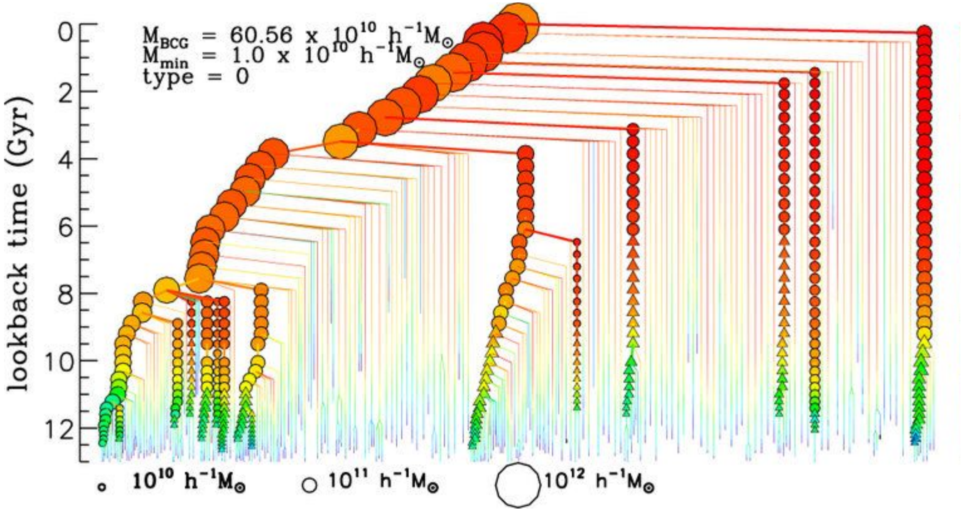
STATISTICAL PROPERTIES OF THE LARGE SCALE STRUCTURES:

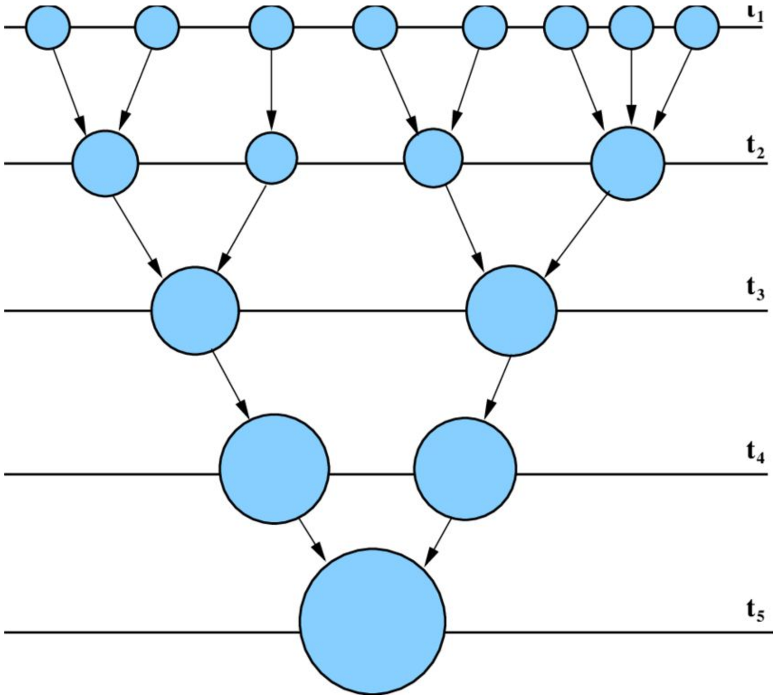
CLUSTER NUMBER COUNTS

For a review: **Allen+2011 or Kravtsov+2012**

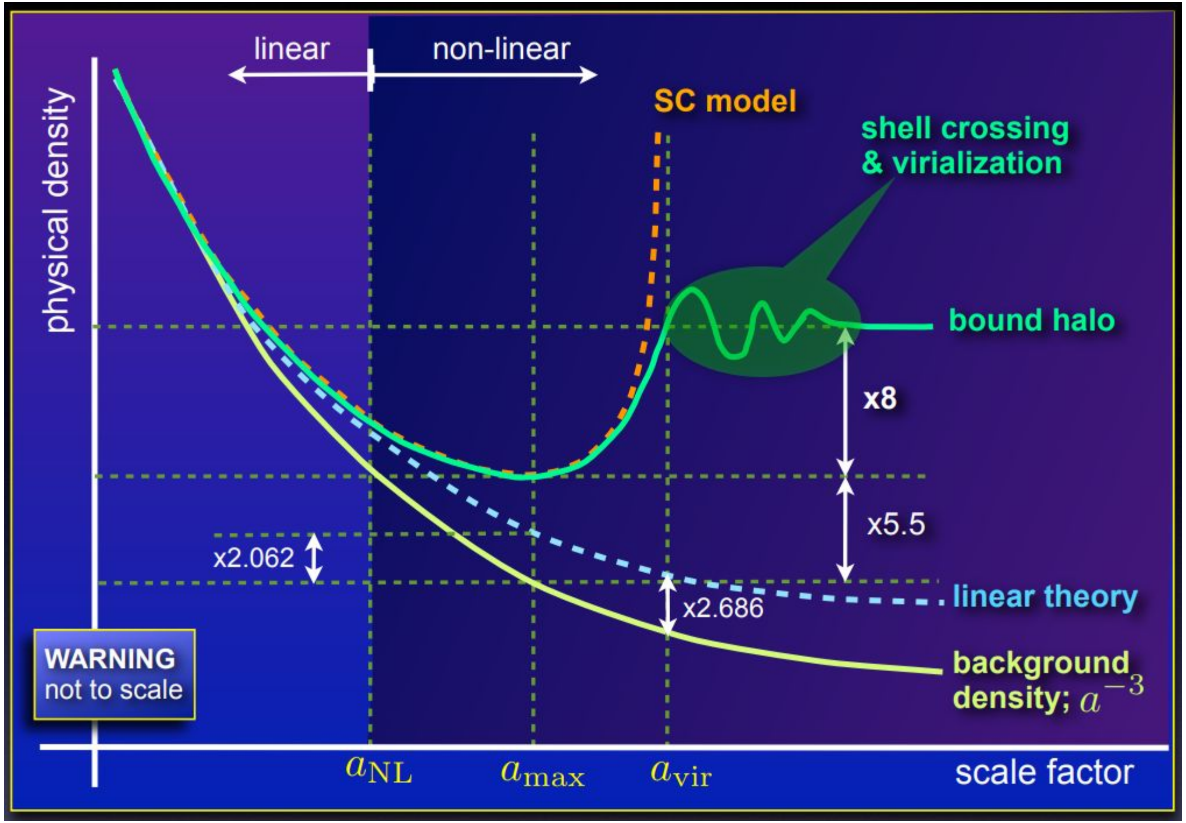
STRUCTURE FORMATION: DARK MATTER HALOS

In the LCDM scenario, structures grow *hierarchically*: Small overdensities are able to overcome the cosmological expansion and collapse first, and the resulting dark matter "halos" merge together to form larger halos which serve as sites of galaxy and galaxy cluster formation





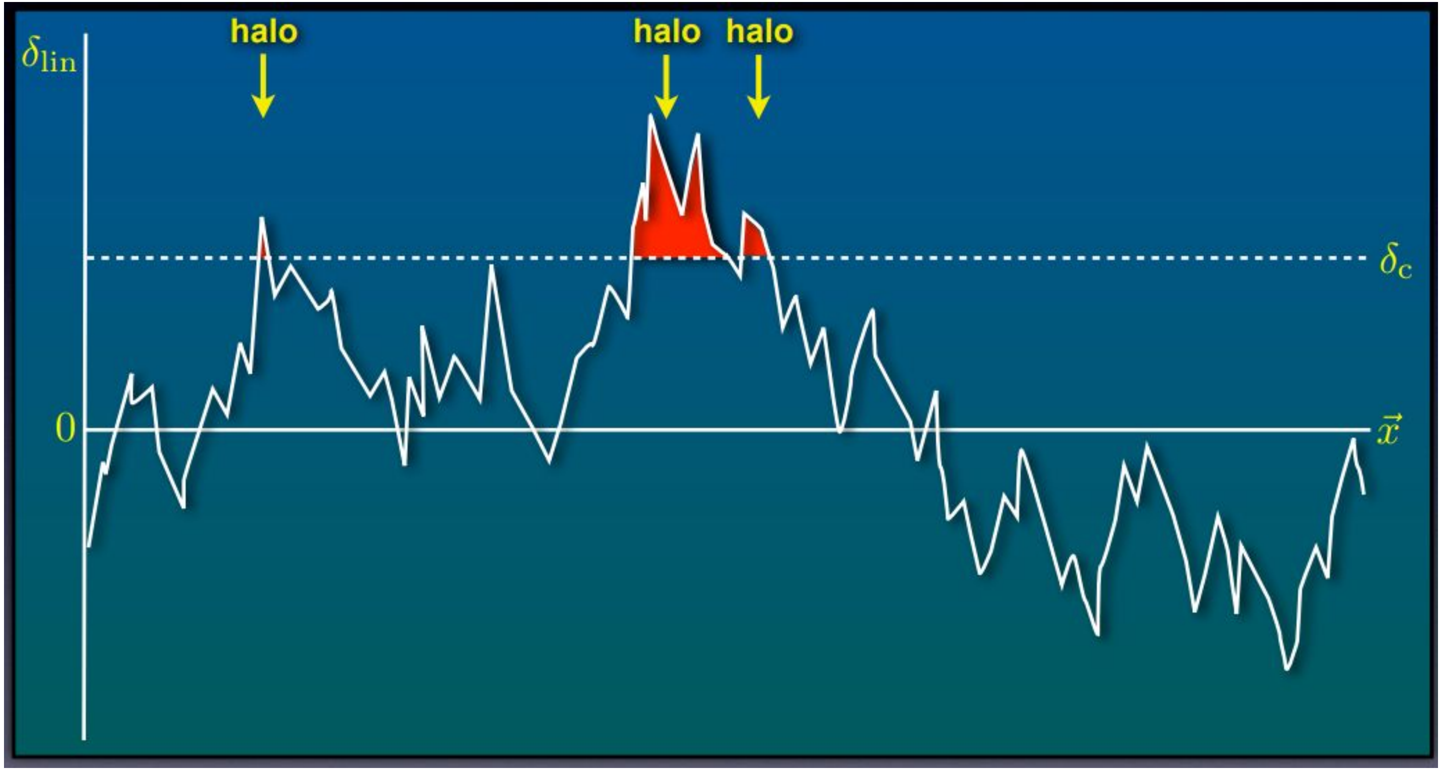
STRUCTURE FORMATION: SPHERICAL COLLAPSE MODEL



We can follow the collapse of a spherical overdensity in a homogeneous universe. SC model becomes inaccurate (brakes down) shortly after turn-around it is still a useful model to identify important epochs in the linearly evolved density field.

- **The linearly extrapolated density field collapses when $\delta_{\rm lin} = \delta_c = 1.686$**
- **Virialized dark matter haloes have an average overdensity of $\Delta_{\rm vir} = 178$**

STRUCTURE FORMATION: SPHERICAL COLLAPSE MODEL



According to the spherical collapse model, regions with $\delta(x,t) > \delta_c \simeq 1.686$ will have collapsed to produce dark matter haloes by time *t*

GALAXY CLUSTERS

- **Most massive bound objects in the Universe**:

  $M \simeq 10^{13} - 10^{15}\,M_\odot$ and $R \simeq 1 - 5$ Mpc

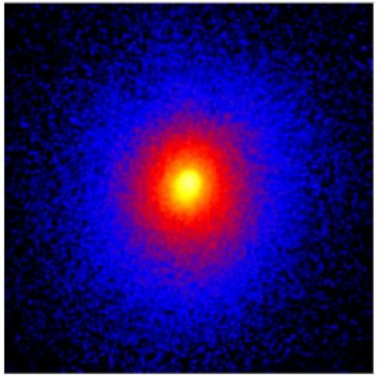
- **Multi-component systems**:

  Galaxies and stars (~5%), ICM (~15%), DM (~80%)

OPTICAL

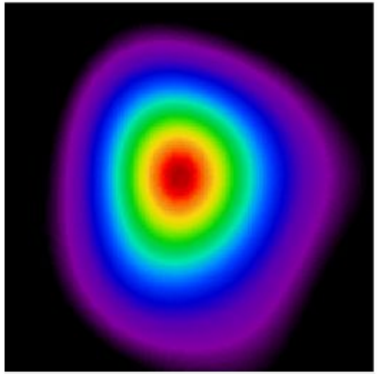
RICHNESS, LENSING EFFECTS

X-RAYS

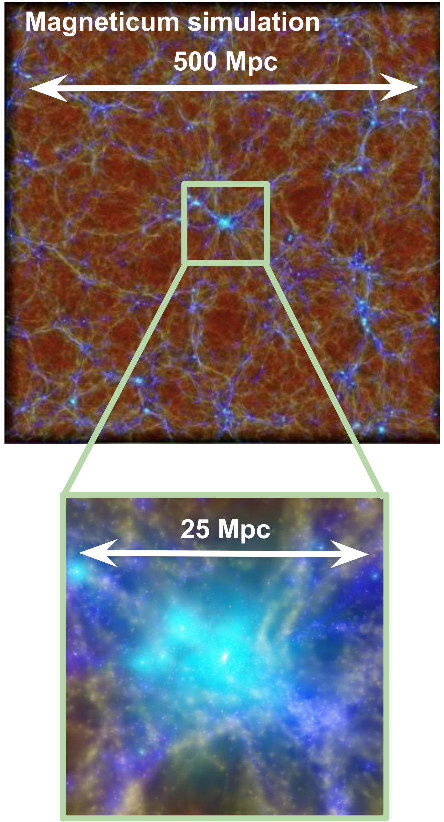


LUMINOUS AND EXTENDED X-RAY SOURCES

MICROWAVES



SUNYAEV-ZEL'DOVICH EFFECT



From Hirschmann+2014

GALAXY CLUSTERS AS COSMOLOGICAL PROBE

The abundance and spatial distribution of galaxy clusters are sensitive to the growth rate of cosmic structures and expansion history of the Universe

σ_8: Amplitude of the matter power spectrum

Ω_m: Present-day total matter density

$$\boxed{S_8 = \sigma_8 \, (\Omega_m / 0.3)^{0.5}}$$

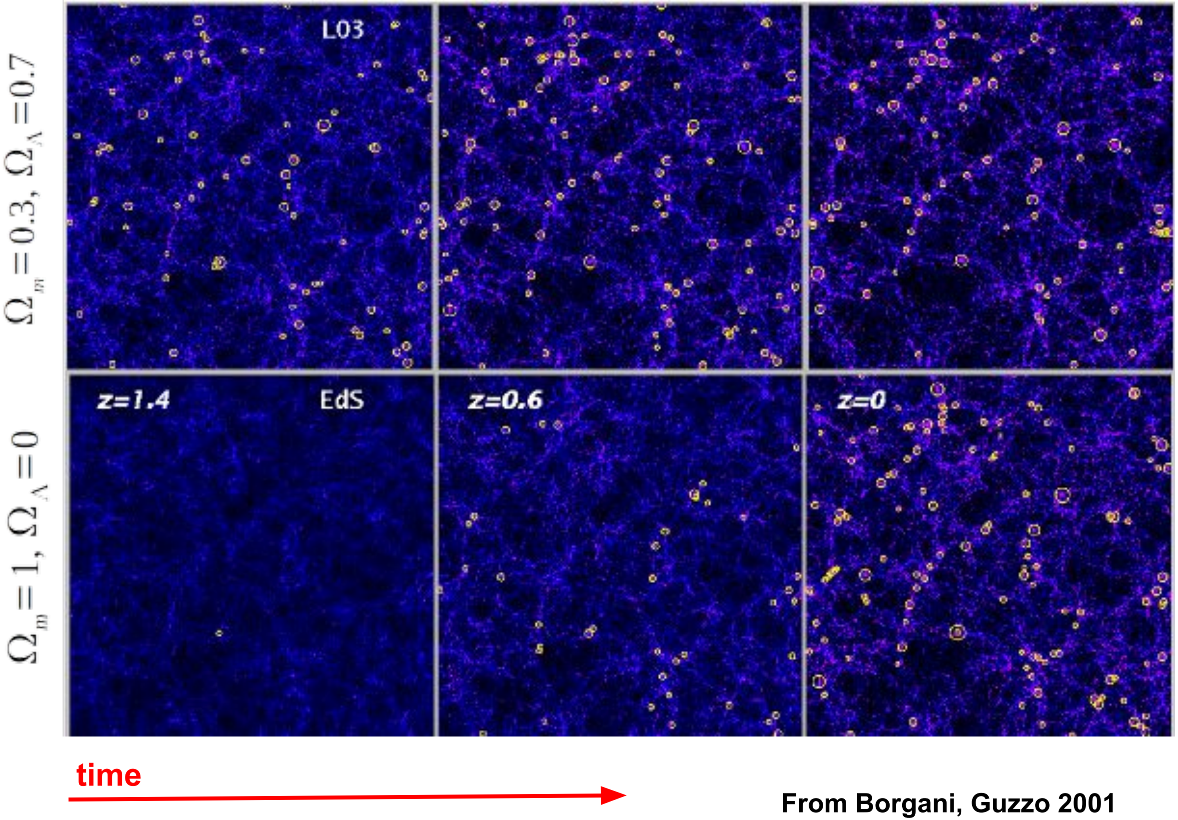
Dark energy equation of state parameter *w*

Total neutrino mass

Deviation from GR

….

Evolution of the clusters population in 2 N-body simulations



From Borgani, Guzzo 2001

THE HALO MASS FUNCTION

Cluster abundance:

$$\frac{dN}{dz d\Omega} = \frac{dV}{dz d\Omega} n(M, z)$$

geometry

growth

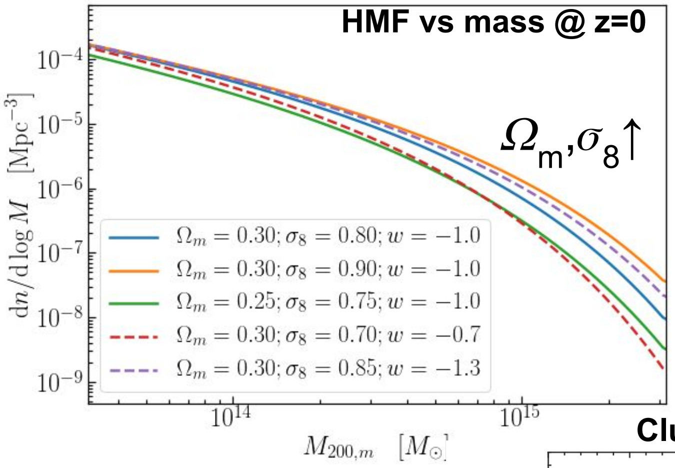
The halo mass function:

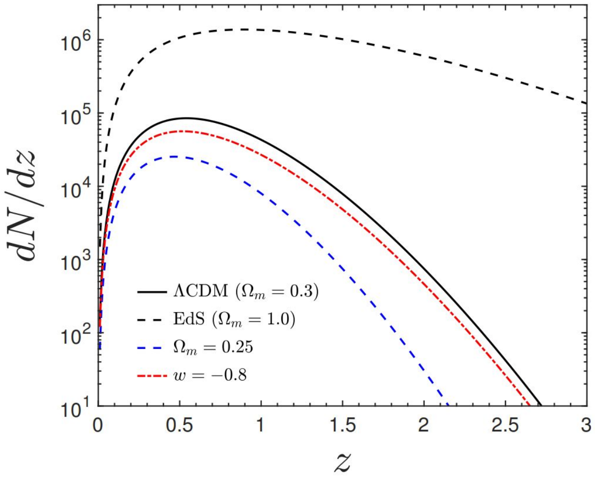
$$n(z, M) = \frac{\rho_m}{M} f(\sigma) \frac{d \ln(\sigma^{-1})}{d M}$$

Variance of the density field:

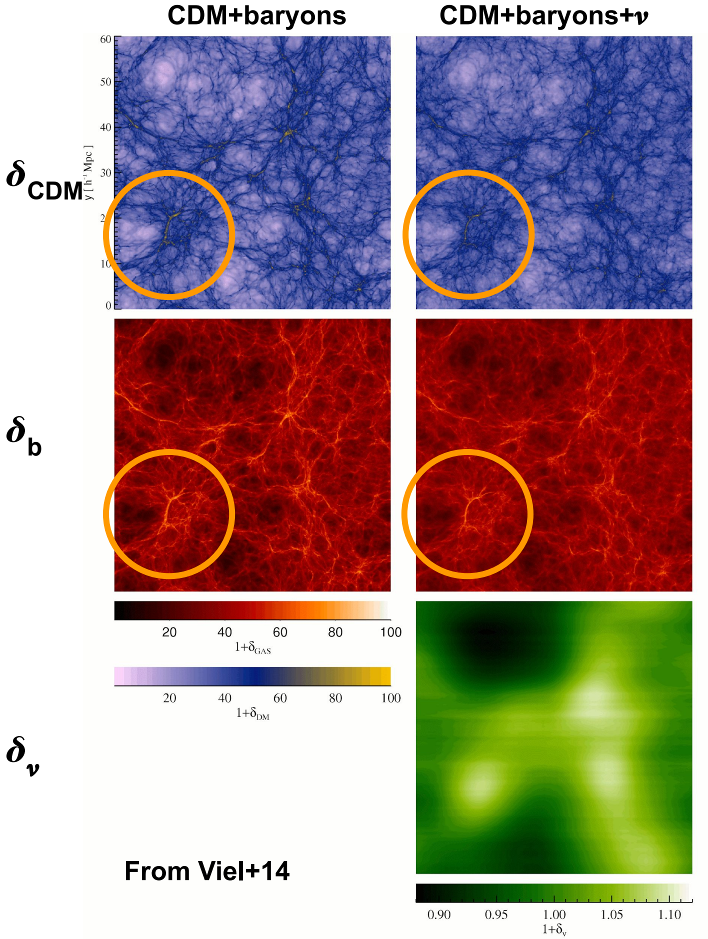
$$\sigma(z, R) = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 \boxed{P_m(z, k)} |W(kR)|^2$$

Matter power spectrum


HMF vs mass @ z=0


Cluster counts vs redshift

THE HALO MASS FUNCTION: MASSIVE NEUTRINOS

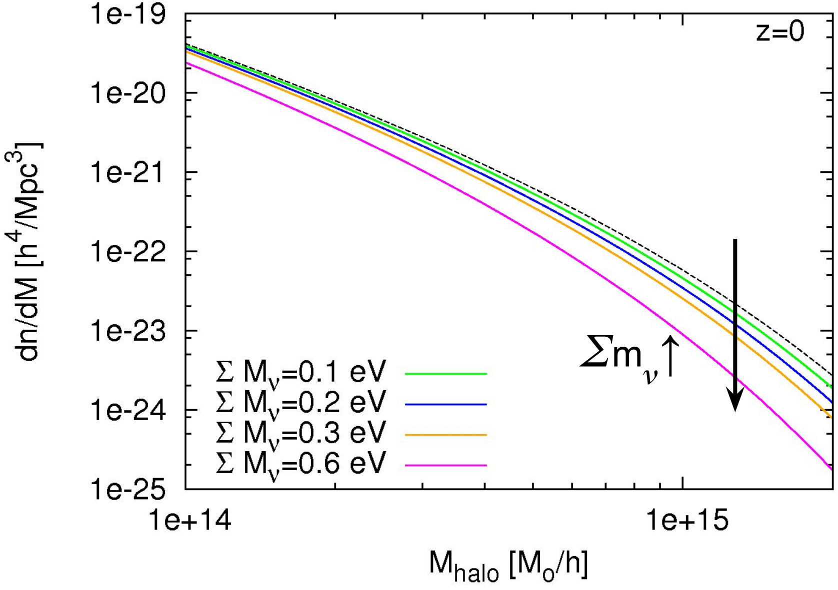


From Viel+14

Massive neutrinos:

- Delay the epoch of matter-radiation equality
- Suppress the growth of density fluctuation on scale smaller than the free-streaming length

Effects on the number density of halos as a function of mass



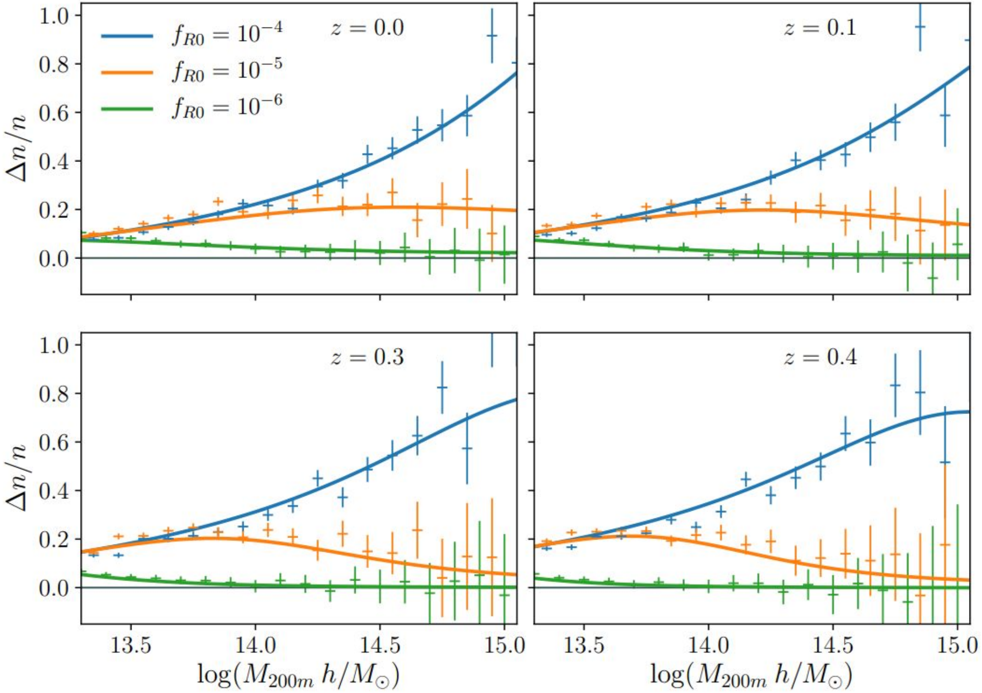
THE HALO MASS FUNCTION: MODIFIED GRAVITY

Modified gravity models, e.g. *f*(R):

$$S = \frac{1}{16\pi G}\int \sqrt{-g}[R + f(R)]d^4x.$$

- Give rise to accelerated expansion and enhance gravity
- Introduce screening mechanism that restores GR in high density environments

Relative effect on the Halo Mass Function compared to ΛCDM

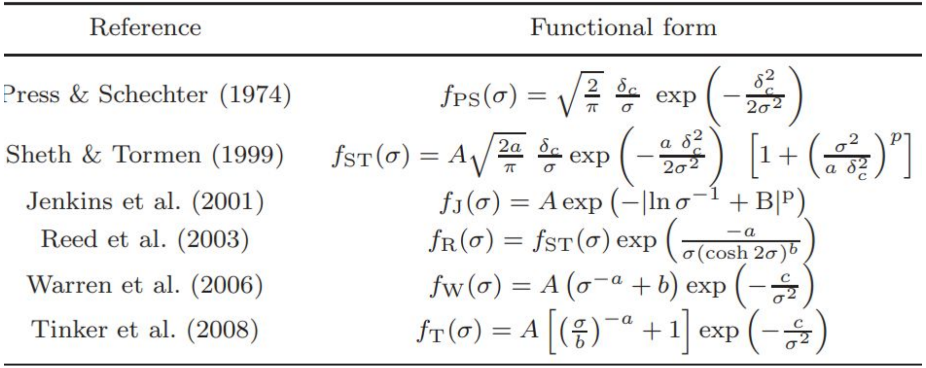


From Hagstotz+18

THE MULTIPLICITY FUNCTION: $f(\sigma)$

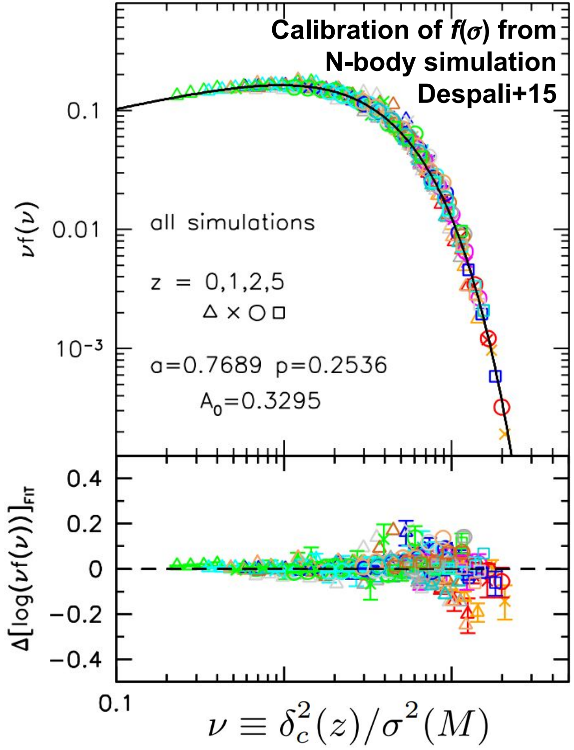
Halo mass function: $n(z,M)=\frac{\rho_m}{M}f(\sigma)\frac{d\ln(\sigma^{-1})}{dM}$

- ***$f(\sigma)$* “universal” function:**
  - **Press & Schechter (1974) approximated from spherical collapse of Gaussian density field**
  - **Improved modeling using ellipsoidal collapse, e.g. Sheth & Tormen (1999)**
  - **Nowadays calibrated against N-body simulations**



Reference	Functional form
Press & Schechter (1974)	$f_{\rm PS}(\sigma) = \sqrt{\frac{2}{\pi}}\ \frac{\delta_c}{\sigma}\ \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$
Sheth & Tormen (1999)	$f_{\rm ST}(\sigma) = A\sqrt{\frac{2a}{\pi}}\ \frac{\delta_c}{\sigma}\exp\left(-\frac{a\ \delta_c^2}{2\sigma^2}\right)\ \left[1+\left(\frac{\sigma^2}{a\ \delta_c^2}\right)^p\right]$
Jenkins et al. (2001)	$f_{\rm J}(\sigma) = A\exp\left(-\lvert\ln\sigma^{-1}+\mathrm{B}\rvert^{\mathrm{P}}\right)$
Reed et al. (2003)	$f_{\rm R}(\sigma) = f_{\rm ST}(\sigma)\exp\left(\frac{-a}{\sigma(\cosh 2\sigma)^b}\right)$
Warren et al. (2006)	$f_{\rm W}(\sigma) = A\left(\sigma^{-a}+b\right)\exp\left(-\frac{c}{\sigma^2}\right)$
Tinker et al. (2008)	$f_{\rm T}(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a}+1\right]\exp\left(-\frac{c}{\sigma^2}\right)$

Pillepich+10



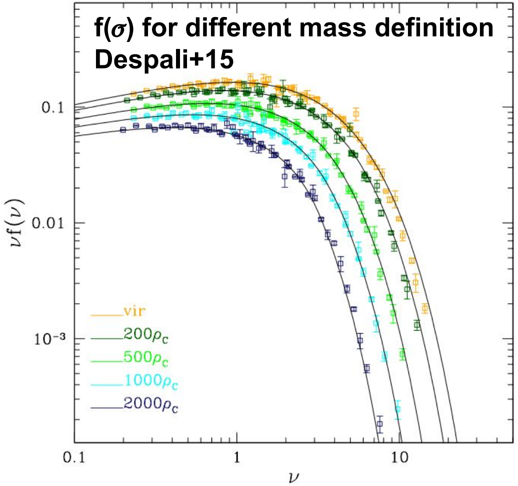
Calibration of *f*(σ) from N-body simulation Despali+15

See also: Despali+15 ; Castro+22

HALO MASS FUNCTION: UNIVERSALITY

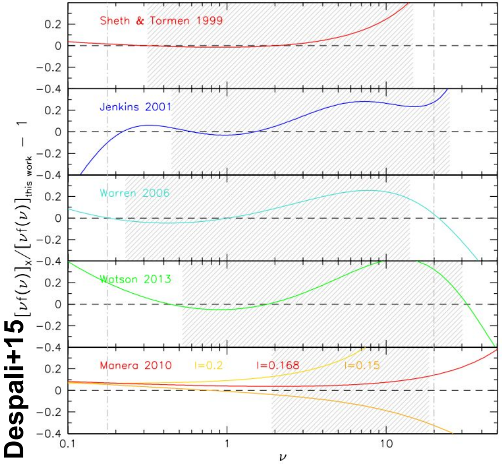
How accurate is the calibration of f(σ)? Is it universal?

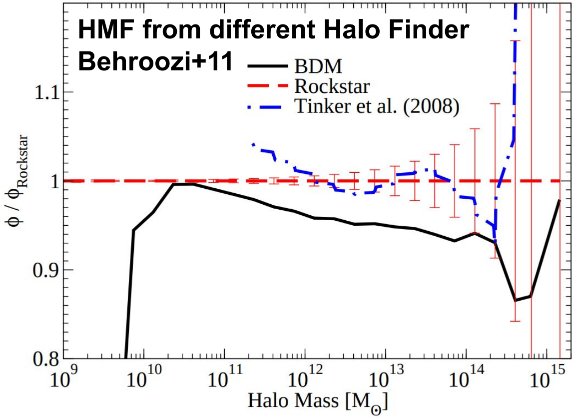
- **Specific of the simulation (e.g. box size, number of particles, softening length)**
- **Halo finder (e.g. linking length, FoF, SO)**
- **Mass definition (e.g. $M_{200,m}$, $M_{500,c}$)**
- **Redshift dependence**
- **Cosmological model (e.g. LCDM, wCDM, massive neutrino)**



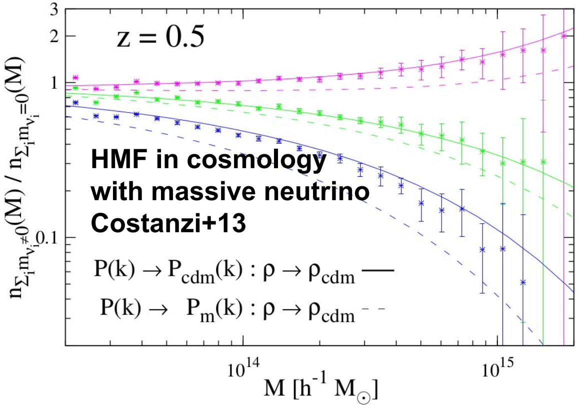
f(σ) for different mass definition Despali+15

Comparison different HMF calibration Despali+15





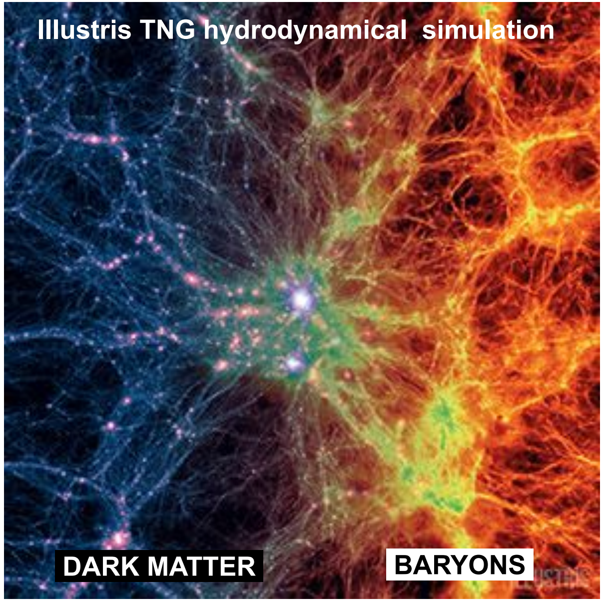
HMF from different Halo Finder Behroozi+11

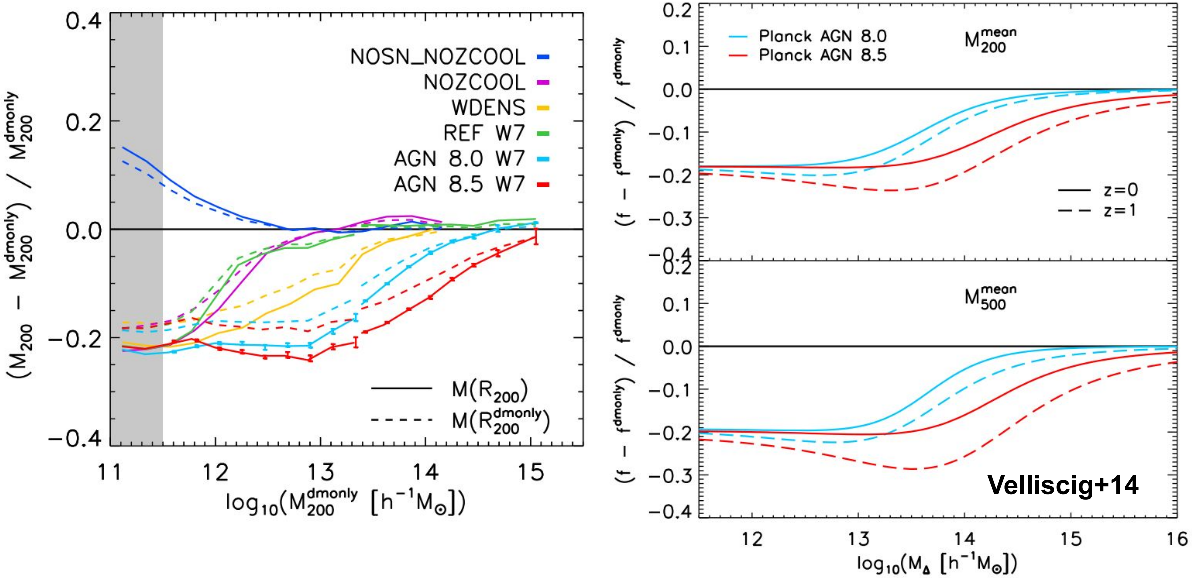


HMF in cosmology with massive neutrino Costanzi+13

HALO MASS FUNCTION: BARYONIC EFFECTS

Baryonic feedbacks (radiative cooling, star formation, AGN feedback) redistribute and expel mass from galaxy clusters





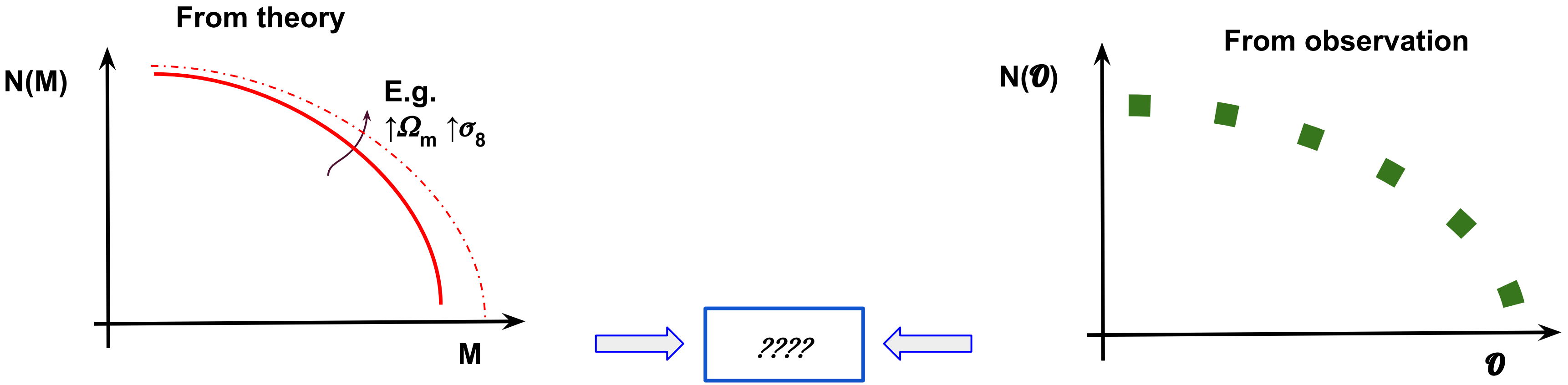
Velliscig+14

Baryonic feedbacks most effective in the inner the regions of the halo and in low mass systems

See also Castro+21

FROM THEORY TO OBSERVATION

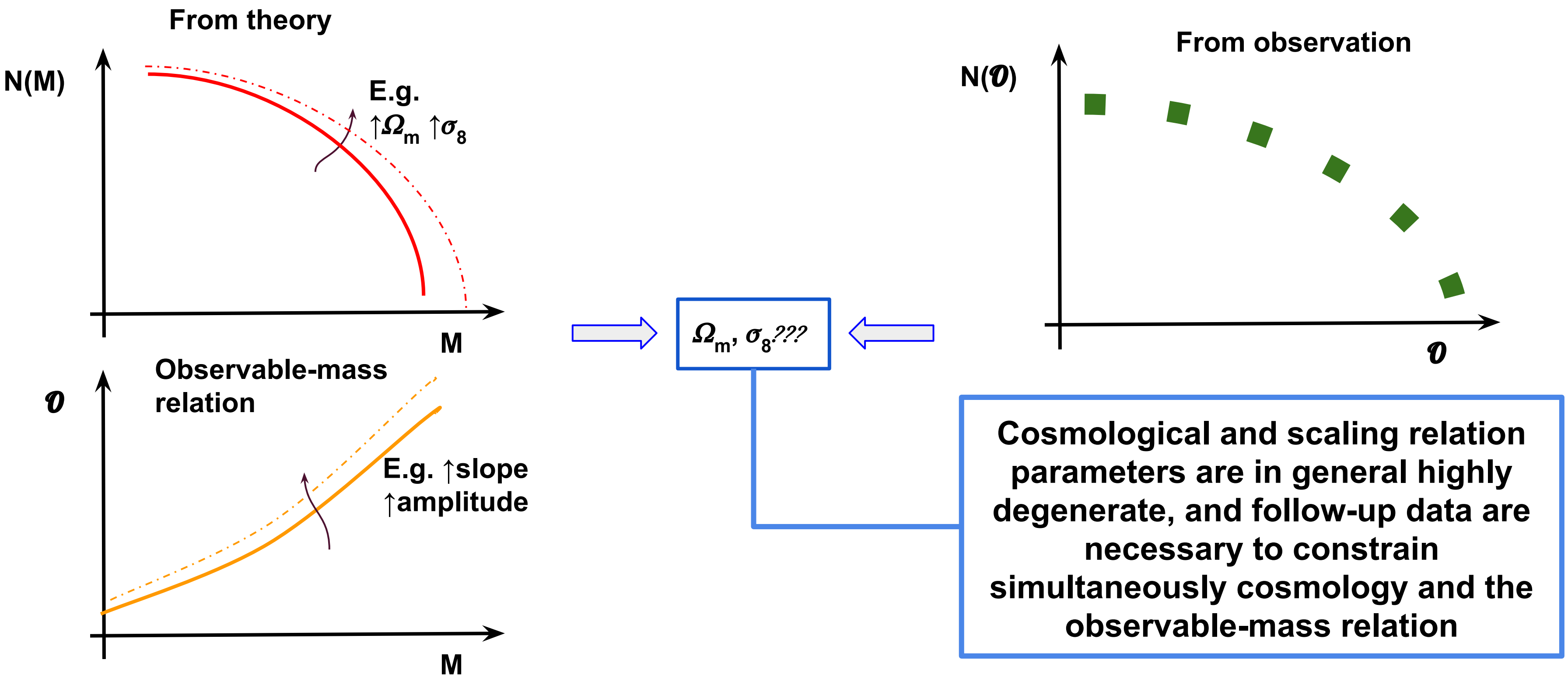
- **Masses are not directly observable. Galaxy clusters are selected according to some observable, in general related to the observational technique, which correlate with the mass.**



$\mathcal{O}$: Observable used to detect/select clusters (e.g. number of galaxies, X-ray luminosity, SZ signal)

FROM THEORY TO OBSERVATION

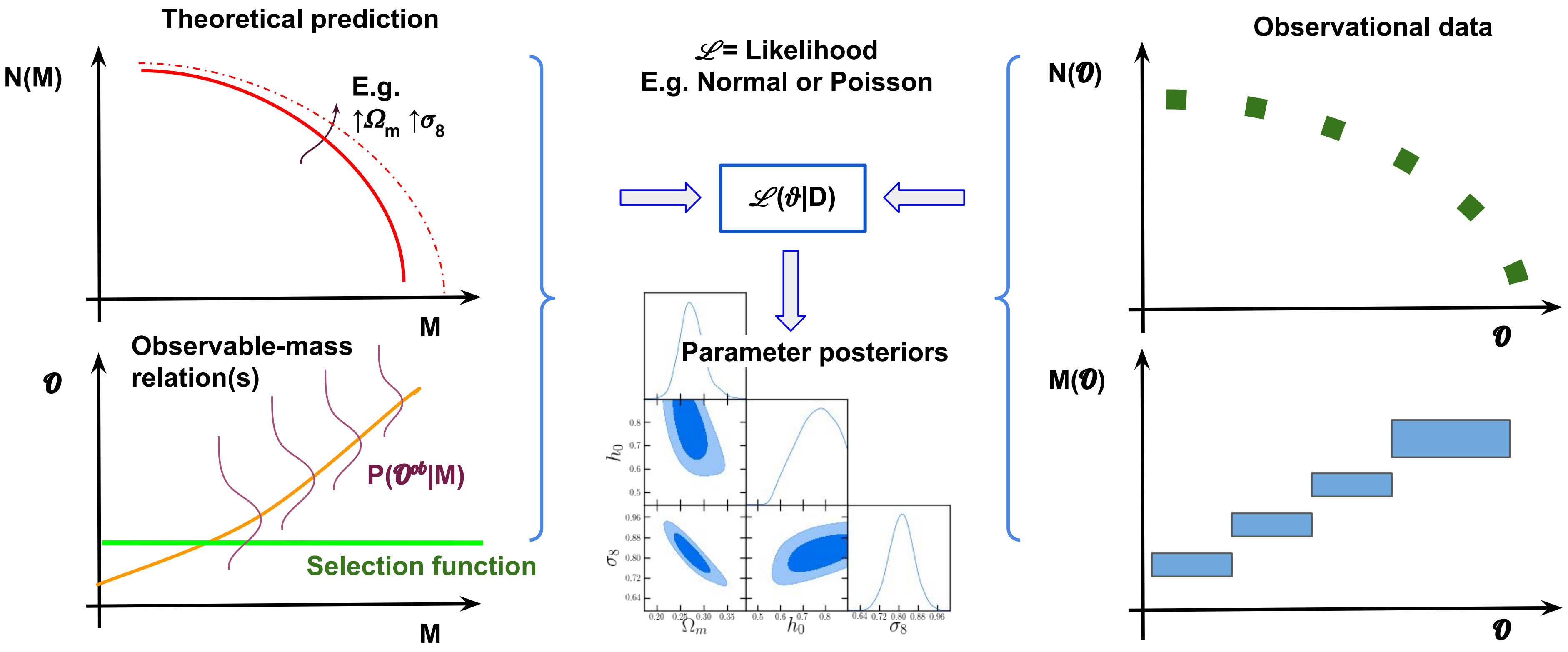
- **Individual mass measurements are expensive and not feasible for cluster survey. We need to rely on mass proxies which are tightly correlated with the halo mass.**



Cosmological and scaling relation parameters are in general highly degenerate, and follow-up data are necessary to constrain simultaneously cosmology and the observable-mass relation

FROM THEORY TO OBSERVATION: CONSTRAINTS

- **Combine cluster abundance and cluster mass estimates data to simultaneously constrain cosmology and the observable-mass relation(s)**



Theoretical prediction

$\mathscr{L}$= Likelihood
E.g. Normal or Poisson

$\mathscr{L}(\vartheta|D)$

Parameter posteriors

Observational data

Observable-mass relation(s)

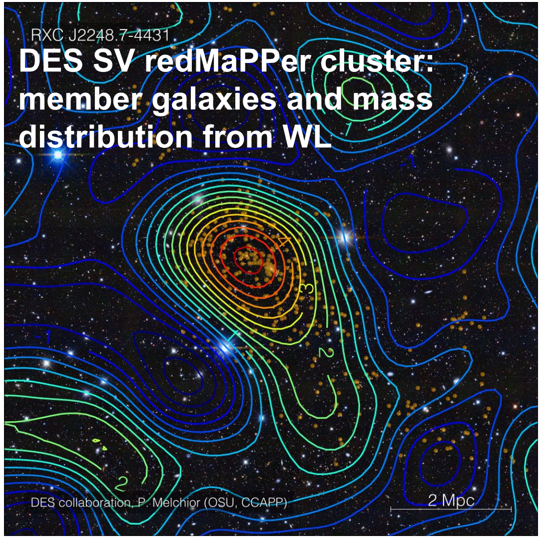
Selection function

$P(\mathcal{O}^{ob}|M)$

CLUSTER DETECTION: PHOTOMETRIC SURVEY

- **Detection:**
  - **Overdensity of (red-sequence) galaxies**
  - **Lensing effect**

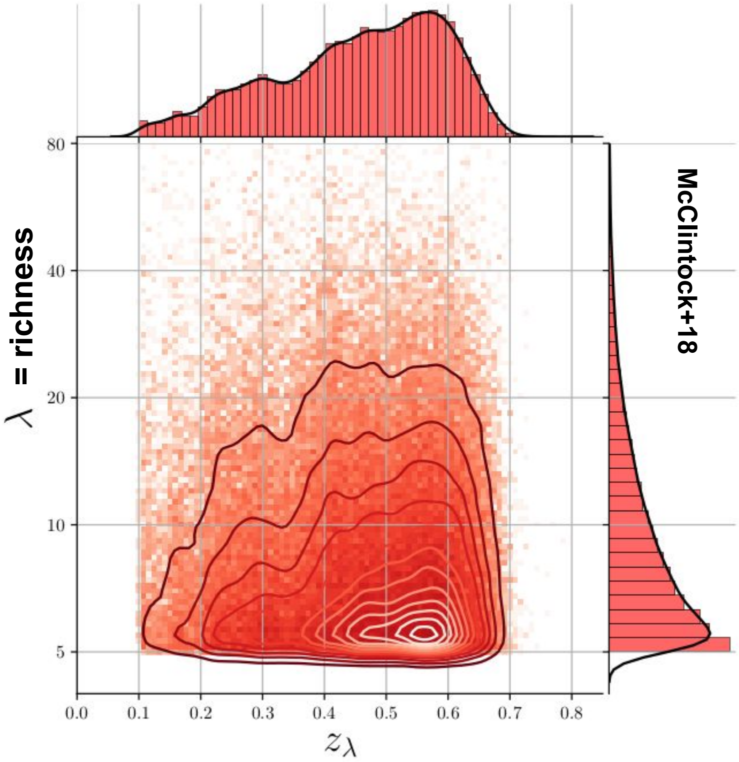
- **Observable/Mass proxy:**
  - **Richness (# member galaxies)**
  - **Luminosity**
  - **Lensing signal**
  - **Velocity dispersion (with spectra)**



DES SV redMaPPer cluster: member galaxies and mass distribution from WL

DES collaboration, P. Melchior (OSU, CCAPP)

***z*-*λ* distribution of redMaPPer clusters in DES Y1**



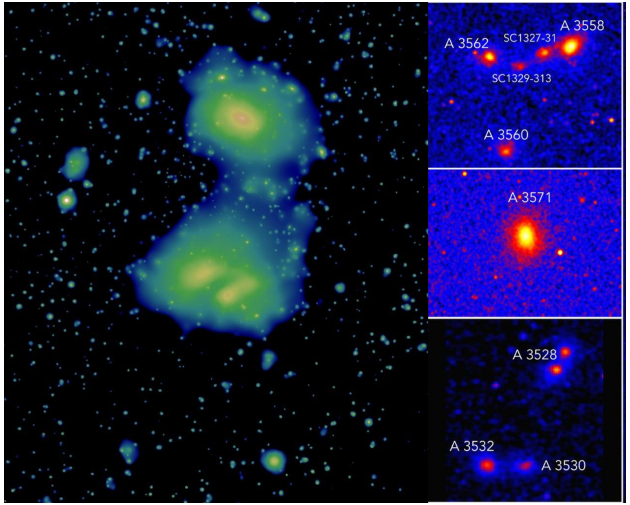
McClintock+18

CLUSTER DETECTION: X-RAY SURVEY

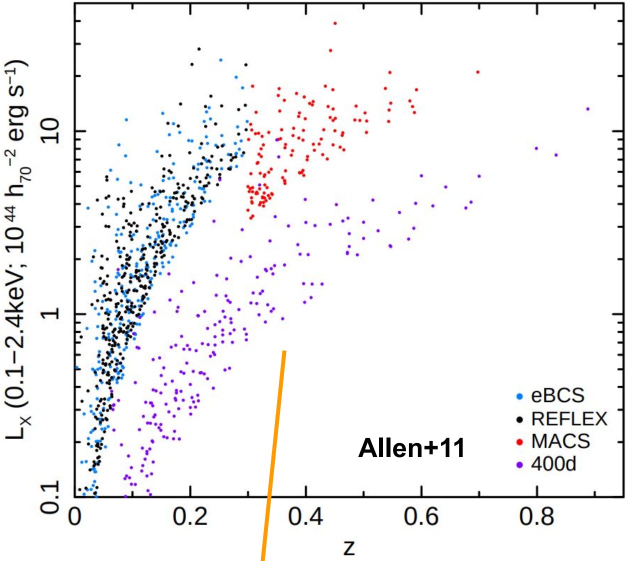
- **Detection:**
  - **Extended x-ray sources**

- **Observable/Mass proxy:**
  - **L_X**
  - **T_X**
  - **Flux**
  - **Y_x=M_{gas} T_X (gas thermal energy)**

X-ray images of clusters from eROSITA



L_X, z distribution of X-ray selected catalogs



Flux limited sample

- **X-ray emissivity from *bremsstrahlung* radiation of the ICM:**

$$\epsilon_\nu \equiv \frac{\mathrm{d}L}{\mathrm{d}V\mathrm{d}\nu} \propto n_e^2 g(\nu,T) T^{-1/2} \exp(-h\nu/k_B T)$$

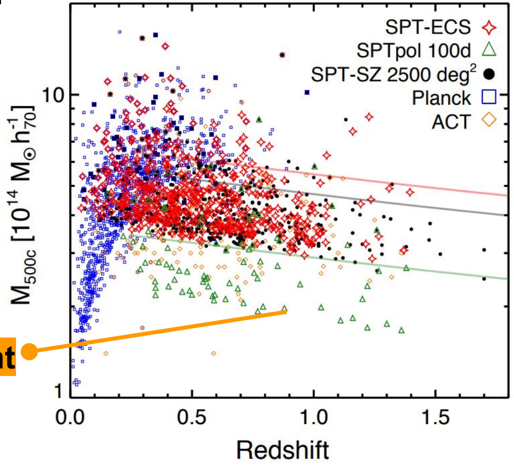
Not very sensitive to projections

CLUSTER DETECTION: SZ SURVEY

- **Detection:**
  Thermal Sunyaev-Zel'dovich effect
  (mm-wavelength)

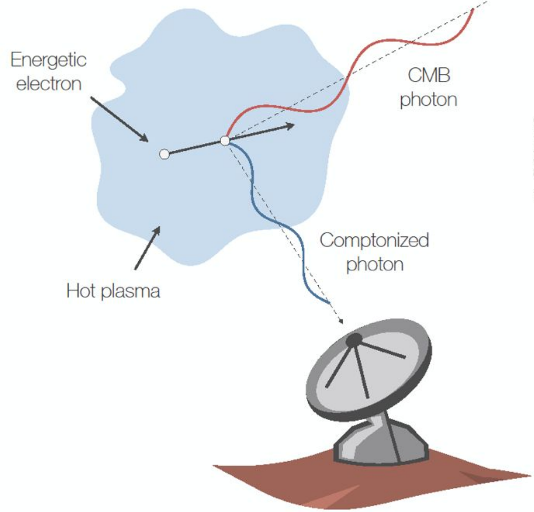
- **Observable/Mass proxy:**
  SZ signal

Mass and redshift distribution of SZ-selected cluster catalogs (Bleem+19)

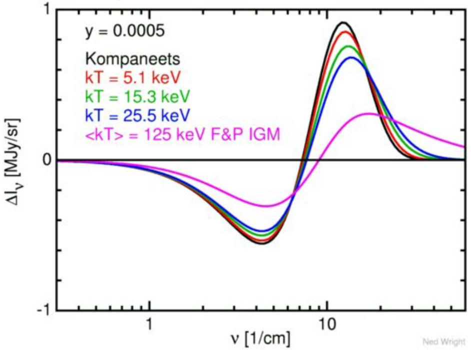


~ redshift independent

Thermal Sunyaev-Zel'dovich effect

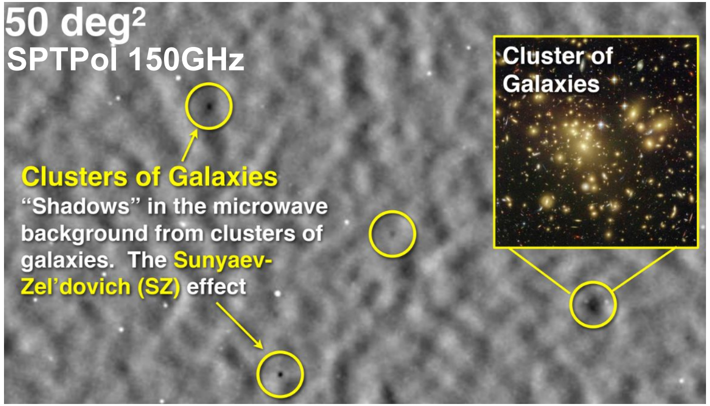


SZ spectral distortion



Compton-y parameter

$$y = \int \mathrm{d}l \frac{k_B T_e}{m_e c^2} n_e \sigma_\mathrm{T}$$



Clusters of Galaxies
"Shadows" in the microwave background from clusters of galaxies. The Sunyaev-Zel'dovich (SZ) effect

Credits B. Benson, SPT Collaboration

CLUSTER CATALOGS

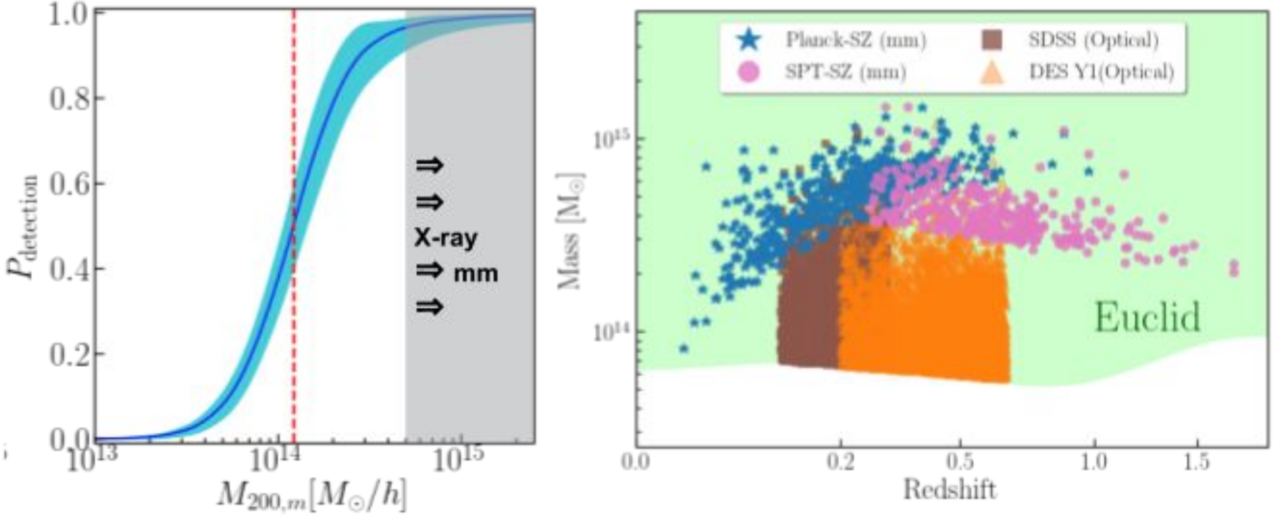


Figure 1: *Left*: redMaPPer DES Y1 cluster catalog detection probability as a function of mass: systems down to ~$5 \cdot 10^{13}$ $M_\odot$ have a non-negligible chance to be included in the optical catalog (λ>20), while clusters selected at different wavelength (X-ray, mm) have masses typically above $5 \cdot 10^{14}$ $M_\odot$ (*gray* area; adapted from [Ab20]). *Right*: Mass and redshift ranges probed by current optical (SDSS, DES Y1) and millimeter (Planck-SZ, SPT-SZ 2500) cluster surveys. The green shaded area marks the mass and redshift range to be covered by the Euclid cluster sample.

Photometric catalogs capable of detecting system down to group mass scale but have a much less cleaner selection function which hamper they cosmological exploitation

MASS MEASUREMENTS FROM X-ray DATA

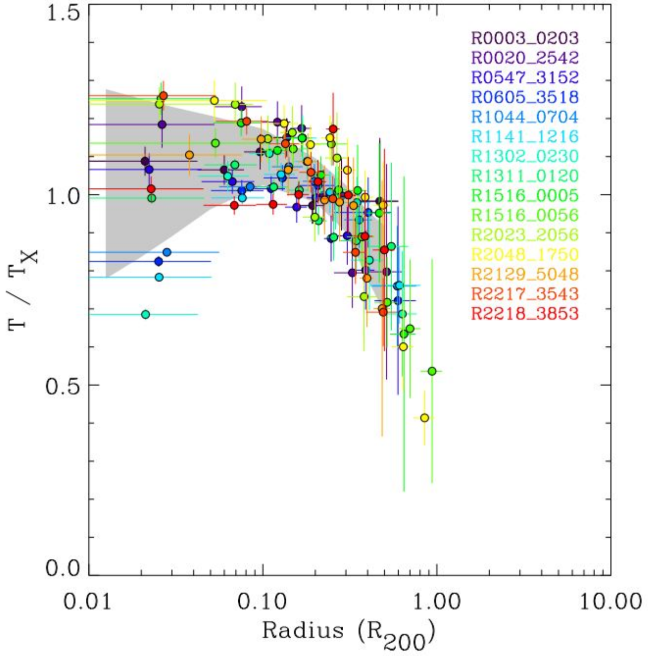
- **From hydrostatic equilibrium:**

$$M(<r) = -\frac{r}{G}\frac{k_B T(r)}{\mu m_p}\left(\frac{\mathrm{d}\ln\rho_{\mathrm{gas}}(r)}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T(r)}{\mathrm{d}\ln r}\right)$$

Assumptions:

- **Hydrostatic equilibrium (Negligible non-thermal pressure support)**
- **Spherical symmetry**

Temperature profiles from *XMM-Newton* observations (Pratt+06)



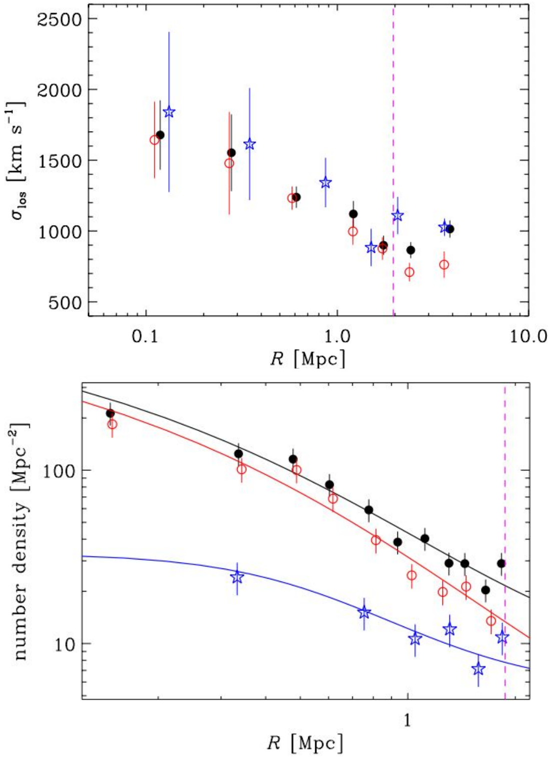
MASS MEASUREMENTS FROM SPECTROSCOPIC DATA

- **Dynamical mass estimates (Jeans equation):**

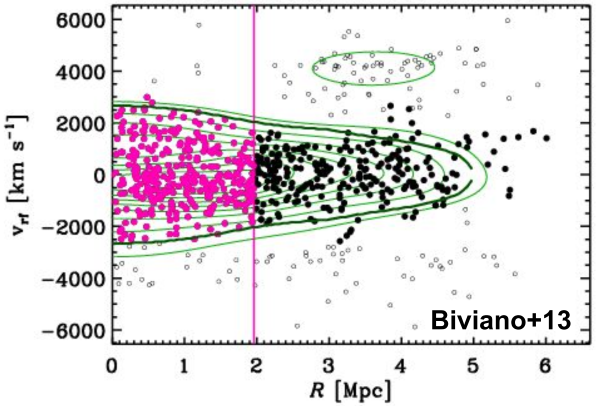
$$M(<r) = -\frac{r\sigma_r^2}{G}\left(\frac{\mathrm{d}\ln\sigma_r^2}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln n_{\mathrm{glx}}}{\mathrm{d}\ln r} + 2\beta\right)$$

Assumptions:
- **Spherical symmetry**
- **Dynamical equilibrium**

L.o.s. velocity dispersion and member galaxy density profiles from VLT/VIMOS (Biviano+13)



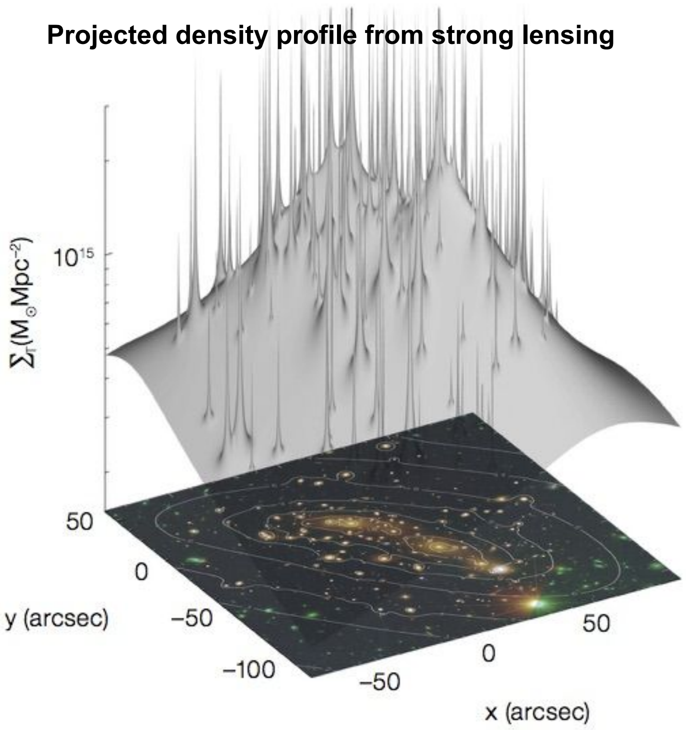
- **Caustic method (projected phase-space distribution):**



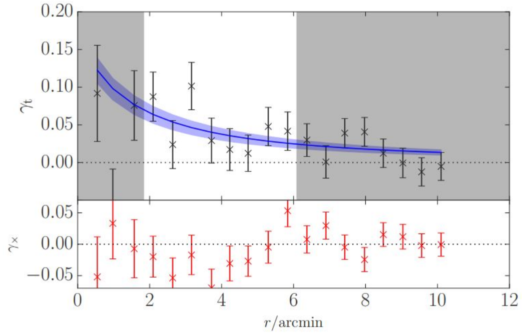
MASS MEASUREMENTS FROM IMAGING

- **Strong and Weak Lensing mass measurements:**

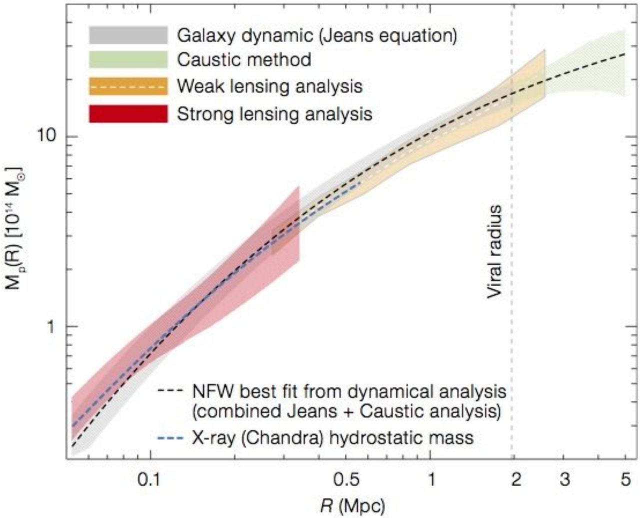
Projected density profile from strong lensing



Tangential shear profile from WL (Dietrich+18)



Cluster mass profile from different techniques (Battaglia+16)



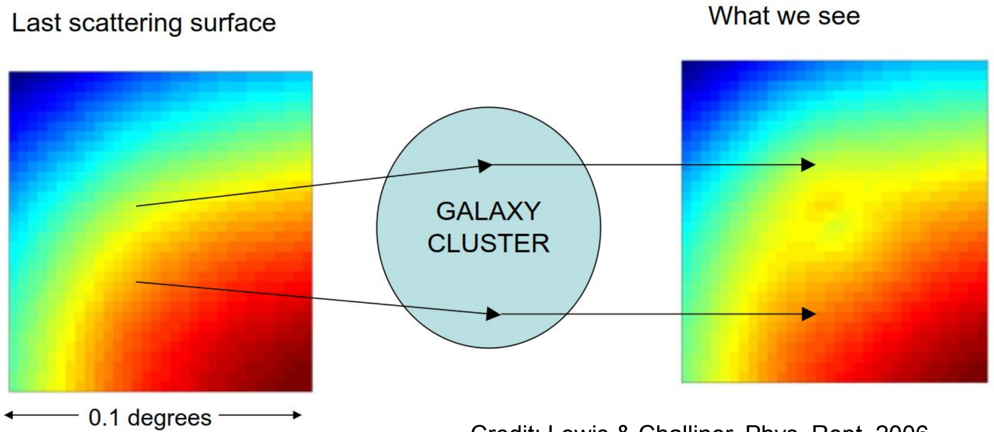
$$\gamma_t(n_s(z), \Sigma(R)) \Rightarrow \Sigma(R) = \int_{-\infty}^{\infty} d\,\chi\,\Delta\rho\left(\sqrt{R^2+\chi^2}\right)$$

Assumption:

- **Parametric form for the halo density profile (e.g. NFW, Einasto profiles; Navarro+97, Einasto 1965) and correlated structures (2-halo term)**

CMB CLUSTER LENSING

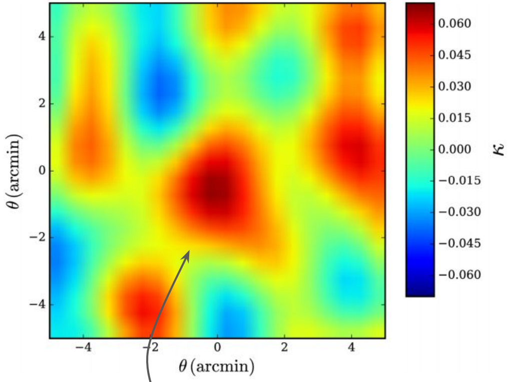
- **Lensing by GC induces a dipole-like distortion in the CMB:**

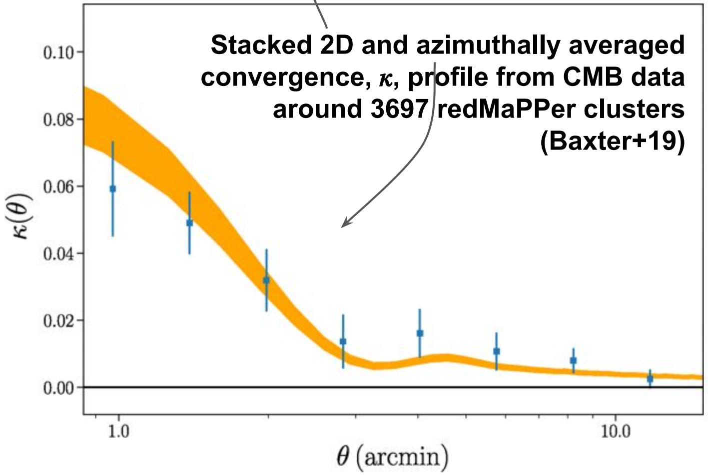


Credit: Lewis & Challinor, Phys. Rept. 2006

The distortion is quite small (~10μK for $10^{15}M_\odot$ halo) but can be used to calibrate the mass of high redshift clusters.

The lensing signal can also be detected in polarization data (see e.g. Raghunathan+19).

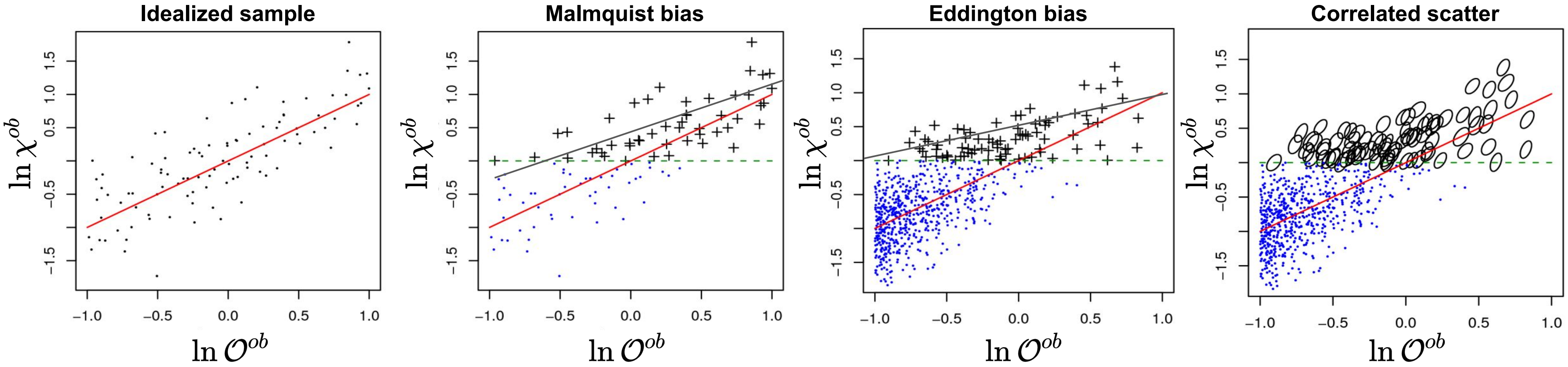




Stacked 2D and azimuthally averaged convergence, κ, profile from CMB data around 3697 redMaPPer clusters (Baxter+19)

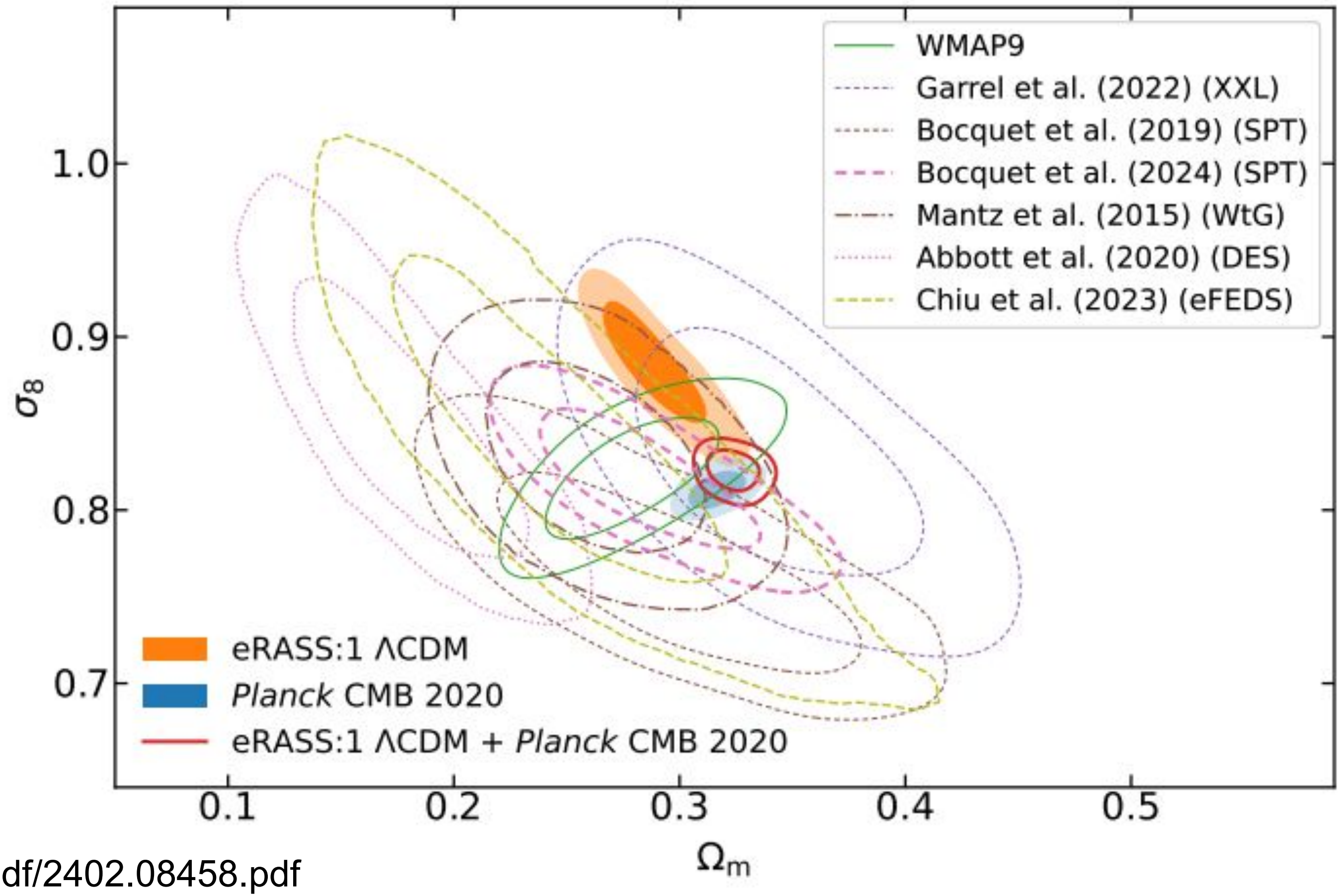
FROM THEORY TO OBSERVATION: SCALING RELATIONS

- **Scaling relation(s) calibration:**



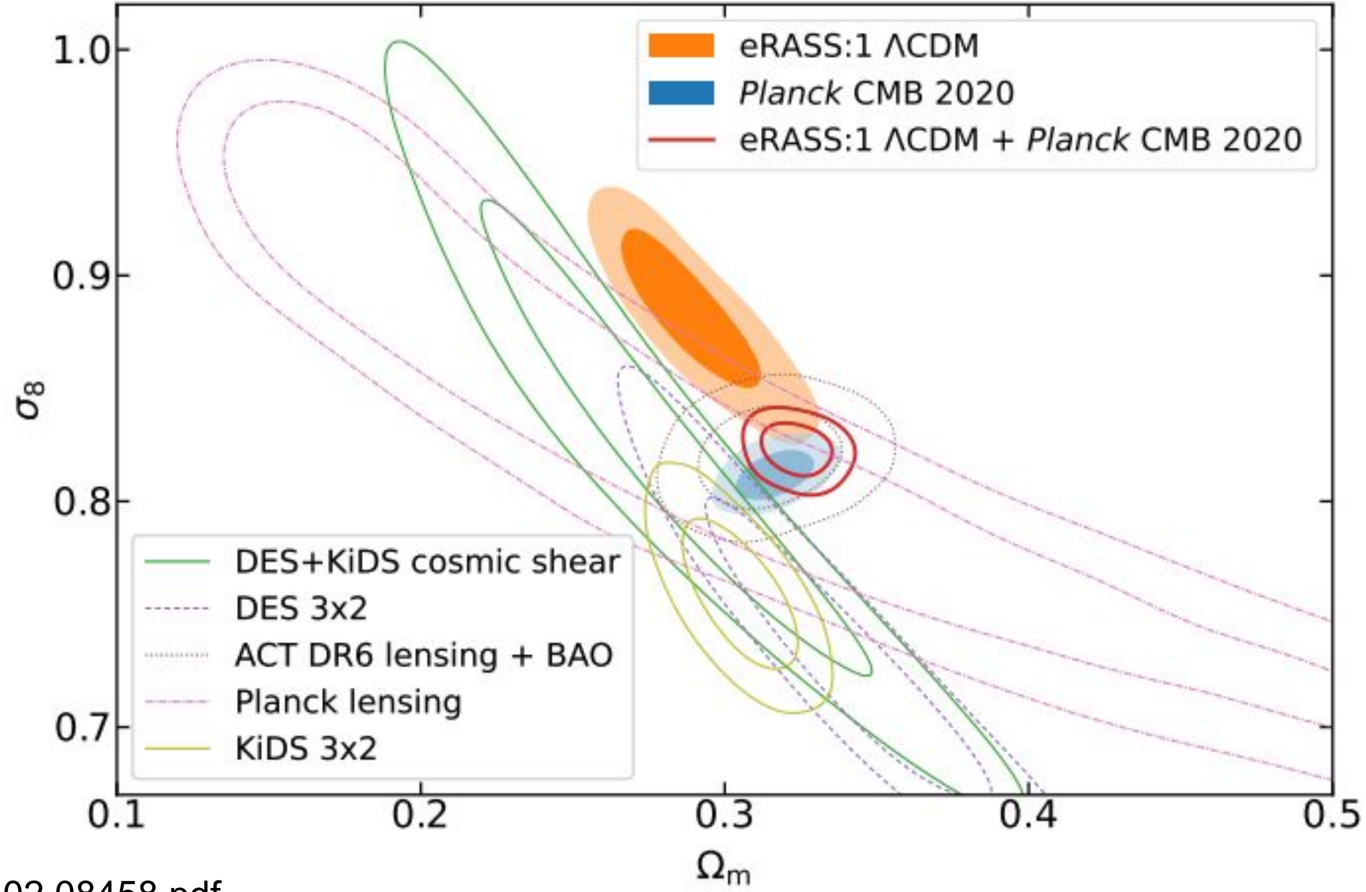
Credit A. Mantz

RECENT CONSTRAINTS FROM CLUSTER NC



https://arxiv.org/pdf/2402.08458.pdf

RECENT CONSTRAINTS FROM CLUSTER NC



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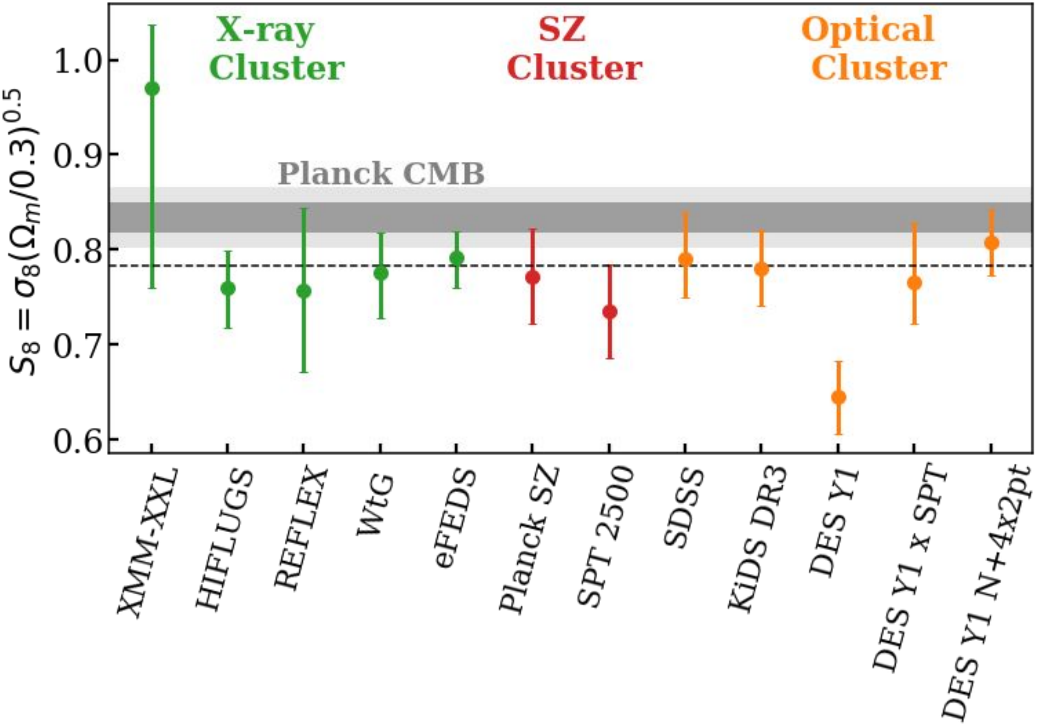
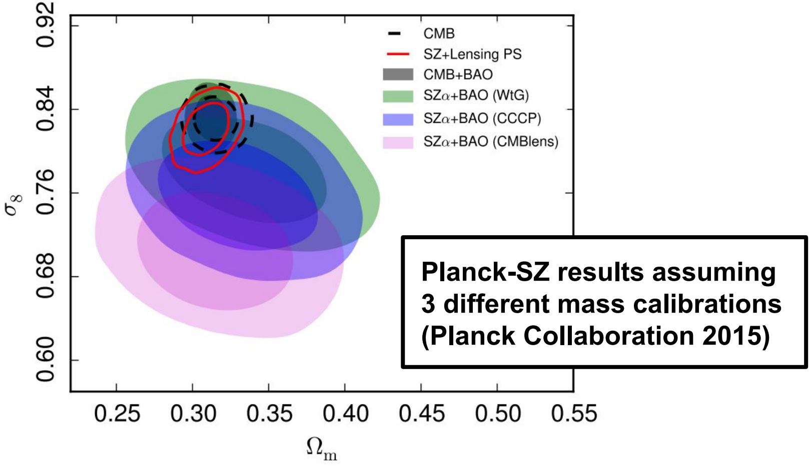
LIMITATIONS FOR CLUSTER COSMOLOGY STUDIES



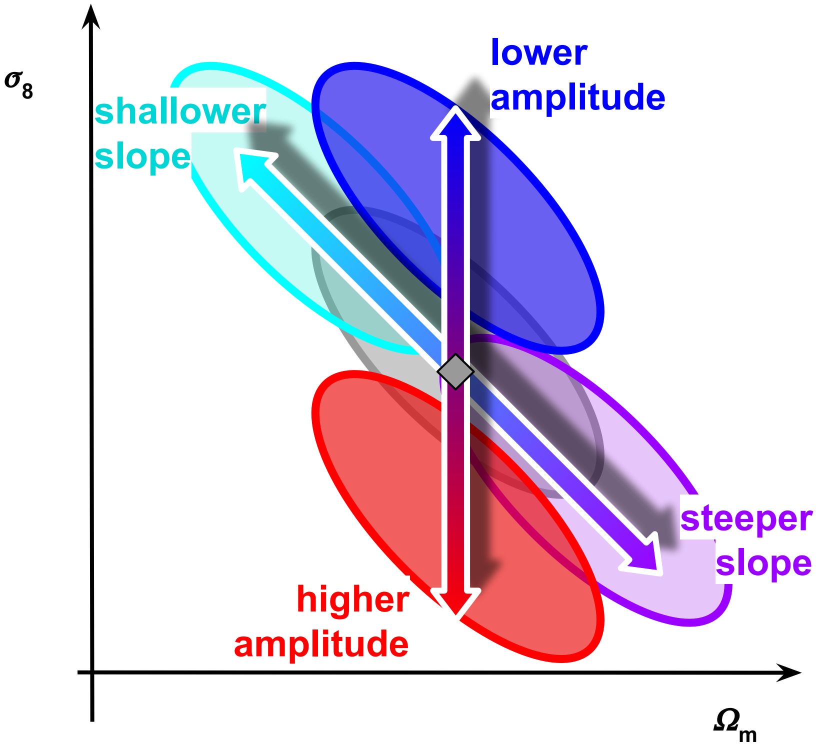
DES Collaboration 2020

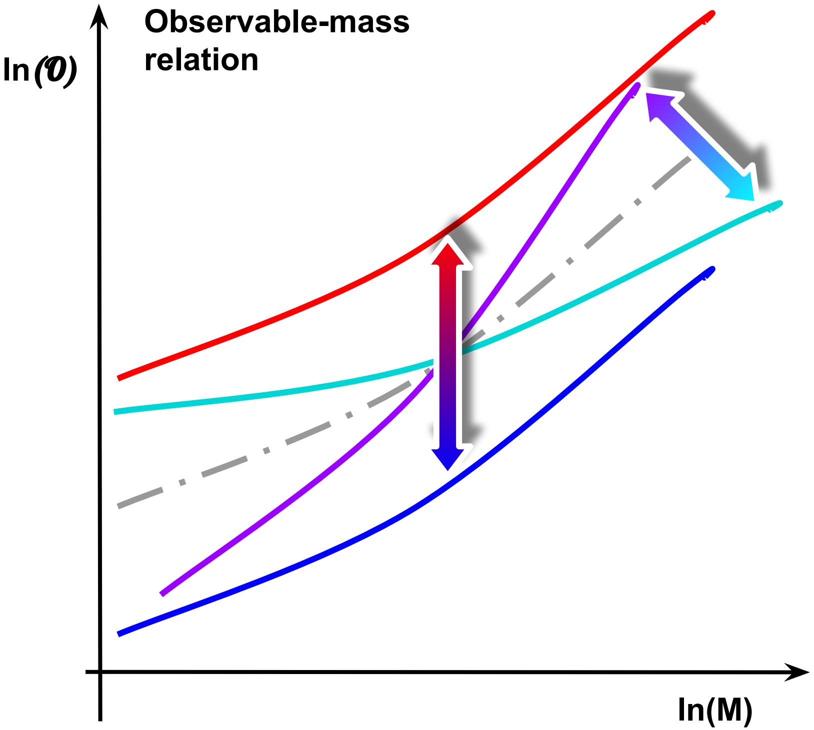
- **Cosmological constraints independent and competitive with other cosmological probes**
- **Slight to moderate tension between different cluster studies**
- **Currently limited by the mass (i.e. scaling relation) calibration**



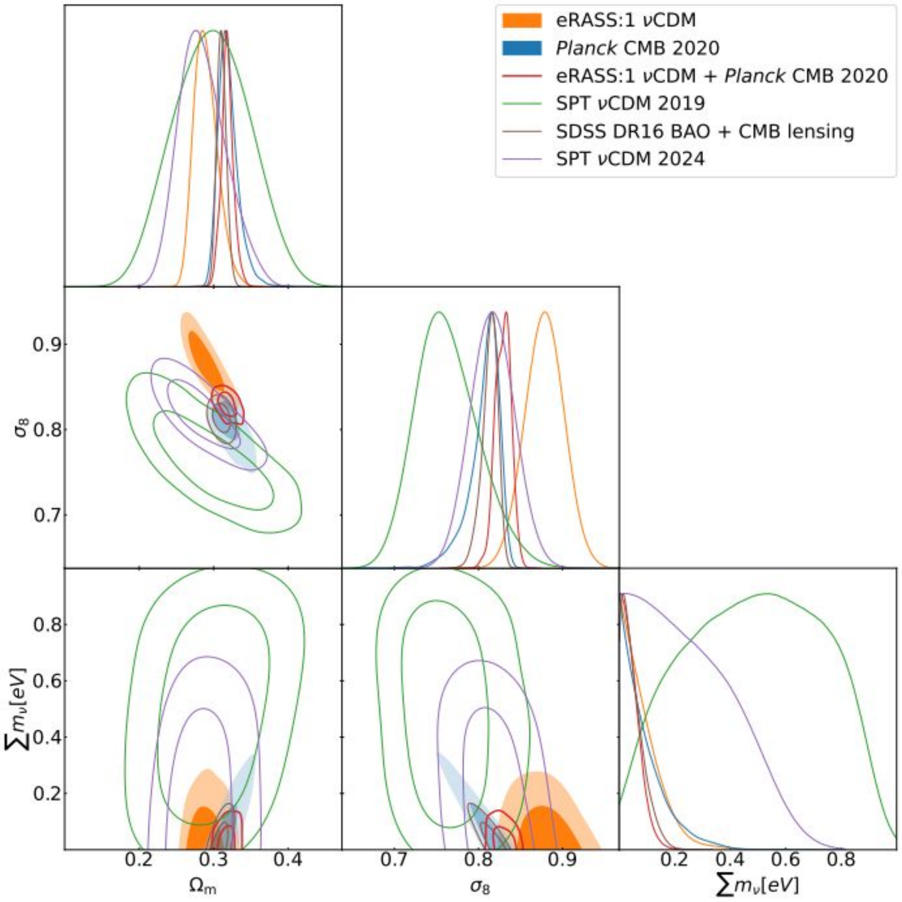
Planck-SZ results assuming 3 different mass calibrations (Planck Collaboration 2015)

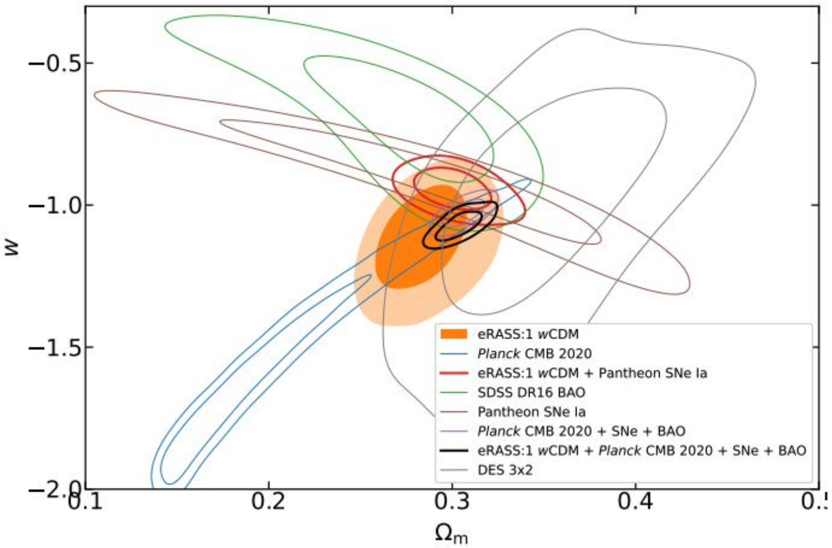
LIMITATIONS FOR CLUSTER COSMOLOGY STUDIES





RECENT CONSTRAINTS FROM CLUSTER NC





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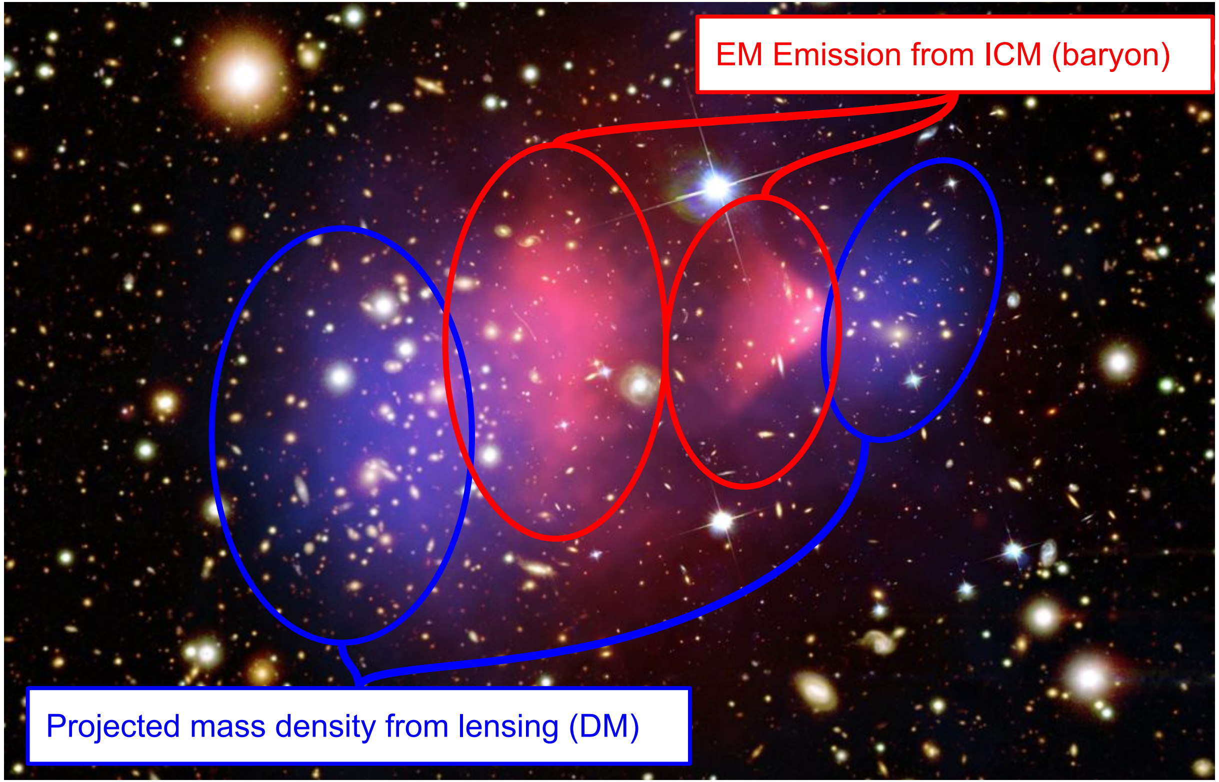
OTHER COSMOLOGICAL TESTS WITH GALAXY CLUSTERS

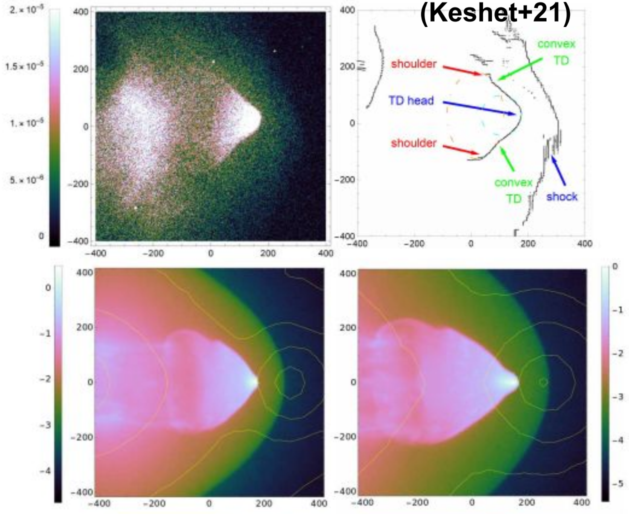
For a review: **Allen+2011 or Kravtsov+2012**

OTHER COSMOLOGICAL TESTS WITH GALAXY CLUSTERS

- **The Bullet Cluster (DM nature)**



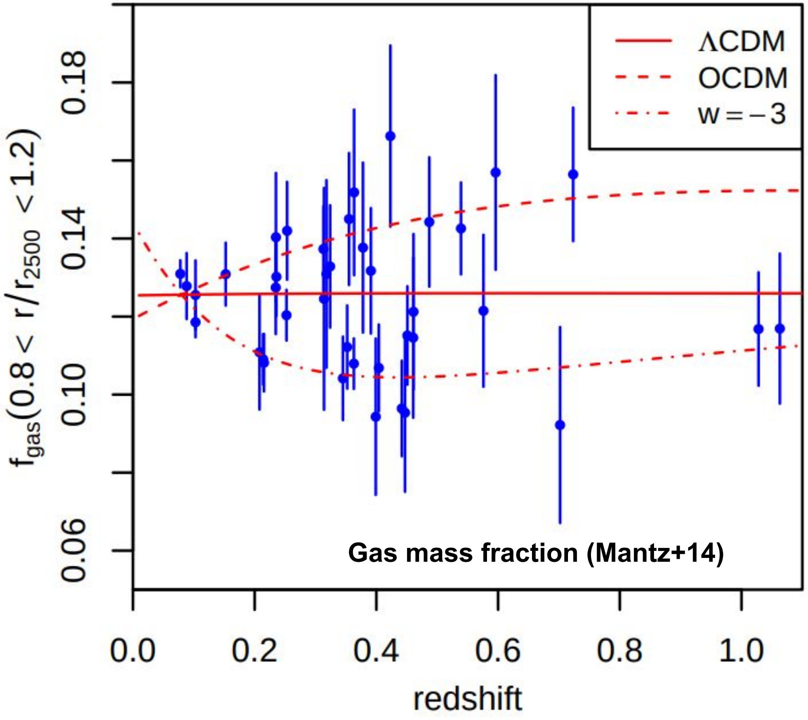
Real vs simulated Bullet-like shock (Keshet+21)



The offset between the EM and WL signal peaks, along with the shape of the shock wave, provide compelling evidence for the presence of dark matter; moreover it allows to place constraints on the dark matter cross-section

OTHER COSMOLOGICAL TESTS WITH GALAXY CLUSTERS

- **Gas mass fraction (Ω_m,Ω_Λ, w):**



Gas mass fraction (Mantz+14)

$$f_{\rm gas} = \frac{M_{\rm gas}}{M_{\rm tot}} \propto \left(\frac{\Omega_b}{\Omega_m}\right) \left[\frac{d^{\rm ref}(z)}{d(z)}\right]^{3/2}$$

With priors on

1. $\Omega_{\rm b}h^2$ (important)
2. h (less important),

the low-z data constrain $\Omega_{\rm m}$:

$$f_{\rm gas}(z \lesssim 0.15) \propto \frac{\Omega_{\rm b}}{\Omega_{\rm m}} h^{3/2}$$

Apparent evolution constrains dark energy:

$$f_{\rm gas}(z) \propto d(z)^{-3/2}$$

Credit A. Mantz

OTHER COSMOLOGICAL TESTS WITH GALAXY CLUSTERS

- **H_0 from X-ray and SZ distance measurements:**

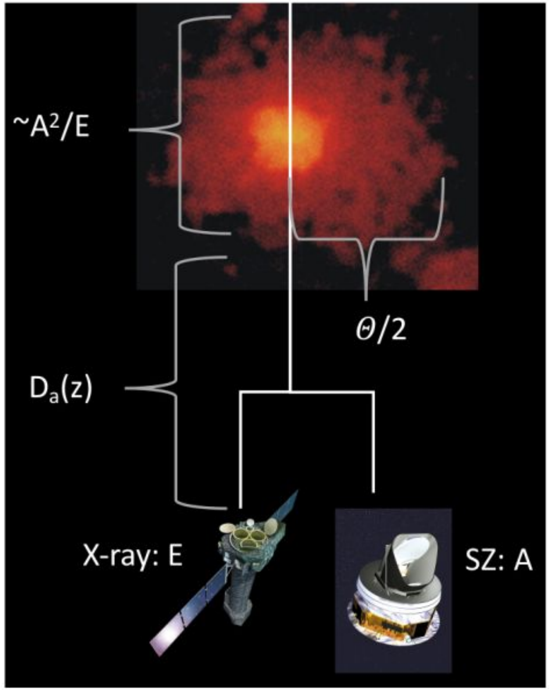
Based on a distance measuring techniques that depend on a comparison of 2 observables (Cavaliere+77):

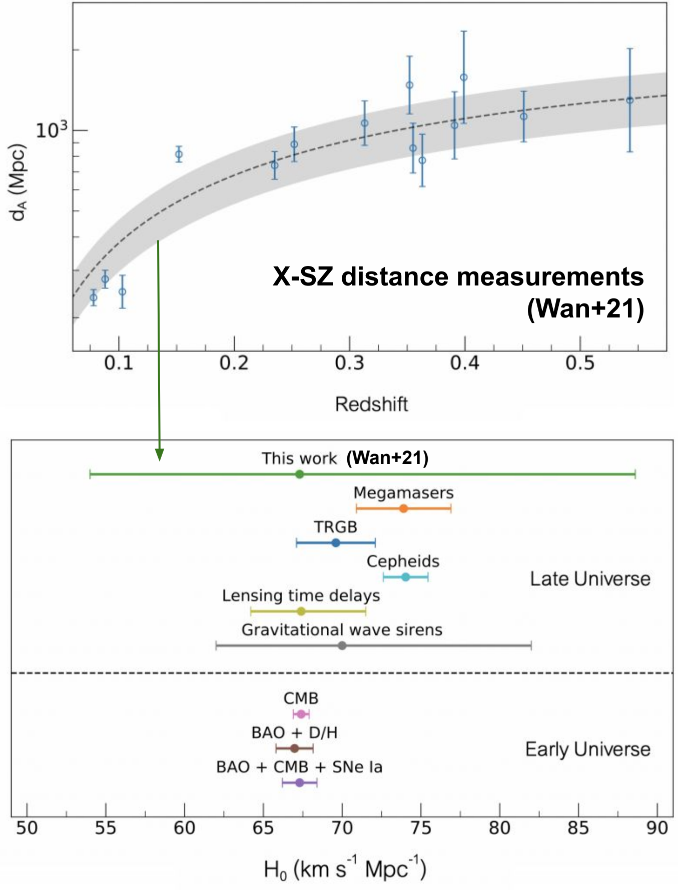
$$E \propto \int n_e^2 dl$$

$$A \propto \int n_e dl$$

If the structure of the gas is known, given the angular size ϑ of the system, the angular diameter distance is given by:

$$D_A(z) = A^2/(E\theta)$$





HALO PROFILE

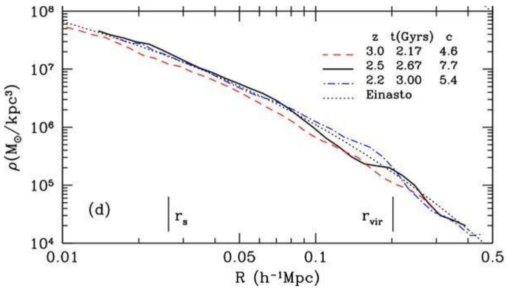
From n-body/hydro simulations we can predict the dark matter/gas halo profiles. For LCDM models E.g. Navarro+97 and Einasto 1965:

$$\rho_{\rm NFW} = \frac{\rho_{\rm s}}{(r/r_{\rm s})(1+r/r_{\rm s})^2}$$

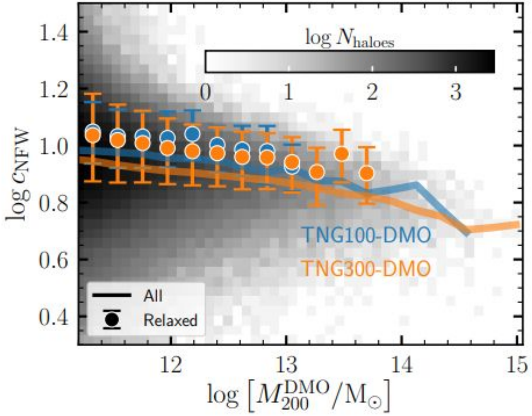
$$\rho_{\rm Ein} = \rho_{-2} \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_{-2}}\right)^{\alpha} - 1\right]\right\}$$

Observationally, cluster profiles can be inferred from strong and weak lensing, galaxy dynamics, and ICM (X-ray,SZ) measurements

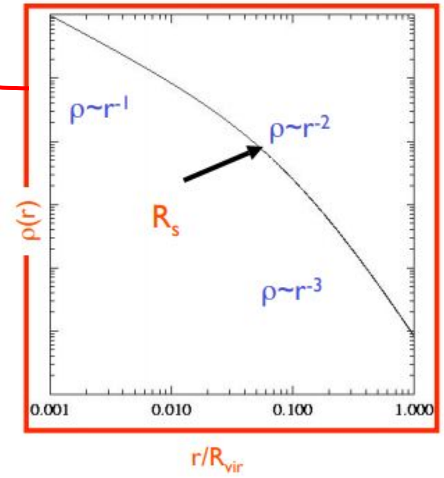
DM halo profiles for halos of different mass and redshift

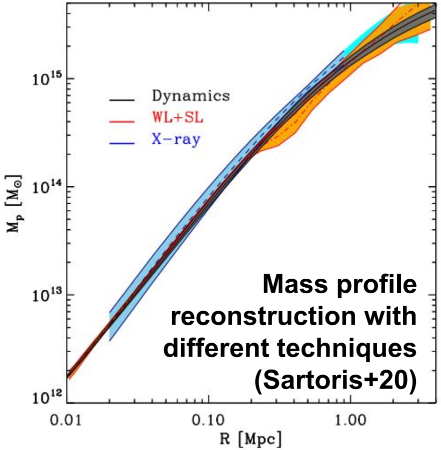


$$c_{\rm NFW} = r_{200}/r_s$$



Concentration-mass relation in TNG simulation (Bose+19)



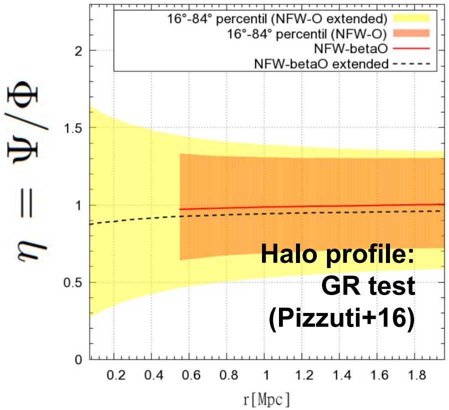


Mass profile reconstruction with different techniques (Sartoris+20)

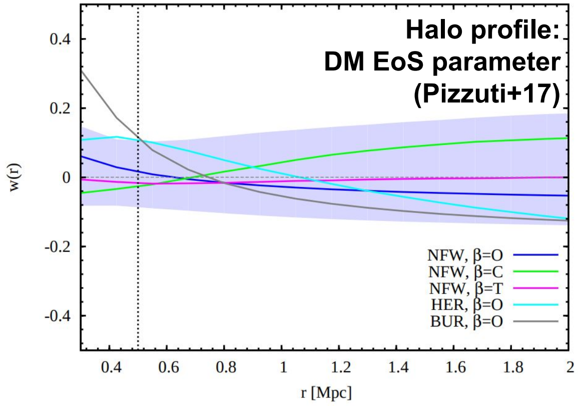
OTHER COSMOLOGICAL TESTS WITH GALAXY CLUSTERS

- **Galaxy cluster mass profile:**

The shape/slope of the halo profile, especially in the inner regions, can be used to test several fundamental physics model, such as the nature of dark matter (e.g. warm vs cold, interacting DM) or GR test.



Halo profile: GR test (Pizzuti+16)



Halo profile: DM EoS parameter (Pizzuti+17)

Halo profile: Interacting DM vs DM (Vega-Ferrero+20)