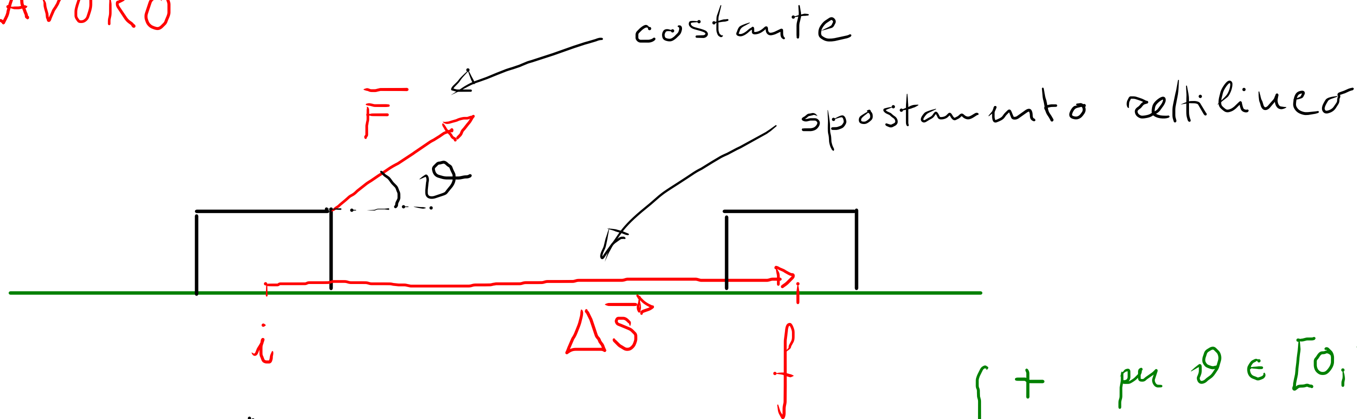
LAVORO

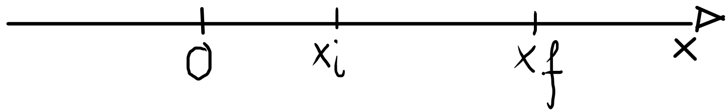


$$\mathcal{L} = \vec F \cdot \Delta\vec S = |\vec F| \cdot |\Delta\vec S| \cos\vartheta \quad \begin{cases} + & \text{per } \vartheta \in \left[0, \frac{\pi}{2}\right[\\ 0 & \text{per } \vartheta = \frac{\pi}{2} \\ - & \text{per } \vartheta = \left]\frac{\pi}{2}, \pi\right] \end{cases}$$

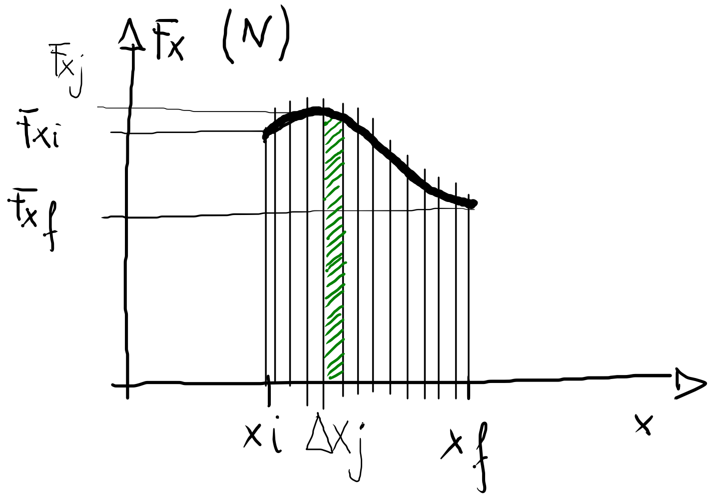
$$[\mathcal{L}] = [\vec F][\Delta\vec S] = 1\,N \cdot 1\,m = 1\,J \quad \text{Joule} \quad (SI)$$

$$10^5\,\text{dyne} \cdot 10^2\,\text{cm} = 10^7 \underbrace{\text{dyne} \cdot \text{cm}}_{\text{erg}}$$

$$1\,J = 10^7\,\text{erg}$$



consideriamo Fx parallela allo spostamento



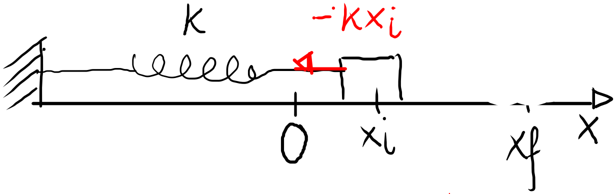
$$\mathcal{L} = ?$$

$$\Delta\mathcal{L}_j \simeq F_{x_j}\cdot\Delta x_j$$

$$\mathcal{L} \simeq \sum_j \Delta\mathcal{L}_j \simeq \sum_j F_{x_j}\,\Delta x_j$$

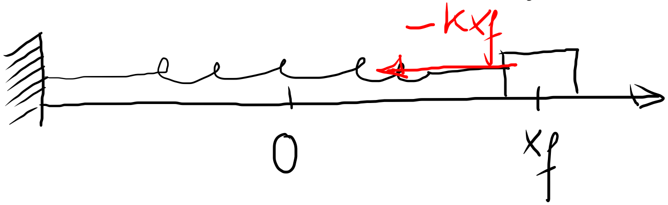
$$\boxed{\mathcal{L} = \lim_{\Delta x_j\to 0} \sum_j F_{x_j}\,\Delta x_j = \int_{x_i}^{x_f} F_x\cdot dx}$$

area sotto la curva

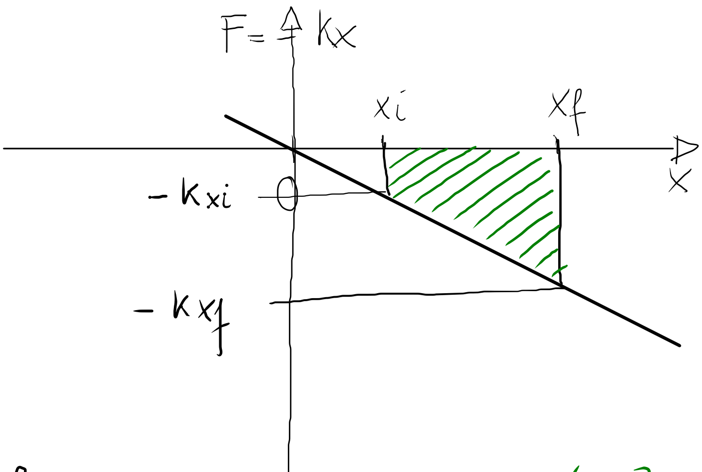


$$\vec{F} = -k\vec{x}$$

$$F = -kx$$

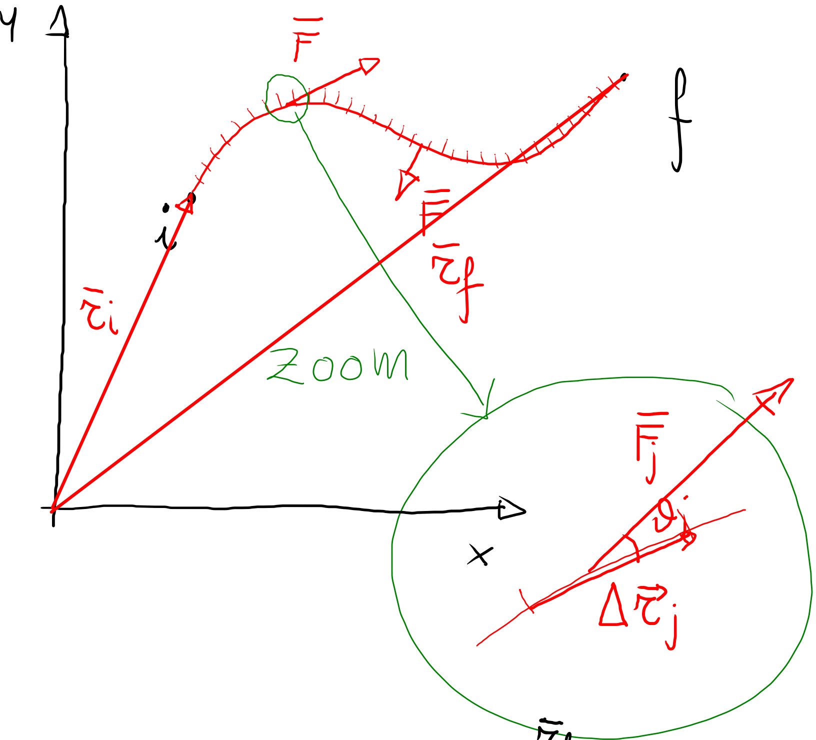


$$\mathcal{L} = \int_{x_i}^{x_f} F_x\,dx = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx = -k\left[\frac{1}{2}x^2\right]_{x_i}^{x_f} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right) = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$$



$\mathcal{L} = -$ area verde $= \frac{1}{2}k\left(x_i^2 - x_f^2\right)$

$$-\frac{1}{2}\left(kx_f + kx_i\right)\left(x_f - x_i\right) = -\frac{1}{2}k\left(x_f + x_i\right)\left(x_f - x_i\right) = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$



$$\Delta\mathcal{L}_j = \vec{F}_j \cdot \Delta\vec{r}_j = |\vec{F}_j| \cdot |\Delta\vec{r}_j| \cos\theta_j$$

$$\mathcal{L} \simeq \sum_j \Delta\mathcal{L}_j$$

$$\mathcal{L} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$ integrale di linea

(il risultato dipende dal particolare percorso che mi porta da $\vec{r}_i$ a $\vec{r}_f$)

ENERGIA (J)

Energia cinetica: $K = \frac{1}{2} m v^2$

$$[K] = [m][v]^2 \stackrel{!}{=} kg \left(\frac{m}{s}\right)^2 = \overset{N}{\frac{kg\,m}{s^2}} \cdot m = J$$

TEOREMA LAVORO ENERGIA

lavoro di ΣF oppure somma del lavoro di ciascuna forza

$$\boxed{\mathcal{L} = \Delta K}$$

variazione en. cinetica del punto materiale su cui ΣF agisce

$$\mathcal{L} = \Delta K = K_f - K_i$$

(dimostrazione in 1D)

$$\mathcal{L} = \int_{x_i}^{x_f} \Sigma F \, dx = \int_{x_i}^{x_f} ma \, dx = \int_{x_i}^{x_f} m \frac{dv}{dt} \, dx$$

$$= \int m \, dv \frac{dx}{dt} = \int_{v_i}^{v_f} m v \, dv = m \left[\frac{1}{2} v^2 \right]_{v_i}^{v_f} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= K_f - K_i = \Delta K$$

II principio

$$\Sigma \vec{F} = m \vec{a}$$

$$\Longleftrightarrow$$

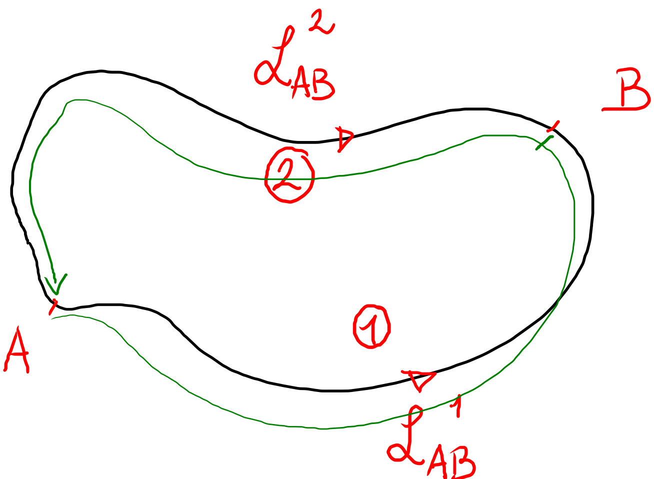
$$\mathcal{L} = \Delta K$$

FORZE CONSERVATIVE

Forza conservativa $\Leftrightarrow$ $\mathcal{L}$ da A a B <u>non</u> dipende dal particolare percorso da A a B

$\Downarrow$

$\mathcal{L}$ forza conservativa su un percorso chiuso è nullo.



F conservativa

$$\mathcal{L}_{AB}^{1} = \mathcal{L}_{AB}^{2}$$

$$\mathcal{L}_{AA} = \mathcal{L}_{AB}^{1} + \mathcal{L}_{BA}^{2} = \mathcal{L}_{AB}^{1} - \mathcal{L}_{AB}^{2} = 0$$

ESEMPI DI FORZE CONSERVATIVE

- forza costante, come $\bar{P} = m\bar{g}$
- forza elastica, $\vec{F} = -K\bar{x}$
- forze radiali con intensità $\propto \frac{1}{r^2}$, come $\vec{F} = -G\,\frac{m_1 m_2}{r^2}\,\hat{r}_{12}$
- MA NON la forza d'attrito $\Rightarrow$ FORZA DISSIPATIVA

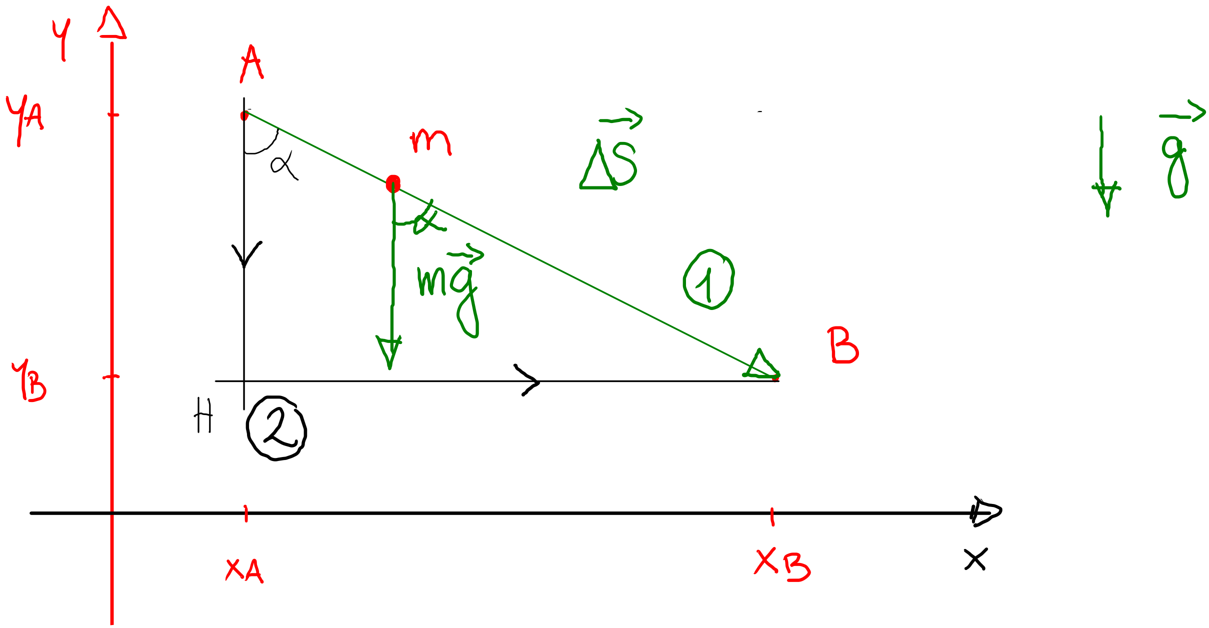
PER LE FORZE CONSERVATIVE, definisco en. potenziale $U(\vec{r})$ tale che

$$\mathcal{L}_{AB} = U(\vec{r}_A) - U(\vec{r}_B)$$

$$\overset{!}{=} U_A - U_B = -\Delta U$$

! attenzione !

ESEMPIO : FORZA PESO



① $\mathcal{L} = \vec{F} \cdot \vec{\Delta S} = |m\vec{g}| \cdot |\vec{\Delta S}| \cos\alpha = mg\Delta s \cos\alpha$

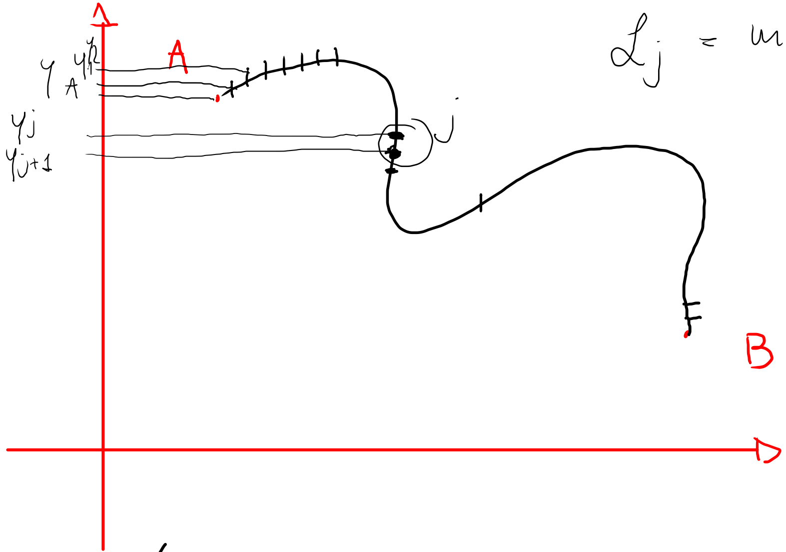
$= mg\,(y_A - y_B)$

② $\mathcal{L} = \mathcal{L}_{AH} + \mathcal{L}_{HB}$

$= mg\,(y_A - y_B) + 0 = mg\,(y_A - y_B)$

Percorso qualsiasi... (approfondimento, non in programma)

$$\mathcal{L}_j = mg\left(y_j - y_{j+1}\right)$$



$$\mathcal{L} = \sum_{j=1}^{N} \mathcal{L}_j = mg\left(y_A - \cancel{y_1}\right) + mg\left(\cancel{y_1} - \cancel{y_2}\right) + \cancel{\ldots}$$

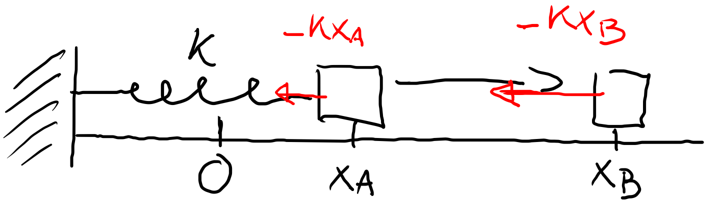
$$+ \ldots\ mg\left(\cancel{y_j} - \cancel{y_{j+1}}\right) + mg\left(\cancel{y_{j+1}} - \cancel{y_{j+2}}\right)$$

$$+ mg\left(\cancel{y_N} - y_B\right) = mg\left(y_A - y_B\right)$$

ULTERIORE ESEMPIO: Forza elastica

$$\vec{F} = -k\vec{x}$$

Ieri abbiamo calcolato il lavoro della forza elastica per andare da x_A a x_B:



$$\mathcal{L} = \frac{1}{2}k\left(x_A^2 - x_B^2\right)$$

Il lavoro dipende solo da x_A e x_B

$\Rightarrow$ La forza elastica è conservativa!

ENERGIA POTENZIALE

Per ogni forza CONSERVATIVA posso definire

$U(\vec{r})$ tale che $\mathcal{L}_{AB} = U(\vec{r}_A) - U(\vec{r}_B) \overset{!}{=} U_A - U_B = -\Delta U$

Esempi precedenti

Forza peso: $\mathcal{L}_{AB} = mg(y_A - y_B)$

$$\boxed{U_g = mgy}$$

$\mathcal{L}_{AB} = U_A - U_B \overset{!}{=} mgy_A - mgy_B$

gravitazionale

Forza elastica $\mathcal{L}_{AB} = \frac{1}{2}k(x_A^2 - x_B^2)$

$$\boxed{U_e = \frac{1}{2}kx^2}$$

$\mathcal{L}_{AB} = U_A - U_B \overset{!}{=} \frac{1}{2}kx_A^2 - \frac{1}{2}kx_B^2$

elastica

ENERGIA MECCANICA

$$\boxed{E_{mecc} = K + U}$$

Sistema conservativo: il $\mathcal{L}$ è fatto da forze conservative

① $\mathcal{L} = \Delta K = K_B - K_A$ (vale sempre)

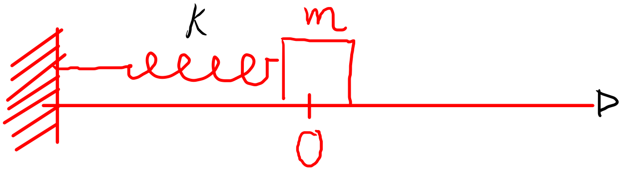
② $\mathcal{L} = -\Delta U = U_A - U_B$ (solo per sistemi conservativi)

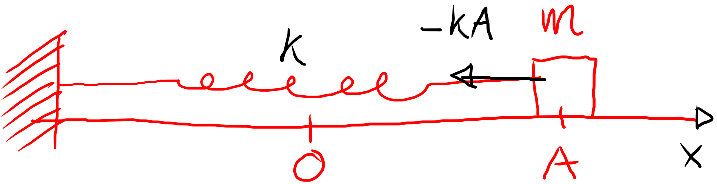
①② $0 = \Delta K + \Delta U = K_B + U_B - (K_A + U_A)$

$$\Delta K + \Delta U = 0$$
$$\Delta(K + U) = 0$$
$$\boxed{\Delta E_{mecc} = 0}$$

$$K_B + U_B - (K_A + U_A) = 0$$
$$E_{mecc\,B} - E_{mecc\,A} = 0$$
$$\boxed{\Delta E_{mecc} = 0}$$

L'E_{mecc} si conserva.





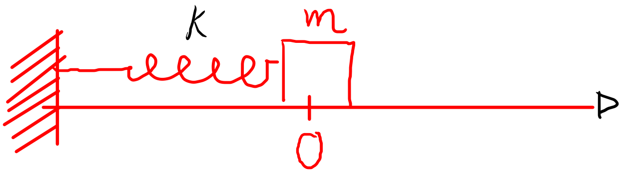
cinematica

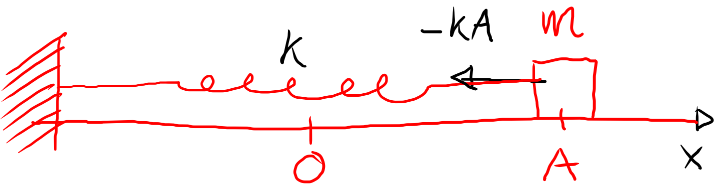
$$
\begin{cases}
x(t) = A\cos(\omega t) & (*) \\
v(t) = -A\omega \,\text{sen}(\omega t) & (**) \\
a(t) = -A\omega^2 \cos(\omega t) = -\omega^2 x(t)
\end{cases}
\qquad \omega = \frac{2\pi}{T}
$$

dinamica : $\omega = \sqrt{\dfrac{k}{m}}$ $\qquad \omega^2 = \dfrac{k}{m}$

energia potenziale elastica : $U_e = \dfrac{1}{2} k x^2$

$$
\begin{aligned}
E_{mecc} = K + U_e &= \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \\
&= \frac{1}{2} m \underbrace{\left[-A\omega\,\text{sen}(\omega t)\right]^2}_{**} + \frac{1}{2} k \underbrace{\left[A\cos(\omega t)\right]^2}_{*} \\
&= \frac{1}{2} \underbrace{m}_{*} A^2 \underbrace{\omega^2}_{**\,*} \text{sen}^2(\omega t) + \frac{1}{2} k A^2 \cos^2(\omega t) \\
&= \frac{1}{2} k A^2 \text{sen}^2(\omega t) + \frac{1}{2} k A^2 \cos^2(\omega t) \\
&= \frac{1}{2} k A^2 \underbrace{\left[\text{sen}^2(\omega t) + \cos^2(\omega t)\right]}_{1} = \frac{1}{2} k A^2
\end{aligned}
$$





$$E_{mecc} = K + U_e = \frac{1}{2} k A^2$$

cinematica

$$\begin{cases} x(t) = A\cos(\omega t) & * \\ v(t) = -A\omega\, \mathrm{sen}(\omega t) & ** \\ a(t) = -A\omega^2 \cos(\omega t) = -\omega^2 x(t) \end{cases} \qquad \omega = \frac{2\pi}{T}$$

dinamica: $\omega = \sqrt{\frac{k}{m}}$ $\quad \omega^2 = \frac{k}{m}$

energia potenziale elastica: $U_e = \frac{1}{2} k x^2$

$x = 0 \Rightarrow U_e = 0$

$$K = \frac{1}{2} k A^2$$

$$\cancel{\tfrac{1}{2}} m v^2 = \cancel{\tfrac{1}{2}} k A^2$$

$$v^2 = \frac{k}{m} A^2 = \omega^2 A^2$$

$$v = \pm \omega A$$

$x \neq 0$

$$\cancel{\tfrac{1}{2}} k A^2 = \cancel{\tfrac{1}{2}} m v^2 + \cancel{\tfrac{1}{2}} k x^2$$

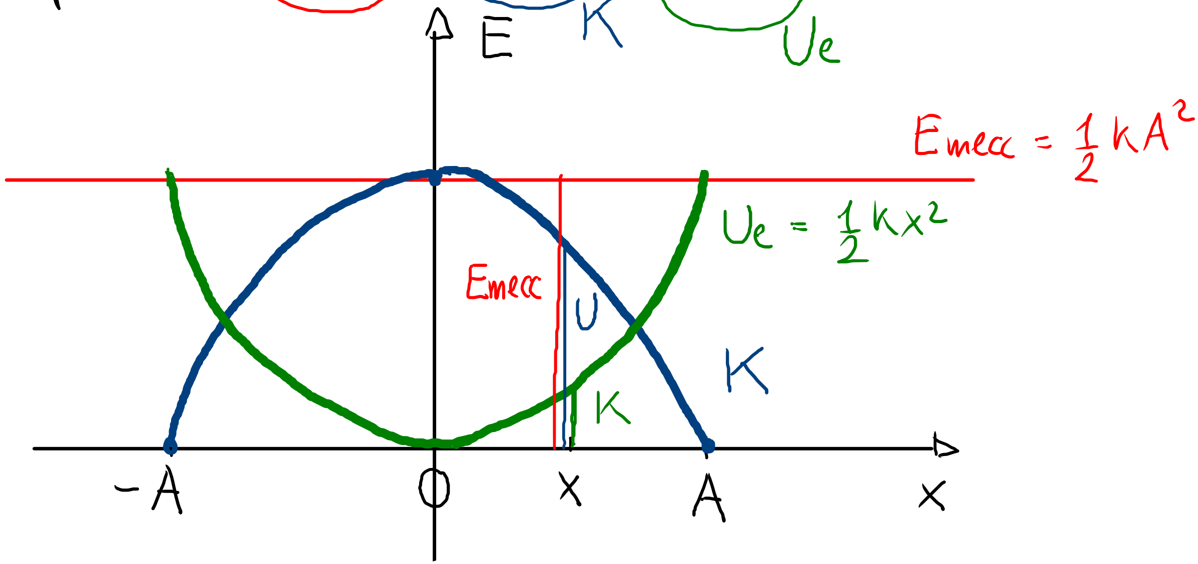
$$\omega^2 A^2 = v^2 + \omega^2 x^2$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v = \pm \omega \sqrt{A^2 - x^2}$$

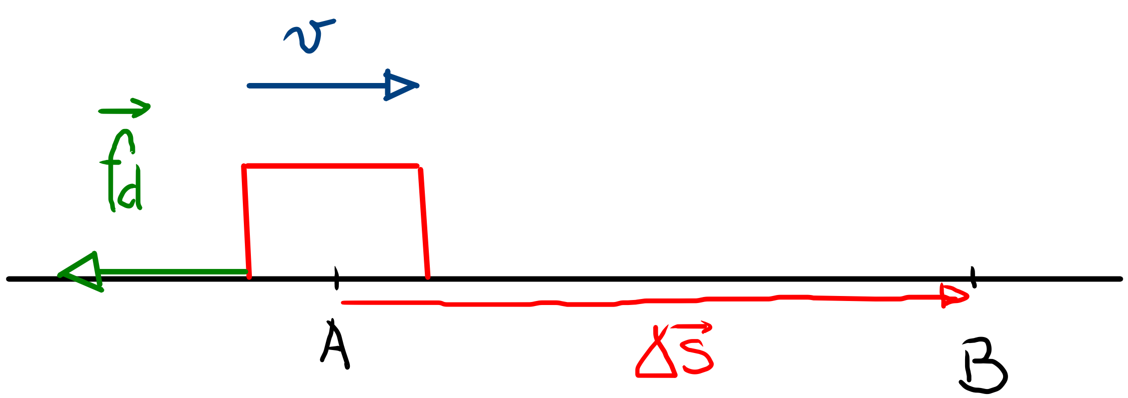
In forma grafica: $\boxed{E_{mecc}} = \underbrace{\frac{1}{2} m v^2}_{K} + \underbrace{\frac{1}{2} k x^2}_{U_e} = \frac{1}{2} k A^2$

$K = E_{mecc} - U_e$



FORZE DISSIPATIVE (ovvero non conservativa)

Es: attrito dinamico



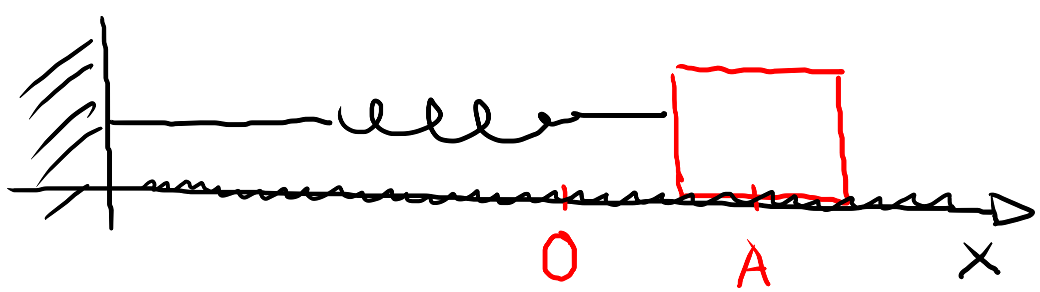
$$\mathcal{L}_d = \vec{f_d} \cdot \Delta\vec{s} = f_d \, \Delta s \cdot \cos\pi = -f_d \, \Delta s$$

negativo!

dipende dal percorso!

SISTEMA CON FORZE CONSERVATIVE E DISSIPATIVE

Es.



piano liscio

$\Rightarrow$ conservativo

$$U = \frac{1}{2} K x^2$$

piano ruvido $\Rightarrow$ c'è attrito

non conservativo

$$\mathcal{L} = \Delta K$$

↑ somma dei lavori fatti da ciascuna forza

assumiamo: una forza conservativa $\mathcal{L}_C$

una forza dissipativa $\mathcal{L}_D$

$$\mathcal{L} = \mathcal{L}_C + \mathcal{L}_D$$

$$\mathcal{L}_C + \mathcal{L}_D = \Delta K \qquad \mathcal{L}_C = -\Delta U$$

$$-\Delta U + \mathcal{L}_D = \Delta K$$

$$\mathcal{L}_D = \boxed{\Delta K + \Delta U = \Delta E_{mecc}} \quad *$$

$$\Delta E_{mecc} = \mathcal{L}_D < 0$$

Emecc NON si conserva ma diminuisce ($\Delta E_{mecc} < 0$)

SISTEMI ISOLATI

Non scambia né materia né energia con l'ambiente che lo circonda

Definisco E_{int}, energia interna del sistema

E_{TOT}, energia totale del sistema. ← si conserva!

$$E_{TOT} = E_{mecc} + E_{int}$$

$$\Delta E_{TOT} = \underbrace{0 = \Delta E_{mecc} + \Delta E_{int}}$$

$$\Delta E_{int} = -\underbrace{\Delta E_{mecc}}_{<0} > 0$$

Quindi da *

$$\boxed{\Delta K + \Delta U = \Delta E_{mecc}}$$

$$\Delta K + \Delta U - \Delta E_{mecc} = 0$$

$$\Delta K + \Delta U + \Delta E_{int} = 0$$

$$\Delta E_{TOT} = 0 \longrightarrow \text{si conserva!}$$

POTENZA

$$P_m = \frac{\mathcal{L}}{\Delta t} \qquad [P] = \frac{J}{s} = W \quad (\text{Watt})$$

MEDIA

$$P = \lim_{\Delta t \to 0} P_m = \lim_{\Delta t \to 0} \frac{\mathcal{L}}{\Delta t}$$

ISTANTANEA

se $\vec{F}$ è costante... $\mathcal{L} = \vec{F} \cdot \vec{\Delta x}$

$$= \lim_{\Delta t \to 0} \frac{\vec{F} \cdot \vec{\Delta x}}{\Delta t} = \vec{F} \cdot \underbrace{\lim_{\Delta t \to 0} \frac{\vec{\Delta x}}{\Delta t}}_{\vec{v}} = \vec{F} \cdot \vec{v}$$

$$1\ kW = 10^3\ W$$

$$1\ kWh = 1\ kW \cdot 1\ h = 10^3\ W \cdot 3600\ s = 3{,}6 \cdot 10^6 \frac{J}{\cancel{s}} \cdot \cancel{s}$$

unità di misura pratica dell'energia

RENDIMENTO

$$\eta = \frac{\mathcal{L}}{E}$$

compiuto dalla macchina

necessaria a farla funzionare

$$\eta \leq 1$$

pertanto si esprime in %

Nota: le slide che seguono sono un approfondimento e non costituiscono materia d'esame

CAMPI VETTORIALI e CAMPI DI FORZE

(nota: queste ultime slide sono un approfondimento e non costituiscono materia d'esame)

$\vec{V}(\vec{r};t)$ campo vettoriale

Esempi: $\vec{E}(\vec{r};t)$ campo elettrico

$\vec{B}(\vec{r};t)$ campo magnetico

Le forze derivano dalla presenza dei campi:

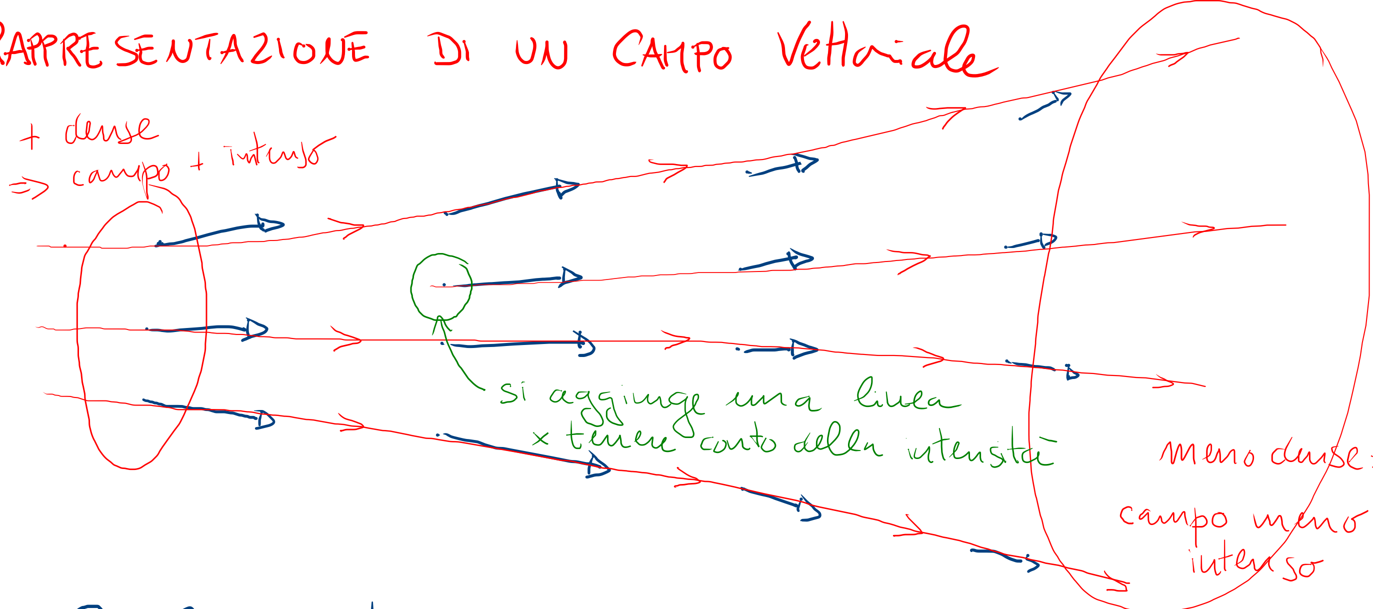
Esempio: forza di Lorentz:

q carica elettrica

$\vec{v}$ velocità

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

RAPPRESENTAZIONE DI UN CAMPO VETTORIALE

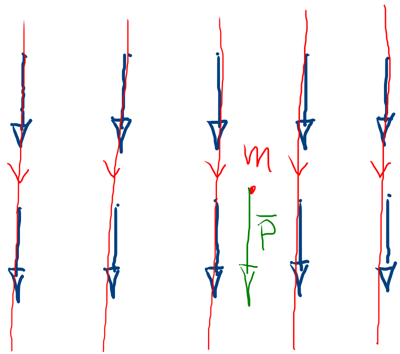


1. Rappresentazione "per punti" (fissato t)

2. mediante linee di campo
   linee di forza

FORZA PESO $\overline{P} = m\vec{g}$

$\vec{g}$ come campo vettoriale :



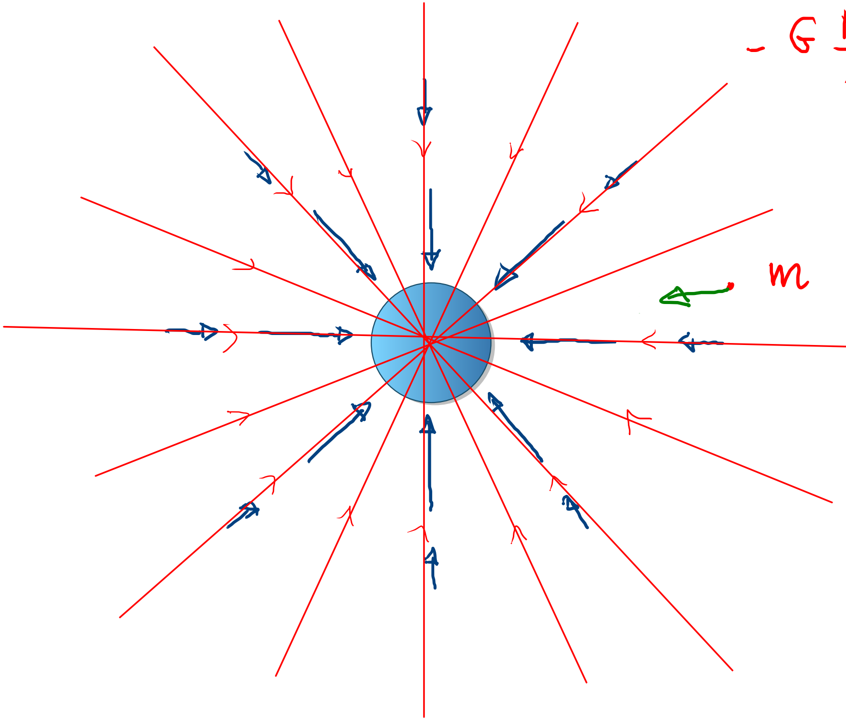
$$\boxed{\overline{P} = m\vec{g}}$$

FORZA GRAVITAZIONALE DI NEWTON (tra la Terra ed m)

$$\vec{F}_g = -G\frac{M_T m}{r^2}\hat{r}$$

campo gravitazionale terrestre

$$-G\frac{M_T}{r^2}\hat{r}$$



$$\vec{F} = -G\frac{M_T m}{r^2}\hat{r}$$

ENERGIA POTENZIALE E CAMPI DI FORZE

$\mathcal{L} = -\Delta U$

$\mathcal{L} = \vec{F} \cdot \vec{\Delta x}$ con $\vec{F}$ costante

$U(\vec{r};t)$ è un campo scalare

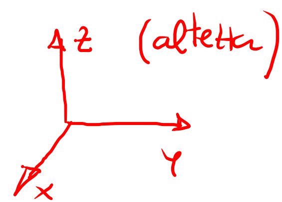
(1D) $\mathcal{L} = \underbrace{F \cdot \Delta x = -\Delta U}$

$F = -\dfrac{\Delta U}{\Delta x}$ $\quad \lim\limits_{\Delta x \to 0}$ $\quad F = -\dfrac{dU}{dx}$

(3D)

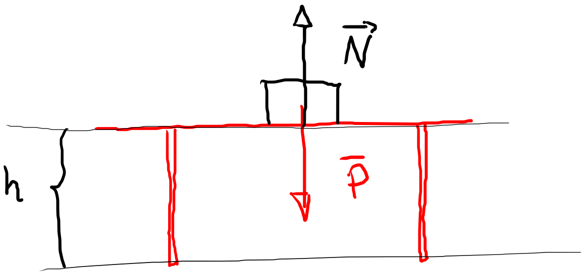
$$\boxed{\vec{F} = -\vec{\nabla} U}$$

$$\vec{\nabla} U = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$



Esempio: $U = mgz$

$$-\vec{\nabla} U = -mg\hat{k} = +m\vec{g} = \vec{P}$$



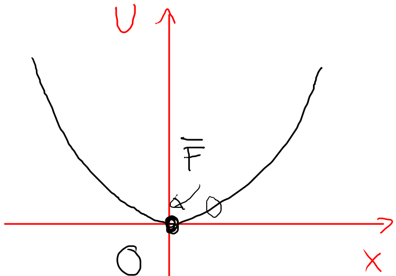
$$\bar{P} + \vec{N} = 0$$

$$U = mgz \Big|_{z=h}$$

$$\bar{\nabla} U = \bar{\nabla}\left(mgz \Big|_{z=h} \right) = \bar{\nabla}\left(mgh \right) = 0 \quad \Rightarrow \sum \bar{F} = 0$$

$$F = -\frac{dU}{dx} \qquad (1D)$$

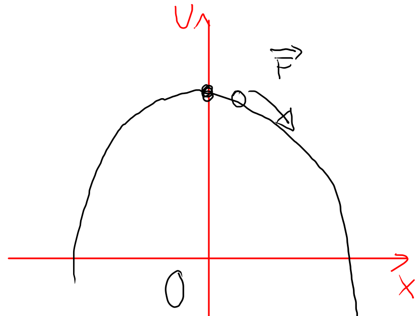
Equilibrio Stabile, Instabile o Indifferente



$$F = -\frac{dU}{dx} = 0$$

U min eq.

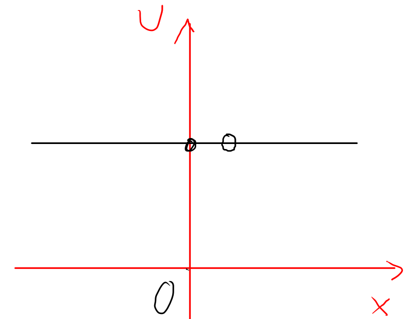
$\Rightarrow$ stabile



$$F = -\frac{dU}{dx} = 0$$

U max eq.

$\Rightarrow$ instabile



$$-\frac{dU}{dx} = 0 = F$$

indifferente