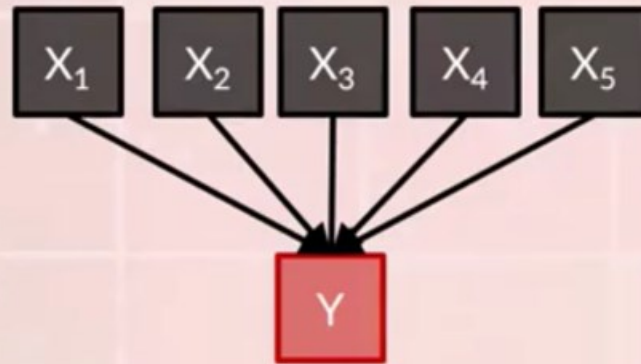
HOW NATURE CREATED IT



Predictive vs causal modelling

PREDICTIVE MODEL

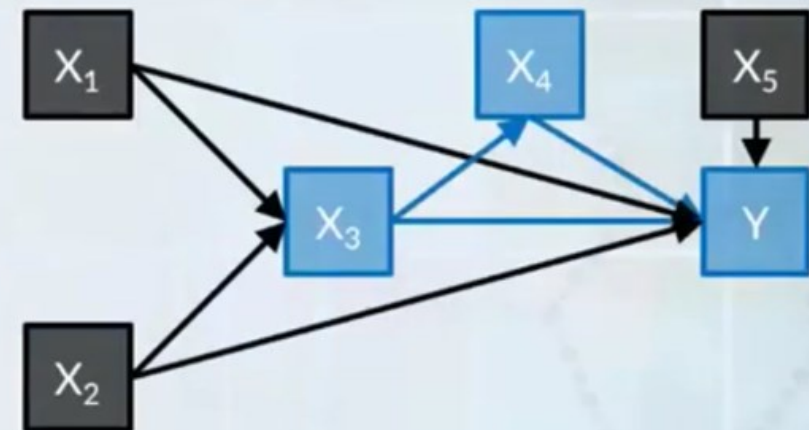
- **Outcome-focused**



X_i is _____ Y
'correlated with'
'a predictor of'
'associated with'

CAUSAL MODEL

- **Effect-focused**



The _____ of X_2 on Y is...
'total causal effect'
'direct causal effect'

Predictive vs causal modelling

Predictive Model

Aim: Predict values of outcome (p)

Maximise: performance measures (R^2 , calibration, AUC...)

Covariates selection focused on:

- Balancing precision & parsimony
- Availability of variables
- Maximising joint information

Coefficients: *associations*

Automation: Favoured

Causal Model

Aim: Estimate a causal effect

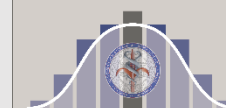
Maximise: accuracy of the effect estimate

Covariates selection focused on:

- External knowledge & judgement
- Role of variables
- Minimizing confounding

Coefficients: *Interpretable in the causal sense*

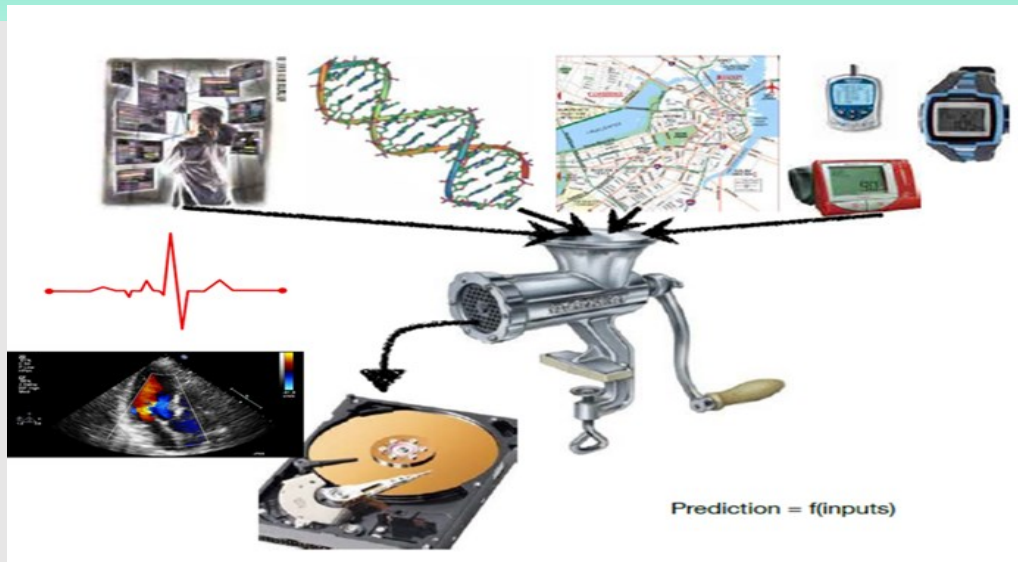
Automation: Not possible



Two extremes in regression models

Prognostic/Predictive modelling

- We are trying to find **predictors** of some outcome
- It is their **joint value** as predictors that is important
- We simply want the **most predictive** model
- We compare entire models to judge which is best



Causal/explanatory analysis

- The putative causal factor **must be** in the model
- Other factors are in the model because **help us understand** the causal factor (they are of no interest in themselves)
- We focus on the **estimation** of the putative causal effect (at a population level)



Smoking & Lung Cancer

A Tale of Two Statisticians

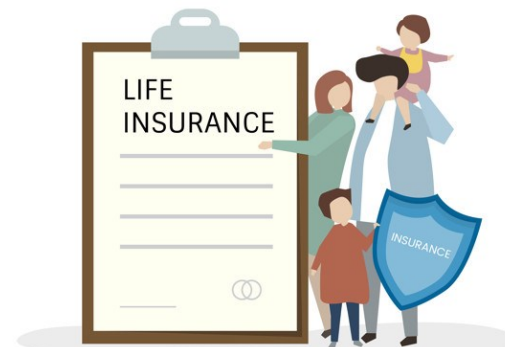
Works in public health (**explanatory**):

- I wish to establish whether it is causal
- If so I can warn smokers **to quit** and this will **benefit** their health [**intervention**]
- It is important for me to rule out possible **confounding** factors



Works in life insurance (**predictive**):

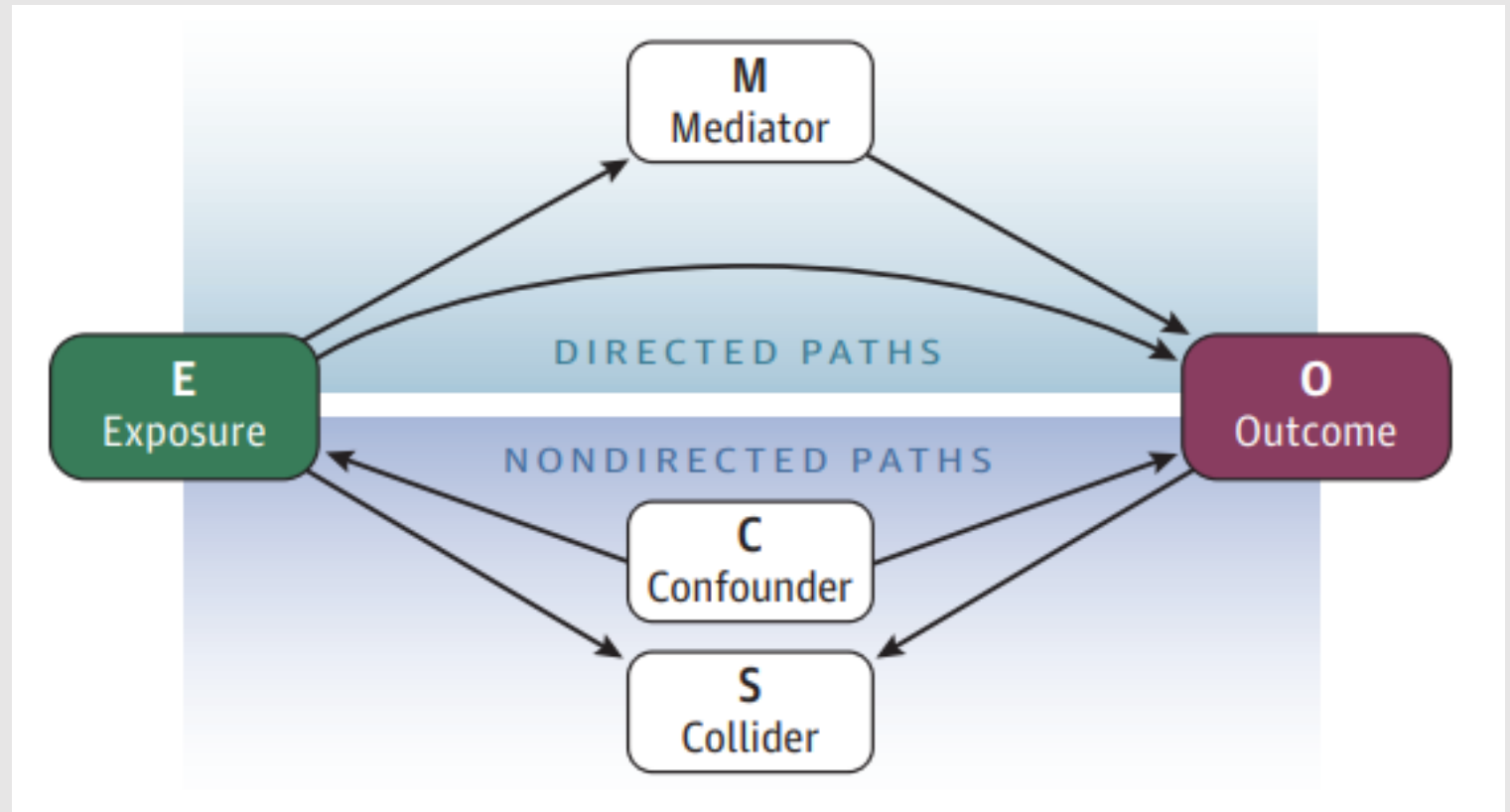
- I don't care if it is causal or not
- The data show that smokers **are much more likely** to get lung cancer
- That's enough for me to take account of it in setting the premiums



...so again: how to *represent* the role of covariates ?

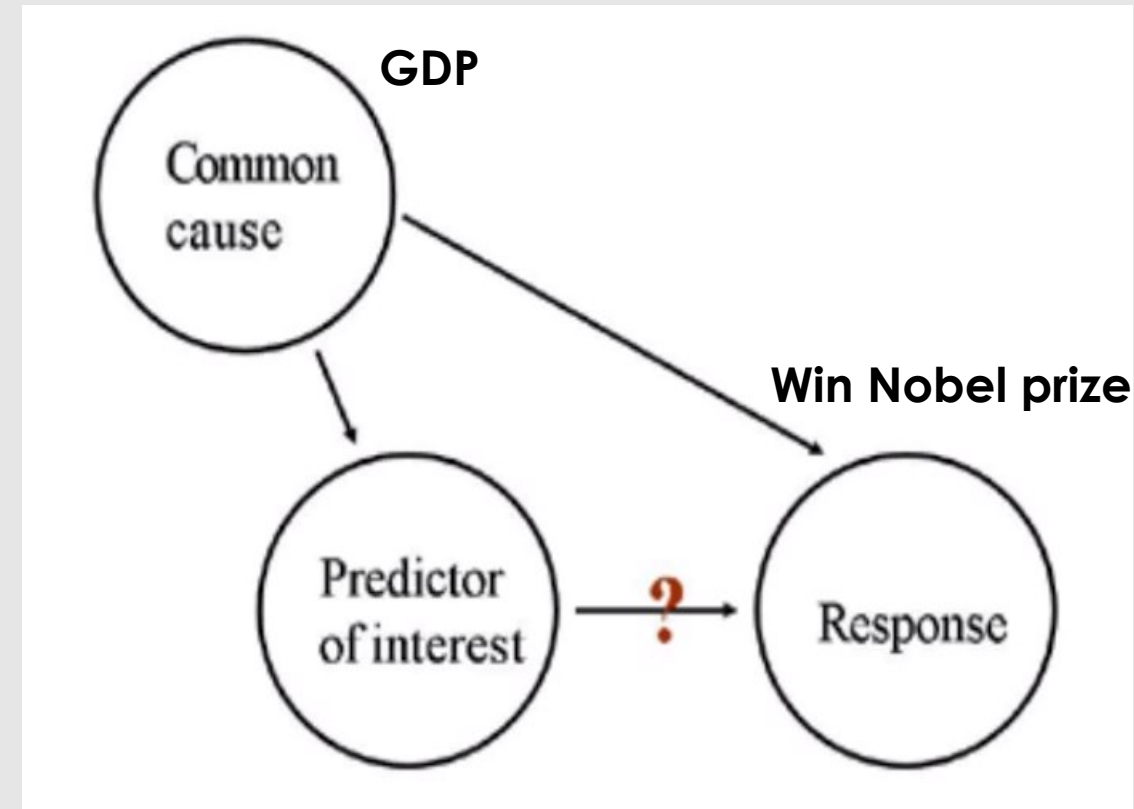
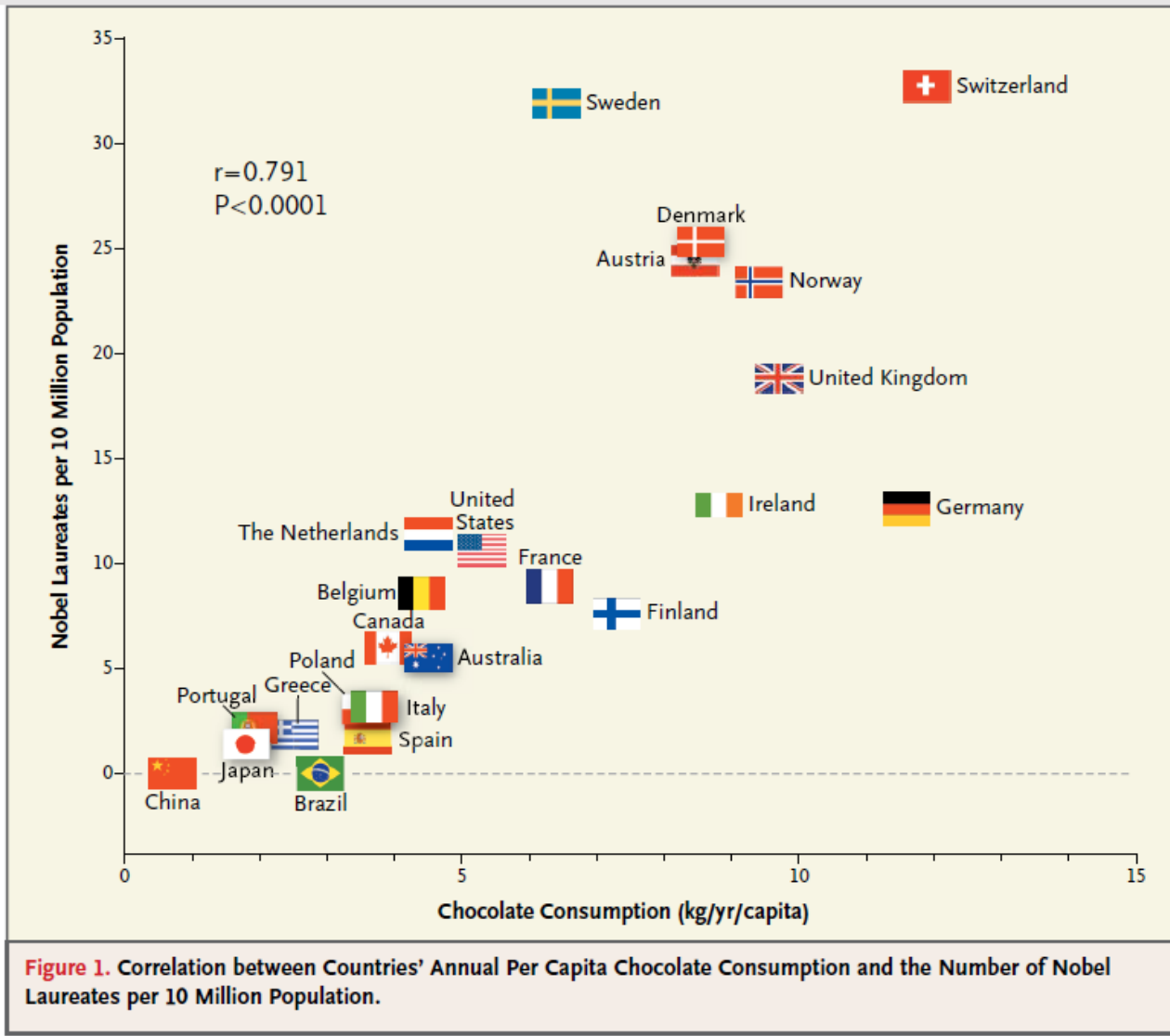
DAGs are visual representations of qualitative causal assumptions

They encode researchers' expert knowledge and beliefs about *how the world works*



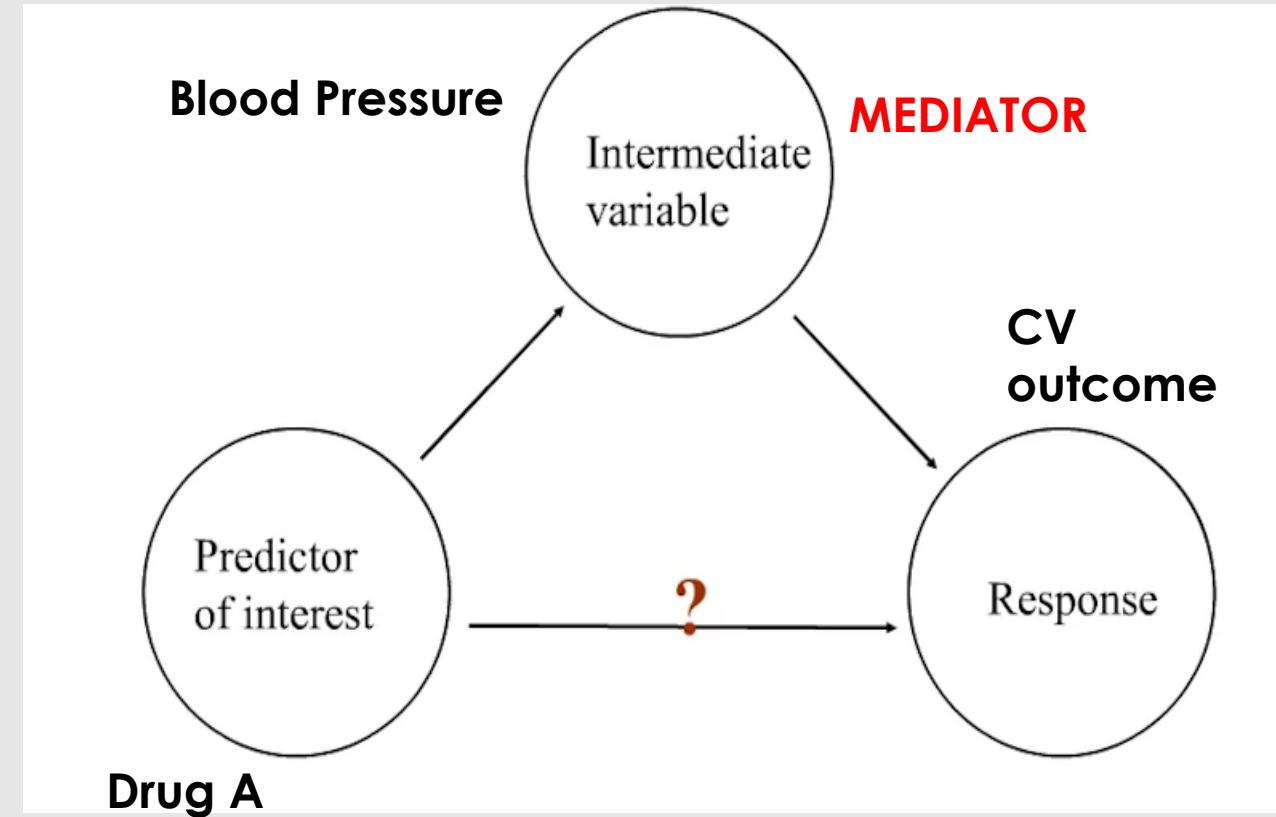
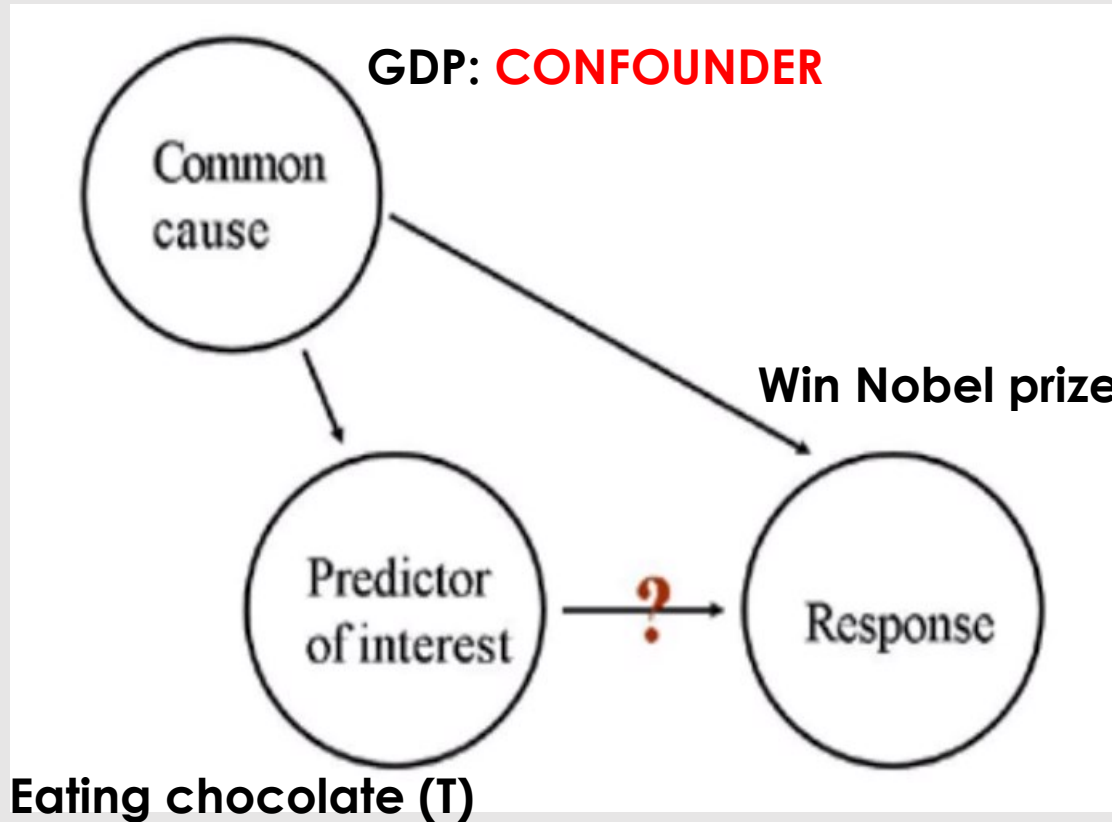
Do you remember? Chocolate Consumption, Cognitive Function, and Nobel Laureates....

N Engl J Med, 2012 Oct 18;367(16).



Eating chocolate (T)

Examples

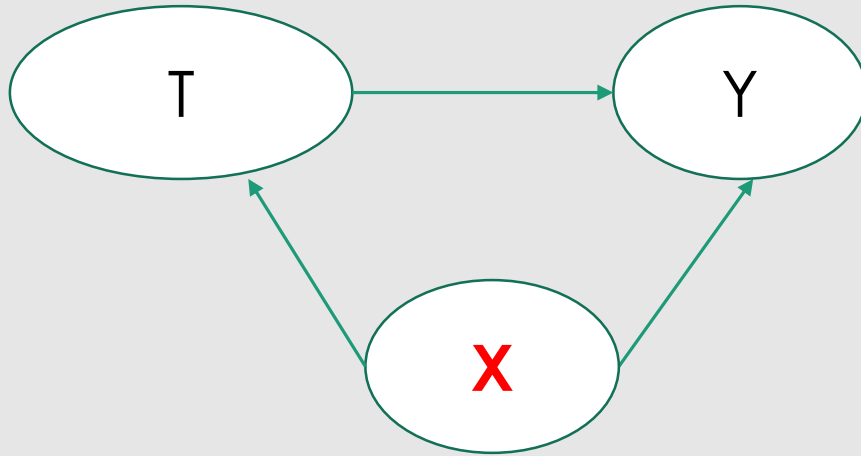


First: **Define relationships** between variables [...a priori knowledge]

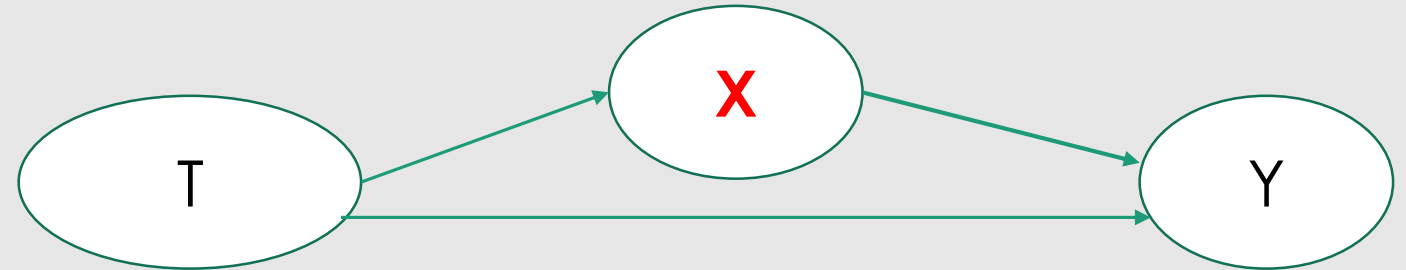
***Intermediate/Mediator vs Confounder: confounder IS NOT on the causal pathway between predictor and outcome**

Examples of different DAGs

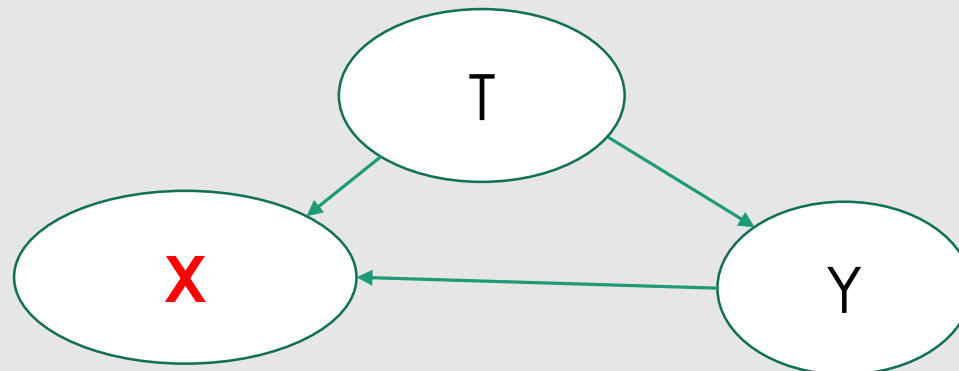
1. Confounder



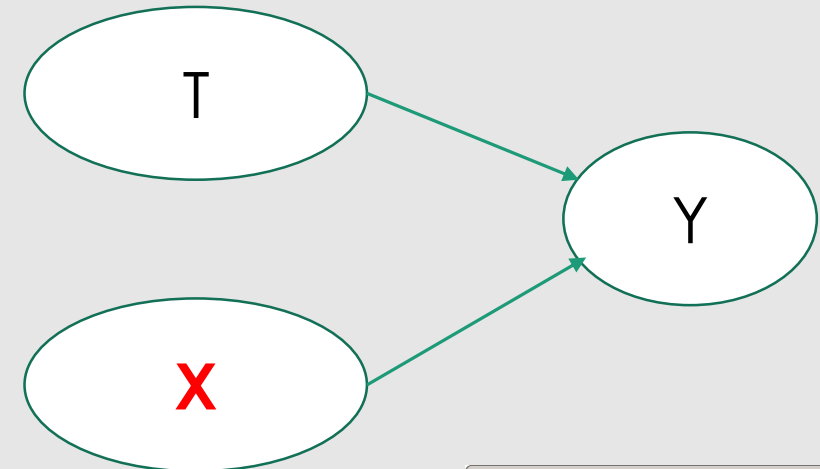
2. Mediator/Intermediate



3. Collider



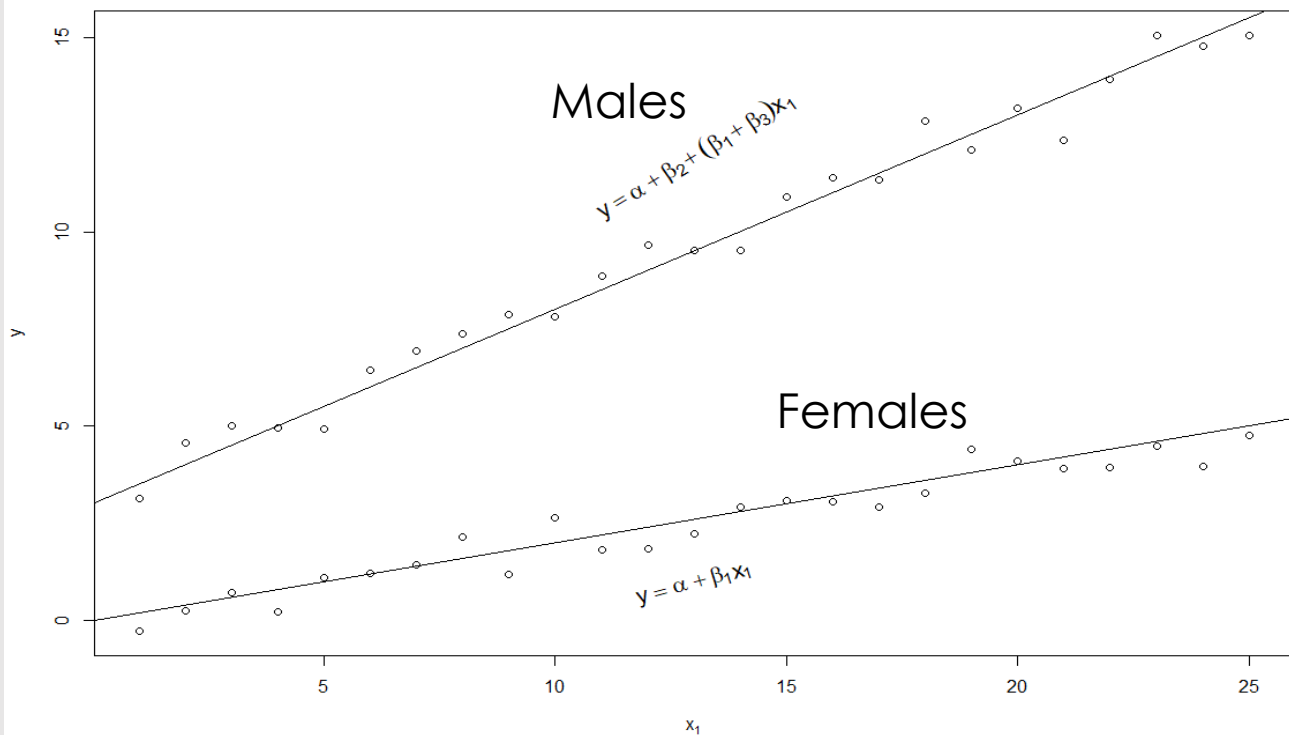
4. Independent Predictor



When a treatment has different effect among different subgroups defined by X, there is **effect modification (moderators)** :

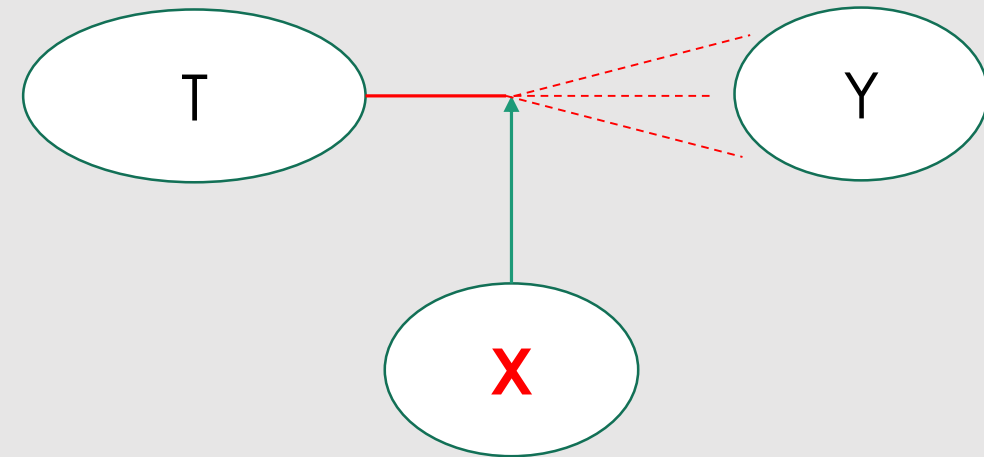
$$E(Y_1 - Y_0|X = x_i) \neq E(Y_1 - Y_0|X = x_j)$$

$$E(y|x) = \alpha + \beta_1 * age + \beta_2 * [sex = m] + \beta_3 * age * [sex = m]$$



! Could not be represented in a DAG !

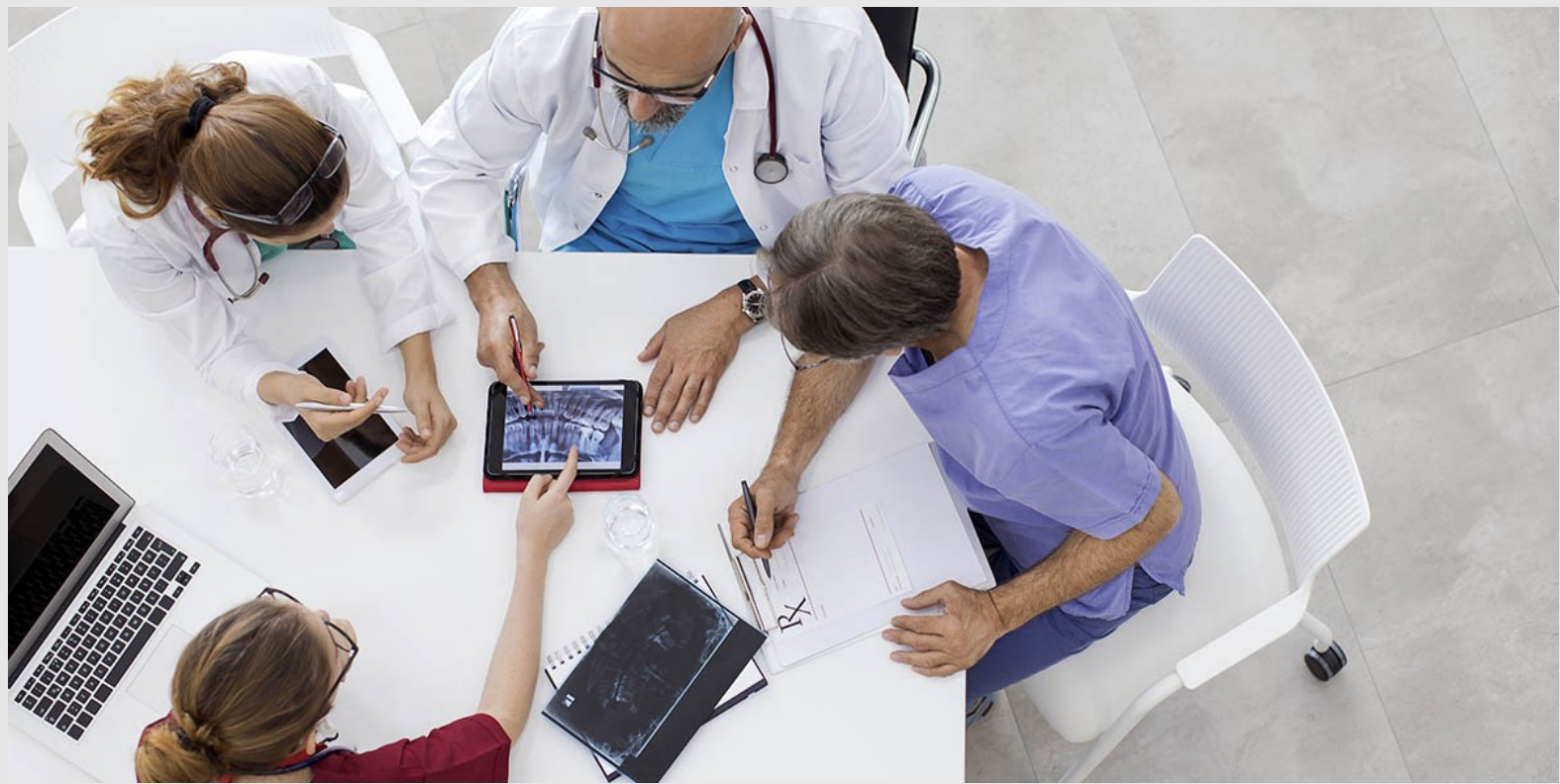
5. Effect modifier



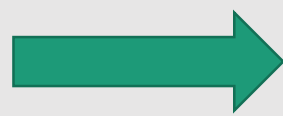
Building a DAG is a team-work...



Not this but....



This !



Basically there are **two types of effects that are of interest** in causal inference:

Average Causal Effect of a treatment or intervention on an outcome *across the entire population (or on the treated)* ATE/ATT

MARGINAL EFFECT



- Standardization*
- IPTW (Inverse Probability of Treatment Weights)

* similar to the concept of std rates in block 1

Causal effect of a treatment *within specific subgroups* or conditions defined by certain covariates (CATE)

CONDITIONAL EFFECT



- Stratification
- Regression modelling

Regression model for the outcome (no interaction)

Suppose a “true” model has a treatment T and a confounder x for outcome y :

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + e_i$$

β_1 : conditional and marginal treatment effect (ATE=CATE)

If we want here to calculate the marginal effect we can **average (standardize)** over values of X:

$$\widehat{ATE} = \beta_0 + \beta_1 + \beta_2 E(X) - \beta_0 - \beta_2 E(X) = \beta_1$$

The conditional effect is equal to the marginal effect when there is no interaction between the treatment variable and any other covariate included in the model.

Regression model if we ignore a confounder

Suppose a “true” model has a treatment T and a confounder x for outcome y :

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + e_i$$

If x is related to the treatment, we could write:

$$x_i = \gamma_0 + \gamma_1 T_i + v_i$$

If we ignore the confounder x , we would fit the model:

$$y_i = \beta_0^* + \beta_1^* T_i + e_i^* \quad \beta_1^* : \text{estimated effect if } x \text{ is omitted}$$

Estimation *without* x is correct only if $\beta_2 \gamma_1 = 0$

- $\beta_2 = 0$ x not associated with y
- $\gamma_1 = 0$ x not associated with T

If we come back to the true model:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 T_i + \beta_2 x_i + e_i = \beta_0 + \beta_1 T_i + \beta_2 (\gamma_0 + \gamma_1 T_i + v_i) + e_i \\ &= \beta_0 + \beta_2 \gamma_0 + (\beta_1 + \beta_2 \gamma_1) T_i + \beta_2 v_i + e_i \end{aligned}$$



$$\beta_1^* = \beta_1 + \beta_2 \gamma_1$$

Regression model for the outcome (with interaction)

Suppose a “true” model has a binary treatment T and a confounder X for outcome y that has an interaction with the treatment (effect modifier):

$$y_i = \beta_0 + \beta_1 T_i + \beta_2 x_i + \beta_3 x_i * T_i + e_i$$

$$\begin{aligned}\widehat{ATE} &= \frac{1}{N} \sum_{i=1}^N [\beta_0 + \beta_1 + \beta_2 x_i + \beta_3 x_i - \beta_0 - \beta_2 x_i] \\ &= \frac{1}{N} \sum_{i=1}^N [\beta_1 + \beta_3 x_i] = \beta_1 + \beta_3 E(X)\end{aligned}$$

Here the conditional effect is not equal to the marginal effect since there is an interaction between the treatment variable and a covariate included in the model.

General basic rules

1. Confounder: include (adjust/stratify)

2. Mediator: exclude

(for *total effect* estimation, then there are more advanced topic: disentangle direct and indirect effects...)

3. Collider: **to be discussed with experts, in general exclude** (but depend on the path...)

4. Independent predictor: include/exclude (in relation to sample size/precision)

5. Effect modifier: include (interaction)

Assumptions behind causal approaches

- **No interference**

- *Stable Unit Treatment Value Assumption*
- The treatment/exposure of one individual **does not affect** the potential outcome of another individual

- **Positivity**

- $0 < \Pr(T = 1|X) < 1$
- $0 < \Pr(T = 1|PS) < 1$
- each subject in the population could be potentially treated/exposed

- **Consistency**

- $Y_t = Y$ with $T = t$ There are no **multiple version** of treatment

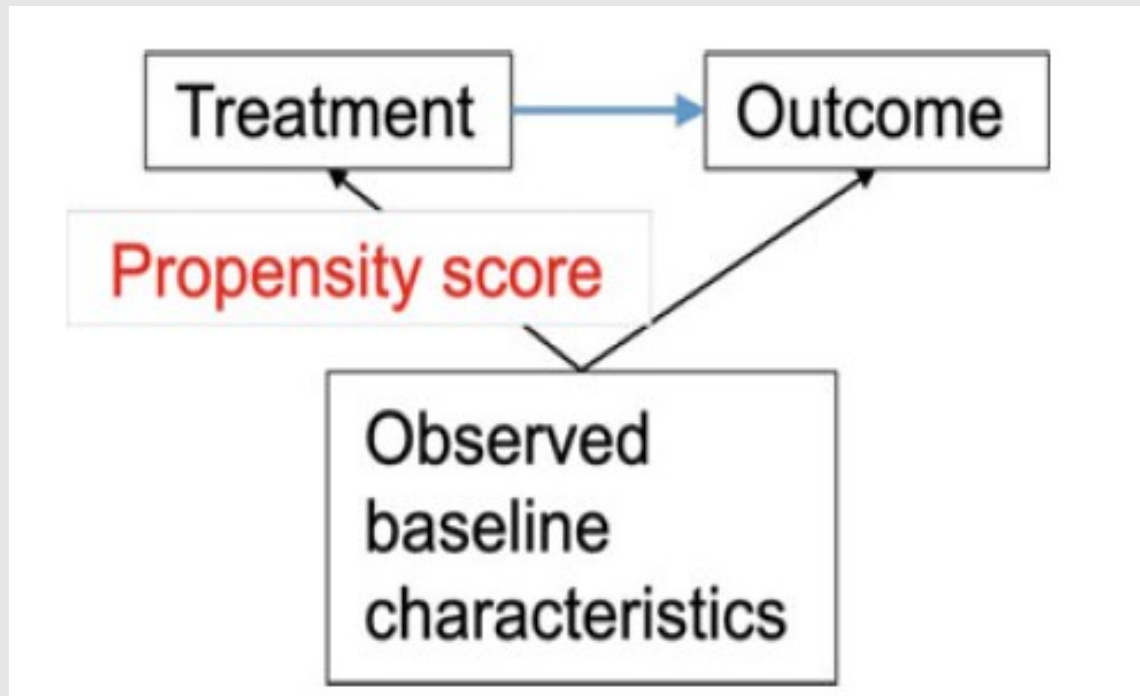
- **No unmeasured confounding**

- $Y_t \perp T|X$
- $Y_t \perp T|PS$
- conditional exchangeability

Regression models for the outcome are used to **estimate treatment effects** *trying to adjust* for **(measured) confounders** between treatment groups.

This approach relies on the underlying assumption that the **specified** model is correct. Another issue is that there is *no warning* if there is no **overlap** (*positivity violation*) between the treated and controls.

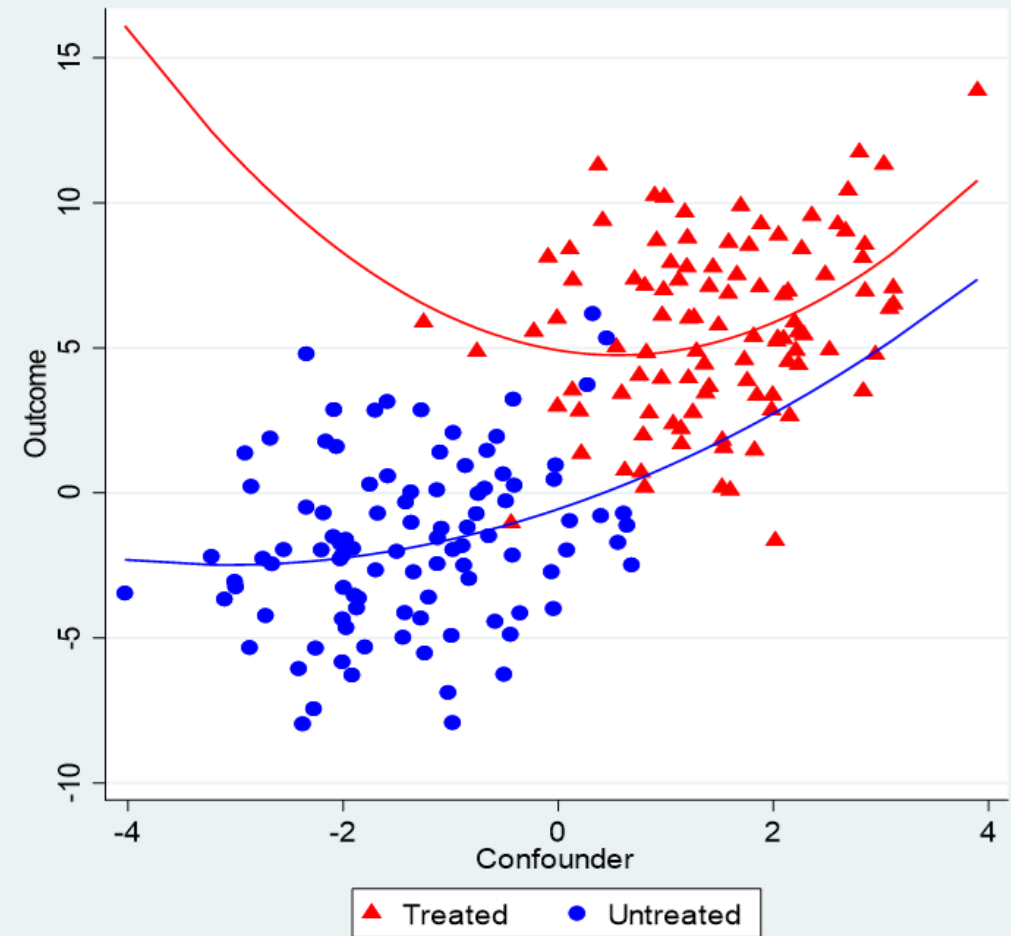
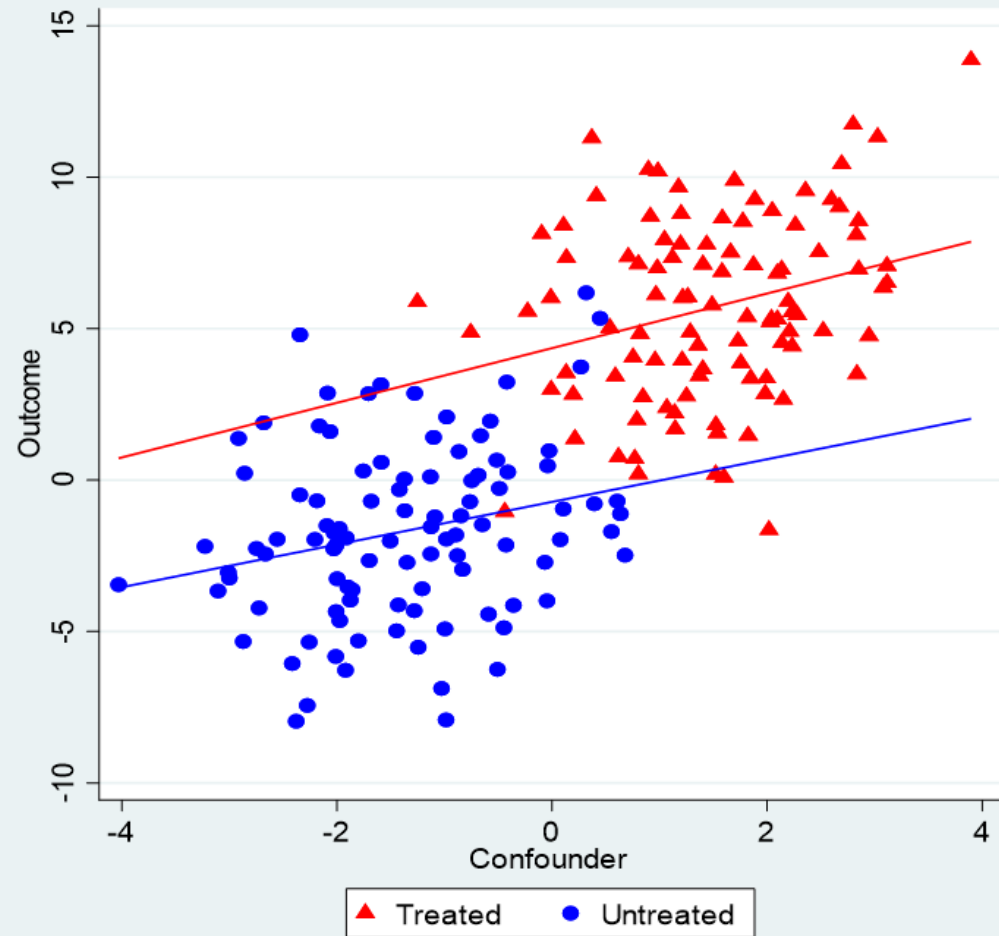
In other words, models could be estimated over regions with no or little data.



Adjustment with regression analysis is moreover **problematic** when the outcome is relatively **rare** or we have a **high-dimensional** confounders set.

An alternative is to use the **propensity score**, which could be especially attractive in the setting of rare outcomes.

No overlap on the confounder range : dangerous extrapolation !



Propensity Score

The Propensity Score (**PS**) for each subject i (Rosembaum & Rubin, 1983) is defined as follows:

$$e_i = \Pr(T_i = 1 | \mathbf{X}_i)$$

The **PS** is a **balancing score**, $e = b(X)$ such that:

$$X \perp T | \mathbf{b}(X)$$

Treated ($T=1$) and control ($T=0$) subjects with the same propensity score $e(x)$ have the same distribution of the observed covariates X

Conditional Exchangeability

$T=0 \mid (PS=p)$



$T=1 \mid (PS=p)$



$$Y_t \perp T \mid X \Rightarrow Y_t \perp T \mid PS$$

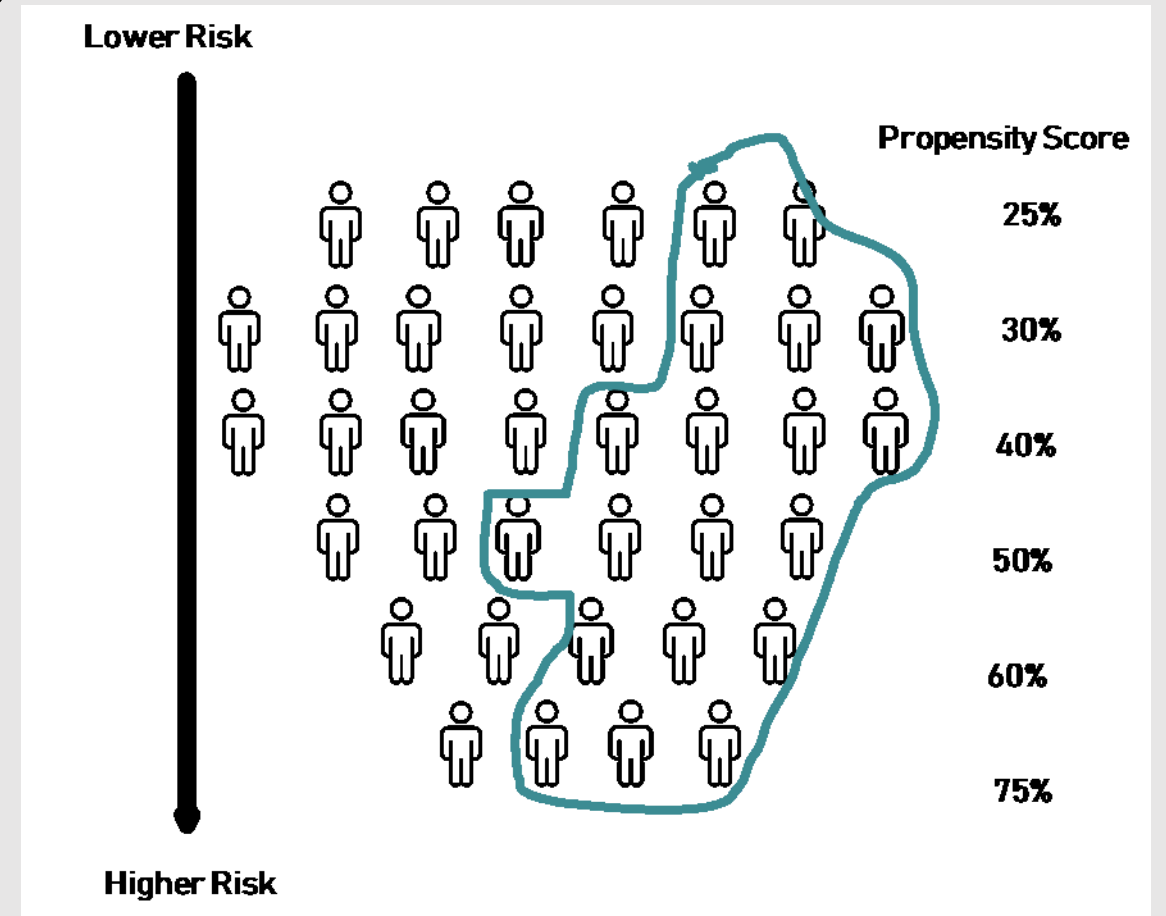
We **can estimate the propensity score** from the data:

e.g. using logistic regression:

$$\log\left(\frac{e_i}{1 - e_i}\right) = X_i\alpha$$

$$e_i = \frac{\exp(X_i\alpha)}{1 + \exp(X_i\alpha)}$$

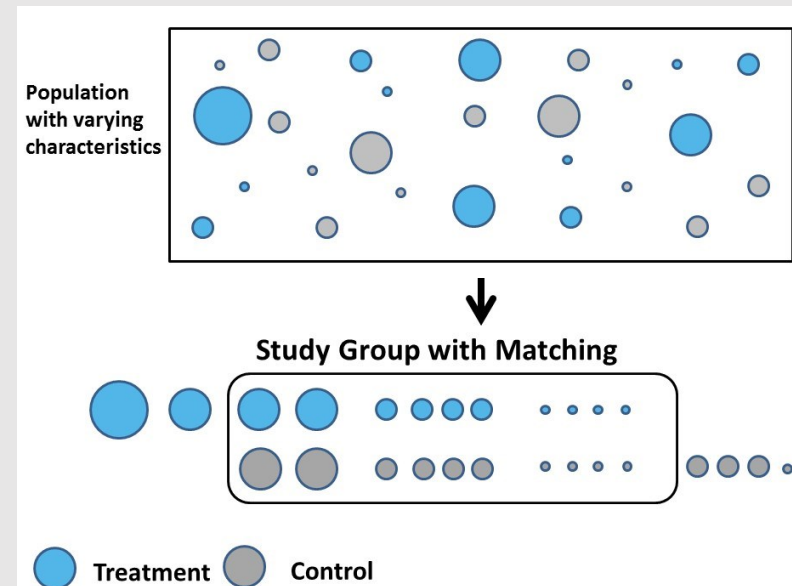
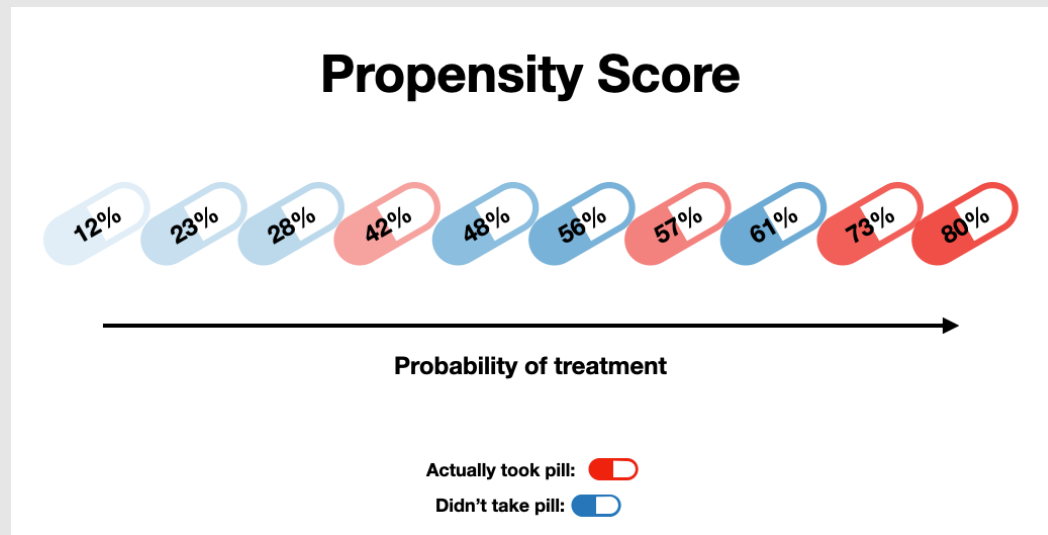
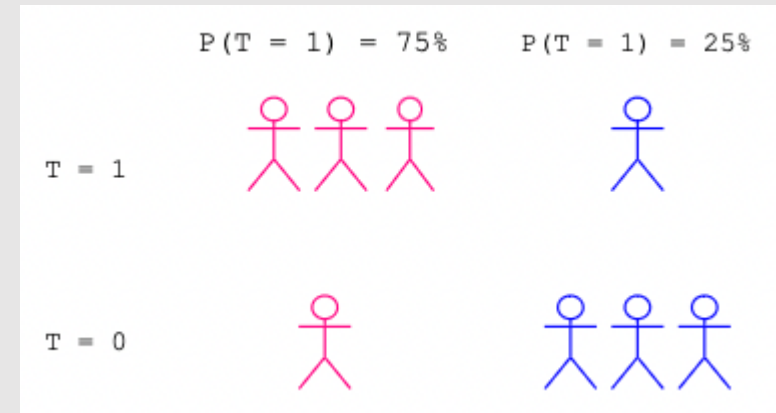
...Or using **machine learning** algorithms !
The important thing here is just to have a predicted probability to receive the treatment



Construction of the control group:

Four *main* approaches:

1. Covariate Adjustment
2. Matching
3. Stratification
4. Inverse **P**robability of **T**reatment **W**eighting (IPTW)



Covariate Adjustment: use PS as a covariate !

Method

$$Y_i = T_i\beta + f(\hat{e}_i)$$

$\hat{\beta}$ is the average[conditional] treatment effect

Assumption of *some* functional relationship between the outcome and the PS

Simple, efficient ...

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**

Matching

Method

We can form matched sets of **treated** and **untreated** subjects who share a **similar value of PS**

In the matched sample we can compare the outcome using **statistical methods** for **paired experiments**

Only the ATT effect could be estimated

Many variants for the matching procedure

It has been argued that appropriate SE are hard to obtain

It discards a lot of data in a non-deterministic manner

Propensity Score



25%

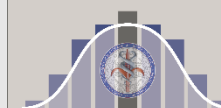


Popular

Intuitive

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**
2. **Characteristics can be compared in the matched sample to assess achievement of balance**

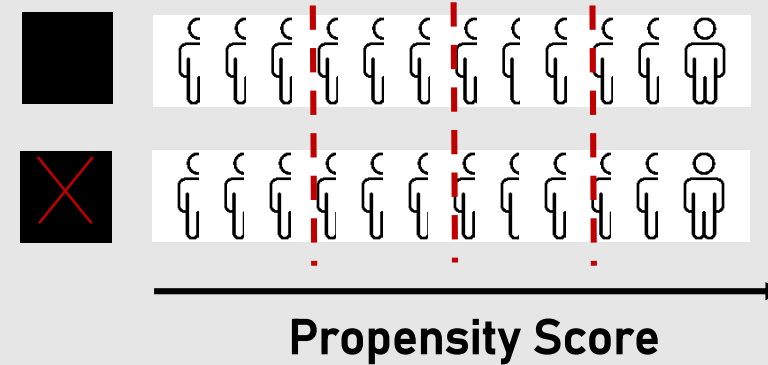


Stratification

Method

We obtain exclusive subsets according to values of the propensity scores

In **each stratum we can** compare the outcome between the two treatment groups (**CATE**) and then eventually use a **weighted mean** to obtain the marginal treatment effect (**ATE**)



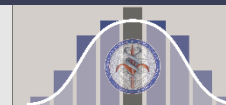
Simple

It can leave residual confounding

We need to be careful to have a sufficient sample size per strata/outcomes

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**
2. **Pre-treatment characteristics can be compared within strata**



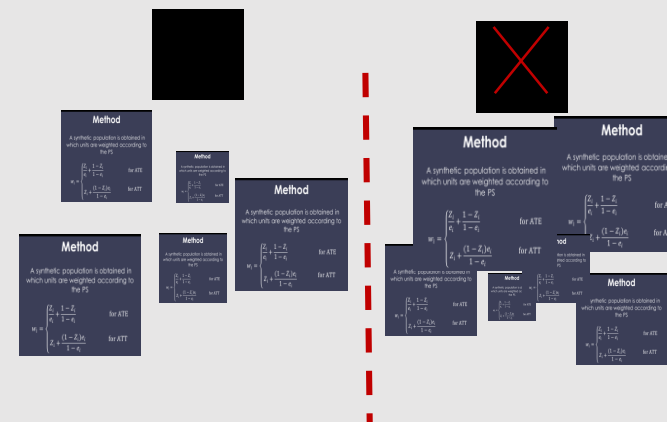
IPTW

Method

A **synthetic** population is obtained in which units are weighted according to the PS

$$w_i = \begin{cases} \frac{T_i}{e_i} + \frac{1 - T_i}{1 - e_i} & \text{for ATE} \\ T_i + \frac{(1 - T_i)e_i}{1 - e_i} & \text{for ATT} \end{cases}$$

Extremes weights → imprecise estimates



Mathematically appealing

Extends to complex scenarios

Diagnostic Tools

1. The distribution of the PS in the two groups **can help verifying the positivity assumption**
2. **Pre-treatment characteristics can be compared in the weighted dataset**

